UNDERWATER TOWED SONAR VEHICLES,
STABILITY DERIVATIVES FOR TWO
THE AN/SQA-13 AND THE HYDROSPACE TOWED BODIES

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1.0 **INTRODUCTION**

1.1 **Purpose**

This memorandum presents data to be used in stability studies of two underwater towed sonar vehicles, the AN/SQA-13, Serial #7 vehicle and the Hydrospace vehicle. Stability derivatives, weights and inertias, and exterior dimensions are presented for both of these vehicles. The stability derivative coefficients and inertias were computed from the preliminary data available. Sources of data are shown below, and methods of computation are given in the text.

1.2 **Background**

To aid study of the characteristics of variable depth sonar systems, the Boeing Company, Seattle, Washington, has initiated analog computer simulation of towed underwater vehicles under Contract N123(953)56341A. One use of this simulation will be to predict tow test performance, including stability characteristics, of vehicles to be tested at later dates. This memorandum presents data to be used in formulating the analog computer representation of two of these vehicles.

1.3 **Data Sources**

The AN/SQA-13, Serial #7 body (built by Telephonics Corporation) was tested by David Taylor Model Basin to determine static and dynamic body characteristics. Many of the stability coefficients were measured and reported in References 9.10 and 9.11. Methods and values used in computing the missing coefficients are from References 9.1, 9.3, 9.6, and 9.9. Weight and inertia information was obtained from References 9.10 and 9.11, and from the drawings of this body (Telephonics drawings 141J1013 and 141J1014).

The Serial #7 body has been previously towed at sea by the Underwater Sound Laboratory, New London, Connecticut. Results of this tow testing will be used to verify the analog computer simulation.
For the Hydrospace vehicle, no testing has been done on the present vehicle configuration. However, Reference 9.5 presents the results of static stability tests made by David Taylor Model Basin on a configuration that is similar except for wing location. Sources for the computed derivatives are explained in the appropriate sections of this report. Data used in weight and inertia calculations is preliminary, and was obtained from Hydrospace personnel. The drawing of the Hydrospace body was formed from a preliminary drawing by Hydrospace Corporation and from photographs of the vehicle.

The nomenclature used throughout this report, wherever possible, conforms to that shown in Reference 9.8. Reduction of the equations of motion to small perturbation equations is consistent with Reference 9.2.
2.0 AXIS SYSTEM AND SIGN CONVENTION

The axis system ox, oy, oz is fixed to the body with the origin at the center of gravity and initially oriented with the x axis parallel to the velocity vector. Velocities u, v, and w are small perturbation velocities measured along the ox, oy, and oz axes respectively. The forces X, Y, and Z are positive forward, along the right wing, and down, respectively. Positive roll (ψ), roll rate (p), and roll moment (K) correspond to right-wing-down motion. Positive yaw (ψ), yaw rate (r), and yaw moment (N) correspond to nose-right motion. Positive pitch (θ), pitch rate (q), and pitch moment (M) correspond to nose-up motion.
3.0 EQUATIONS OF MOTION

\[ \Sigma X = m(\ddot{u} + qv - rv) \]
\[ \Sigma Y = m(\dot{v} + ru - pw) \]
\[ \Sigma Z = m(\ddot{w} + pv - qu) \]
\[ \Sigma L = I_x (\dot{p}) + (I_z - I_y)qr - I_{xz} (\dot{\phi}) - I_{xz} (\dot{p}) \]
\[ \Sigma M = I_y (\dot{q}) + (I_x - I_z)pr - I_{xz} (r)^2 + I_{xz} (p)^2 \]
\[ \Sigma N = I_x (\dot{\phi}) + (I_y - I_x)pq - I_{xz} (\dot{\phi}) + I_{xz} (qr) \]

3.1 For small perturbations these equations reduce to:

\[ \Sigma X = m(\ddot{u}) \]
\[ \Sigma Y = m(\dot{v} + ru_o) \]
\[ \Sigma Z = m(\ddot{w} - qu_o) \]
\[ \Sigma K = I_x \dot{p} - I_{xz} \dot{\phi} \]
\[ \Sigma M = I_y \dot{q} \]
\[ \Sigma N = I_z \dot{\phi} - I_{xz} \dot{\phi} \]

3.2 Expanded Equations of Motion

\[ \Sigma X = X_u u + X_v v + X_w w + X_u \dot{u} + X_v \dot{v} + X_w \dot{w} + X_p \dot{p} + X_q \dot{q} + X_r \dot{r} + X_{\dot{p}} \dot{p} + X_{\dot{q}} \dot{q} + X_{\dot{r}} \dot{r} \]
\[ \Sigma Y = Y_u u + Y_v v + Y_w w + Y_u \dot{u} + Y_v \dot{v} + Y_w \dot{w} + Y_p \dot{p} + Y_q \dot{q} + Y_r \dot{r} + Y_{\dot{p}} \dot{p} + Y_{\dot{q}} \dot{q} + Y_{\dot{r}} \dot{r} \]
\[ \Sigma Z = Z_u u + Z_v v + Z_w w + Z_u \dot{u} + Z_v \dot{v} + Z_w \dot{w} + Z_p \dot{p} + Z_q \dot{q} + Z_r \dot{r} + Z_{\dot{p}} \dot{p} + Z_{\dot{q}} \dot{q} + Z_{\dot{r}} \dot{r} \]
\[ \Sigma K = K_u u + K_v v + K_w w + K_u \dot{u} + K_v \dot{v} + K_w \dot{w} + K_p \dot{p} + K_q \dot{q} + K_r \dot{r} + K_{\dot{p}} \dot{p} + K_{\dot{q}} \dot{q} + K_{\dot{r}} \dot{r} \]
\[ \Sigma M = M_u u + M_v v + M_w w + M_u \dot{u} + M_v \dot{v} + M_w \dot{w} + M_p \dot{p} + M_q \dot{q} + M_r \dot{r} + M_{\dot{p}} \dot{p} + M_{\dot{q}} \dot{q} + M_{\dot{r}} \dot{r} \]
\[ \Sigma N = N_u u + N_v v + N_w w + N_u \dot{u} + N_v \dot{v} + N_w \dot{w} + N_p \dot{p} + N_q \dot{q} + N_r \dot{r} + N_{\dot{p}} \dot{p} + N_{\dot{q}} \dot{q} + N_{\dot{r}} \dot{r} \]
4.0 DERIVATIVE COEFFICIENTS

