Hoglund
ASD Company file

A951955

ESA-TN-72-12

PREDICTIONS OF SUBMARINE WAKE SIZE AND CONTAMINANT CONCENTRATIONS IN AN OCEAN ENVIRONMENT

A

APPROVED F[illegible] RELEASE
DI[illegible]

AUGUST 1972

TECHNICAL NOTE

Environmental
Science
Associates

83 06 23 063

ESA-TN-72-12

TECHNICAL NOTE

PREDICTIONS OF SUBMARINE WAKE SIZE AND CONTAMINANT CONCENTRATIONS IN AN OCEAN ENVIRONMENT

By

K. W. Kaulum

For the

Nuclear Monitoring Research Office

Defense Advanced Research Projects Agency

Contract No. N00014-72-C-0358
(DARPA Order No. 2199)

August 1972

Environmental Science Associates
770 Airport Boulevard
Burlingame, California 94010
(415) 342-9407

CONTENTS

INTRODUCTION

This report will very briefly consider the problem of predicting the wake size and concentration distribution of a continuously introduced contaminant within the wake of a submerged submarine as a function of time after passage or downwake distance. The only environmental condition which will be considered is vertical density stratification since it is known to influence both the vehicle generated wake dispersion and the natural ocean dispersion in the vertical direction.

Model experiments have shown that a self-propelled submersible vehicle produces a well-defined turbulent wake which maintains identity for long distances as it disperses or grows in cross-section. In addition, these experiments have shown that the wake growth is influenced by density stratification in a predictable manner. In contrast, only rather crude full-scale experiments at sea have been attempted because of the practical difficulties. An important conclusion resulting from one of the most elaborate of these full-scale experiments is that natural ocean turbulent dispersion processes appear to become the dominant mechanism for wake dispersion very shortly after passage of the submarine.* This result is indicative of only a specific set of operating conditions.

To review, the self-generated wake initially disperses by entrainment of ambient water as a result of its internal turbulence. This process continues, subject to density stratification effects, until the internal turbulence decays sufficiently so that natural ocean turbulent diffusion becomes the dominant process for further dispersion.

---

* References available from the author.

At present there is adequate formulation, based on model data, to make predictions of the wake growth; but as yet no scheme has been proposed to determine the conditions when the internal wake dispersion processes are superseded by natural ocean dispersion. In this analysis we will avoid the problem by simply making separate predictions of wake growth and natural dispersion and superimposing the results. It is then assumed that the dominant process for wake dispersion is that which results in the largest wake size and hence lowest contaminant concentration. Future theoretical development or further full-scale experiments should provide higher quality proedictions of overall wake dispersion; however, this approach should provide a most useful first-order prediction.

THE TURBULENT WAKE

The turbulent wake of a submerged submarine in a stratified ocean has been recently analyzed by several DARPA contractors (Ko, 1971; Ko and Alber, 1972; Kuo and Grosch, 1972; Merritt, 1972). The review by Merritt (1972) provides the most readily usable formulation for the desired predictions and, unless noted, the following is taken from his report.

For unstratified conditions, the coaxial wake growth is

$$\frac{b_r}{D_o} = 1.25 \left(\frac{X}{D_o}\right)^n \tag{1}$$

where b_r is the circular wake diameter
D_o is the submarine diameter
X is the down-wake distance
n is the exponent ($n = 0.22$) .

For conditions of stratification (a natural density gradient) the wake initially maintains a circular cross-section until a critical time is reached when the uniformly mixed wake can no longer maintain itself in the ambient density gradient. At this time, the wake starts to collapse and spread horizontally. Its cross-sectional shape first becomes elliptical and then finally approaches that of a flat rectangle. The critical time for the onset of wake collapse has been found by experiments to be T/3, where T is the Brunt-Vaisala period, which is defined as

$$T = \frac{2\pi}{\left[\frac{g}{\rho_o} \frac{\partial \rho}{\partial z}\right]^{\frac{1}{2}}} \tag{2}$$

where g is the gravitational constant

ρ_o is the ambient density

$\partial\rho/\partial z$ is the ambient vertical-density gradient .

The vertical size of the wake after the onset of collapse is given as

$$\frac{b_z}{b_r} = \frac{.40\ n}{(\frac{t}{T})\ (1- \frac{\beta}{\alpha})^{\frac{1}{2}}} \qquad (3)$$

where b_z is the vertical size

t/T is the ratio of time since sumbarine passage to the Brunt-Vaisala period

n is the exponent for the unstratified growth function (Equation 1).

β/α is the ratio of the ambient to the wake density gradient .

From experimental results, $n = 1/4$ and $\beta/\alpha = 0.9$. Thus, the expression for vertical size (Equation 3) reduces to

$$\frac{b_z}{b_r} = \frac{.32}{(\frac{t}{T})} \qquad (4)$$

Also Equations 1 and 4 can be combined to give

$$b_z = .040\ TU\ (\frac{X}{D_o})^{-3/4} \qquad (5)$$

where U is the constant submarine speed in the X direction.

During wake collapse, the horizontal size grows at a rate greater than that for the unstratified case; however, Merritt gives the horizontal size for only one condition. van de Watering (1966) gives considerably more experimental data for horizontal wake size. This data is summarized as follows:

b_y/b_o	t' (time since collapse)
1.5	$3T/2\pi$
2.0	$10T/2\pi$

where b_o is wake width at the start of collapse and t' is the time since the start of collapse, expressed in terms of T, the Brunt-Vaisala period. The data also indicates that the horizontal size ratio probably does not exceed the value of $b_y/b_o = 2.0$ for most conditions of stratification.

In order to eventually compare the wake growth with the natural turbulent-dispersion processes in the ocean, it was necessary to choose a suitable range of conditions for calculation of the wake growth. The value $D_o = 10$m, and submarine speeds of 2, 10 and 30 knots were arbitrarily chosen. However, the values of the Brunt-Vaisala period, T, depend on natural ocean density gradients, which can range from 10^{-4} to 10^{-7}/m or less in the ocean. For these calculations, gradients of 10^{-5}, 10^{-6} and 10^{-7}/m were felt to be representative of operating conditions. The corresponding values for T are approximately 6, 18 and 60 minutes. Here, T = 6 min would represent conditions for a highly stratified condition, while T = 60 min indicates virtual absence of stratification. The results of these calculations are shown graphically in Figures 1 and 2 with vertical size b_z and horizontal size b_y given separately as functions of time.

NATURAL TURBULENT DISPERSION

Next we considered the natural dispersion processes acting on the wake. The assumption was made that the natural turbulent dispersion processes are independent of the internal wake turbulence, at least for the size scale of the initial wake or larger. Thus the appropriate size of a naturally dispersing volume source (taken as the initial cross-wake size) was calculated and superimposed on the time history of the turbulent wake to find the times and conditions when each process would dominate. Horizontal and vertical dispersion were considered separately because the horizontal turbulence processes are though to be independent of density stratification, which greatly influences vertical dispersion.

