AD A951960

3c.

ROCK ISLAND ARSENAL
RESEARCH & ENGINEERING DIVISION
DEVELOPMENT ENGINEERING BRANCH

TECHNICAL NOTES

FIRST APPROXIMATION TO FLUID FLOW IN RECOIL SYSTEM
SPECIFICALLY APPLIED TO THE XM37 RECOIL MECHANISM

DTIC
ELECTE
JUL 20 1983
S D
A

A.M.S. CODE NO.	5523.11.44001.01
D.A. PROJECT NO.	1-W-5-23801-A-290
REPORT NO.	16-65
AUTHOR	Robert H. Coberly and Jerry W. Frantz
DATE	November 1965

REPORT NO. 16-65

83 07 20 000

FIRST APPROXIMATION TO FLUID FLOW IN RECOIL SYSTEMS SPECIFICALLY APPLIED TO THE XM37 RECOIL MECHANISM

BY

Robert H Coberly
Robert H. Coberly

and

Jerry W Frantz
Jerry W. Frantz

Approved by

Arnold A. Kester
ARNOLD A. KESTER
Chief, development Engineering Branch

Accession For
NTIS GRA&I
DTIC TAB
Unannounced
Justification
Distribution/
Availability Codes
Avail and/or
Dist Special
A

November 1965

DTIC COPY INSPECTED 2

A.M.S. Code No. 5523.11.44001.01

D.A. Project No. 1-W-5-23801-A-290

UNANNOUNCED

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

Rock Island Arsenal
Rock Island, Illinois

[redacted]

ABSTRACT

This Technical Note presents the results ~~of~~ the first in a series ~~of investigations to be conducted by the Research~~ and Analysis Section on the analysis and design of recoil mechanisms. This note deals with the approximation of fluid flow in a recoil system. The M102 Howitzer, XM37 Recoil Mechanism, is the model studied in this investigation. The M102 Howitzer is a lightweight, towed-typed, 105mm field artillery weapon suited for airborne and air assault operations. The XM37 Recoil Mechanism is a hydro-pneumatic dependent type of recoil mechanism.

TABLE OF CONTENTS

	Page
CONCLUSIONS AND RECOMMENDATIONS	iv
INTRODUCTION	v
THEORY	1
COMPUTER ANALYSIS	12
A. Recoil	12
B. Counterrecoil	17
ACKNOWLEDGEMENT	19
APPENDICES	
Appendix A - Graphs	A1 - A6
Appendix B - Analysis Program	B1 - B13
Appendix C - Design Program	C1 - C9

Conclusions and Recommendations

The recoil stroke and counterrecoil stroke for short and long recoil were closely duplicated. The discharge coefficient for short and long recoil were found to be .75 and .85 respectively. Tolerancing (leakage) was shown to have a definite effect upon rod pull. It is felt that for a preliminary analysis, the results are very good.

It is recommended that the error analysis upon the technique of control rod design be continued.

Introduction

Currently, a digital computer is being utilized to solve a system of differential equations which are the equations of motion for fluid flow through any Puteaux recoil system. This study encompasses both the recoil and counterrecoil stroke. This computer program has two definite purposes: Analysis and design. In the analysis phase, parameters may be varied within a known recoil system geometry in order to ascertain differential effects upon recoil lengths, recoil forces, in-battery velocities, cycle times, etc. The design program takes a known system geometry and generates groove areas for the control rod. The length of recoil and the recoil force are required inputs to this program. These items are interdependent and their values are based on the momentum of the recoiling parts and the combined momentum of projectile and propelling charge. The recoil force, at any point along the stroke, depends upon the recoil velocity and orifice area at that point. Therefore the design program computes the necessary orifice area from point to point in order to meet the design requirements. Based on these two programs, a control rod was designed and manufactured. Experimental testing showed a close agreement between the firing results and the computer predictions. This rod was subsequently accepted for production.

This note deals only with the analysis program, although both the analysis and the design programs, are listed in the Appendices B and C.

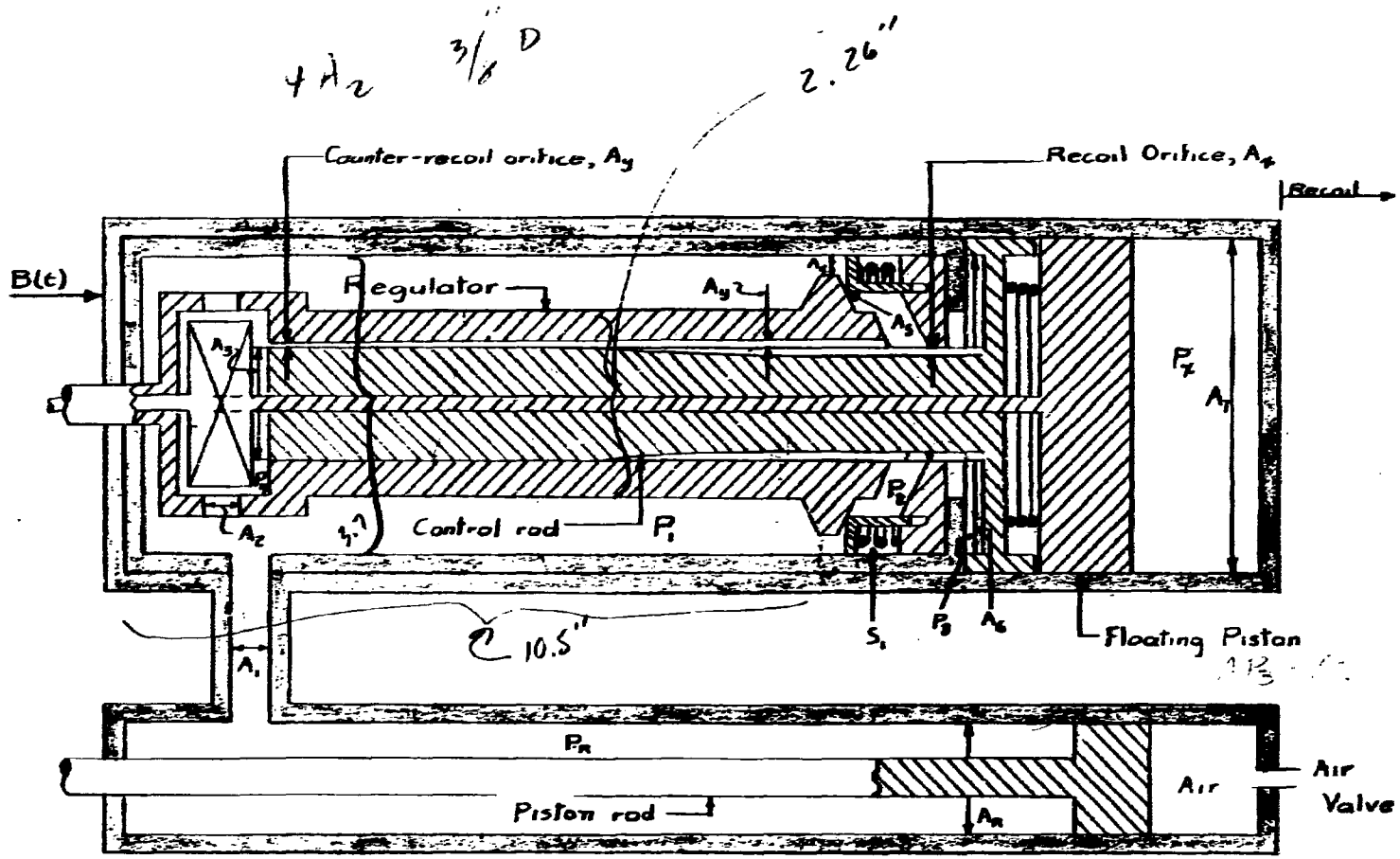

4 A2
3/8" D
2.26"
Counter-recoil orifice, Ay
Recoil Orifice, Ax
Recoil
B(t)
Regulator
Ay
A3
A5
Px
AT
A2
Control rod
P1
P2
S1
P3
A6
Floating Piston
10.5"
A1
PR
Piston rod
AR
Air
Air
Valve

FIGURE 1

First Approximation to Fluid Flow in Recoil Systems

1. Theory

The first mechanism to be studied is the XM37 Recoil Mechanism for the XM102 (105 mm) Howitzer.

Figure 1 is a schematic drawing of the XM37 Recoil Mechanism. The operation of this mechanism is described in the following paragraphs.

In the XM37, the piston rod is attached rigidly to a non recoiling part. The remaining parts of the mechanism are attached to the Howitzer via a yoke arrangement, constituting the Recoiling Parts.

Upon firing, the Recoiling Parts are moved to the rear, forcing the piston rod and piston to move forward relative to the recoil mechanism. The oil is forced under pressure P_R through Area A_1 into the recuperator cylinder at pressure P_1. This oil is forced into the front of the regulator through Area A_2 under pressure P_4. This pressure acts on the end of the control rod over Area A_3. Also, the oil under pressure P_1 is forced through Area A_4 and through the spring loaded valve (S_1), via Area A_5, into the throttling chamber at pressure P_2. The oil then passes through the throttling Area A_x where it acts against the diaphragm (Areas A_6 and A_7) under pressure P_3. The pressure P_3, acting on the diaphragm, plus the pressure P_4, acting on the control rod, compresses the springs and forces the floating piston against the gas pressure P_x. As the diaphragm and floating piston are forced to the rear, the control rod moves with them, thus varying the orifice Area A_x. Note $A_3 + A_6 = A_7$.

During counter-recoil, the gas pressure P_x forces the floating piston, diaphragm, and control rod forward, throttling the oil back through the orifice Area A_x. The spring loaded valve is closed during counter-recoil, thus forcing the oil to be throttled between the control rod and the regulator through Area A_y. The oil then returns to the recoil cylinder, forcing the recoil piston back into battery.

2. Analysis of the Recoil Stroke

This study is based on the following assumptions

(a) Uni-directional flow

(b) Friction is assumed constant

(c) The fluid is assumed to be incompressible

(d) The gas pressure follows the adiabatic gas law

(e) No pressure drop across the yoke (A_1)

(f) The coefficient of discharge is assumed constant

By Newton's third law, the resistance to recoil due to the recoil mechanism is equivalent to the force exerted on the carriage by the recoil mechanism (which is the force exerted on the piston rod) here referred to as the "rod pull" and defined in this problem as $A_R P_R + (\text{sgn}\,\dot{x}) F_p$

The equation of motion for the recoiling parts is:

$$M_R \ddot{x} = B(t) - A_R P_R - (\text{sgn}\,\dot{x})\cdot(F + F_p) + W_R \sin\xi \qquad (1)$$

where

M_R = Mass of Recoiling Parts (W_R = weight)

$B(t)$ = Breech Force

x = Recoil Displacement

$\text{Sgn}\,\dot{x}$ = Algebraic sign of $\dot{x}$ ($\text{sgn}\,\dot{x} = 1, \dot{x} > 0$: $\text{sgn}\,\dot{x} = -1, \dot{x} > 0$)

F = Guide Friction

F_p = Recoil Piston and Stuffing Box Friction

ξ = Angle of Elevation

The equation of motion for the floating piston is:

$$M_p \ddot{y} = P_4 A_3 + P_3 A_6 - P_x A_7 - (P_2 - P_3) A_x - (\text{sgn}\,\dot{y}) F_{fp} + W_p \sin\xi \qquad (2)$$

where

M_p = Mass of floating piston (W_p = Weight)

y = Floating piston displacement (absolute)

F_{fp} = Floating piston friction

Equations (1) and (2) are the desired equations of motion. The problem now is one of evaluating the various pressures. We consider two cases: (a) $P_3 > 0$ and (b) $y > (1 + \frac{A_B}{A_7}) x$.

CASE 1 - $P_3 > 0$

Define: $\Delta P_3 = P_3 - P_1$

$\Delta P_1 = P_{B_1} - P_4$ = Pressure Drop across A_2

$\Delta P_2 = P_2 - P_3$ = Pressure Drop across A_x

Define the orifice area for two short recoil grooves as A_{rs}, and for two long recoil grooves as A_{rL}. In long recoil both A_{rs} and A_{rL} are acting; while in short recoil only A_{rs} is acting. NOTE: the inch slug-inch-sec system is used throughout the study.