\[
\begin{align*}
X'_{u} &= \frac{X_u}{1/2 \rho \xi^2 U_0} \\
Y'_{u} &= \frac{Y_u}{1/2 \rho \xi^2 U_0} \\
Z'_{u} &= \frac{Z_u}{1/2 \rho \xi^2 U_0} \\
X'_{p} &= \frac{X_p}{1/2 \rho \xi^3 U_0} \\
Y'_{p} &= \frac{Y_p}{1/2 \rho \xi^3 U_0} \\
Z'_{p} &= \frac{Z_p}{1/2 \rho \xi^3 U_0} \\
X'_{\dot{u}} &= \frac{X_{\dot{u}}}{1/2 \rho \xi^3} \\
Y'_{\dot{u}} &= \frac{Y_{\dot{u}}}{1/2 \rho \xi^3} \\
Z'_{\dot{u}} &= \frac{Z_{\dot{u}}}{1/2 \rho \xi^3} \\
X'_{\dot{p}} &= \frac{X_{\dot{p}}}{1/2 \rho \xi^4} \\
Y'_{\dot{p}} &= \frac{Y_{\dot{p}}}{1/2 \rho \xi^4} \\
X'_{\dot{q}} &= \frac{X_{\dot{q}}}{1/2 \rho \xi^4} \\
\end{align*}
\]
\[
\begin{align*}
Z'_p &= \frac{Z_p}{1/2 \rho l^4} \\
Z'_q &= \frac{Z_q}{1/2 \rho l^4} \\
Z'_r &= \frac{Z_r}{1/2 \rho l^4}
\end{align*}
\]

\[
\begin{align*}
K'_u &= \frac{K_u}{1/2 \rho l^3 u_o} \\
K'_v &= \frac{K_v}{1/2 \rho l^3 u_o} \\
K'_w &= \frac{K_w}{1/2 \rho l^3 u_o}
\end{align*}
\]

\[
\begin{align*}
M'_u &= \frac{M_u}{1/2 \rho l^3 u_o} \\
M'_v &= \frac{M_v}{1/2 \rho l^3 u_o} \\
M'_w &= \frac{M_w}{1/2 \rho l^3 u_o}
\end{align*}
\]

\[
\begin{align*}
N'_u &= \frac{N_u}{1/2 \rho l^3 u_o} \\
N'_v &= \frac{N_v}{1/2 \rho l^3 u_o} \\
N'_w &= \frac{N_w}{1/2 \rho l^3 u_o}
\end{align*}
\]

\[
\begin{align*}
K'_\bar{u} &= \frac{K_{\bar{u}}}{1/2 \rho l^4} \\
K'_\bar{v} &= \frac{K_{\bar{v}}}{1/2 \rho l^4} \\
K'_\bar{w} &= \frac{K_{\bar{w}}}{1/2 \rho l^4}
\end{align*}
\]

\[
\begin{align*}
M'_\bar{u} &= \frac{M_{\bar{u}}}{1/2 \rho l^4} \\
M'_\bar{v} &= \frac{M_{\bar{v}}}{1/2 \rho l^4} \\
M'_\bar{w} &= \frac{M_{\bar{w}}}{1/2 \rho l^4}
\end{align*}
\]

\[
\begin{align*}
N'_\bar{u} &= \frac{N_{\bar{u}}}{1/2 \rho l^4} \\
N'_\bar{v} &= \frac{N_{\bar{v}}}{1/2 \rho l^4} \\
N'_\bar{w} &= \frac{N_{\bar{w}}}{1/2 \rho l^4}
\end{align*}
\]

\[
\begin{align*}
K'_\bar{p} &= \frac{K_{\bar{p}}}{1/2 \rho l^4 u_o} \\
K'_\bar{q} &= \frac{K_{\bar{q}}}{1/2 \rho l^4 u_o} \\
K'_\bar{r} &= \frac{K_{\bar{r}}}{1/2 \rho l^4 u_o}
\end{align*}
\]

\[
\begin{align*}
M'_\bar{p} &= \frac{M_{\bar{p}}}{1/2 \rho l^4 u_o} \\
M'_\bar{q} &= \frac{M_{\bar{q}}}{1/2 \rho l^4 u_o} \\
M'_\bar{r} &= \frac{M_{\bar{r}}}{1/2 \rho l^4 u_o}
\end{align*}
\]

\[
\begin{align*}
N'_\bar{p} &= \frac{N_{\bar{p}}}{1/2 \rho l^4 u_o} \\
N'_\bar{q} &= \frac{N_{\bar{q}}}{1/2 \rho l^4 u_o} \\
N'_\bar{r} &= \frac{N_{\bar{r}}}{1/2 \rho l^4 u_o}
\end{align*}
\]

\[
\begin{align*}
K'_\bar{\bar{u}} &= \frac{K_{\bar{\bar{u}}}}{1/2 \rho l^5} \\
K'_\bar{\bar{v}} &= \frac{K_{\bar{\bar{v}}}}{1/2 \rho l^5} \\
K'_\bar{\bar{w}} &= \frac{K_{\bar{\bar{w}}}}{1/2 \rho l^5}
\end{align*}
\]

\[
\begin{align*}
M'_\bar{\bar{u}} &= \frac{M_{\bar{\bar{u}}}}{1/2 \rho l^5} \\
M'_\bar{\bar{v}} &= \frac{M_{\bar{\bar{v}}}}{1/2 \rho l^5} \\
M'_\bar{\bar{w}} &= \frac{M_{\bar{\bar{w}}}}{1/2 \rho l^5}
\end{align*}
\]

\[
\begin{align*}
N'_\bar{\bar{u}} &= \frac{N_{\bar{\bar{u}}}}{1/2 \rho l^5} \\
N'_\bar{\bar{v}} &= \frac{N_{\bar{\bar{v}}}}{1/2 \rho l^5} \\
N'_\bar{\bar{w}} &= \frac{N_{\bar{\bar{w}}}}{1/2 \rho l^5}
\end{align*}
\]

\[
\begin{align*}
K'_{\bar{\bar{p}}} &= \frac{K_{\bar{\bar{p}}}}{1/2 \rho l^5} \\
K'_{\bar{\bar{q}}} &= \frac{K_{\bar{\bar{q}}}}{1/2 \rho l^5} \\
K'_{\bar{\bar{r}}} &= \frac{K_{\bar{\bar{r}}}}{1/2 \rho l^5}
\end{align*}
\]

\[
\begin{align*}
M'_{\bar{\bar{p}}} &= \frac{M_{\bar{\bar{p}}}}{1/2 \rho l^5} \\
M'_{\bar{\bar{q}}} &= \frac{M_{\bar{\bar{q}}}}{1/2 \rho l^5} \\
M'_{\bar{\bar{r}}} &= \frac{M_{\bar{\bar{r}}}}{1/2 \rho l^5}
\end{align*}
\]

\[
\begin{align*}
N'_{\bar{\bar{p}}} &= \frac{N_{\bar{\bar{p}}}}{1/2 \rho l^5} \\
N'_{\bar{\bar{q}}} &= \frac{N_{\bar{\bar{q}}}}{1/2 \rho l^5} \\
N'_{\bar{\bar{r}}} &= \frac{N_{\bar{\bar{r}}}}{1/2 \rho l^5}
\end{align*}
\]
\[ \dot{M}'_p = \frac{N_p}{1/2 \rho \dot{t}^5} \quad \dot{M}'_q = \frac{N_q}{1/2 \rho \dot{t}^5} \quad \dot{M}'_r = \frac{N_r}{1/2 \rho \dot{t}^5} \]