First, consider horizontal dispersion. There are large quantities of experimental data for the growth of instantaneous point-source releases of various tracers. Generally the results of these experiments are presented as the variance of the pool size distribution, σ_r^2 , vs. time after release. Okubo (1962) has made a least-squares fit of the available data between 10^3 and 10^8 seconds. His result is shown on Figure 3 with the curve extrapolated down to six minutes. Foxworthy (1965) presents a more detailed description of his experiments off the southern California coast in that he gives both σ_x^2 (down-current) and σ_y^2 (cross-current) for both instantaneous and continuous releases for times down to 10 minutes or less. These results are also shown on Figure 3. An inspection of Figure 3 reveals that the agreement at times below 100 minutes is certainly not striking. However, Foxworthy's curve for σ_y^2 (point-source) was selected for the following reasons: (a) Foxworthy's data is concentrated in the time period of interest (0 to 300 minutes); (b) the increased dispersion resulting from current shear is absent from the σ_y^2 data;* and (c) the resulting pool size is larger during the period of interest and thus gives a conservative result.

The horizontal size of a naturally dispersing volume source ($4\sigma_y$) could then be computed on the basis of the curve selected from Figure 3, with a fictitious upstream point source to generate appropriate volume source with a 4σ Gaussian size distribution. The result has been included in Figure 2 with the previously calculated turbulent-wake sizes. It is most interesting to see that the natural dispersion begins to dominate the wake as early as 400 seconds after submarine passage at the lowest speed.

For vertical dispersion in the oceans, it is generally assumed that the diffusivity is constant (Okubo and Pritchard, 1969) and thus the standard deviation of the vertical size distribution is given as

$$\sigma_z = 2\,(K_z\,t)^{\frac{1}{2}} \qquad (6)$$

where K_z is the vertical diffusion coefficient and t is time.

* Okubo's data is given as σ_r^2, which is based on a round pool of area equal to the actual elongated pools usually observed as a result of down-current shear.

Based on the recommendations of Okubo and Pritchard (1969) we computed the vertical wake size using values for K_z of 1, 10 and 100 cm^2/sec. These values of K_z are approximately related to the density gradients (10^{-5}, 10^{-6} and 10^{-7} m^{-1}) previously used to calculate the values of the Brunt-Vaisala period (the stratification parameter). Again, with a fictitious upstream point-source to generate the initial volume source, a $4\sigma_z$ vs. time curve was calculated for each value of K_z. These are shown on Figure 1, superimposed on the vertical wake size. Each of the three natural dispersion curves is coded to the appropriate wake size according to the degree of stratification as indicated by the Brunt-Vaisala period, T. Thus the K_z = 100 dispersion curve is associated with the T_3 = 60 minute wake-collapse curves, and would be taken as the condition (no stratification) encountered in the surface layer. Similarly the K_z = 1 curve and the T_1 = 6 minute wake curves represent conditions in a significant thermocline. The remaining K_z = 10 curve and the T_2 = 18 minute wake curves would suggest conditions in a deep layer. It is apparent from Figure 1 that for highly stratified conditions, natural vertical dispersion will not influence the vertical size of the turbulent wake during the period of interest.

CONTAMINANT CONCENTRATIONS IN A WAKE

Now that we had computed: (1) the turbulent wake size; and (2) the size of a competing volume source dispersing naturally, both as functions of time, the calculation of down-wake contaminant concentration was straightforward. The same approach was taken in that the down-wake concentration was calculated separately for turbulent wake growth and natural dispersion, and the results were superimposed. Considering first the natural dispersion of a volume source, Foxworthy (1965) suggests the following expression for the centerline concentration of a continuous volume source:

$$C_{max} = \frac{Q}{2\pi U \sigma_y(0)\, \sigma_z(0) \left[\frac{2\sigma_y^2}{\sigma_y^2(0)} + 1\right]^{\frac{1}{2}} \left[\frac{2\sigma_z^2}{\sigma_z^2(0)} + 1\right]^{\frac{1}{2}}} \quad (7)$$

where Q is the quantity of contaminant introduced per second

U is submarine speed

$\sigma_y(0)$ and $\sigma_z(0)$ are the plume sizes, standard deviations at X = 0

σ_y and σ_z are the horizontal and vertical point-source pool sizes, functions of time or X/U. The x direction is taken as down-plume or down-wake.

At x = 0,

$$C_{max}(0) = C_o = \frac{Q}{2\pi U \sigma_y(0)\, \sigma_z(0)} \quad (8)$$

The initial wake size, D_o, which has been taken as 10 m, is assumed to represent diameter of 4σ, so

$$\sigma_y(0) = \sigma_z(0) = \frac{10 \text{ m}}{4} = 2.5 \text{ m}$$

C_{max}/Q has been calculated for each of the three values of K_z (Equation 7) and the same function of K_y previously chosen from Figure 3. The results are shown on Figures 4-6 as functions of down-wake distance (X) at each of the submarine speeds used previously.

For the calculation of C_{max} for the turbulent wake, the contaminant distribution was then to be Gaussian so that

$$C_{max} = \frac{Q}{2\pi U \sigma_y \sigma_z}$$

Prior to the time of wake collapse at T/3,

$$\sigma_y = \sigma_z = \frac{b_r}{4}$$

so

$$\frac{C_{max}}{Q} = \frac{8}{\pi U \quad b_r^2} .$$

Substituting for b_r from Equation 1 gave

$$\frac{C_{max}}{Q} = \frac{1.65}{U\, D_o^{1.66}\, X^{.44}} \tag{9}$$

where D_o is the initial wake size
X is the down-wake distance .

For down-wake distances greater than $X = U(T/3)$ (or after wake collapse) the wake concentration C_{max} can be calculated by taking values of b_y and b_z from Figures 2 and 1 respectively. In practice this procedure does not work out well since the resulting values of C_{max} tend to increase immediately after wake collapse and then subsequently decrease again. This rather unpredictable growth of the wake area (and hence the concentration) after collapse is evident from van de Watering's (1966) data where the wake area continues to increase but at a rate very much reduced from the initial rate. Because of this uncertainty, it appeared best for now to assume the value of C_{max} to remain constant after reaching $X = U(T/3)$.

C_{max}/Q was calculated as a function of X for the three Brunt-Vaisala periods and speeds used previously. The results are superimposed on the natural dispersion concentrations in Figures 4, 5 and 6.

Several conclusions are apparent from these graphs of C_{max}/Q vs. down-wake distance X. They are:

(a) At low submarine speeds (2 knots) natural turbulence becomes the dominant dispersal mechanism at down-wake distances $> 10^3$ m or times $>$ 15 minutes.

(b) At high submarine speeds (30 knots), wake turbulence dominates dispersal to distances of the order of 10 miles (n).

(c) The uncertainty associated with the rate of increase in wake area after collapse begins (T/3) effects only results for highly stratified conditions ($K_z = 1$) where T/3 occurs well before natural dispersion dominates.

FIGURES

Figures (cont'd)

Figure 4. Maximum Contaminant Concentration vs. Down-wake Distance for a Submarine Wake with T = 6 min and for a Naturally Dispersing Volume Source with K_z = 1 cm^2/sec at Three Speeds.

5. Maximum Contaminant Concentration vs. Down-wake Distance for a Submarine Wake with T = 18 min and for a Naturally Dispersing Volume Source with K_z = 10 cm^2/sec at Three Speeds.