Q_2 = Rate of Flow through Orifice A_2 (V_2 = Velocity of Flow)
Q_{rs} = Rate of Flow through Orifice A_{rs} (V_{rs} = Velocity of Flow)
Q_{rL} = Rate of Flow through Orifice A_{rL} (V_{rL} = Velocity of Flow)
C_D = Coefficient of Discharge for A_2
C'_D = Coefficient of Discharge for A_{rs}
C''_D = Coefficient of Discharge for A_{rL}

From Reference 2, repeating the analysis:
The pressure (ΔP) resulting from a head (h) of oil with a specific weight (σ) is:

$$\Delta P = h\sigma$$

The velocity (V) of oil through the orifice (A) is:

$$V = c\sqrt{2gh}$$

thus

$$V^2 = c^2 \cdot 2g \cdot \frac{\Delta P}{\sigma}$$

So

$$\Delta P = \frac{V^2 \sigma}{2 g c^2}$$

The flow (Q) through the orifice is given by the relation

$$AV = Q$$

The above analysis when applied to Fig. 1 gives:

$$\Delta P_1 = \frac{V_2^2 \cdot \sigma}{2 \cdot g \cdot C_D^2} \qquad \Delta P_2 = \frac{V_{rs}^2 \cdot \sigma}{2 \cdot g \cdot C_D'^2} = \frac{V_{rL}^2 \cdot \sigma}{2 \cdot g \cdot C_D''^2} \qquad (3)$$

From the second of equations 3:

$$C'_D V_{rL} = C''_D V_{rs} \qquad (4)$$

Also, from the above analysis

$$Q_2 = A_2 V_2 \qquad Q_{rs} = A_{rs} V_{rs} \qquad Q_{rL} = A_{rL} V_{rL} \qquad (5)$$

Now

$$Q_2 + Q_{rs} + Q_{rL} = A_R \dot{x} \qquad (6)$$

but

$$Q_2 = A_3(\dot{y} - \dot{x}) \qquad Q_{xs} + Q_{xL} = A_6(\dot{y} - \dot{x}) \tag{7}$$

Substituting equations 7 into equation 6

$$(A_3 + A_6)(\dot{y} - \dot{x}) = A_R \dot{x}$$

or

$$(A_R + A_3 + A_6)\dot{x} = (A_3 + A_6)\dot{y}$$

So

$$\dot{y} = \left(1 + \frac{A_R}{A_3 + A_6}\right)\dot{x} \qquad \ddot{y} = \left(1 + \frac{A_R}{A_3 + A_6}\right)\ddot{x} \tag{8}$$

and

$$\dot{y} - \dot{x} = \frac{A_R}{A_3 + A_6}\dot{x} \tag{9}$$

Substituting equation 9 into equations 7

$$Q_2 = A_3 \cdot \frac{A_R}{A_3 + A_6} \cdot \dot{x} \qquad Q_{xs} + Q_{xL} = A_6 \cdot \frac{A_R}{A_3 + A_6} \cdot \dot{x} \tag{10}$$

Now from equation 5

$$Q_{xs} + Q_{xL} = A_{xs}V_{xs} + A_{xL}V_{xL}$$

Substituting from equation 4 for V_{xL}

$$Q_{xs} + Q_{xL} = V_{xs}\left(A_{xs} + A_{xL} \cdot \frac{C_D''}{C_D'}\right) \tag{11}$$

From equation 5 and 11, write equation 10 as:

$$A_2V_2 = A_3 \cdot \frac{A_R}{A_3 + A_6} \cdot \dot{x}$$

$$\left(A_{xs} + A_{xL} \cdot \frac{C_D''}{C_D'}\right)V_{xs} = A_6 \cdot \frac{A_R}{A_3 + A_6} \cdot \dot{x}$$

Now we find

$$V_2 = \frac{A_3}{A_2} \cdot \frac{A_R}{A_3 + A_6} \cdot \dot{x}$$

$$V_{xs} = \frac{A_6 A_R C_D'}{(A_3 + A_6)(C_D' A_{xs} + C_D'' A_{xL})} \cdot \dot{x} \tag{12}$$

$$V_{xL} = \frac{A_6 A_R C_D''}{(A_3 + A_6)(C_D' A_{xs} + C_D'' A_{xL})} \cdot \dot{x}$$

Substitution of equations 12 into equations 3 give

$$\Delta P_1 = \left[\frac{A_3 A_R}{A_2(A_3+A_6)}\right]^2 \cdot \frac{\sigma}{2g C_D^2} \cdot \dot{x}^2 \qquad (13)$$

$$\Delta P_2 = \left[\frac{A_6 A_R}{(A_3+A_6)(C_D' A_{xs} + C_D'' A_{xL})}\right]^2 \cdot \frac{\sigma}{2g} \cdot \dot{x}^2 \qquad (14)$$

If we assume a leakage area (A_K) with a discharge coefficient (C_K) in parallel with the recoil orifices, we may write eq. 14 as:

$$\Delta P_2 = \left[\frac{A_6 A_R}{(A_3+A_6)(C_D' A_{xs} + C_D'' A_{xL} + C_K A_K)}\right]^2 \cdot \frac{\sigma}{2g} \cdot \dot{x}^2 \qquad (14A)$$

Now from Fig. 1

$$\begin{aligned} P_1 &= P_R - \Delta P_1 \\ P_2 &= P_1 - S_1/A_5 = P_R - S_1/A_5 \\ P_3 &= P_2 - \Delta P_2 = P_R - S_1/A_5 - \Delta P_2 \\ P_4 &= P_1 - \Delta P_1 = P_R - \Delta P_1 \end{aligned} \qquad (15)$$

Substitution of equations 15 and 8 into equation 2 gives:

$$M_p\left(1 + \frac{A_R}{A_3+A_6}\right)\ddot{x} = A_3(P_R - \Delta P_1) + A_6(P_R - S_1/A_5 - \Delta P_2) - P_x A_7 - \Delta P_2 A_x - (\operatorname{sgn} \dot{x}) F_{fp} + W_p \sin \xi$$

Rewriting

$$M_p\left(1 + \frac{A_R}{A_3+A_6}\right)\ddot{x} = (A_3+A_6)P_R - A_3\Delta P_1 - (A_6+A_x)\Delta P_2 - P_x A_7 - S_1/A_5 \cdot A_6 - (\operatorname{sgn} \dot{x}) F_{fp} + W_p \sin \xi \qquad (16)$$

Assume

$$A_6 + A_x \simeq A_6$$

Making this substitution and multiplying thru by $\frac{A_R}{A_3+A_6}$ eq. 16 becomes:

$$M_p\left[\frac{A_R(A_3+A_6+A_R)}{(A_3+A_6)^2}\right]\ddot{x} = A_R P_R + \frac{A_R}{A_3+A_6}\left[-A_3\Delta P_1 - A_6\Delta P_2 - P_x A_7 - \frac{S_1}{A_5}A_6 - (\operatorname{sgn}\dot{x})F_{fp} + W_p \sin\xi\right]$$

Substituting from equations 13 and 14A

$$A_R P_R = \frac{A_3 A_R}{A_3+A_6}\left[\frac{A_3 A_R}{A_2(A_3+A_6)}\right]^2 \cdot \frac{\sigma}{2g C_D^2} \cdot \dot{x}^2 + M_p\left[\frac{A_R(A_3+A_6+A_R)}{(A_3+A_6)^2}\right]\ddot{x}$$
$$\frac{A_6 A_R}{A_3+A_6}\left[\frac{A_6 A_R}{(A_3+A_6)(C_D' A_{xs} + C_D'' A_{xL} + C_K A_K)}\right]^2 \cdot \frac{\sigma}{2g} \cdot \dot{x}^2 + \frac{A_R}{A_3+A_6}(\operatorname{sgn}\dot{x})F_{fp}$$
$$- \frac{A_R}{A_3+A_6} \cdot W_p \sin\xi + \frac{A_R}{A_3+A_6} A_7 P_x + \frac{A_R A_6}{A_5(A_3+A_6)} S_1 \qquad (17)$$

Let

V_N = Initial Volume of Gas in the Recuperator

P_N = Initial Gas Pressure

V_x = Volume of Gas at any Travel x

P_x = Pressure of Gas at any Travel x

Now the motion of the floating piston relative to the cylinder is $y-x$. From equation 9, on integrating

$$y - x = \frac{A_R}{A_3 + A_6} x$$

Define

$$A_3 + A_6 = A_7 \tag{18}$$

Since

$$V_N = A_7 (y_0 - x_0) = A_7 \cdot \frac{A_R}{A_3 + A_6} \cdot x_0 = A_R x_0$$

$$V_x = V_N - A_7 (y - x) = A_7 \cdot \frac{A_R}{A_3 + A_6} (x_0 - x) = A_R (x_0 - x)$$

then

$$P_x = P_N \left(\frac{V_N}{V_x}\right)^k = P_N \left(\frac{x_0}{x_0 - x}\right)^k \tag{19}$$

where

$$x_0 = V_N / A_R$$

Substitution of equations (17), (18), and (19) into equation (1) gives the desired equation of motion,

$$M_R \ddot{x} = B(t) - \left[\left(F + F_p + \frac{A_R}{A_7} F_{sp}\right)(\operatorname{sgn} \dot{x}) + \frac{A_R A_4}{A_5 A_7} S_1\right] + \left(W_R + \frac{A_R}{A_7} W_p\right) \sin \xi$$

$$- \frac{\sigma}{2g} \left[\frac{A_6^3 A_R^3}{A_7^3 (C_D' A_{rs} + C_D'' A_{rL} + C_K A_K)^2} + \frac{A_3^3 A_R^3}{A_2^2 A_7^3 C_D^2}\right] \dot{x}^2 - A_R P_N \left(\frac{x_0}{x_0 - x}\right)^k$$

$$- M_p \left[\frac{A_R (A_7 + A_R)}{A_7^2}\right] \ddot{x} \qquad \frac{A_R^3}{A_1^2 \bar{C}_D^2} \tag{20}$$

Define

$$U_1 = F (\operatorname{sgn} \dot{x}) \qquad U_2 = F_p (\operatorname{sgn} \dot{x}) \qquad U_3 = \frac{A_R}{A_7} F_{sp} (\operatorname{sgn} \dot{x})$$

$$U_4 = \frac{A_R A_4}{A_5 A_7} S_1 \qquad U_5 = \frac{A_R}{A_7} W_p \sin \xi \qquad G = A_R P_N \left(\frac{x_0}{x_0 - x}\right)^k$$

$$I = M_p \left[\frac{A_R (A_7 + A_R)}{A_7^2}\right] \qquad H_1 = \frac{\sigma}{2g} \left[\frac{A_6^3 A_R^3}{A_7^3 (C_D' A_{rs} + C_D'' A_{rL} + C_K A_K)^2}\right]$$

$$H_2 = \frac{\sigma}{2g} \cdot \frac{A_3^3 A_R^3}{A_2^2 A_7^3 C_D^2} \qquad H_4 = \frac{\sigma}{2g} \cdot \frac{A_R^3}{A_1^2 \bar{C}_D^2} \tag{21}$$

Equation 20 is now written as:

$$M_R \ddot{x} = B(t) - (U_1 + U_2 + U_3 + U_4 - U_5) - G + W_R \sin\xi - (H_1 + H_2)\dot{x}^2 - I\ddot{x} \qquad (22)$$

Equation 22 is now the desired equation of motion.

From equation 17

$$A_R P_R = (H_1 + H_2)\dot{x}^2 + I\ddot{x} + (U_3 + U_4 - U_5) + G \qquad (23)$$

Thus

$$\text{Rod Pull} = A_R P_R + U_2 \qquad (24)$$

$$\text{Total Resistance} = \text{Rod Pull} + U_1 - W_R \sin\xi \qquad (25)$$

and from eq.'s 15

$$P_1 = P_R - \frac{H_1}{A_R}\dot{x}^2$$

$$P_2 = P_R - S_1/A_5 \qquad (26)$$

$$P_3 = P_2 - \frac{A_7 H_1}{A_6 A_R}\dot{x}^2 \qquad (27)$$

$$P_4 = P_R - \frac{A_7 H_2}{A_5 A_R}\cdot\dot{x}^2 \qquad (28)$$

This completes the analysis of Case 1.