**Definition Example**

\[ X_u = \text{Force along the X-axis due to a velocity perturbation (u) in X direction (ft. lbs.)} \]

\[ X_u = \frac{2X}{\dot{u}} \]

\[ X'_u = \text{non-dimensional form of } X_u \]

\[ \dot{t} = \text{body length (ft)} \]

\[ U_0 = \text{steady state velocity in X direction (ft/sec)} \]

\[ \rho = \text{mass density of sea water (standard) = 2.0 (slug/ft}^3) \]
5.0 DATA FOR THE AN/SQA-13 TOWED BODY
5.1 EQUATIONS USED FOR COMPUTED COEFFICIENTS

(a) \[ X_u = \frac{1}{2} \rho U_o^2 (X'_{u, body})^3 \frac{b}{U_o^2} \]

(b) \[ Y_p = \frac{1}{2} \rho U_o^2 (Y'_{p, wing}) \cdot S_{wing} \cdot I_{wing}^2 + \frac{(Y'_{p, wing})}{S_{wing}} \cdot \frac{b}{2} \]

- \[ C_{L\delta tail} \cdot S_{tail} \cdot I_{ztail} \cdot \frac{b}{U_o} \]

(c) \[ Y_v = \frac{1}{2} \rho U_o^2 (Y'_{v, body}) \cdot S_{body} \cdot I_{body}^2 + \frac{(Y'_{v, tail})}{S_{tail}} \cdot \frac{b_{tail}}{2} \cdot I_{ztail} \cdot \frac{b}{U_o} \] (negligible)

(d) \[ K_p = \frac{1}{2} \rho U_o^2 (K'_{p, wing}) \cdot S_{wing} \cdot I_{wing}^2 + \frac{(K'_{p, wing})}{S_{wing}} \cdot \frac{b}{2} \]

- \[ C_{L\delta tail} \cdot S_{tail} \cdot (I_{ztail})^2 \]

+ \[ (K'_{p, tail}) \cdot S_{tail} \cdot \frac{b_{tail}}{2} \cdot (I_{ztail})^2 \]

- \[ C_{Lpbody} \cdot S_{body} \cdot (I_{zbody})^2 \cdot \frac{b}{U_o} \]

(e) \[ N_p = \frac{1}{2} \rho U_o^2 [(N'_{p, wing}) \cdot S_{wing} \cdot I_{wing}^2 - (Y'_{v, wing}) \cdot S_{wing} \cdot I_{ztail} \cdot I_{xwing} \]
\( + C_{\text{L, tail}} S_{\text{tail}} X_{\text{tail}} + C_{\text{L, body}} S_{\text{body}} X_{\text{body}} \frac{P}{U_o} \)

\((f)\ K_p = [(m + m_n)\bar{r}_y(\ddot{\phi})]\)

\((g)\ N_p = [(m + m_n)\bar{r}_x(\ddot{\phi})]\)

Where

\[ \begin{align*}
\alpha &= \text{angle of attack (deg)} \\
S &= \text{area (ft.}^2\text{)} \\
L &= \text{body length} = 8.12' \\
b &= \text{span (ft)} \\
U_o &= \text{steady state velocity in X direction (ft/sec)} \\
C_{L\delta} &= \text{coefficient of lift due to a change in tail surface angle} \\
C_{L\alpha} &= \text{coefficient of lift due to a change in angle of attack} \\
C_{Lp} &= \text{coefficient of lift due to a rotation about the X-axis} \\
m &= \text{mass in ft lb sec}^2 \\
m_n &= \text{hydrodynamic mass}
\end{align*} \)

Subscripts (tail, wing, body) refer to particular definitions, i.e., \(S_{\text{wing}} = \text{area of wing.}\)

\[ \begin{align*}
X_{\text{tail}} &= \text{distance from C.G. to center of tail area in respective direction (ft)} \\
X_{\text{wing}} &= \text{distance from C.G. to center of wing area in respective direction (ft)} \\
X_{\text{body}} &= \text{characteristic length of body only} = 7.87' \\
Z_{\text{body}} &= \text{characteristic height of body only} = 3.52'
\end{align*} \]

10
5.1.1 Values Used in Computed Coefficients

(a) All length, area and volume values are referenced to the drawing of section 6.0.

(b) TERM  VALUE  REF.

\( (X'_v)_\text{body} \)  
\(-.015\)  
9.1

\( (Y'_v)_\text{wing} \)  
\( .016\alpha^2 \)  
9.1

\( (Y'_p)_\text{wing} \)  
\( .14\alpha \)  
9.1

\( C_{L_{\text{tail}}}/\text{deg.} \)  
\( .687 \)  
9.1

\( (Y'_v)_\text{body} \)  
negligible  
---

\( (Y'_v)_\text{tail} \)  
negligible  
---

\( (K'_p)_\text{wing} \)  
\( .26 \)  
9.1

\( (K'_p)_\text{tail} \)  
\( .2208 \)  
9.1

\( C_{L_p}\text{body} \)  
\( .04 \)  
9.3

\( (N'_{p})_\text{wing} \)  
\( .2155\alpha \)  
9.1

\( C_{L_{\text{tail}}}/\text{deg} \)  
\( 1.02 \) upper, \( .938 \) lower  
9.9

\( CL_{\omega}\text{body}/\text{deg} \)  
\( .04 \)  
9.3

(c) \[ K_p = \hat{p} \left( (m + m_h)z \right) \]
\[ \text{upper tail} = 43.5 \]
\[ \text{lower tail} = 14.2 \]
\[ \text{upper wing} = 19.4 \]
\[ \text{lower wing} = 15.0 \]
\[ 92.1 \hat{p} \text{ lb ft} \]

\[ \text{where} \]
\[ m \text{ upper tail} = 3.49 \]
\[ m \text{ lower tail} = 2.36 \]
\[ m \text{ upper wing} = 1.535 \]
\[ m \text{ lower wing} = 2.12 \]
\[ m^2_{\text{upper tail}} = \pi a^2 = 30.2 \]
\[ m^2_{\text{lower tail}} = \pi a^2 = 13.9 \]
\[ m^2_{\text{upper wing}} = 1.7 \pi a^2 = 5.94 \]
\[ m^2_{\text{lower wing}} = 1.85 \pi a^2 = 6.45 \]
\[ a_{\text{upper tail}} = 1.29 \]
\[ a_{\text{lower tail}} = -0.8745 \]
\[ a_{\text{upper wing}} = 0.4375 \]
\[ a_{\text{lower wing}} = -0.4375 \]
\[ I_z \text{ upper tail} = 1.29 \]
\[ I_z \text{ lower tail} = -0.8745 \]
\[ I_z \text{ upper wing} = 2.581 \]
\[ I_z \text{ lower wing} = -1.749 \]