6. Maximum Contaminant Concentration vs. Down-wake Distance for a Submarine Wake with T = 60 min and for a Naturally Dispersing Volume Source with K_z = 100 cm^2/sec at Three Speeds.

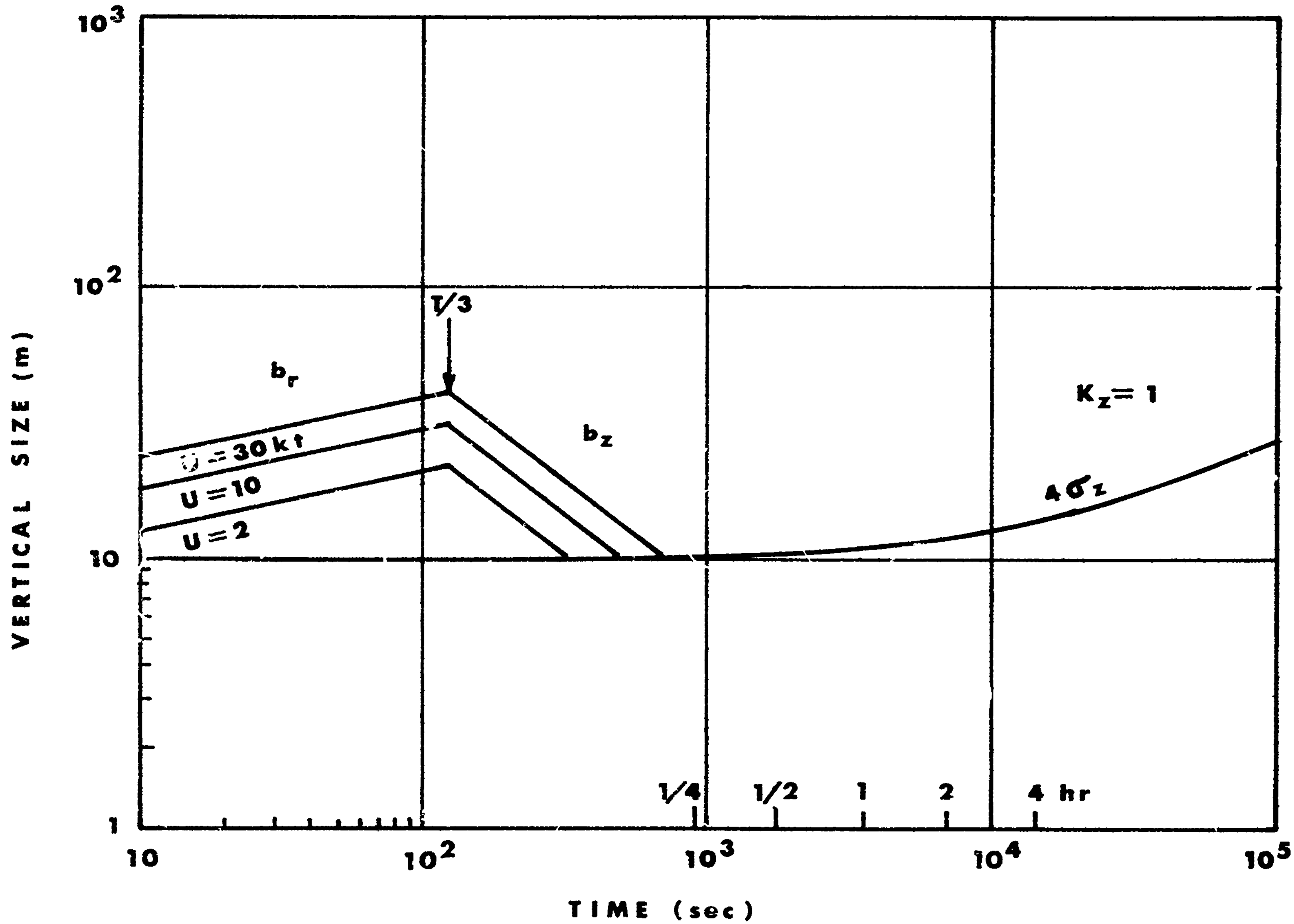

Figure 1a. Vertical Wake Size (b_r and b_z) for a Brunt-Vaisala Period of T = 6 min and Vertical Size ($4\sigma_z$) for a Naturally Dispersing Volume Source with K_z = 1 cm^2/sec as Functions of Time.

