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AN ANALYSIS OF THE EFFECT OF CORE STRUCTURE
AND PERFORMANCE ON VOLUME AND SHAPE OF
CROSS-FLOW TUBULAR INTERCOOLERS

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SUMMARY

An analysis of cross-flow tubular intercoolers having a staggered-tube arrangement has been made for turbulent-flow conditions. Charts are presented that show how the over-all dimensions of the intercooler vary with the tube diameter, the transverse spacing, and the wall thickness without affecting the operation of the intercooler. These charts are applicable to both the charge-across-tube and the charge-through-tube intercoolers. Charts and equations relating the intercooler dimensional characteristics and the operating conditions are also given.

The charts show how, for any given set of operating conditions, the tubular intercooler may be altered in size and shape to meet a given set of space requirements. When the intercooler operating conditions are kept constant, a reduction in either tube diameter or transverse spacing is shown to decrease the flow dimensions and to increase the no-flow dimension, the net result being a definite reduction in intercooler volume. For given transverse spacing and for constant operating conditions, the charge-through-tube intercooler and the charge-across-tube intercooler are about equal in volume, heat transfer surface, and width within the usual range of aircraft application. Under the same conditions, however, the frontal area of the charge-across-tube intercooler is much less than that of the charge-through-tube intercooler.

INTRODUCTION

The increased supercharging made necessary by engine operation at high-critical altitudes has resulted in demands for more charge-air cooling. When intercoolers
with a fixed core structure are used, these demands can be met only by increasing the intercooler volume. Consideration should be given, however, to the revision of the core structure for the purpose not only of increasing the cooling accomplished per unit volume, but also of proportioning the intercooler dimensions to suit the space requirements in the airplane.

In reference 1, an analysis was made of the plate-type intercooler showing, for any set of constant operating conditions, the variety of shapes and sizes of this type of intercooler made possible by changing the core structure. The purpose of this paper is to show, in a similar manner, the effect of change in the core structure of the tubular intercooler on the over-all dimensions when the operating conditions are fixed. Both the charge-across-tube—and the charge-through-tube intercoolers are considered.

This work was done at the Langley Memorial Aeronautical Laboratory during the spring of 1942.

SYMBOLS

d  outside tube diameter, feet

\(d_i\)  inside tube diameter, feet

\(t\)  tube-wall thickness, inch

\(s\)  tube transverse spacing (minimum distance between walls of adjacent tubes in a bank), feet

\(s_b\)  minimum distance between walls of adjacent tubes in adjacent banks, feet

\(l\)  tube length, feet

\(L\)  intercooler dimension in the direction of flow across tube banks, feet

\(w\)  intercooler dimension measured in no-flow direction, feet

\(m\)  number of tube banks

\(S\)  total outside tube surface area, square feet
\( S_i \)  total inside tube surface area, square feet
\( v \)  intercooler volume, cubic feet
\( A \)  intercooler face area perpendicular to direction of flow across tube banks, square feet (lw)
\( a \)  intercooler face area perpendicular to direction of flow through the tubes, square feet (lw)
\( N \)  total number of intercooler tubes
\( W \)  weight rate of air flow, pounds per second
\( \eta \)  cooling effectiveness, ratio of temperature drop of charge air to temperature difference between charge air and cooling air at entrance
\( \Delta P \)  total pressure drop across the intercooler, inches of water
\( \Delta P_f \)  pressure drop through tubes caused by skin friction or pressure drop across tubes caused by skin friction and form drag, inches of water
\( \Delta P_e \)  pressure drop due to entrance-exit losses, including vena-contracta loss, for air flowing through the tubes, inches of water
\( \Delta P_v \)  pressure drop of air flowing through the tubes due to the changes in velocity distribution in the tubes, inches of water
\( \Delta P_h \)  pressure change of air due to momentum change caused by heat exchange and pressure loss in intercooler, inches of water
\( \beta \)  ratio of change in temperature of air in passing through the intercooler to absolute temperature of air at intercooler entrance
\( p \)  pressure of intercooler entrance, inches of mercury
\( f \)  ratio of total cross-sectional flow area to area of core face (areas taken perpendicular to the direction of flow through the tubes)
\( h \)  surface heat-transfer coefficient, Btu per second per square foot per degree Fahrenheit
\( A_r \) over-all effective heat-transfer area, square feet

\( U \) over-all heat-transfer coefficient based on over-all effective heat-transfer area, Btu per second per square foot per degree Fahrenheit

\( \rho \) air weight density, pounds per cubic foot

\( \rho_0 \) standard atmospheric density (0.0765 lb per cu ft)

\( \sigma \) density of air relative to standard atmosphere \((\rho/\rho_0)\)

\( V \) velocity of air flow, velocity through transverse spacing \( s \) for flow across tube banks and average velocity for flow through the tubes, feet per second