Ref/ 9.6
\[
\begin{array}{c}
\frac{\text{lb sec}^2}{\text{ft}} \\
\text{ft. } \rho = 5.78 \frac{\text{lb sec}}{\text{ft}^4} \\
\end{array}
\]
### 5.3 NONDIMENSIONAL COEFFICIENTS

**Reference:** DTMB Report 153-H-01

- **$X'_u$** See Fig. 5.3
- **$Y'_u$**
- **$Z'_u$**

<table>
<thead>
<tr>
<th>$X'_v$</th>
<th>$Y'_v$</th>
<th>$Z'_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X'_w$</td>
<td>$Y'_w$</td>
<td>$Z'_w$</td>
</tr>
<tr>
<td>$X'_\dot{u}$</td>
<td>$Y'_\dot{u}$</td>
<td>$Z'_\dot{u}$</td>
</tr>
<tr>
<td>$X'_\dot{v}$</td>
<td>$Y'_\dot{v}$</td>
<td>$Z'_\dot{v}$</td>
</tr>
<tr>
<td>$X'_\dot{w}$</td>
<td>$Y'_\dot{w}$</td>
<td>$Z'_\dot{w}$</td>
</tr>
<tr>
<td>$X'_p$</td>
<td>$Y'_p$</td>
<td>$Z'_p$</td>
</tr>
<tr>
<td>$X'_q$</td>
<td>$Y'_q$</td>
<td>$Z'_q$</td>
</tr>
<tr>
<td>$X'_r$</td>
<td>$Y'_r$</td>
<td>$Z'_r$</td>
</tr>
<tr>
<td>$X'_\dot{p}$</td>
<td>$Y'_\dot{p}$</td>
<td>$Z'_\dot{p}$</td>
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<tr>
<td>$X'_\dot{q}$</td>
<td>$Y'_\dot{q}$</td>
<td>$Z'_\dot{q}$</td>
</tr>
<tr>
<td>$X'_\dot{r}$</td>
<td>$Y'_\dot{r}$</td>
<td>$Z'_\dot{r}$</td>
</tr>
</tbody>
</table>

| $K'_{\dot{u}}$ | $M'_{\dot{u}}$ | $N'_{\dot{u}}$ |
| $K'_v$ | $M'_v$ | $N'_v$ |
| $K'_w$ | $M'_w$ | $N'_w$ |
| $K'_\dot{u}$ | $M'_\dot{u}$ | $N'_\dot{u}$ |
| $K'_\dot{v}$ | $M'_\dot{v}$ | $N'_\dot{v}$ |
| $K'_\dot{w}$ | $M'_\dot{w}$ | $N'_\dot{w}$ |
| $K'_p$ | $M'_p$ | $N'_p$ |
| $K'_q$ | $M'_q$ | $N'_q$ |
| $K'_r$ | $M'_r$ | $N'_r$ |
| $K'_\dot{p}$ | $M'_\dot{p}$ | $N'_\dot{p}$ |
| $K'_\dot{q}$ | $M'_\dot{q}$ | $N'_\dot{q}$ |
| $K'_\dot{r}$ | $M'_\dot{r}$ | $N'_\dot{r}$ |

*Computed Coefficients; assumed negligible for small perturbations*
Figure 5.3 - Longitudinal Force Coefficient at a Zero Pitch Angle as a Function of Reynolds Number Based on Length (body)
5.3.1 TRIM TAB COEFFICIENTS

\[ \delta_{\text{Horizontal tab}} = 0.004 \]

\[ M_{\delta_{\text{Horizontal tab}}} = -0.649 \]

where \( \delta \) = tab angle

5.3.2 TRANSDUCER COEFFICIENTS

\[ M_q = 68.196 q^2 \]

\[ M_q = 68.45 q \]

7.3.1 \( M_q = D_{\text{total}}(\overline{t}) \)

\[ = D_1\overline{t}_3 + D_2\overline{t}_4 \]

where \( D_1 = \ell_1^2 q^2 (d_1)(\ell_2 - \ell_1) = 0.354 q^2 \text{ lb sec} \over rad \)

\[ \ell_3 = \ell_1 + \left( \frac{\ell_2 - \ell_1}{2} \right) = 1.682 \text{ ft} \]

\[ D_2 = \left[ \frac{\ell_2^2 q^2 - \ell_1^2 q^2}{2} \right] [d_2(\ell_2 - \ell_1)] = 22.313 q^2 \text{ lb sec} \over rad \]

\[ \ell_4 = \ell_1 + \frac{2}{3}(\ell_2 - \ell_1) = 2.16 \text{ ft} \]

\[ M_q = F_2 \]

\[ = [(M_{\text{transducer}} + M_{\text{transducer}})q] \overline{t} \]

where \( M_{\text{rt. cylinder}} = \frac{\rho \pi r^2}{g} \)

\[ \overline{t} = \ell_1 + \left( \frac{\ell_2 - \ell_1}{2} \right) \]

Where

\( l = \text{length in ft} \) \hspace{1cm} \( q = \text{rotational velocity about Y-axis} \)

\( D = \text{drag force} \) \hspace{1cm} \( M_h = \text{hydrodynamic mass} \)

\( S = \text{area in ft}^2 \) \hspace{1cm} \( M = \text{mass in ft lb sec}^2 \)

\( v = \text{velocity} \) \hspace{1cm} \( g = \text{acceleration due to gravity} \)

\( r = \text{radius of cylinder} \)

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### 5.4 WEIGHTS AND INERTIAS

<table>
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<td>$I_{xz}$</td>
<td>11.62</td>
<td>11.62</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

Inertia computed about the X axis through the body center line.