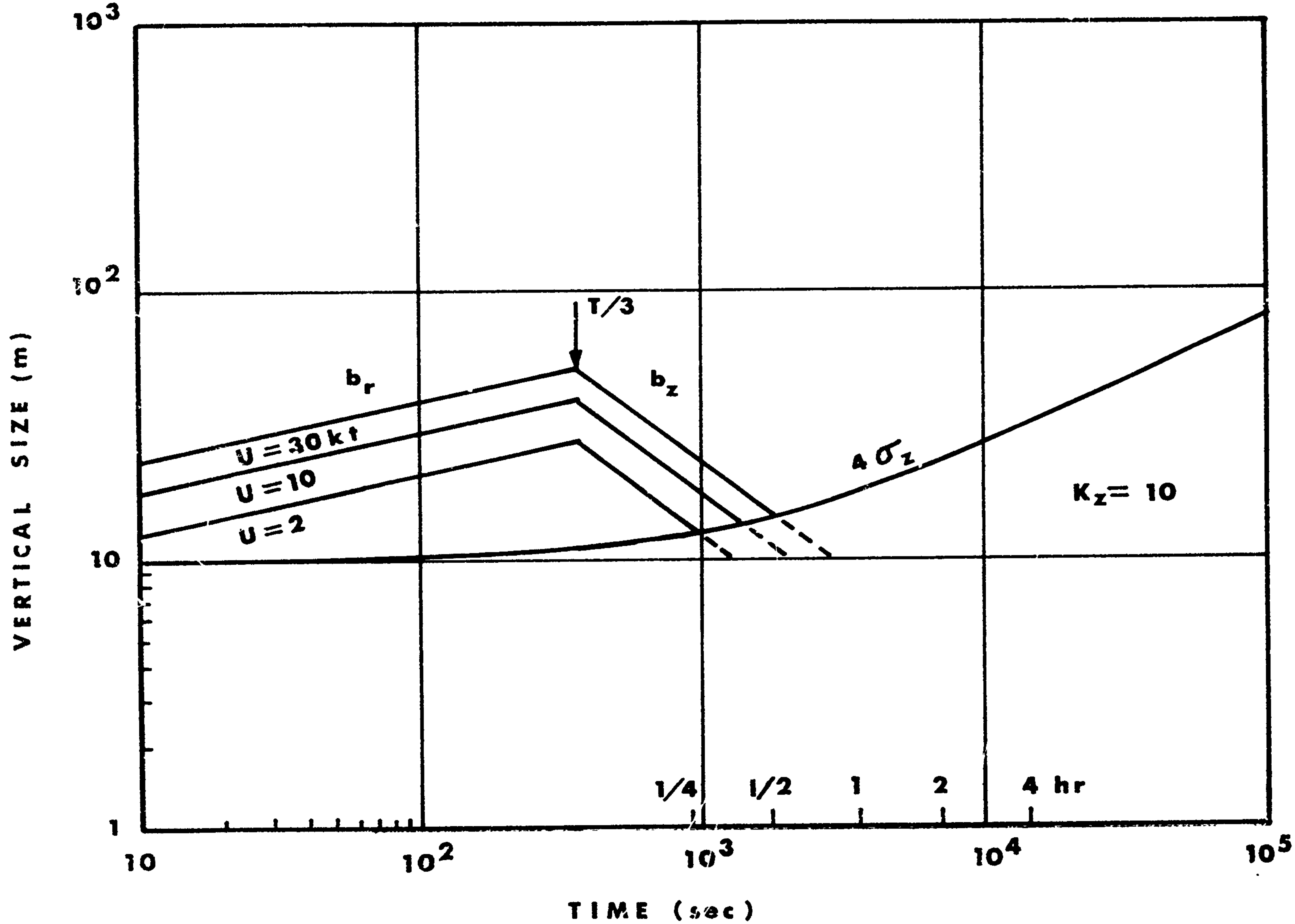

Figure 1b. Vertical Wake Size (b_r and b_z) for a Brunt-Vaisala Period of $T = 18$ min and Vertical Size ($4\sigma_z$) for a Naturally Dispersing Volume Source with $K_z = 10\ cm^2/sec$ as Functions of Time.

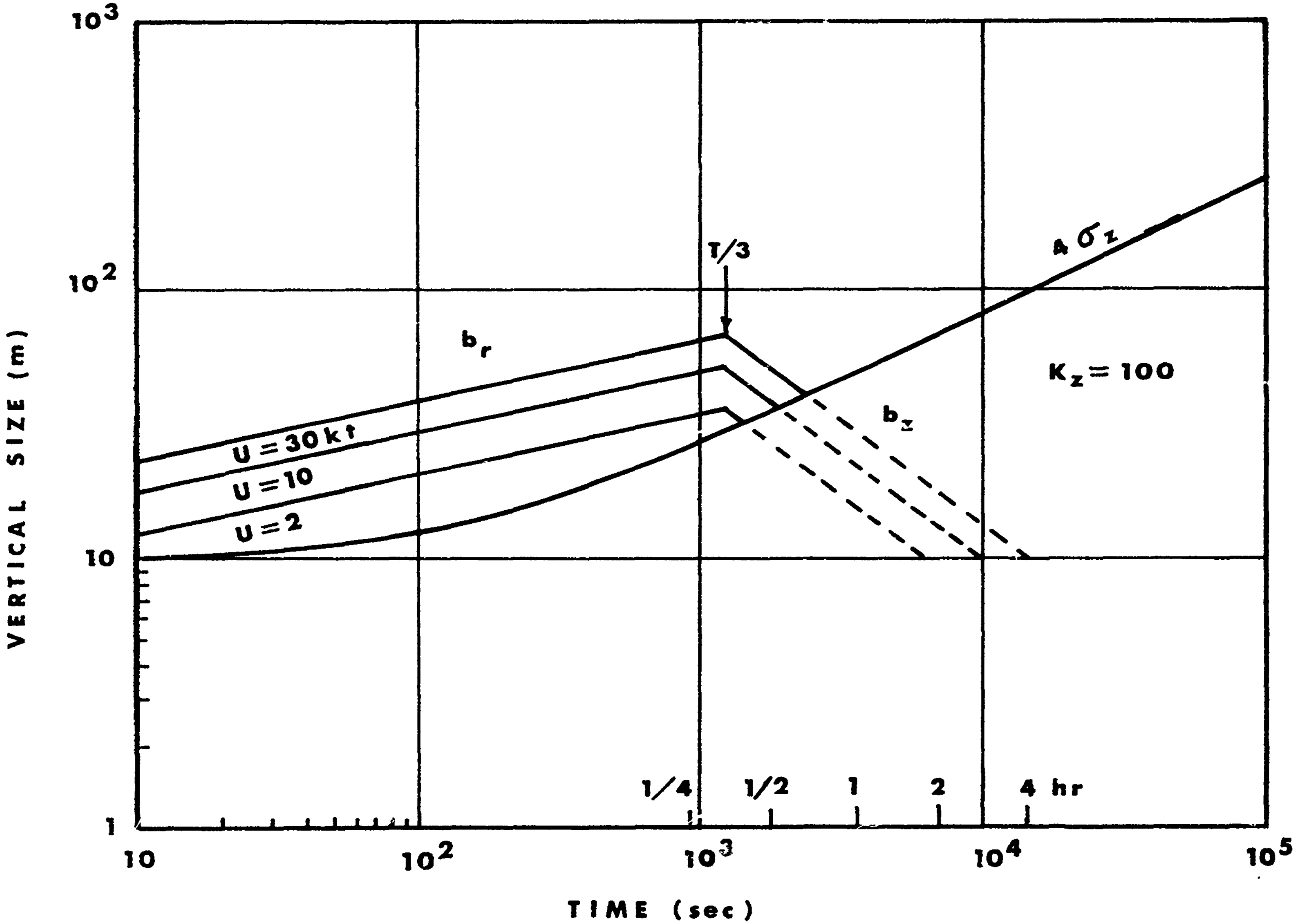

Figure 1c. Vertical Wake Size (b_r and b_z) for a Brunt-Vaisala Period of T = 60 min and Vertical Size ($4\sigma_z$) for a Naturally Dispersing Volume Source with K_z = 100 cm^2/sec as Functions of Time.