CASE 2 - $y > (1 + \frac{A_8}{A_7})x$

We now consider that P_3 drops to 0 psi. This implies a vacuum has been created which in turn implies that the reservoir is not filled with oil. Thus, one may reason that the volume displaced by the motion of the floating piston exceeds the volume displaced by the recoil piston resulting in the criteria $y > (1 + \frac{A_8}{A_7})x$

Now, returning to equation 5 which may be written (by use of eq. 3) as:

$$Q_{xs} = C_D' A_{xs}\sqrt{\frac{2g\Delta P_2}{\sigma}} \qquad Q_{xL} = C_D'' A_{xL}\sqrt{\frac{2g\Delta P_2}{\sigma}}$$

but

$$Q_2 = A_3(\dot{y} - \dot{x})$$

So, on substituting into equation 6 (noting that $P_2 = \Delta P_2$)

$$A_3(\dot{y} - \dot{x}) + \sqrt{\frac{2gP_2}{\sigma}}\left(C_D' A_{xs} + C_D'' A_{xL} + C_K A_K\right) = A_R\dot{x}$$

where we have included flow due to leakage. Thus

$$P_2 = \left(\frac{A_R\dot{x} - A_3(\dot{y} - \dot{x})}{C_D' A_{xs} + C_D'' A_{xL} + C_K A_K}\right)^2 \cdot \frac{\sigma}{2g} \qquad (29)$$

The desired equations of motion for Case 2 are

$$M_R \ddot{x} = B(t) - A_R P_R - (\operatorname{sgn} \dot{x}) \cdot (F + F_p) + W_R \sin \xi \tag{30}$$

$$M_P \ddot{y} = P_4 A_3 - P_x A_7 - (\operatorname{sgn} \dot{y}) F_{fp} + W_p \sin \xi \tag{31}$$

where

$$P_2 = \left(\frac{A_R \dot{x} - A_3 (\dot{y} - \dot{x})}{C_D' A_{2S} + C_D'' A_{2L} + C_K A_K} \right)^2 \cdot \frac{\sigma}{2g} \tag{32}$$

$$P_1 = P_R = P_2 + {}^{S_1}/_{A_3} \tag{33}$$

$$P_4 = P_1 - \Delta P_1 = P_1 - \left[\frac{A_3 (\dot{y} - \dot{x})}{A_2 C_D} \right]^2 \cdot \frac{\sigma}{2g} \tag{34}$$

Note: equations (24) and (25) still hold. That is:

$$\text{Rod Pull} = A_R P_R + U_2$$

$$\text{Total Resistance} = \text{Rod Pull} + U_1 - W_R \sin \xi$$

Also

$$P_x = P_N \left(\frac{V_N}{V_N - A_7 (y - x)} \right)^k$$

3. Analysis of the Counter-Recoil Stroke

For the counter recoil stroke, the equations of motion (1) and (2) are essentially unchanged; but, the breech force $B(t)$ is no longer present. Thus we write

$$M_R \ddot{x} = -A_R P_R - (\operatorname{sgn} \dot{x})(F + F_p) + W_R \sin \xi \tag{35}$$

$$M_P \ddot{y} = P_4 A_3 + P_3 A_6 - P_x A_7 - (P_2 - P_3) A_x - (\operatorname{sgn} \dot{y}) F_{fp} + W_p \sin \xi \tag{36}$$

As before

$$\Delta P_1 = P_R - P_4 = \text{Pressure Drop across } A_2$$

$$\Delta P_2 = P_2 - P_3 = \text{Pressure Drop across } A_x$$

Define

$$\Delta P_3 = P_4 - P_2 = \text{Pressure Drop across } A_y$$

but

$$\Delta P_1 = \frac{V_2^2 \sigma}{2g C_D^2} \qquad \Delta P_2 = \frac{V_{xs}^2 \sigma}{2g C_D'^2} = \frac{V_{xL}^2 \sigma}{2g C_D''^2} \qquad \Delta P_3 = \frac{V_y^2 \sigma}{2g C_y^2}$$

and since in counter recoil

$$Q_y = Q_x$$

we find

$$A_y V_y = A_{xs} V_{xs} + A_{xL} V_{xL}$$

by equations 12

$$V_y = \frac{A_6 A_R}{A_7 A_y} \dot{x}$$

thus

$$\Delta P_3 = \left(\frac{A_6 A_R}{A_7 C_y A_y} \right)^2 \cdot \frac{\sigma}{2g} \cdot \dot{x}^2 \tag{30}$$

The pressure drops ΔP_1, ΔP_2, and ΔP_3 are positive quantities irregardless of the direction of flow. In counter recoil, these quantities must be entered into the equations of motion as negative quantities. Thus for counter recoil:

$$P_R = P_4 - \Delta P_1$$

$$P_3 = P_2 + \Delta P_2$$

$$P_2 = P_4 + \Delta P_3$$

or

$$P_4 = P_R + \Delta P_1$$

$$P_3 = P_R + \Delta P_1 + \Delta P_2 + \Delta P_3 \tag{31}$$

$$P_2 = P_R + \Delta P_1 + \Delta P_3$$

Recalling that

$$\ddot{y} = \left(1 + \frac{A_R}{A_7}\right) \ddot{x} \tag{32}$$

then on substituting eq.'s 31 and 32 into eq.'s (28) and (29):

$$M_R \ddot{x} = -A_R P_R - (\operatorname{sgn} \dot{x})(F + F_p) + W_R \sin \xi \tag{33}$$

$$M_p \left(1 + {}^{A_R}/_{A_7}\right) \ddot{y} = (P_R + \Delta P_1) A_3 + (P_R + \Delta P_1 + \Delta P_2 + \Delta P_3) A_6 - P_x A_7 - (\operatorname{sgn} \dot{x}) F_{fp} + W_p \sin \xi \tag{34}$$

Substituting into eq 34 from eq.'s 13, 14A, and 30

$$M_p \left(1 + {}^{A_R}/_{A_7}\right) \ddot{y} = (A_3 + A_6) P_R + (A_3 + A_6) \left[\frac{A_3 A_R}{A_2 (A_3 + A_6)} \right]^2 \cdot \frac{\sigma}{2g C_D^2} \cdot \dot{x}^2$$

$$+ A_6 \left[\frac{A_6 A_R}{(A_3 + A_6)(C_D' A_{xs} + C_D'' A_{xL} + C_x A_y)} \right]^2 \cdot \frac{\sigma}{2g} \cdot \dot{x}^2 + A_6 \left(\frac{A_6 A_R}{A_7 C_y A_y} \right)^2 \cdot \frac{\sigma}{2g} \cdot \dot{x}^2$$

$$- A_7 P_x - (\operatorname{sgn} \dot{x}) F_{fp} + W_p \sin \xi$$

Solving for $A_R P_R$

$$A_R P_R = M_c\left[\frac{A_R(A_R+A_T)}{A_T}\right]^2 \ddot{x} + A_R P_x + \frac{A_R}{A_T}(\operatorname{sgn}\dot{x})F_{sp} - \frac{A_R}{A_T}W_p \sin\xi$$
$$-\left[\frac{A_3^2 A_R^3}{A_2^2 A_T^2} + \frac{A_6^3 A_R^3}{A_T^3(C_b' A_{s1} + C_b'' A_{s2} + C_K A_K)^2} + \frac{A_6^3 A_R^3}{A_T^3(C_y A_y + C_K' A_K')^2}\right]\cdot\frac{\sigma}{2g}\cdot\dot{x}^2 \quad (35)$$

where $C_K' A_K'$ is the effective area associated with leakage parallel to A_y. Substituting from eq. 21

$$A_R P_R = I\ddot{x} + G + U_3 - U_5 - \frac{A_T}{A_3}H_2\dot{x}^2 - H_1\dot{x}^2 - H_3\dot{x}^2 \quad (36)$$

where

$$H_3 = \frac{A_6^3 A_R^3}{A_T^3(C_y A_y + C_K' A_K')^2}\cdot\frac{\sigma}{2g} \quad (37)$$

Now, the desired equation of motion (33) is written as:

$$M_R\ddot{x} = -(U_1 + U_2 + U_3 - U_5) - G + W_R \sin\xi + \left(\frac{A_T}{A_3}H_2 + H_1 + H_3\right)\dot{x}^2 - I\ddot{x} \quad (38)$$

Thus:

$$\text{Rod Pull} = (U_2 + U_3 - U_5) + G + I\ddot{x} - \left(H_1 + \frac{A_T}{A_3}H_2 + H_3\right)\dot{x}^2$$

$$P_R = \frac{1}{A_R}\left[U_3 - U_5 + G + I\ddot{x} - \left(H_1 + \frac{A_T}{A_3}H_2 + H_3\right)\dot{x}^2\right]$$

$$P_2 = \frac{1}{A_R}\left[U_3 - U_5 + G + I\ddot{x} - \left(H_1 - \frac{A_3}{A_6}H_3\right)\dot{x}^2\right]$$

$$P_3 = \frac{1}{A_R}\left[U_3 - U_5 + G + I\ddot{x} + \frac{A_3}{A_6}(H_1 + H_3)\dot{x}^2\right]$$

$$P_4 = \frac{1}{A_R}\left[U_3 - U_5 + G + I\ddot{x} - (H_1 + H_3)\dot{x}^2\right]$$

To review: the equation of motion is:

<u>For Recoil</u>:

$$M_R\ddot{x} = B(t) - (U_1 + U_2 + U_3 + U_4 - U_5) - G + W_R \sin\xi - (H_1 + H_2)\dot{x}^2 - I\ddot{x}$$

<u>For Counter Recoil</u>:

$$M_R\ddot{x} = -(U_1 + U_2 + U_3 - U_5) - G + W_R \sin\xi + \left(H_1 + \frac{A_T}{A_3}H_2 + H_3\right)\dot{x}^2 - I\ddot{x}$$

where U_1, U_2, U_3, U_4, U_5, G, H_1, H_2, H_3, and I are defined by equations 21 and 37.

These two equations may be written as one equation as follows:

$$M_R \ddot{x} = B(t) - (U_1 + U_2 + U_3 - U_5) - G + W_R \sin \xi - I \ddot{x} - (\operatorname{sgn} \dot{x})(H_1 + H_2)\dot{x}^2$$
$$+ E(\dot{x}) \left[\frac{A_6}{A_3} H_2 + H_3 \right] \dot{x}^2 + F(\dot{x}) U_4$$

where

$$E(\dot{x}) \begin{matrix} = 0 & \dot{x} \geq 0 \\ = 1 & \dot{x} < 0 \end{matrix} \qquad F(\dot{x}) = \begin{matrix} 1 & \dot{x} \geq 0 \\ 0 & \dot{x} < 0 \end{matrix}$$

Computer Model Analysis - Case 1

Beginning with equation 22 and basic input parameters of the M102 Howitzer, XM37 Recoil Mechanism, a preliminary computer study was initiated. This specifically dealt with Zone 7, 100% charge. The desired goal was to duplicate as close as possible the rod-pull time curves of actual carriage firings (See graphs A and B). This also means a duplication in recoil length, recoil time, and cycle time. It must be kept in mind, however, that graphs A and B were carriage firings using a simulated M482 round. The chances are that the breech force used in the computer solution is not quite the same as actually fired.

Computer runs were made both in long and short recoil. The rate of flow or coefficient of discharge was held constant throughout a run. First, a number of runs were made in short recoil to determine its coefficient of discharge. Next, using the determined short groove coefficient and varying the long groove coefficient, long recoil runs were made until a suitable value was obtained for the long groove discharge coefficient. Maximum leakage (tolerancing between the control rod and regulator) was considered present in all runs.

Input Parameters

The parameters defined below were the basic input parameters in all runs unless otherwise noted.