*Other cross products are zero by symmetry.*
6.0 DATA FOR THE HYDROSPACE TOWED BODY
6.1 COMPUTATION OF COEFFICIENTS

Added Mass and Inertia Derivatives

Added mass and inertia derivatives were estimated using the following representation of the Hydrospace fish: the body was approximated by two semi-ellipsoids of revolution, the wing and tail surfaces were approximated by rectangular plates as shown in the following sketch.

Linear and rotary acceleration derivatives for the body ellipsoids, and linear acceleration derivatives for the rectangular plates were computed using the methods presented in Reference 6. These basic acceleration derivatives were then combined to form the acceleration derivatives for the complete fish.

The following expressions define the basic component acceleration derivatives.

Semi-ellipsoid, linear acceleration
\[ X_\alpha = \left( k_1 \frac{4}{3} \pi ab^2 \right) / 2 \]
\[ Z_\alpha = Y_\alpha = \left( k_2 \frac{4}{3} \pi ab^2 \right) / 2 \]
Semi-ellipsoid, angular acceleration
\[ M_\alpha = \left[ k' \frac{4}{15} \pi ab^2 (a^2 + b^2) \right] / 2, \]
where \( a \) and \( b \) are long and short semi axis lengths, respectively and \( k_1, k_2 \) and \( k' \) are constants defined in Lamb, *Hydrodynamics*, Chapter 5 section 115.

Flat plate, linear acceleration
\[ Z_\alpha = k_z \frac{\pi \rho}{4} \frac{a^2 b}{a^2 + b^2} \]
where \( a \) and \( b \) are the lengths of the short and long sides of the rectangle, respectively, and \( k_z \) is evaluated in Reference 6.

Using these relationships, the following values of elemental acceleration derivatives were computed.

Elemental acceleration derivatives:

Body:
\[ X_{UB} = -2.13 \quad X_{UB_1} = -1.22 \quad X_{UB_2} = -0.91 \]
FORWARD BODY ELLIPSOID

AFT BODY ELLIPSOID

WING PANEL

.79' x 1.31'

HORIZONTAL TAIL PANEL

.83' x 1.31'

3.7'

VERTICAL FIN PANEL

78' x 1.34'

REPRESENTATION OF HYDROSPACE FISH USED

IN ESTIMATING ADDED MASS DERIVATIVES
Z_{WB} = -20.95 \quad Z_{WB_1} = -5.95 \quad Z_{WB_2} = -15.0

M_{qB} = -59.2 \quad M_{qB_1} = -3.28 \quad M_{qB_2} = -55.9

M_{WB} = -26.4 \quad M_{WB_1} = +5.85 \quad M_{WB_2} = -32.2

Wing:
Z_{ww} = -1.36 \quad Z_{ww_1} = Z_{ww_2} = -0.93

Horizontal:
Z_{wHT} = -2.04 \quad Z_{wHT_1} = Z_{wHT_2} = -1.02

Vertical:
Y_{vvt} = -1.23

Coordinate locations of the centers of the flat plate areas are designated \( Y_{w1}, Y_{w2}, Y_{HT_1}, Y_{HT_2}, Z_{vt}, \) etc., where the subscripts \( w1 \) and \( w2 \) indicate the two wing panels, \( HT_1, HT_2 \) indicate the horizontal tail panels, and \( vt \) indicates the vertical tail panel.

Typical units for these elemental acceleration derivatives are:
\( X_u, \) lb. sec.\(^2/\)ft.
\( M_q, \) ft. lb. sec.
\( M_v, \) lb. sec.\(^2\)
6.1.1 Formulation of Acceleration Derivatives

Using the foregoing elemental derivatives, the complete acceleration derivatives for the fish were computed using the following relationships.

\[
\begin{align*}
X'_\dot{u} &= \frac{X\ddot{u}}{1/2 \rho l^3} = X_{\ddot{u}B} \frac{1}{2} \rho l^3 \\
Y'_\dot{v} &= \frac{Y\ddot{v}}{1/2 \rho l^3} = \left( Z_{\dot{w}B} + Y_{\ddot{u}vt} \right) \frac{1}{2} \rho l^3 \\
Z'_\dot{w} &= \frac{Z\ddot{w}}{1/2 \rho l^3} = \left( Z_{\dot{w}B} + Z_{\ddot{w}HT} + Z_{\ddot{w}v} \right) \frac{1}{2} \rho l^3 \\
K'_{\ddot{v}} &= \frac{K\ddot{v}}{1/2 \rho l^4} = \left( -Y_{\ddot{w}vt} \left( Z_{vt} \cos \alpha_0 + X_{vt} \sin \alpha_0 \right) \right) \frac{1}{2} \rho l^4 = \left\{ [-Y_{\ddot{w}vt}] [\ddot{Z}_{vt}(\alpha_0)] \right\} \frac{1}{2} \rho l^4 \\
M'_{\ddot{v}} &= \frac{M\ddot{v}}{1/2 \rho l^4} = \left( M_{\dot{w}B} - Z_{\ddot{w}HT} X_{HT} \right) \frac{1}{2} \rho l^4 \\
N'_{\ddot{v}} &= \frac{N\ddot{v}}{1/2 \rho l^4} = \left( -M_{\dot{w}B} + Y_{\ddot{w}vt} X_{vt} \right) \frac{1}{2} \rho l^4 \\
Z'_q &= \frac{Z\ddot{q}}{1/2 \rho l^4} = \left( -Z_{\dot{w}B} X_{B_1} - Z_{\dot{w}B} X_{B_2} - Z_{\ddot{w}HT} X_{HT} \right) \frac{1}{2} \rho l^4 \\
Y'_p &= \frac{Y\ddot{p}}{1/2 \rho l^4} = \left( Y_{\ddot{w}vt} \left( \ddot{Z}_{vt} \alpha_0 \right) \right) /_{2} \rho l^4 \\
Y'_f &= \frac{Y\ddot{f}}{1/2 \rho l^4} = \left( -Z_{\dot{w}B} X_{B_1} + Z_{\dot{w}B} X_{B_2} + Y_{\ddot{w}vt} X_{vt} \right) \frac{1}{2} \rho l^4 \\
K'_p &= \frac{K\ddot{p}}{1/2 \rho l^5} = \left\{ Z_{\ddot{w}v}(Y_{\ddot{v}t})^2 + Z_{\ddot{w}HT}(Y_{\ddot{w}ht})^2 + Y_{\ddot{w}vt} \left( Z_{vt} \alpha_0 \right) \right\} /_{2} \rho l^5 
\end{align*}
\]
\[
N' \hat{p} = \frac{N_\hat{p}}{\frac{1}{2} \rho \xi^5} = \left( Y_{vvt} \left[ \bar{X}_{vt}(\alpha_0) \right] \right) / \frac{1}{2} \rho \xi^5
\]