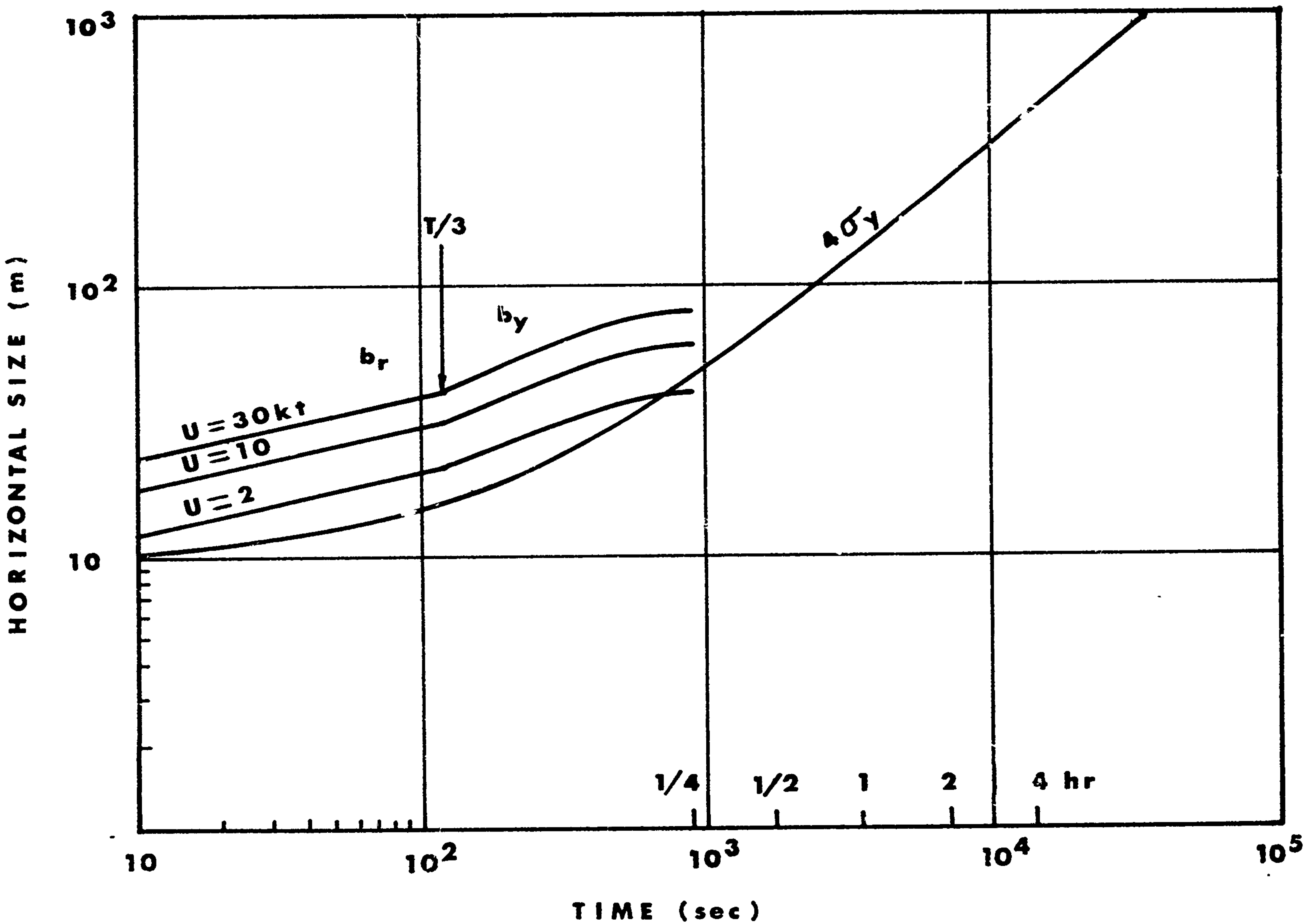

Figure 2a. Horizontal Wake Size (b_r and b_y) for a Brunt-Vaisala Period of T = 6 min and Horizontal Size ($4\sigma_y$) for a Naturally Dispersing Volume Source as Functions of Time.

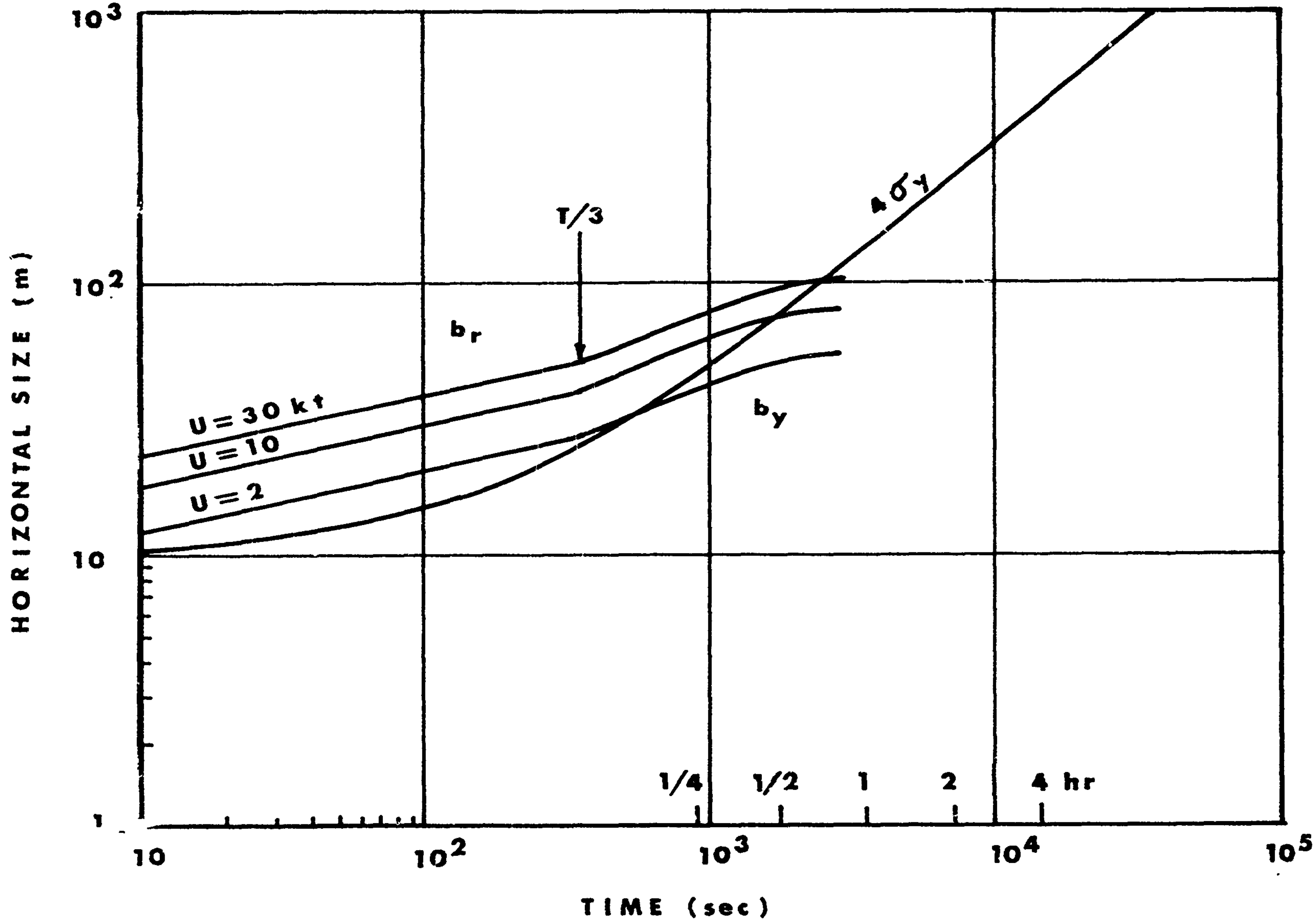

Figure 2b. Horizontal Wake Size (b_r and b_y) for a Brunt-Vaisala Period of T = 18 min and Horizontal Size ($4\sigma_y$) for a Naturally Dispersing Volume Source as Functions of Time.