Symbol	Pneumonic Code	Description	Value
F	F	Guide Friction	263.88 pounds
F_{fp}	FFP	Floating Piston Friction	658. pounds
F_p	FP	Recoil Piston & Stuffing Box Friction	263. pounds
A_R	AR	Area of Recoil Piston	$2.971 in^2 = .0206 ft^2$
A_7	A7	See Figure 1	$14.186 in^2 = .0985 ft^2$
A_6	A6	See Figure 1	$11.781 in^2 = .0818 ft^2$
A_5	A5	See Figure 1	$3.620 in^2 = .0251 ft^2$
A_3	A3	See Figure 1	$2.405 in^2 = .0167 ft^2$
A_2	A2	See Figure 1	$.4418 in^2 = .0031 ft^2$
S_1	S1	Spring Loaded Valve Force	81 pounds
W_p	WP	Floating Piston Weight	25 pounds
ζ	ZETA	Angle of Elevation	Varied
P_N	PN	Initial Gas Pressure	$1150 psi = 165600 lbs/ft^2$
X_o	XO	Quantity of Length Needed	168.07" = 14.00 ft
k	XK	To Satisfy equation 19 Adiabatic Gas Exponent	1.40

W_R	WR	Weight of Recoiling Parts	1466	pounds
σ	SIGMA	Density of Oil	53.3	lbs/ft^3
g	G	Acceleration of Gravity	32.17	ft/sec^2
C_D	CD	Discharge Coefficient Through A		0.75
C'_D	CDP	Short Groove Discharge Coefficient		Varied
C''_D	CDPP	Long Groove Discharge Coefficient		Varied
C_K	CK	Recoil Leakage Coefficient		Varied
A_K	AK	Recoil Leakage Area	$.0134in^2 = .00009306ft^2$	
C_Y	CY	Counterrecoil Discharge Coefficient		Varied
C'_K	CKP	Counterrecoil Leakage Coefficient		Varied
A'_K	AKP	Counterrecoil Leakage Area	$.0669in^2 = .00004636ft^2$	
M_R	XMR	Mass of Recoiling Parts	45.57	slugs
M_P	XMP	Mass of Floating Piston	0.78	slugs

In addition to these constants and variables, the theoretical breech force-time B(t) curve for Zone 7 was a necessary input. The technique of developing this force-time relationship is fully discussed in reference 2. The B(t) curve consisted of the following points.

M442 Projectile T36E1 Propellant

B (t) (#)	Time-milliseconds	B(t) (#)	Time-milliseconds
0	0	55024	9.657
16290	.830	46700	10.170
75815	2.006	46249	10.236
138959	2.526	44494	10.498
235115	3.069	42403	10.826
301003	3.404	38538	11.481
345271	3.655	35059	12.137
373976	3.859	31922	12.793
391412	4.034	29092	13.448
400664	4.188	26537	14.104
398823	4.576	24227	14.760
355812	5.087	22136	15.415
304705	5.512	18527	16.726
258984	5.892	12038	20.005
220822	6.242	7966	23.283
164023	6.886	5362	26.561
125687	7.484	3665	29.839
99038	8.053	1786	36.895
79904	8.601	915	42.951
72322	8.869	489	49.508
65756	9.135	272	56.064
		205	59.342
		156	62.620
		92	69.176
		56	75.732
		0	78.000

Another integral part of the input data was the relationship between X (recoil travel) and A_x (corresponding orifice area). The total groove area is computed by the relationship $A_x = \frac{\text{Depth} - .00434}{1.666}$. The areas for the long and short grooves are listed below.

2 - Short Grooves

Travel(ft)	Area(ft^2)	Travel(ft)	Area (ft^2)	Travel(ft)	Area(ft^2)
.0000	.00056736	.5715	.00069861	1.6265	.00046111
.0149	.00060625	.7112	.00069236	1.6662	.00044653
.0348	.00064167	.7510	.00068611	1.7060	.00043194
.0447	.00065347	.7908	.00067986	1.7458	.00041736
.0547	.00066250	.8316	.00067361	1.7856	.00040278
.0646	.00066806	.8704	.00066528	1.8254	.00038611
.0746	.00066944	.9102	.00065694	1.8652	.00035944
.0845	.00066806	.9500	.00064861	1.9050	.00035278
.0945	.00066250	.9898	.00064028	1.9448	.00033611
.1044	.00065347	1.0296	.00063125	1.9846	.00031736
.1144	.00065111	1.0694	.00062153	2.0244	.00029861
.1541	.00068611	1.1092	.00061187	2.0642	.00027778
.1940	.00069861	1.1490	.00060278	2.1040	.00025694
.2337	.00070694	1.1887	.00059236	2.1437	.00023194
.2735	.00071111	1.2285	.00058194	2.1835	.00020694
.3133	.00071250	1.2683	.00057153	2.2233	.00017778
.3531	.00071389	1.3081	.00056111	2.2631	.00014652
.3929	.00071528	1.3479	.00054861	2.3029	.00011111
.4327	.00071528	1.3877	.00053611	2.3427	.00006944
.4725	.00071528	1.4275	.00052361	2.3825	.00001944
.5123	.00071319	1.4673	.00051111	2.4024	.00000000
.5521	.00071111	1.5071	.00049861	5.0000	.00000000
.5919	.00070694	1.5461	.00048611		
.6317	.00070278	1.5867	.00047361		

2 - Long Grooves

Travel(ft)	Area(ft^2)	Travel(ft)	Area(ft^2)	Travel(ft)	Area(ft^2)
.0000	.00020069	.1443	.00042431	.7114	.00040278
.0149	.00026181	.1542	.00041528	.7909	.00041111
.0348	.00032708	.1940	.00038819	.8705	.00041736
.0547	.00037708	.2338	.00036528	.9501	.00042361
.0746	.00041250	.2786	.00033819	1.0297	.00043194
.0846	.00042431	.3134	.00035278	1.1093	.00044028
.0945	.00043333	.3532	.00036111	1.1889	.00044861
.1045	.00043889	.3930	.00036736	1.2685	.00045694
.1144	.00044028	.4726	.00037778	1.3481	.00046528
.1244	.00043889	.5522	.00038611	1.4277	.00047361
.1343	.00043333	.6318	.00039444	1.5073	.00048403

Travel(ft)	Area(ft^2)	Travel(ft)	Area(ft^2)	Travel(ft)	Area(ft^2)
1.5869	.00049444	2.3927	.00076389	3.2583	.00051111
1.6665	.00050486	2.4027	.00076528	3.3379	.00047986
1.7461	.00051736	2.4126	.00076389	3.4175	.00044653
1.8257	.00053194	2.4226	.00076319	3.4971	.00040903
1.9052	.00054861	2.4624	.00075278	3.5767	.00036736
1.9848	.00056528	2.5420	.00073403	3.6563	.00032361
2.0644	.00058403	2.6216	.00071319	3.7359	.00027153
2.1440	.00061111	2.7012	.00069028	3.8155	.00021111
2.2235	.00064444	2.7808	.00066944	3.8951	.00013194
2.3032	.00068819	2.8604	.00064653	3.9348	.00007917
2.3430	.00071736	2.9399	.00062153	3.9547	.00004375
2.3629	.00073750	3.0195	.00059444	3.9746	.00000000
2.3728	.00074931	3.0991	.00056944	5.0000	.00000000
2.3828	.00075833	3.1787	.00054028		

Counterrecoil Grooves

Travel (ft)	Area (ft^2)
.0000	.00002083
.1740	.00002361
.5720	.00007361
.9699	.00011111
1.5668	.00016667
3.9841	.00017361
5.0000	.00017361

Computer Run Results - Case 1

Runs number 1-3 were short recoil runs in which only the short groove discharge coefficient (C'_D) was varied. The leakage coefficient (C_K) was held constant. Values of C'_D = .80, .75, and .70 were used. C_K was equal to .75.

Runs No. 1-3 **Short Recoil** **Computed Maximum Values** C_K = .75

	#1 C'_D=.75	#2 C'_D=.80	#3 C'_D =.70
Time To End Of Recoil (seconds)	.1355	.1305	.1425
Time To Maximum Recoiling Parts Velocity (sec)	.0171	.0176	.0185
Recoiling Parts Velocity (ft/sec)	36.68	37.05	36.27
Recoil Length (inches)	29.16	29.30	28.97
Time At Which Maximum Rod Pull Occurs (sec)	.0495	.0961	.0155
Maximum Rod Pull (pounds)	19436	24476	20900

Graph 1 - This graph is a comparison of the rod-pull time curves for runs 1-3. It can be seen that a small change in C'_D can completely alter the shape of the rod pull-time curve. This graph was compared to graph A and it was felt that run 1 was the closest approximation.

Run No. 4 was then made with $C'_D = .75$ and $C_K = .95$. It was hoped that the rod pull shape could be more closely approximated by varying the leakage coefficient. But this produced a shape very similar to run no. 2 although not as severe. It was then concluded that varying the leakage coefficient has the same effect as varying the discharge coefficient. Of course, the results will never be quite the same because of the difference in areas associated with these coefficients.

Run No. 4 Short Recoil Computed Maximum Values $C'_D = .75$ $C_K = .95$

Time To End Of Recoil (seconds)	.7375
Time To Maximum Recoiling Parts Velocity (sec)	.0172
Recoiling Parts Velocity (ft/sec)	36.88
Recoil Length (inches)	29.62
Time At Which Maximum Rod Pull Occurs (sec)	.0980
Maximum Rod Pull (pounds)	22157

Now, some long recoil runs were made. $C'_D = C_K = .75$ were held constant while the long groove discharge coefficient (C''_D) was varied.

Runs No. 5-7 Long Recoil Computed Maximum Values $C'_D = C_K = .75$

	#5 $C''_D = .85$	#6 $C''_D = .90$	#7 $C''_D = .75$
Time To End Of Recoil (seconds)	.1985	.1915	.2035
Time To Maximum Recoiling Parts Velocity (sec)	.0195	.0205	.0200
Recoiling Parts Velocity (ft/sec)	38.48	38.57	38.32
Recoil Length (inches)	47.64	47.91	46.26
Time At Which Maximum Rod Pull Occurs (sec)	.0130	.1730	.0130
Maximum Rod Pull (pounds)	11355	12185	11900

Graph 2 - This graph is a comparison of the rod pull-time curves for runs 5-7. This graph was compared to graph B and it was felt that run no. 5 was the best approximation.

At this point, it was concluded that the flow coefficients for the long and short grooves were .85 and .75, respectively, until a further refinement of the program could be made.

A counterrecoil study was then begun. A representative cycle time for short recoil is 1.82 seconds. Now letting $C'_D = C_K = .75$, the only variables are the flow coefficient through the counterrecoil groove (C_y) and the leakage coefficient associated with the counterrecoil groove (C'_K). Two methods of approach were used. One method was to vary both C_y and C'_K until a reasonable cycle time was obtained; the other was to vary only C_y and set $C'_K = 0$. A number of runs were made to determine the best value for each method. These are listed as runs 8-9.

Runs No. 8-9 **Counterrecoil (Short)** $C'_D = C_K = .75$

	#8 $C_y = C'_K = .50$	#9 $C_y = .95, C'_K = 0$
Maximum Counterrecoil Velocity (ft/sec)	-2.31	-3.26
In-Battery Velocity (ft/sec)	-0.61	-0.36
Cycle Time (seconds)	1.87	1.85

Using the values determined in runs 8 and 9, two long recoil runs were made. A representative cycle time for long recoil is 2.43 seconds. $C'_D = C_K = .75$ and $C''_D = .85$ were the criteria for the recoil stroke.

Runs No. 10-11 **Counterrecoil (Long)** $C'_D = C_K = .75$ $C''_D = .85$

	#10 $C_y = C'_K = .50$	#11 $C_y = .95, C'_K = 0$
Maximum Counterrecoil Velocity (ft/sec)	-2.78	-4.91
In-Battery Velocity (ft/sec)	-0.68	-0.50
Cycle Time (seconds)	2.36	1.73

Run no. 10 was the better approximation because of the comparatively close cycle time. This now completes the analysis of the recoil and counterrecoil stroke for Zone 7. Reviewing the results, it was found that during the recoil stroke C'_D and C_K were equal to .75 while C''_D was equal to .85. Of course, C''_D was only active in long recoil. The counterrecoil analysis showed that $C_y = C'_K = .50$ was the best approximation.