\[
M' \bar{q} = \frac{M_{\bar{q}}}{\frac{1}{2} \rho \xi^5} = \left[ M_{qB} + Z_{wht} \left( X_{nt} \right)^2 \right] / \frac{1}{2} \rho \xi^5
\]

\[
K' \bar{x} = \frac{K_{\bar{x}}}{\frac{1}{2} \rho \xi^5} = \left\{ -Y_{vvt} \left[ \bar{Z}_{vt}(\alpha_0) \right] \right\} / \frac{1}{2} \rho \xi^5
\]

\[
N' \hat{n} = \frac{N_\hat{n}}{\frac{1}{2} \rho \xi^5} = \left[ M_{qB} + X_{vvt} \left( X_{vt} \right)^2 \right] / \frac{1}{2} \rho \xi^5
\]

\[
z' \bar{q} = \frac{Z_{\bar{q}}}{\frac{1}{2} \rho \xi^5} = \left( Z_{\bar{q}u_0} \right) / \frac{1}{2} \rho \xi^3 u_0
\]

\[
y' \bar{r} = \frac{Y_{\bar{r}}}{\frac{1}{2} \rho \xi^5} = \left( Y_{\bar{r}u_0} \right) / \frac{1}{2} \rho \xi^3 u_0
\]

Certain assumptions are incorporated in the foregoing expressions for the derivatives. It is assumed that the derivatives \( X_\psi, X_\psi, Y_\psi, Y_\psi, Z_\psi, Z_\psi, K_\psi, K_\psi, M_\psi, M_\psi, N_\psi, N_\psi, X_{\bar{q}}, X_{\bar{q}}, Y_{\bar{q}}, Z_{\bar{q}}, Z_{\bar{q}}, K_{\bar{q}}, M_{\bar{q}}, M_{\bar{q}}, N_{\bar{q}}, N_{\bar{q}} \) are zero.

It is also assumed that body contributions to roll moment, side force, and yaw moment do not change with \( \alpha_0 \), and that the tail surface moment length in the X direction changes negligibly with changes in \( \alpha_0 \). Effects of vertical wing location from the axis origin are assumed small.
6.1.2 Hydrodynamic Derivatives

Hydrodynamic (q dependent) stability derivatives for the Hydrospace fish were computed using, wherever possible, the methods presented in Reference 9, and using the basic lift, drag, and moment data presented in Reference 5.

Reference 5 concerns a fish identical to the fish considered here except for wing location. The changes in lift, drag and moment coefficients, total and tail-off, due to the difference in wing location have been ignored. Down wash angle effects and interference effects associated with the correct wing location have been included in rotary derivative estimates.

The following pages show the equations relating the dimensional derivatives (i.e., \( X_u, X_v \), etc.) to the nondimensional derivatives used in conventional aircraft analysis (i.e., \( C_x, C_{xy} \), etc.), and the methods used in calculating each of these latter derivatives is discussed.
Hydrodynamic (q dependent) derivatives

\[ X_u = \frac{3X}{3u} = \frac{1}{2} \rho U_o S(2c_x) \]

\[ X_v = \frac{3X}{3v} = 0 \]

\[ X_w = \frac{3X}{3w} = \frac{1}{2} \rho U_o^2 S(c_{x0}) \frac{1}{U_o} \]

\[ Y_u = \frac{3Y}{3u} = 0 \]

\[ Y_v = \frac{3Y}{3v} = \frac{1}{2} \rho U_o^2 S(c_{y0}) \frac{1}{U_o} \]

\[ Y_w = \frac{3Y}{3w} = 0 \]

\[ Z_u = \frac{3Z}{3u} = \frac{1}{2} \rho U_o S(2c_z) \]

\[ Z_v = \frac{3Z}{3v} = 0 \]

\[ Z_w = \frac{3Z}{3w} = \frac{1}{2} \rho U_o^2 S(c_{z0}) \frac{1}{U_o} \]

\[ K_u = \frac{3K}{3u} = 0 \]

\[ K_v = \frac{3K}{3v} = \frac{1}{2} \rho U_o^2 S(b(c_{kB}) \frac{1}{U_o} \]

\[ K_w = \frac{3K}{3w} = 0 \]
\[ M_u = \frac{3M}{a} = \frac{1}{2} \rho U_o \bar{S}(2C_m) \]

\[ M_v = \frac{3M}{a} = 0 \]

\[ M_w = \frac{3M}{a} = \frac{1}{2} \rho U_o \bar{2S}(C_{ma}) \frac{1}{U_o} \]

\[ N_u = \frac{2M}{a} = 0 \]

\[ N_v = \frac{2M}{a} = \frac{1}{2} \rho U_o \bar{2S}(C_{n2}) \frac{1}{U_o} \]

\[ N_w = \frac{2M}{a} = 0 \]

\[ X_p = \frac{3X}{a} = 0 \]

\[ X_q = \frac{3X}{a} = 0 \]

\[ X_r = \frac{3X}{a} = 0 \]

\[ Y_p = \frac{3Y}{a} = \frac{1}{2} \rho U_o \bar{2S}(C_{yf}) \frac{b}{2U_o} \]

\[ Y_q = \frac{3Y}{a} = 0 \]

\[ Y_r = \frac{3Y}{a} = \frac{1}{2} \rho U_o \bar{2S}(C_{yr}) \frac{b}{2U_o} \]

\[ Z_p = \frac{3Z}{a} = 0 \]