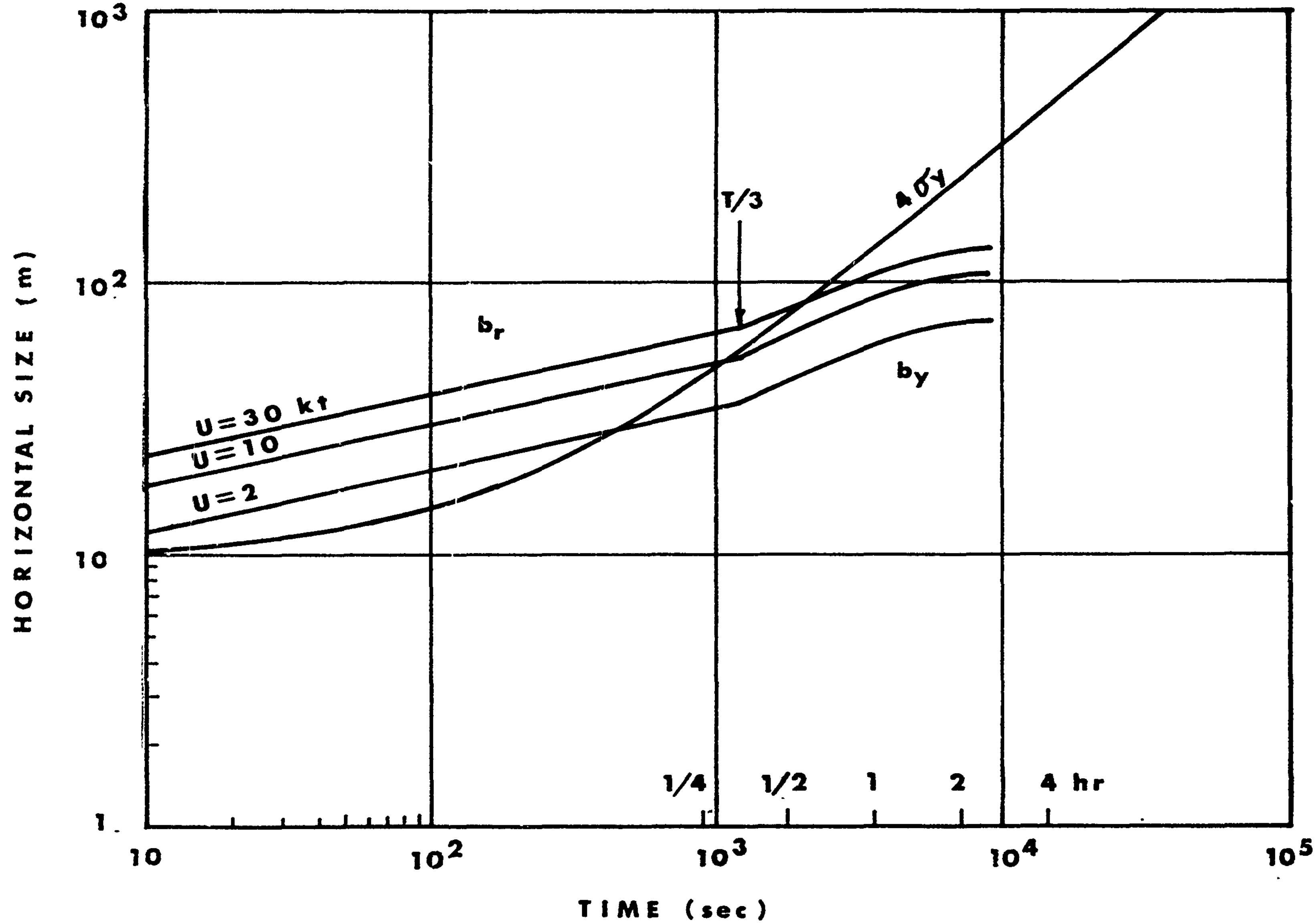

Figure 2c. Horizontal Wake Size (b_r and b_y) for a Brunt-Vaisala Period of T = 60 min and Horizontal Size ($4\sigma_y$) for a Naturally Dispersing Volume Source as Functions of Time.

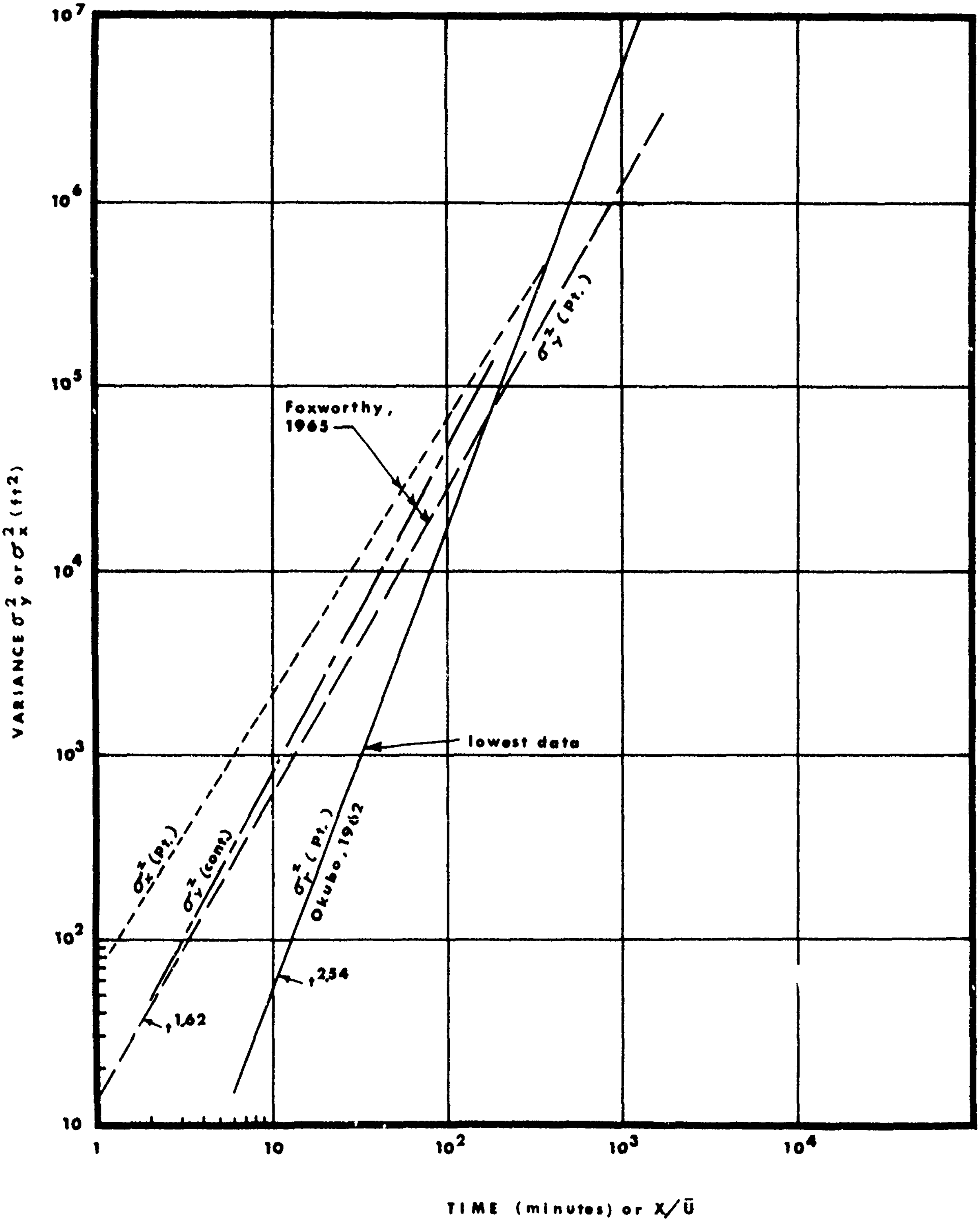

Figure 3. Size Variance of Naturally Dispersing Point Instantaneous and Continuous Dye Tracer Releases vs. Time or X/$\overline{U}$ for Continuous Plume.