Computer Model Analysis - Case 2

Now using equations 30 and 31, and utilizing the same input parameters as case 1 except for the breech force-time curve, an analysis was conducted. Case 2 is an undesirable design condition. It only occurs when the floating piston moves at such a velocity that there is a void created between the oil and the floating piston. Now, in order to generate enough momentum to obtain this condition, a Zone 8 breech force-time curve was used in the solution. A run was made in both long and short recoil. A 5k muzzle brake with an efficiency rating of 90% was considered active in these runs. A muzzle brake only acts during the gas ejection period. This analysis is strictly mathematical as there is no firing data to compare against. However, runs were made under these conditions in order to approximate mathematically what force the recoil mechanism would feel.

The Zone 8 breech force-time curve consisted of the following points.

B (t) #	Time(millisec)	B(t) #	Time(millisec)	B(t) #	Time(millisec)
0	0	123328	1.728	378723	2.570
31414	.403	176728	1.967	482102	2.836
26576	.799	225264	2.140	519827	2.942
64594	1.325	309412	2.388	550175	3.036

B (t) #	Time(millisec)	B(t) #	Time(millisec)	B(t) #	Time(millisec)
593154	3.198	80983	7.901	27224	14.210
598101	3.221	68290	8.350	17689	17.154
618194	3.338	67960	8.379	11706	20.099
630370	3.462	66980	8.468	7878	23.044
633360	3.562	65381	8.615	5539	25.988
633321	3.573	63824	8.762	2624	31.878
621355	3.772	62308	8.909	1344	37.767
547660	4.183	59394	9.204	719	43.656
464902	4.526	56629	9.500	533	46.601
392528	4.833	51516	10.087	399	49.545
332952	5.116	49152	10.382	301	52.490
245434	5.640	46907	10.676	229	55.435
187075	6.127	42749	11.265	135	61.324
146834	6.590	42137	11.357	0	62.000
131324	6.816	38994	11.845		
118110	7.038	35599	12.443		
106768	7.257	32844	12.969		
96966	7.474	32528	13.032		

Runs No. 12-13 Zone 8

	# 12 Short Recoil	# 13 Long Recoil
Time To End Of Recoil (sec)	.1283	.1805
Time To Maximume Recoiling Parts Velocity (sec)	.0085	.0085
Recoiling Parts Velocity (ft/sec)	43.63	44.47
Recoil Length (inches)	29.20	47.94
Time At Which Maximum Rod Pull Occurs (sec)	.0065	.0105
Maximum Rod Pull (pounds)	24494	14187

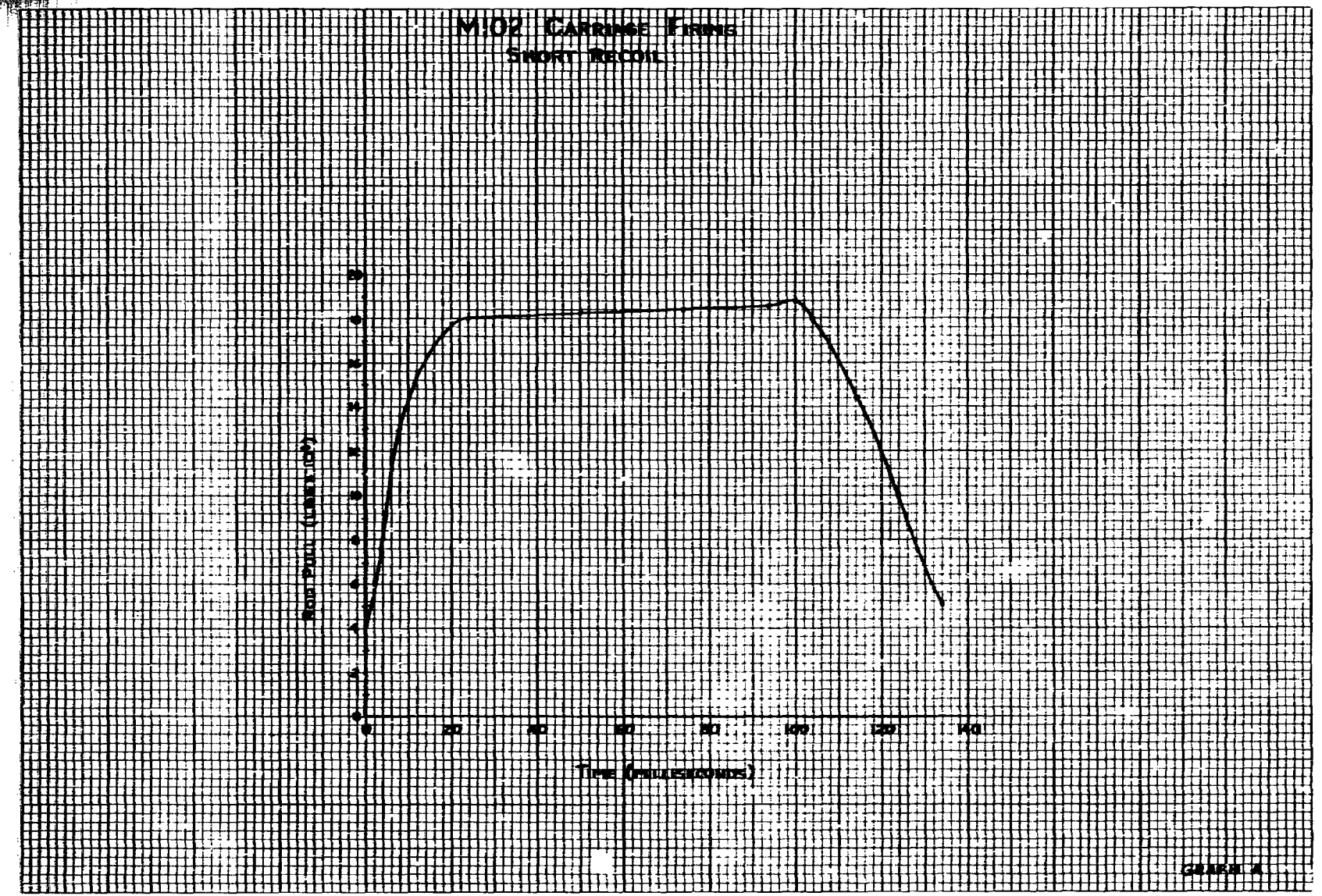
Graph 3 - This graph is a plot of the rod pull-time curves for runs 12-13. The uniqueness of run no. 12 requires some explanation. Run no. 12 has a discontinuity in it due in part from shifting out of case 2 back into case 1 and because certain system flexibilities were ignored in this model. Run no. 13 never went into case 2 and therefore has no discontinuity.

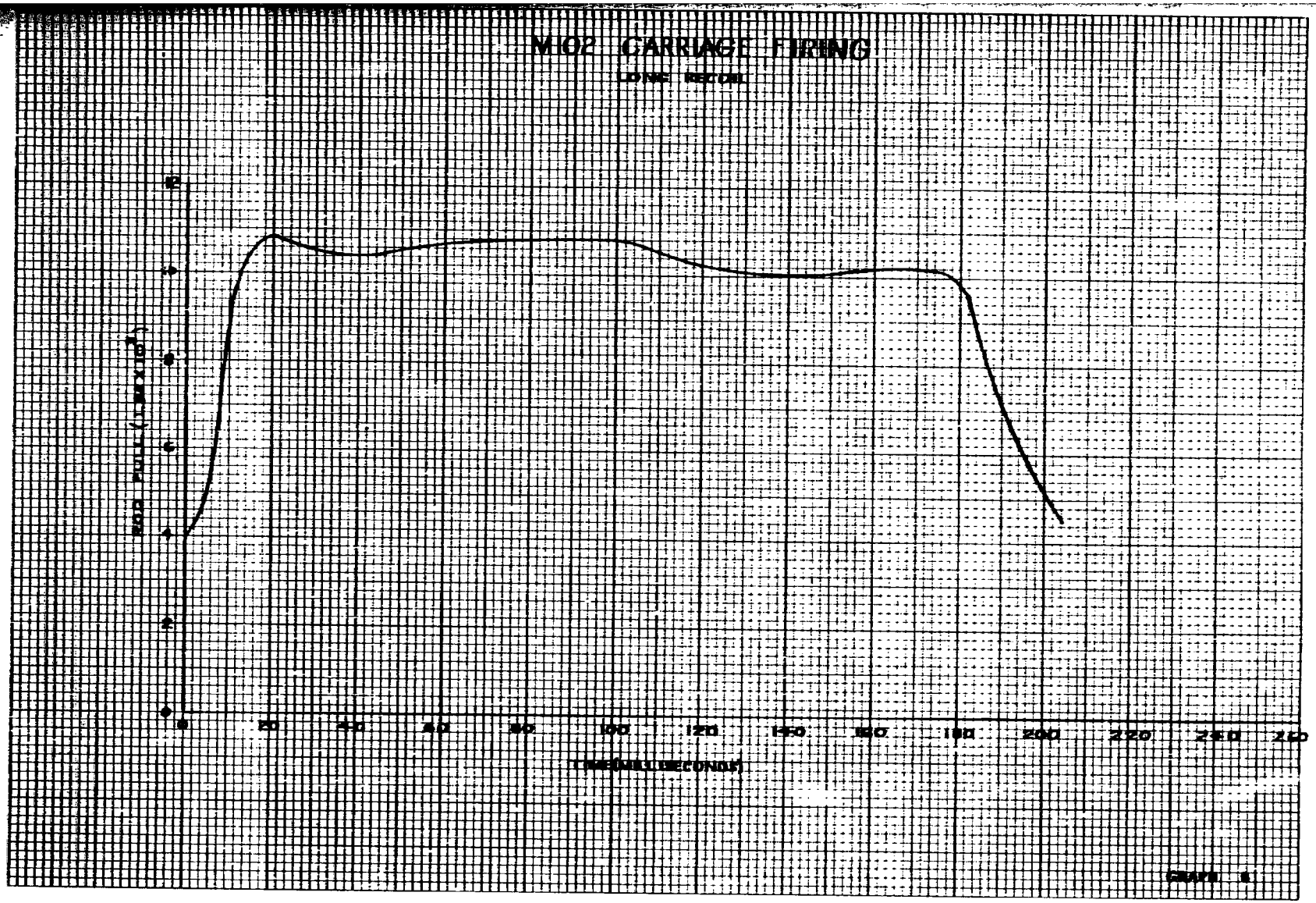
Acknowledgement

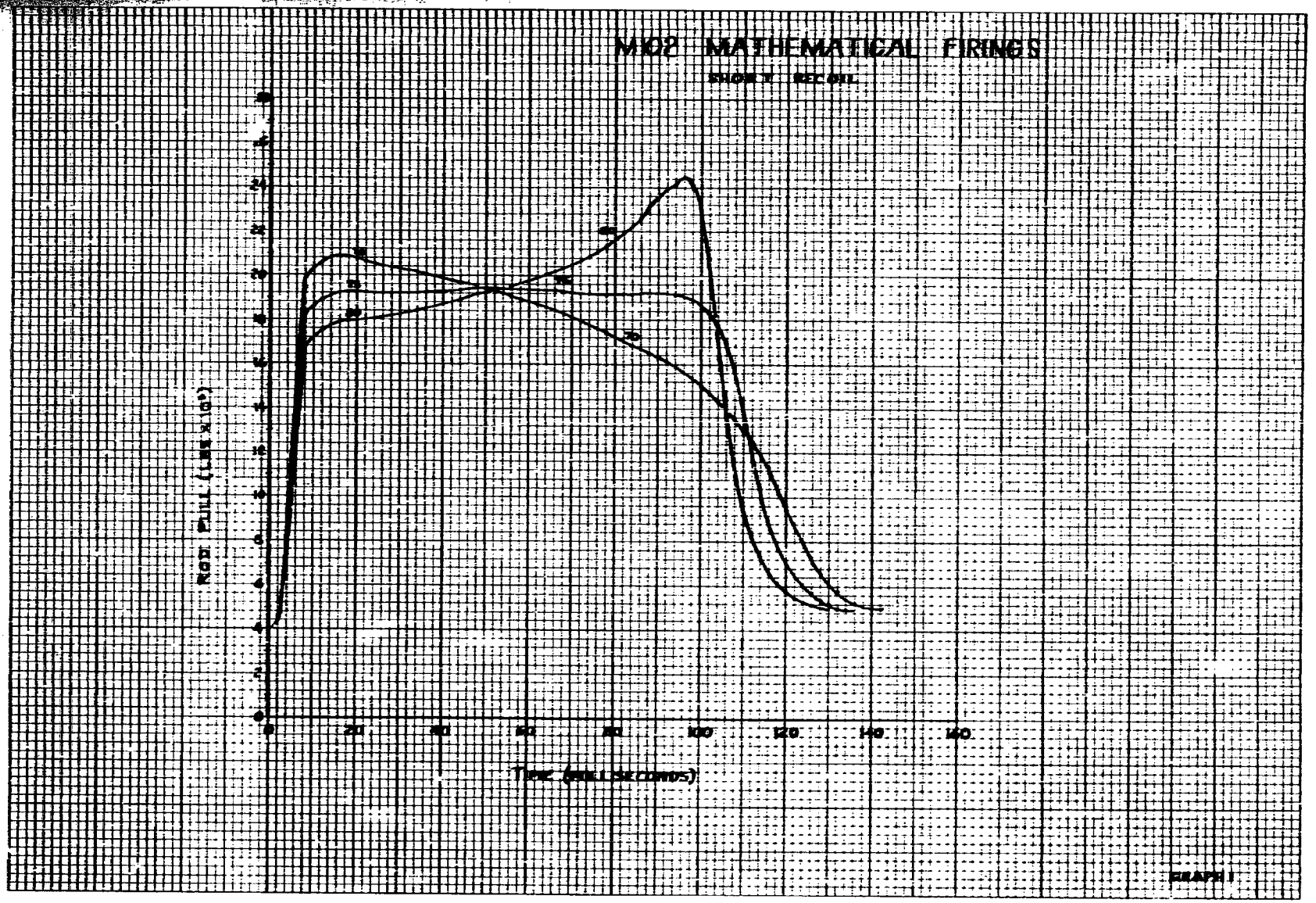
The authors of this technical note wish to acknowledge the computer programming done by Thomas D. Streeter.