\[ Z_q = \frac{3Z}{a} = \frac{1}{2} \rho U_o \bar{2S}(C_{yq}) \frac{c}{2U_o} \]
\[
\begin{align*}
\mathbf{z}_r &= \frac{\partial Z}{\partial r} = 0 \\
K_p &= \frac{\partial K}{\partial p} = \frac{1}{2} \rho U_o^2 S_b(C_{p_r}) \frac{b}{2U_o} \\
K_q &= \frac{\partial K}{\partial q} = 0 \\
K_r &= \frac{\partial K}{\partial r} = \frac{1}{2} \rho U_o^2 S_b(C_{r_r}) \frac{b}{2U_o} \\
M_p &= \frac{\partial M}{\partial p} = 0 \\
M_q &= \frac{\partial M}{\partial q} = \frac{1}{2} \rho U_o^2 \overline{S_C(C_{m_q})} \frac{\overline{C}}{2U_o} \\
M_r &= \frac{\partial M}{\partial r} = 0 \\
N_p &= \frac{\partial N}{\partial p} = \frac{1}{2} \rho U_o^2 \overline{S_C(C_{p_q})} \frac{b}{2U_o} \\
N_q &= \frac{\partial N}{\partial q} = 0 \\
N_r &= \frac{\partial N}{\partial r} = \frac{1}{2} \rho U_o S_b(C_{n_r}) \frac{b}{2U_o} \\
\mathbf{z}_w &= \frac{\partial Z}{\partial w} = \frac{1}{2} \rho U_o^2 S(C_{l_w}) \left( \frac{\overline{C}}{2U_o} \right) \left( \frac{1}{U_o} \right) \\
M_w &= \frac{\partial M}{\partial w} = \frac{1}{2} \rho U_o^2 \overline{S_C(C_{m_w})} \frac{\overline{C}}{2U_o} \\
X_{de} &= \frac{\partial X}{\partial \delta e} = \frac{1}{2} \rho U_o^2 S(C_{\delta e})
\end{align*}
\]
\[ Z_{\delta e} = \frac{3Z}{3\delta e} = \frac{1}{2} \rho U_o^2 S(\chi_{\delta e}) \]

\[ M_{\delta e} = \frac{3M}{3\delta e} = \frac{1}{2} \rho U_o^2 SC(\chi_{\delta e}) \]

\[ Y_{\delta r} = \frac{3Y}{3\delta r} = \frac{1}{2} \rho U_o^2 S(\chi_{\delta r}) \]

\[ K_{\delta r} = \frac{3K}{3\delta r} = \frac{1}{2} \rho U_o L_{Sb} (\chi_{\delta r}) \]

\[ N_{\delta r} = \frac{3N}{3\delta r} = \frac{1}{2} \rho U_r^2 Sb(\chi_{\delta r}) \]
6.1.3 Methods of Derivation of Aerodynamic Derivatives

$C_x, C_z$. These derivatives were resolved from the lift and drag data of Reference 5. The somewhat peculiar shape of the $C_x$ vs $\alpha$ curve arised from the component of lift resolved to the X axis, and reflects the stall of the wing at about 18° angle of attack for the wing (with an angle of incidence of 11°) while the relatively large tail surface remains unstalled.

Data from Reference 5 were extrapolated to obtain information for an 11° incidence angle. The test data in Reference 5 includes data at 0°, 5°, and 10° incidence.

$C_m$. Values of this coefficient were extrapolated from the data of Reference 5.

$C_{x\alpha}, C_{z\alpha}, C_{m\alpha}$. These derivatives are the slopes of the $C_x, C_z, C_m$ vs $\alpha$ data. Since $C_{m\alpha}$ and $C_{z\alpha}$ are approximately constant for a large range of $\alpha$, these constants are used in computation of $M'_w$ and $Z'_w$.

$CL_\alpha = \frac{3CL}{\alpha(2U)}$; $Cm_\alpha = \frac{3C_m}{\alpha(2U)}$. These damping derivatives were evaluated for wing-body and tail components using the methods of Reference 9.

$CL_\alpha = \frac{3CL}{\alpha(2U)}$; $Cm_\alpha = \frac{3C_m}{\alpha(2U)}$. The tail contribution to these derivatives, a downwash-lag effect, was evaluated using the methods of Reference 9. Wing-body contributions to these derivatives as evaluated in Reference 9 are added mass contributions, and are not included in the hydrodynamic
derivatives. The added mass derivatives $Z'_w$ and $M'_w$ constitute an evaluation of these quantities by other methods. There is, incidentally, an alarming disagreement between the added mass contributions to $Z'_w$ and $M'_w$ as computed by the methods of Reference 9 and Reference 6.

$$C_{y_B} = \frac{3C_y}{2(\frac{V}{U})}; \quad C_{n_B} = \frac{3C_n}{2(\frac{V}{U})}$$

These derivatives were evaluated for wing-body and vertical form contributions using the methods of Reference 9.

$$C_{y_n} = \frac{3C_y}{2(\frac{n^b}{2U})}$$

Body and vertical form contributions to these derivatives were derived using Reference 9. Wing contribution was ignored.

$$C_{n_r} = \frac{3C_n}{2(\frac{n^b}{2U})}$$

Body and vertical form contribution, in accordance with Reference 9 were utilized. The wing contribution $C_{D^2}$ is small in comparison with the other contributions.

$$C_{\ell_B} = \frac{3C_B}{2(\frac{V}{U})}$$

Reference 9 methods were used for vertical fin and for wing contributions to this derivative.

$$C_{\ell_f} = \frac{3C_2}{2(\frac{n^b}{2U})}$$

Wing contribution, a function of $C_L$, was evaluated using the data from Reference 3. The vertical fin contribution, a function of $C_2_B$ of the fin was also included.
$$C_{Y_p} = \frac{3C_y}{a_p}$$  Wing-body and vertical fin contributions were computed using Reference 9.

$$C_{L_p} = \frac{3C}{a_p^{\alpha}}$$  Wing contribution, a function of $C_{\alpha}$, and vertical fin contribution were derived from Reference 9.

$$C_{N_p} = \frac{3C_{N}}{a_p^{\alpha}}$$  Wing-body contribution was computed using the value suggested in Reference 7. Vertical tail contribution was based on the side force coefficient $C_{Y_p}$ and the side slip angle resulting from the rotation $p$. 
6.2 Drawing of Hydrospace Vehicle

Nose section: 2:1 ellipse

Tow point coincides with 0.25 MAC

Center of tank

Tail section: ~3.75:1 ellipse flattened on "top"

Wing incidence = 11°

Wing area = 4.14 sq. ft.

Frontal area = 2.64 sq. ft.

All dimensions in inches
6.3. Summary of Values of Derivative Coefficients

6.3.1 Non-dimensional Coefficients, Hydrospace Fish

Summary of Stability Derivatives, Force

\[ X'_u = (0.0845)(2) \left[ f_4(a_o) \right] \]
\[ X'_v = (0.0845) f_{10}(a_o) \]
\[ X'_\hat{u} = -0.00675 \]
\[ Y'_v = -0.324 \]
\[ Y'_\hat{v} = -0.0645 \]
\[ Y'_p = (0.0275) f_8(a_o) \]
\[ Y'_r = -0.0645 + 0.247 \]
\[ Y'_\hat{p} = -0.000535 f_{15}(a_o) \]
\[ Y'_\hat{r} = +0.0126 \]
\[ Z'_u = (0.0845)(2) f_{11}(a_o) \]
\[ Z'_v = -0.425 \]
\[ Z'_\hat{v} = -0.0722 + (0.0055) f_3(a_o) \]
\[ Z'_p = -0.0645 - 0.214 \]
\[ Z'_\hat{p} = -0.0139 \]

Underlined quantities are q dependent, other quantities are added mass terms.