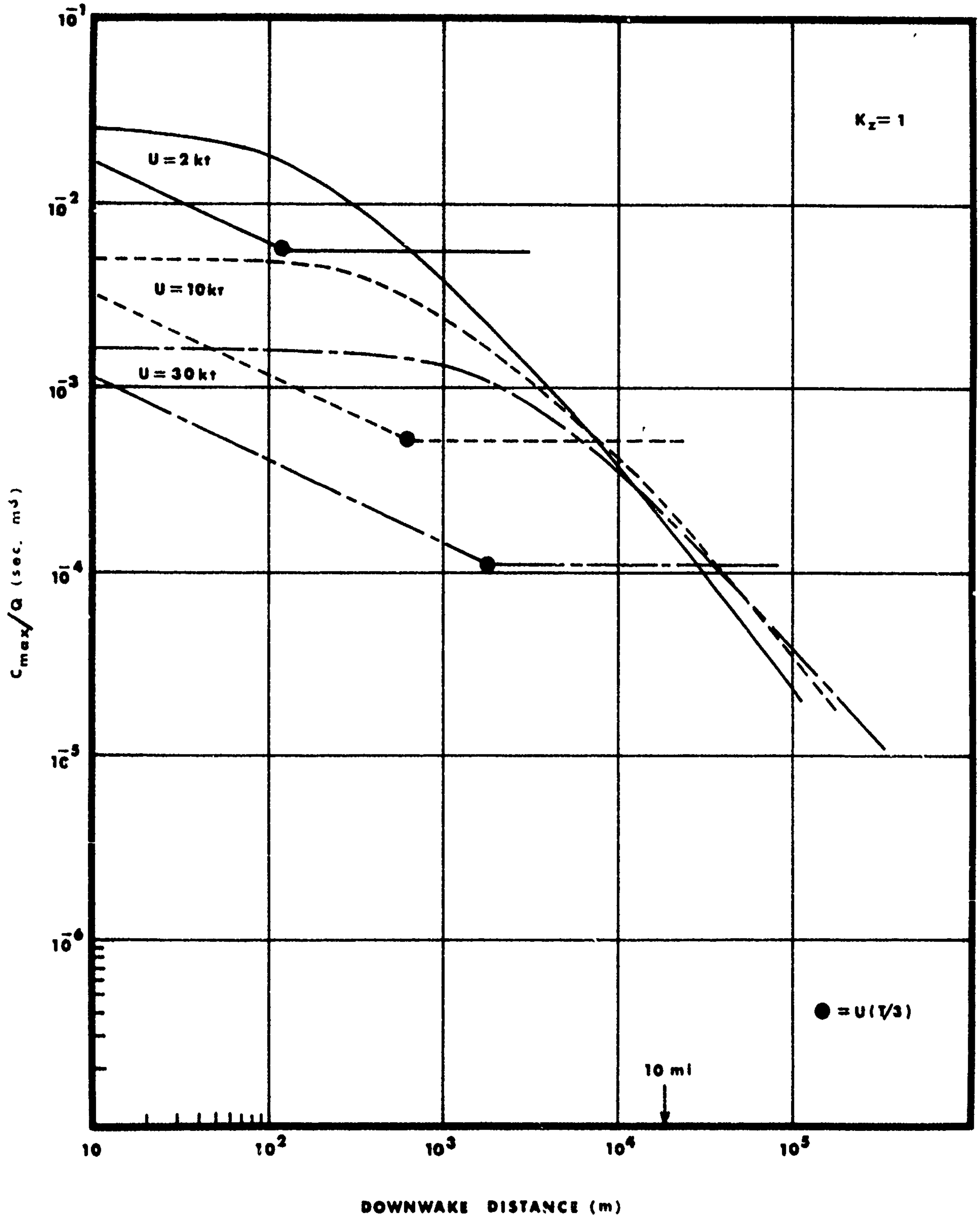

Figure 4. Maximum Contaminant Concentration vs. Downwake Distance for a Submarine Wake with T = 6 min and for a Naturally Dispersing Volume Source with K_z = 1 cm^2/sec at Three Speeds.