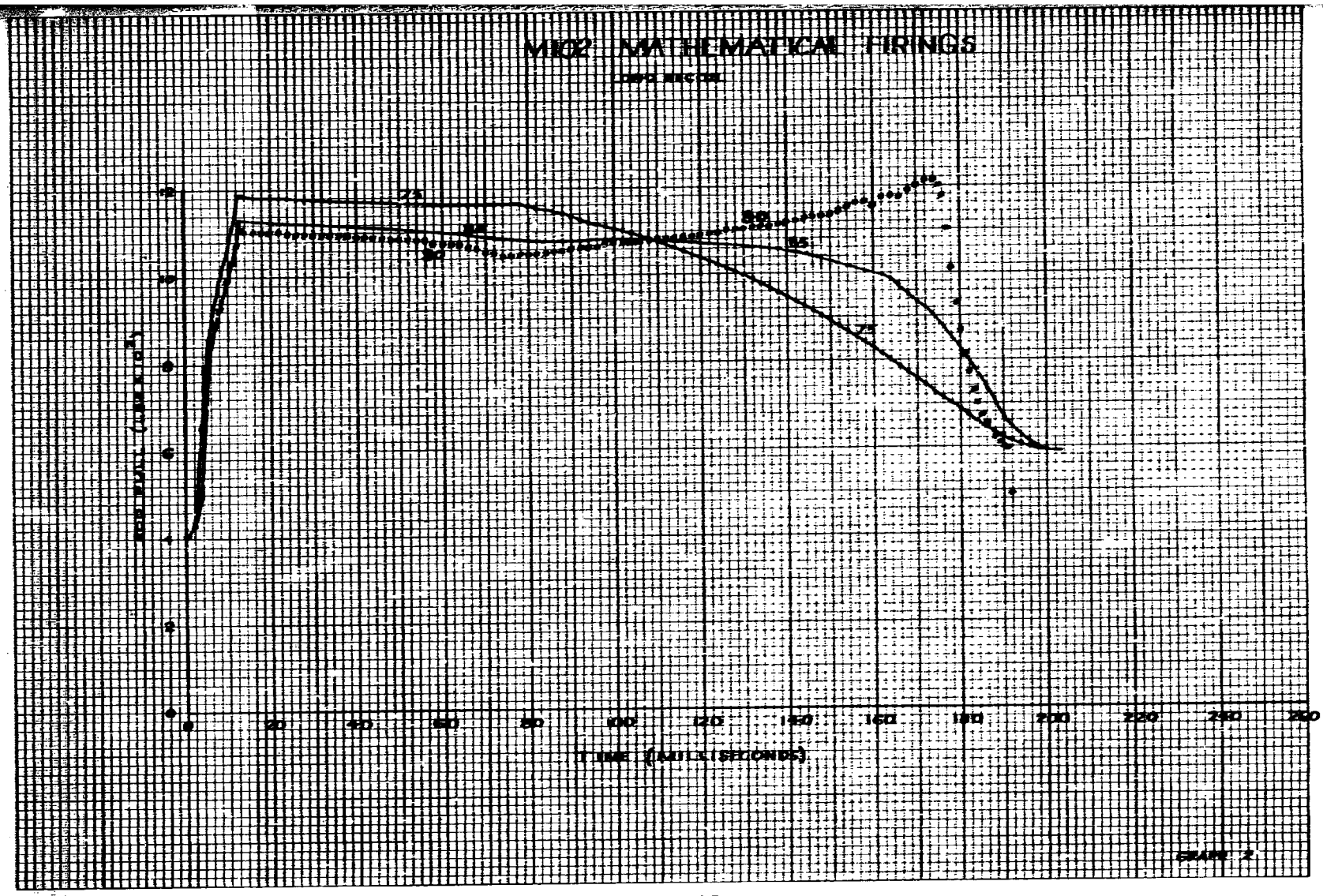
APPENDIX A

GRAPHS


M102 Carriage Firing
Short Recoil
Rod Force (Lbs x 10³)
0
2
4
6
8
10
12
14
16
18
20
0
20
40
60
80
100
120
140
Time (milliseconds)
Graph A


M102 CARRIAGE FIRING
LONG RECOIL
ROD PULL (LBS X 10³)
0
2
4
6
8
10
12
0
20
40
60
80
100
120
140
160
180
200
220
240
260
(MILLISECONDS)
CHART 6


M102 MATHEMATICAL FIRINGS
SHORT RECOIL
ROD PULL (LBS X 10³)
TIME (MILLISECONDS)
0
20
40
60
80
100
120
140
160
GRAPH 1


M102 MATHEMATICAL FIRINGS
LONG RECOIL
TIME (MILLISECONDS)
0
20
40
60
80
100
120
140
160
180
200
220
240
260
2
4
6
8
10
12
75
80
85
90
GRAPH 2