For \( f_4 \), see Figure 6.3.3
For \( f_{10} \), see Figure 6.3.6
For \( f_8 \), see Figure 6.3.5
For \( f_{15} \), see Figure 6.3.9
For \( f_{11} \), see Figure 6.3.7
For \( f_3 \), see Figure 6.3.1
6.3.2 Non-dimensional Coefficients, Hydrospace Fish

Summary of Stability Derivatives - Moment.

\[ K'_{v} = (0.055) f_{1}(a_{o}) \]
\[ K'_{\gamma} = (-0.000542) f_{15}(a_{o}) \]
\[ K'_{p} = (0.0186) f_{4}(a_{o}) \]
\[ K'_{r} = (0.0186) f_{5}(a_{o}) \]
\[ K'_{u} = (-0.00382 - 0.00076) f_{15}(a_{o}) \]
\[ K'_{\gamma} = -0.00286 f_{15}(a_{o}) \]
\[ K'_{\eta} = -0.0110(2) f_{2}(a_{o}) \]
\[ M'_{u} = (0.0110) f_{2}(a_{o}) \]
\[ M'_{w} = -0.0378 \]
\[ M'_{\eta} = -0.0108 + 0.00742 f_{7}(a_{o}) \]
\[ M'_{q} = -0.122 \]
\[ M'_{\eta} = -0.00540 \]
\[ N'_{v} = -0.067 \]
\[ N'_{\gamma} = +0.0134 \]
\[ N'_{p} = (0.0186) f_{6}(a_{o}) \]
\[ N'_{r} = -0.216 \]
\[ N'_{u} = (-0.000286) f_{15}(a_{o}) \]
\[ N'_{\eta} = -0.00472 \]

Underlined quantities are \( q \) dependent, other quantities are mass terms.

For \( f_{1} \), see Figure 6.3.1
For \( f_{2} \), see Figure 6.3.2
For \( f_{15} \), see Figure 6.3.9
For \( f_{7} \), see Figure 6.3.4
For \( f_{4} \), see Figure 6.3.3
For \( f_{5} \), see Figure 6.3.3

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6.3.3 Non-dimensional Coefficients, Hydrospace Fish

Control Surface Coefficients

\[
\begin{align*}
X_{\text{elevator}} &= +0.0845 \, U_0 \, [f_{13}(\alpha_0)] \\
Z_{\text{elevator}} &= +0.0314 \, U_0 \\
M_{\text{elevator}} &= +0.0183 \, U_0 \\
Y_{\text{rudder}} &= -0.0316 \, U_0 \\
N_{\text{rudder}} &= +0.0167 \, U_0 \\
K_{\text{rudder}} &= +0.055 \, U_0 \, [f_{14}(\alpha_0)]
\end{align*}
\]

Sign convention: 
\[+\delta e \rightarrow +M \]
\[+\delta r \rightarrow +N\]
Figure 6.3.4
6.4 WEIGHTS AND INERTIAS, HYDROSPACE FISH

The following weights were used as a basis for the mass and inertia estimates:

- Weight of body (without tank and electronics) in air: 440 lbs.
- Weight of body with tank and electronics in air: 570 lbs.

It was assumed that the wing and tail surfaces have a density of 10 lb per sq. ft. of planform area, giving weights for the wing, horizontal fin and vertical fin of 41.4 lbs, 32.0 lbs, and 19.4 lbs, respectively. The remaining body weight, 347.2 lbs, was assumed distributed uniformly over an ellipsoidal surface of length and maximum width equal that of the body.

The volume of entrained water was assumed to be the volume of the body shell less the volume of the tank. A better approximation than the ellipsoidal shape was used in calculating body volume (15.3 cu ft). For the inertia estimates, the mass of entrained water was assumed to have the inertia characteristics of the ellipsoid (but solid) used in representing body shell inertia minus the inertia characteristic of a volume of water equal to the approximate tank shape (cylindrical with flat end) and size.

The tank and electronics weight was assumed distributed uniformly over the tank volume.

Inertia data is presented for an axis system with the x axis coincident with the body center line.

No attempt was made to rationalize the anomaly of having the center of gravity on the body center line and at the same time having the vertical tail and wing masses not symmetrically distributed. Instead, the cross-product of inertia that would result from the isolated tail mass is presented.

Uncertainties in the weight distributions (and other uncertainties arising in calculation of the stability derivatives) will be resolved by precise physical measurements later in the program.
6.4.1 Hydrospace Fish

Weights and Inertias

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<th>TOTAL</th>
<th>STRUCTURE</th>
<th>ENTRAined WATER</th>
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<tr>
<td>Weight in seawater (lbs.)</td>
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</tr>
<tr>
<td>$I_x$</td>
<td>19.78</td>
<td>11.58</td>
<td>8.2</td>
</tr>
<tr>
<td>$I_y$</td>
<td>206.73</td>
<td>109.93</td>
<td>96.8</td>
</tr>
<tr>
<td>$I_z$ (lb. ft. sec$^2$)</td>
<td>207.96</td>
<td>111.16</td>
<td>96.8</td>
</tr>
<tr>
<td>$I_{xz}$</td>
<td>-2.32</td>
<td>-2.32</td>
<td>0</td>
</tr>
</tbody>
</table>

Inertias computed about the X axis through body CENTER LINE.

Other cross products are zero by symmetry.

$I_{xz}$ includes only vertical fin (+ rudder) contribution.
7.0 **SCOPE**

Data in this report is preliminary in nature, and may be modified as testing proceeds.

8.0 **COMMENTS**

The dynamic damping terms $K_q$, $Z_q$, and $N_q$ (presented in Reference 9.10 and listed on Page 14 of this report) are of such sign as to give negative damping. The location of the reference point for the derivatives for the AN/SQA-13 is approximately the center of gravity, and is well aft of the tow point. This aft location of reference for the dynamic derivatives may account for the seemingly incorrect signs of these derivatives.
9.0 REFERENCES


9.3 Etkin, B; Dynamics of Flight; March, 1963.

9.4 Gertler, Morton; The Hydrodynamic Coefficients of Cable Towed Bodies.


9.8 Technical and Research Bulletin No. 1-5, Nomenclature for Treating the Motion of a Submerged Body Through a Fluid, April 1952; Society of Naval Architects and Marine Engineers.

9.9 USAF Stability and Control Datcon; Air Force Flight Dynamics Laboratory; Wright-Patterson Air Force Base, Ohio.