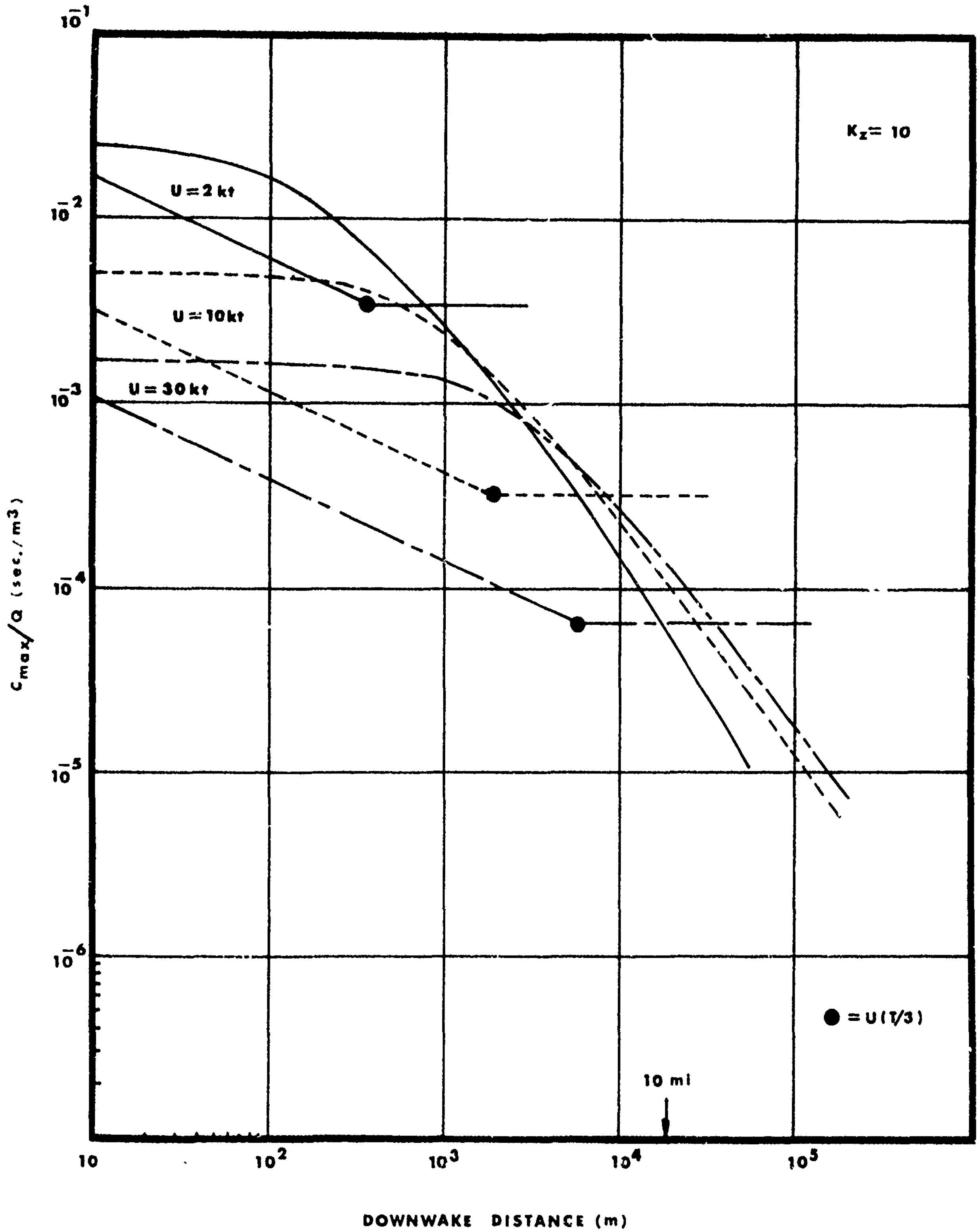

Figure 5. Maximum Contaminant Concentration vs. Downwake Distance for a Submarine Wake with T = 18 min and for a Naturally Dispersing Volume Source with K_z = 10 cm^2/sec at Three Speeds.

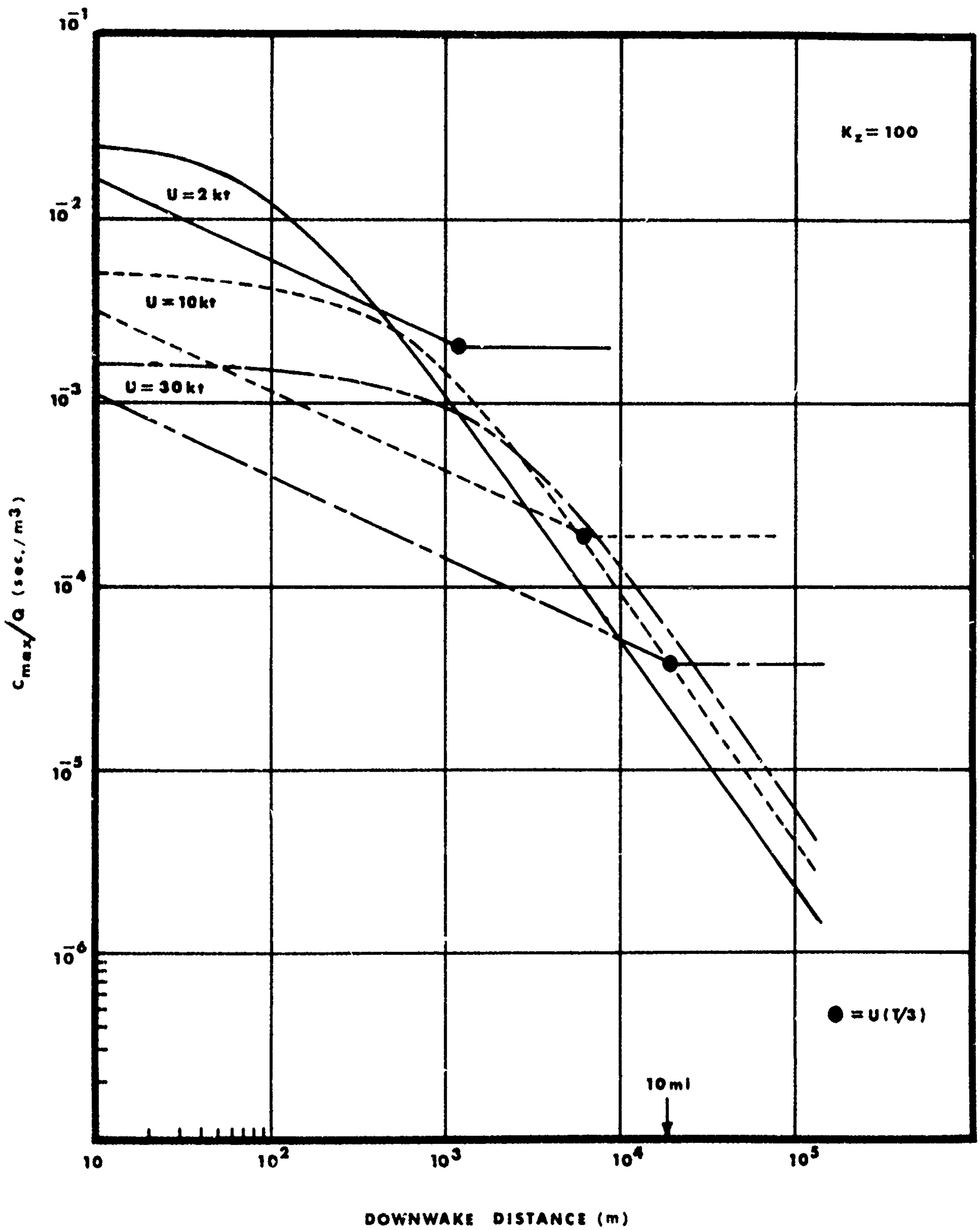

Figure 6. Maximum Contaminant Concentration vs. Downwake Distance for a Submarine Wake with T = 60 min and for a Naturally Dispersing Volume Source with K_z = 100 cm^2/sec at Three Speeds.

REFERENCES

FOXWORTHY, J.E., "Dilution and Dispersion of a Waste Field in the Sea" in "An Investigation on the Fate of Organic and Inorganic Wastes Discharged into the Marine Environment and Their Effects on Biological Productivity," State of California Resources Agency Publication No. 29, 1965.

KO, Denny R.S., "Vol. II, Collapse of a Turbulent Wake in a Stratified Medium," TRW Report No. 18202-6001-120-00, Nov. 1971.

KO, Denny R.S. and ALBER, I.E., "Diffusion of a Passive Scaler, Quarterly Technical Report," TRW Report No. 20086-6001-RO-00, Feb. 1972.

KUO, E.Y.T. and GROSCH, C.E., "Some Comments on the Modeling of the Turbulent Wake of a Self-Propelled Body in a Stratified Fluid," Ocean and Atmospheric Science, Inc., Report No. TR-72-073, March 1972.

MERRITT, Gordon E., "Wake Laboratory Experiment, Quarterly Technical Report No. 1," Cornell Aeronautical Lab., Jan. 1972.

OKUBO, Akira, "Horizontal Diffusion from an Instantaneous Point-Source due to Oceanic Turbulence," Chesapeake Bay Institute, Johns Hopkins University Technical Report 32, Dec. 1962.

OKUBO, Akira and PRITCHARD, D.W., "Summary of our Present Knowledge of the Physical Processes of Mixing in the Ocean and ...," Chesapeake Bay Institute, Johns Hopkins University Report No. 69-1, Sept. 1969.

van de WATERING, Walter, P.M., "The Growth of a Turbulent Wake in a Density-Stratified Fluid," Hydronautics Inc., Technical Report No. 231-12, Nov. 1966.