APPENDIX B

ANALYSIS PROGRAM

```
C     FIRST APPROXIMATION TO FLUID FLOW IN RECOIL SYSTEMS
      DIMENSION AREASX(75),AREASY(75),AREALX(75),AREALY(75),AYX(20),AYY(
     120),BRCHX(65),BRCHY(65)
      COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
     1FXD,CONST2,AREASX,AREASY,AXS,AREALX,AREALY,AXL,AXCD,CDP,CDPP,CK,AK
     2,CONST3,AYX,AYY,AY,AYCD,CY,CKP,AKP,H,HH,BOFT1,BOFT2,BOFT3,A7,DELX,
     3DELY,DELXD,DELYD,DX,DY,T,BRCHX,BRCHY,S1,A5,SIGMA,CD,XMR,ZETA,WP,FF
     4P,A2,P3H,XMP,Y,YD
      READ1,M1,(BRCHX(I),BRCHY(I),I=1,M1)
      READ1,M1,(AREASX(I),AREASY(I),I=1,M1)
      READ1,M1,(AREALX(I),AREALY(I),I=1,M1)
      READ1,M1,(AYX(I),AYY(I),I=1,M1)
    1 FORMAT (I10/(8F10.0))
      READ2,T,H,X,XD,F,FP,AR,FFP,A7,A6,S1,A5,WP,ZETA,PN,XO,XK,WR,SIGMA,C
     1DP,CDPP,CK,AK,A3,A2,CD,CY,CKP,AKP,XMR,XMP,HHH,H3,SGNXD,Y,YD
    2 FORMAT(8F10.0)
      P3H=H
      AREA1=0.
      HH=H/2.
      XI=XMP*AR*(A7+AR)/(A7**2)
      CONST1=WR*SINF(ZETA)
      CONST2=XMR+XI
      CONST3=SIGMA*A6**3*AR**3/(2.*32.17*A7**3)
      U1=F
      UU1=U1
```

```
      U2=FP
      UU2=U2
      U3=AR*FFP/A7
      UU3=U3
      U4=AR*A6*S1/(A5*A7)
      U5=AR*WP*SINF(ZETA)/A7
      H2=SIGMA*A3**3*AR**3/(2.*32.17*A2**2*A7**3*CD**2)
      A1=AR*(FFP/A7-WP*SINF(ZETA)/A7+PN)+U2+U1-WR*SINF(ZETA)
      IBRCH=1
   11 CALL LINEAR(T,BRCHX,BRCHY,BOFT1,IBRCH)
      T=T+HH
      CALL LINEAR(T,BRCHX,BRCHY,BOFT2,IBRCH)
      T=T+HH
      CALL LINEAR(T,BRCHX,BRCHY,BOFT3,IBRCH)
   12 CALL KUTTA(X,XD)
      IF(XD)3,4,4
    3 U1=-UU1
      U2=-UU2
      U3=-UU3
      SGNXD=-1.
      GO TO 5
    4 U1=UU1
      U2=UU2
      U3=UU3
      SGNXD=1.
```

```
5 XDD=DFUNC(BOFT3,X,XD)
  IF(XD)6,7,7
6 GG=AR*PN*(XO/(XO-X))**XK
  PR=(XI*XDD+GG+U3-U5-A7*H2*XD**2/A3-H1*XD**2-H3*XD**2)/AR
  R=PR*AR+U2
  P2=(U3-U5+GG+XI*XDD-(H1-A3*H3/A6)*XD**2)/AR
  P3=(U3-U5+GG+XI*XDD+A3*(H1+H3)*XD**2/A6)/AR
  P4=(U3-U5+GG+XI*XDD-(H1+H3)*XD**2)/AR
  H=HHH
  HH=H/2.
  GO TO 8
7 IARES=1
  IAREL=1
  CALL LINEAR(X,AREASX,AREASY,AXS,IARES)
  CALL LINEAR(X,AREALX,AREALY,AXL,IAREL)
  AXCD=CDP*AXS+CDPP*AXL+CK*AK
  H1=CONST3/(AXCD**2)
  PR=((H1+H2)*XD**2+XI*XDD+U3+U4-U5+AR*PN*(XO/(XO-X))**XK)/AR
  R=PR*AR+U2
  P2=PR-S1/A5
  P3=P2-A7*H1*XD**2/(A6*AR)
  P4=PR-A7*H2*XD**2/(A3*AR)
8 TR=R+U1-CONST1
  IF(P3+2116.8)31,32,32
32 CALL AREA(AREA1,A1,TR,H)
```

```
      PUNCH 9,T,X,XD,R,PR,P2,P3,P4,AREA1,TR
    9 FORMAT(F16.7,4F16.4/5F16.4)
      IF(SENSE SWITCH 1 )201,202
  201 H=P3H
      HH=H/2.
      GO TO 35
  202 IF(P3-10000.)135,135,181
  135 H=P3H/100.
      HH=H/2.
      GO TO 35
  181 IF(P3-50000.)81,81,35
   81 H=P3H/10.
      HH=H/2.
      GO TO 35
   31 PRINT 33
   33 FORMAT(6HP3 NEG)
      PRINT 9,T
      T=T-H
      X=X-DX
      XD=XD-DY
      CALL P3NEG(X,XD,AREA1,A1)
   35 IF(BOFT1)10,10,11
   10 T=T+H
      GO TO 12
      END
```

```
      SUBROUTINE KUTTA(X,XD)
      DIMENSION AREASX(75),AREASY(75),AREALX(75),AREALY(75),AYX(20),AYY(
     120)
      COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
     1FXD,CONST2,AREASX,AREASY,AXS,AREALX,AREALY,AXL,AXCD,CDP,CDPP,CK,AK
     2,CONST3,AYX,AYY,AY,AYCD,CY,CKP,AKP,H,HH,BOFT1,BOFT2,BOFT3,A7,DELX,
     3DELY,DELXD,DELYD,DX,DY,T,BRCHX,BRCHY,S1,A5,SIGMA,CD,XMR,ZETA,WP,FF
     4P,A2
      IARES=1
      CALL LINEAR(X,AREASX,AREASY,AXS,IARES)
      IAREL=1
      CALL LINEAR(X,AREALX,AREALY,AXL,IAREL)
      AXCD=CDP*AXS+CDPP*AXL+CK*AK
      H1=CONST3/(AXCD**2)
      IF(XD)1,2,2
    1 EXD=1.
      FXD=0.
      IAY=1
      CALL LINEAR(X,AYX,AYY,AY,IAY)
      AYCD=CY*AY+CKP*AKP
      H3=CONST3/(AYCD**2)
      GO TO 3
    2 EXD=0.
      FXD=-1.
    3 AK1=H*DFUNC(BOFT1,X,XD)
```

```
      IARES=1
      CALL LINEAR(X+HH*XD+H*AK1/8.,AREASX,AREASY,AXS,IARES)
      IAREL=1
      CALL LINEAR(X+HH*XD+H*AK1/8.,AREALX,AREALY,AXL,IAREL)
      AXCD=CDP*AXS+CDPP*AXL+CK*AK
      H1=CONST3/(AXCD**2)
      IF(XD)4,5,5
    4 IAY=1
      CALL LINEAR(X+HH*XD+H*AK1/8.,AYX,AYY,AY,IAY)
      AYCD=CY*AY+CKP*AKP
      H3=CONST3/(AYCD**2)
    5 AK2=H*DFUNC(BOFT2,X+HH*XD+H*AK1/8.,XD+AK1/2.)
      AK3=H*DFUNC(BOFT2,X+HH*XD+H*AK1/8.,XD+AK2/2.)
      IARES=1
      CALL LINEAR(X+H*XD+HH*AK3,AREASX,AREASY,AXS,IARES)
      IAREL=1
      CALL LINEAR(X+H*XD+HH*AK3,AREALX,AREALY,AXL,IAREL)
      AXCD=CDP*AXS+CDPP*AXL+CK*AK
      H1=CONST3/(AXCD**2)
      IF(XD)6,7,7
    6 IAY=1
      CALL LINEAR(X+H*XD+HH*AK3,AYX,AYY,AY,IAY)
      AYCD=CY*AY+CKP*AKP
      H3=CONST3/(AYCD**2)
    7 AK4=H*DFUNC(BOFT3,X+H*XD+HH*AK3,XD+AK3)
```

```
      DX=H*(XD+(AK1+AK2+AK3)/6.)
      X=X+H*(XD+(AK1+AK2+AK3)/6.)
      DY =(AK1+2.*(AK2+AK3)+AK4)/6.
      XD=XD+(AK1+2.*(AK2+AK3)+AK4)/6.
      RETURN
      END
      FUNCTION DFUNC (BOFT,X,XD)
      COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
     1FXD,CONST2
      G=AR*PN*(XO/(XO-X))**XK
      DFUNC=(BOFT-(U1+U2+U3-U5)-G+CONST1-SGNXD*(H1+H2)*XD**2+EXD*(A6*H2/
     1A3+H3)*XD**2+FXD*U4)/CONST2
      RETURN
      END
      SUBROUTINE LINEAR (A,X,Y,VV,I)
      DIMENSION X(75),Y(75)
    2 IF(A-X(I)) 3,1,1
    1 I=I+ 1
      GO TO 2
    3 I = I-1
      VV=Y(I)*(A-X(I+1))/(X(I)-X(I+1))+Y(I+1)*(A-X(I))/(X(I+1)-X(I))
      RETURN
      END
      SUBROUTINE AREA(AREA1,A1,TR,H)
      A2=TR
```

```
AREA1=AREA1+H*(A1+A2)/2.
A1=A2
RETURN
END
FUNCTION DFUNC1(XD,YD,BOFT)
DIMENSION AREASX(75),AREASY(75),AREALX(75),AREALY(75),AYX(20),AYY(
120),BRCHX(65),BRCHY(65)
COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
1FXD,CONST2,AREASX,AREASY,AXS,AREALX,AREALY,AXL,AXCD,CDP,CDPP,CK,AK
2,CONST3,AYX,AYY,AY,AYCD,CY,CKP,AKP,H,HH,BOFT1,BOFT2,BOFT3,A7,DELX,
3DELY,DELXD,DELYD,DX,DY,T,BRCHX,BRCHY,S1,A5,SIGMA,CD,XMR,ZETA,WP,FF
4P,A2
P2=((AR*XD-A3*(YD-XD))/AXCD)**2*SIGMA/2./32.17
PR=P2+S1/A5
DFUNC1=(BOFT-AR*PR-(U1+U2)+CONST1)/XMR
RETURN
END
SUBROUTINE P3NEG(X,XD,AREA1,A1)
DIMENSION AREASX(75),AREASY(75),AREALX(75),AREALY(75),AYX(20),AYY(
120),BRCHX(65),BRCHY(65)
COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
1FXD,CONST2,AREASX,AREASY,AXS,AREALX,AREALY,AXL,AXCD,CDP,CDPP,CK,AK
2,CONST3,AYX,AYY,AY,AYCD,CY,CKP,AKP,H,HH,BOFT1,BOFT2,BOFT3,A7,DELX,
3DELY,DELXD,DELYD,DX,DY,T,BRCHX,BRCHY,S1,A5,SIGMA,CD,XMR,ZETA,WP,FF
4P,A2,P3H,XMP,Y,YD
```

```
      IBRCH=1
      IF(SENSE SWITCH 3)4,200
  200 YD=(1.+AR/A7)*XD
      Y=(1.+AR/A7)*X
    4 CALL LINEAR(T,BRCHX,BRCHY,BOFT1,IBRCH)
      T=T+HH
      CALL LINEAR(T,BRCHX,BRCHY,BOFT2,IBRCH)
      T=T+HH
      CALL LINEAR(T,BRCHX,BRCHY,BOFT3,IBRCH)
      CALL K(X,XD,YD,Y)
      X=X+DELX
      Y=Y+DELY
      XD=XD+DELXD
      YD=YD+DELYD
    1 P2=((AR*XD-A3*(YD-XD))/AXCD)**2*SIGMA/2./32.17
      P1=P2+S1/A5
      PR=P1
      P4=PR-(A3*(YD-XD)/(A2*CD))**2*SIGMA/2./32.17
      R=AR*PR+U2
      TR=R+U1-CONST1
      CALL AREA(AREA1,A1,TR,H)
      CC=(1.+AR/A7)*X
      DIFFC=Y-CC
      PUNCH 3,T,X,XD,Y,YD,P1,P2,P4,R,TR,AREA1,DIFFC
    3 FORMAT(F13.7,5F13.4/6F13.4)
```

```
      IF(SENSE SWITCH 1)201,202
201   ACCEPT 203,H
203   FORMAT(F10.0)
      HH=H/2.
202   IF(YD-CC)2,8,4
  8   CCC=(1.+AR/A7)*XD
      IF(YD-CCC)2,2,4
  2   RETURN
      END
      SUBROUTINE K(X,XD,YD,Y)
      DIMENSION AREASX(75),AREASY(75),AREALX(75),AREALY(75),AYX(20),AYY(
     120),BRCHX(65),BRCHY(65)
      COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
     1FXD,CONST2,AREASX,AREASY,AXS,AREALX,AREALY,AXL,AXCD,CDP,CDPP,CK,AK
     2 CONST3,AYX,AYY,AY,AYCD,CY,CKP,AKP,H,HH,BOFT1,BOFT2,BOFT3,A7,DELX,
     3DELY,DELXD,DELYD,DX,DY,T,BRCHX,BRCHY,S1,A5,SIGMA,CD,XMR,ZETA,WP,FF
     4P,A2,P3H,XMP,Y
      IARES=1
      CALL LINEAR(X,AREASX,AREASY,AXS,IARES)
      IARES=1
      CALL LINEAR(X,AREALX,AREALY,AXL,IARES)
      AXCD=CDP*AXS+CDPP*AXL+CK*AK
      AK1=H*DFUNC1(XD,YD,BOFT1)
      AL1=H*DFUNC2(XD,YD,X,Y)
```

```
Z=X+HH*XD+H*AK1/8.
ZY=Y+HH*YD+H*AL1/8.
IARES=1
CALL LINEAR(Z,AREASX,AREASY,AXS,IARES)
IARES=1
CALL LINEAR(Z,AREALX,AREALY,AXL,IARES)
AXCD=CDP*AXS+CDPP*AXL+CK*AK
AK2=H*DFUNC1(XD+AK1/2.,YD+AL1/2.,BOFT2)
AL2=H*DFUNC2(XD+AK1/2.,YD+AL1/2.,Z,ZY)
AK3=H*DFUNC1(XD+AK2/2.,YD+AL2/2.,BOFT2)
AL3=H*DFUNC2(XD+AK2/2.,YD+AL2/2.,Z,ZY)
Z=X+H*XD+HH*AK3
ZY=Y+H*YD+HH*AL3
IARES=1
CALL LINEAR(Z,AREASX,AREASY,AXS,IARES)
IARES=1
CALL LINEAR(Z,AREALX,AREALY,AXL,IARES)
AXCD=CDP*AXS+CDPP*AXL+CK*AK
AK4=H*DFUNC1(XD+AK3,YD+AL3,BOFT3)
AL4=H*DFUNC2(XD+AK3,YD+AL3,Z,ZY)
DELX=H*(XD+(AK1+AK2+AK3)/6.)
DELY=H*(YD+(AL1+AL2+AL3)/6.)
DELXD=(AK1+2.*(AK2+AK3)+AK4)/6.
DELYD=(AL1+2.*(AL2+AL3)+AL4)/6.
RETURN
```

```
 END
 FUNCTION DFUNC2(XD,YD,X,Y)
 DIMENSION AREASX(75),AREASY(75),AREALX(75),AREALY(75),AYX(20),AYY(
120),BRCHX(65),BRCHY(65)
 COMMON U1,U2,U3,U4,U5,AR,PN,XO,XK,CONST1,SGNXD,H1,H2,H3,EXD,A6,A3,
1FXD,CONST2,AREASX,AREASY,AXS,AREALX,AREALY,AXL,AXCD,CDP,CDPP,CK,AK
2,CONST3,AYX,AYY,AY,AYCD,CY,CKP,AKP,H,HH,BOFT1,BOFT2,BOFT3,A7,DELX,
3DELY,DELXD,DELYD,DX,DY,T,BRCHX,BRCHY,S1,A5,SIGMA,CD,XMR,ZETA,WP,FF
4P,A2,P3H,XMP,Y
 P2=((AR*XD-A3*(YD-XD))/AXCD)**2*SIGMA/2./32.17
 PR=P2+S1/A5
 P4=PR-(A3*(YD-XD)/(A2*CD))**2*SIGMA/2./32.17
 PX=PN*(XO/(XO-A7*(Y-X)/AR))**XK
 DFUNC2=(P4*A3-PX*A7-FFP+WP*SINF(ZETA))/XMP
 RETURN
 END
```

APPENDIX C

DESIGN PROGRAM

```
C     CONTROL ROD AREA ANALYSIS SHORT RECOIL TOM STREETER MAR 1965
      DIMENSION BRCHX(100),BRCHY(100),RODX(100),RODY(100)
      COMMON CONST1,CONST2,CONST3,CONST4,ARPN,XZERO,XK,H1,H2,H,HH,CDP,
     1 CDPP,XKS,XKL
      PRINT 44
   44 FORMAT(5X40HTURN SWITCH 2 ON FOR PUNCHED CARD OUTPUT//)
      READ 1 , M , ( BRCHX ( I ) , BRCHY ( I ) , I = 1 , M )
    1 FORMAT (I10/(8F10.0) )
      READ 1,N,(RODX(I),RODY(I),I=1,N)
      READ 3,S1,AR,A5,A7,A3,WR,WP,ZETA,XMR,F,FP,FFP,PN,XZERO,XK,SIGMA,
     1 G,A2,CD,CDP,CDPP,T,H,FACTOR,ERROR,X,XDOT,XMP,AXL,AXS,XKS,XKL
    3 FORMAT(8F10.0)
      XI=XMP*AR*(A7+AR)/A7**2
      CONST1=(WR+AR/A7*WP)*SINF(ZETA)
      CONST2=XMR+XI
      CONST3=S1*AR*(A7-A3)/A5/A7
      CONST4=F+FP+AR*FFP/A7
      ARPN=AR*PN
      H1=SIGMA*A3**3*AR**3/(2.*G*A2**2*A7**3*CD**2)/144.
      H2=SIGMA*(A7-A3)**3*AR**3/(2.*G*A7**3)/144.
      U=S1*AR*(A7-A3)/A5/A7+F+FP+  AR*FFP/A7
      DO 4 I = 1 , M
    4 BRCHY (I) = BRCHY (I) * FACTOR
      IBRCH = 1
      IROD=1
```

```
      PUNCH 30, U,H1,H2,XI
   30 FORMAT(12X1HU14X2HH114X2HH214X2HX1//4F16.4)
      PUNCH 18
   18 FORMAT(//7X1HT12X1HX11X4HXDOT10X1HR10X5HREAL10X3HAXS//)
      AXCD=AXS*CDP+XKS
      HH = H * .5
    9 CALL LINEAR ( T, BRCHX, BRCHY, BOFT1, IBRCH)
      T = T + HH
      CALL LINEAR ( T, BRCHX, BRCHY, BOFT2, IBRCH)
      T = T + HH
      CALL LINEAR ( T, BRCHX, BRCHY, BOFT3, IBRCH )
      CALL LINEAR(T,RODX,RODY,RREAL,IROD)
    5 IERR=0
    6 IERR = IERR + 1
      CALL KUTTA(BOFT1,BOFT2,BOFT3,X,XDOT,DELX,DELXD,AXS,AXL,AXCD)
      ZZZ=X+DELX
      ZXDOT=XDOT+DELXD
      ZXDD=DFUNC(BOFT3,ZZZ,ZXDOT,AXCD)
      G=AR*PN*(XZERO/(XZERO-ZZZ))**XK
      R=U+G+XI*ZXDD+(H1+H2/AXCD**2)*ZXDOT**2
      ZZ=ABSF(RREAL-R)
      IF(ZZ-ERROR)102,103,103
  103 DENOM=( RREAL-U-G-XI*ZXDD)/ZXDOT**2
      AXCD=SQRTF(H2/DENOM)
      IF(IERR-5)6,51,51
```

```
 51 ACCEPT 52,ERROR
 52 FORMAT(F10.0)
    GO TO 5
102 CALL KUTTA(BOFT1,BOFT2,BOFT3,X,XDOT,DELX,DELXD,AXS,AXL,AXCD)
    IF(SENSE SWITCH 3)7,99
  7 ACCEPT 52, ERROR
 99 X=X+DELX
    XDOT = XDOT + DELXD
    XDD=DFUNC(BOFT3,X,XDOT,AXCD)
    G=AR*PN*(XZERO/(XZERO-X))**XK
    R=U+G+XI*XDD+(H1+H2/AXCD**2)*XDOT**2
    ABC=(H1+H2/AXCD**2)*XDOT**2
    AXS=(AXCD-XKS)/CDP
    IF(SENSE SWITCH 1)8,100
  8 PRINT 21,IERR,ERROR,ZZ,AXS
 21 FORMAT(I10,2F10.2,F15.7)
100 IF ( SENSE SWITCH 2 ) 11, 12
 11 PUNCH 19, T,X,XDOT,R,RREAL,AXS
 19 FORMAT(5F13.4,F13.7)
 12 IF(T-RODX(N))9,10,10
 10 STOP
    END
    FUNCTION DFUNC(BOFT,X,XDOT,AXCD)
    COMMON CONST1,CONST2,CONST3,CONST4,ARPN,XZERO,XK,H1,H2,H,HH,CDP,
   1 CDPP,XKS,XKL
    DFUNC=(BOFT-(CONST3+      CONST4)-ARPN*(XZERO/(XZERO-X))**XK
```

```
1 -(H1+H2/AXCD**2)*XDOT**2+CONST1)/CONST2
  RETURN
  END
  SUBROUTINE KUTTA(BOFT1,BOFT2,BOFT3,X,XDOT,DELX,DELXD,AXS,AXL,AXCD)
  COMMON CONST1,CONST2,CONST3,CONST4,ARPN,XZERO,XK,H1,H2,H,HH,CDP,
 1 CDPP,XKS,XKL
  AK1=H*DFUNC(BOFT1,X,XDOT,AXCD)
  AK2=H*DFUNC(BOFT2,X+HH*XDOT+H/8.*AK1,XDOT+AK1/2.,AXCD)
  AK3=H*DFUNC(BOFT2,X+HH*XDOT+H/8.*AK1,XDOT+AK2/2.,AXCD)
  AK4=H*DFUNC(BOFT3,X+H*XDOT+HH*AK3,AK3+XDOT,AXCD)
  DELX = H* (XDOT+ (AK1 + AK2 +AK3)/6. )
  DELXD = (AK1 + 2.*AK2 + 2.*AK3 + AK4)/6.
  RETURN
  END
  SUBROUTINE LINEAR (A,X,Y,VV,I)
  DIMENSION X(100),Y(100)
2 IF(A-X(I)) 3,1,1
1 I=I+ 1
  GO TO 2
3 I = I-1
  VV=Y(I)*(A-X(I+1))/(X(I)-X(I+1))+Y(I+1)*(A-X(I))/(X(I+1)-X(I))
  RETURN
  END
```

```
C     CONTROL ROD AREA ANALYSIS LONG RECOIL TOM STREETER MAR 1965
      DIMENSION BRCHX(100),BRCHY(100),RODX(100),RODY(100),AREASX(100),
     1 AREASY(100)
      COMMON CONST1,CONST2,CONST3,CONST4,ARPN,XZERO,XK,H1,H2,H,HH,CDP,
     1 CDPP,XKS,XKL
      PRINT 44
   44 FORMAT(5X40HTURN SWITCH 2 ON FOR PUNCHED CARD OUTPUT//)
      READ 1 , M , ( BRCHX (I ) , BRCHY ( I ) , I = 1 , M )
    1 FORMAT (I10/(8F10.0) )
      READ 1,N,(RODX(I),RODY(I),I=1,N)
      READ 1,MM,(AREASX(I),AREASY(I),I=1,MM)
      READ 3,S1,AR,A5,A7,A3,WR,WP,ZETA,XMR,F,FP,FFP,PN,XZERO,XK,SIGMA,
     1 G,A2,CD,CDP,CDPP,T,H,FACTOR,ERROR,X,XDOT,XMP,AXL,AXS,XKS,XKL
    3 FORMAT(8F10.0)
      XI=XMP*AR*(A7+AR)/A7**2
      CONST1=(WR+AR/A7*WP)*SINF(ZETA)
      CONST2=XMR+XI
      CONST3=S1*AR*(A7-A3)/A5/A7
      CONST4=F+FP+AR*FFP/A7
      ARPN=AR*PN
      H1=SIGMA*A3**3*AR**3/(2.*G*A2**2*A7**3*CD**2)/144.
      H2=SIGMA*(A7-A3)**3*AR**3/(2.*G*A7**3)/144.
      U=S1*AR*(A7-A3)/A5/A7+F+FP+  AR*FFP/A7
      DO 4 I = 1 , M
    4 BRCHY (I) = BRCHY (I) * FACTOR
```

```
      IAREA=1
      IBRCH = 1
      IROD=1
      PUNCH 30, U,H1,H2,XI
   30 FORMAT(12X1HU14X2HH114X2HH214X2HX1//4F16.4)
      AXCD=AXS*CDP+XKS+AXL*CDPP+XKL
      HH = H * .5
    9 CALL LINEAR ( T, BRCHX, BRCHY, BOFT1, IBRCH)
      T=T+HH
      CALL LINEAR ( T, BRCHX, BRCHY, BOFT2, IBRCH)
      T = T + HH
      CALL LINEAR ( T, BRCHX, BRCHY, BOFT3, IBRCH )
      CALL LINEAR(T,RODX,RODY,RREAL,IROD)
    5 IERR=0
    6 IERR = IERR + 1
      CALL KUTTA(BOFT1,BOFT2,BOFT3,X,XDOT,DELX,DELXD,AXS,AXL,AXCD)
      ZZZ=X+DELX
      ZXDOT=XDOT+DELXD
      ZXDD=DFUNC(BOFT3,ZZZ,ZXDOT,AXCD)
      G=AR*PN*(XZERO/(XZERO-ZZZ))**XK
      R=U+G+XI*ZXDD+(H1+H2/AXCD**2)*ZXDOT**2
      ZZ=ABSF(RREAL-R)
      IF(ZZ-ERROR)102,103,103
  103 DENOM=( RREAL-U-G-XI*ZXDD)/ZXDOT**2
      AXCD=SQRTF(H2/DENOM)
```

```
      IF(IERR-5)6,51,51
   51 ACCEPT 52,ERROR
   52 FORMAT(F10.0)
      GO TO 5
  102 CALL KUTTA(BOFT1,BOFT2,BOFT3,X,XDOT,DELX,DELXD,AXS,AXL,AXCD)
      IF(SENSE SWITCH 3)7,99
    7 ACCEPT 52, ERROR
   99 X=X+DELX
      XDOT = XDOT + DELXD
      XDD=DFUNC(BOFT3,X,XDOT,AXCD)
      G=AR*PN*(XZERO/(XZERO-X))**XK
      R=U+G+X1*XDD+(H1+H2/AXCD**2)*XDOT**2
      ABC=(H1+H2/AXCD**2)*XDOT**2
      CALL LINEAR(X,AREASX,AREASY,AXS,IAREA)
      AXL=(AXCD-AXS*CDP-XKS-XKL)/CDPP
      IF(SENSE SWITCH 1)8,100
    8 PRINT 21,IERR,ERROR,ZZ,AXS,AXL
   21 FORMAT(I10,2F10.2,2F15.7)
  100 IF ( SENSE SWITCH 2 ) 11, 12
   11 PUNCH 19, T,X,XDOT,R,RREAL,AXS,AXL
   19 FORMAT(3F10.4,2F12.3,2F13.7)
   12 IF(T-RODX(N))9,10,10
   10 STOP
      END
```

```
      SUBROUTINE KUTTA(BOFT1,BOFT2,BOFT3,X,XDOT,DELX,DELXD,AXS,AXL,AXCD)
      COMMON CONST1,CONST2,CONST3,CONST4,ARPN,XZERO,XK,H1,H2,H,HH,CDP,
     1 CDPP,XKS,XKL
      AK1=H*DFUNC(BOFT1,X,XDOT,AXCD)
      AK2=H*DFUNC(BOFT2,X+HH*XDOT+H/8.*AK1,XDOT+AK1/2.,AXCD)
      AK3=H*DFUNC(BOFT2,X+HH*XDOT+H/8.*AK1,XDOT+AK2/2.,AXCD)
      AK4=H*DFUNC(BOFT3,X+H*XDOT+HH*AK3,AK3+XDOT,AXCD)
      DELX = H* (XDOT+ (AK1 + AK2 +AK3)/6. )
      DELXD = (AK1 + 2.*AK2 + 2.*AK3 + AK4)/6.
      RETURN
      END
      FUNCTION DFUNC(BOFT,X,XDOT,AXCD)
      COMMON CONST1,CONST2,CONST3,CONST4,ARPN,XZERO,XK,H1,H2,H,HH,CDP,
     1 CDPP,XKS,XKL
      DFUNC=(BOFT-(CONST3+      CONST4)-ARPN*(XZERO/(XZERO-X))**XK
     1 -(H1+H2/AXCD**2)*XDOT**2+CONST1)/CONST2
      RETURN
      END
      SUBROUTINE LINEAR (A,X,Y,VV,I)
      DIMENSION X(100),Y(100)
    2 IF(A-X(I)) 3,1,1
    1 I=I+ 1
      GO TO 2
    3 I = I-1
      VV=Y(I)*(A-X(I+1))/(X(I)-X(I+1))+Y(I+1)*(A-X(I))/(X(I+1)-X(I))
      RETURN
      END
```