\( R \) Reynolds number of the air flowing through the tubes based on the inside tube diameter \((\rho V d_1/\mu)\)

\( R_s \) Reynolds number of the air flowing across the tubes based on the tube transverse spacing \((\rho V s/\mu)\)

\( \lambda \) friction factor:
\[ 0.049 R^{-0.2} \] for flow through tubes
\[ 0.8 R_s^{-0.23} \] for flow across tube banks

\( c_p \) specific heat of air at constant pressure (0.24 Btu per lb per deg F)

\( k \) thermal conductivity of air, Btu per second per square foot per degree Fahrenheit gradient per foot

\( \mu \) absolute viscosity of air, pounds per second per foot

\( Z \) a correction factor to be applied to intercooler dimensional characteristics

\[ \theta = \frac{M_1 \sigma_{1av} \Delta P_{1f_1}}{M_2 \sigma_{2av} \Delta P_{2f_2}} \]

Subscripts:

1 cooling air

2 charge air

en condition at intercooler entrance
av  average condition in intercooler

\[ t \]  charge-through-tube intercooler

\[ m \]  charge-across-tube intercooler

eq  equilateral staggered tube arrangement

ev  equivelocity staggered tube arrangement

c  intercooler with reference core structure

Throughout this paper the term "operating conditions" refers to the values of \( \eta, \dot{m}_1, \dot{m}_2, \sigma_{1av}\Delta p f_1, \) and \( \sigma_{2av}\Delta p f_2. \)

The core structure of a tubular intercooler is defined by the values of \( d, t, s, \) and \( \frac{a_p}{a}. \) The reference core structure has the following dimensions:

\[
\begin{align*}
d & = 0.25 \text{ inch} \\
t & = 0.01 \text{ inch} \\
s & = 0.25 \text{ inch} \\
\frac{a_p}{a} & = \frac{1}{2}
\end{align*}
\]

All linear dimensions are given in feet except where designated in inches.

METHOD

The cooling effectiveness of an intercooler can be determined from the over-all heat-transfer coefficient, the heat-transfer surface, and the weight flows of the charge and cooling air. This relationship is plotted in figure 1 of reference 1 for cross-flow heat exchangers and is used in this analysis.

As in reference 1 for the flow through channels, only the pressure drop caused by surface-friction is considered in the analysis; the remaining pressure changes must therefore be algebraically added to the surface-friction loss to obtain the total pressure loss through the tubes. The
origin of these additional pressure changes and an indication of their orders of magnitude is discussed in reference 1. If the intercooler entrance and exit sections are streamlined, the total pressure loss will be very nearly equal to the surface-friction loss except at high altitude, where the pressure change due to density variation in the intercooler may be appreciable. In this paper the surface-friction loss through tubes is given in equation (8) of appendix A; equations for the remaining pressure changes are given in appendix B.

For the flow across the tube banks only the surface-friction and the form-drag pressure losses are used in the analysis. To these losses must be algebraically added the pressure change due to the momentum change of the air as it flows across the tube banks. This momentum change arises from the air density change accompanying heat exchange and pressure drop. The surface-friction and form-drag losses are given in equation (9) of appendix A; the equation for the loss caused by the momentum change is given in appendix B.

The heat-transfer coefficients for flow through and across tubes are given in equations (6) and (7) in appendix A.

Two staggered-tube arrangements are considered in this paper, namely, the equi-velocity and the equilateral arrangements. These arrangements are compared in appendix C on the basis of heat-transfer and pressure-drop data obtained from references 2 and 3.

Based on the previously mentioned heat-transfer and pressure-drop equations, fundamental equations relating the operating conditions and the basic design parameters of the tubular intercooler are derived in appendix A. From these fundamental equations and from the geometry of the tubular intercooler, all the dimensional characteristics of the tubular intercooler can be definitely determined when the core structure and the operating conditions are known. These relationships are given in appendix B and are also presented in the form of curves for convenient calculation of intercooler design. The following plan has been used in order to simplify the presentation:

(1) A reference intercooler is defined as one having the reference core structure. The core of the reference structure consists of tubes of 0.25-inch outside diameter and 0.01-inch wall thickness. These tubes are spaced transversely
0.25 inch and are staggered in adjacent banks in such a manner that there is a minimum change in the velocity of the air as it flows from bank to bank. This staggered arrangement is called the equivelocity arrangement and is characterized by a value of \( \frac{s_b}{s} \) of 1/2. (See fig. 1.) The reference core structure of the charge-across-tube intercooler is identical with that of the charge-through-tube intercooler.

(2) Curves are drawn that show how the dimensional characteristics of the tubular intercooler vary with change in core structure from the reference core structure when the operating conditions are kept constant (fig. 2). These curves are nearly independent of the operating conditions and apply for both the charge-across-tube and the charge-through-tube intercoolers. The small effects of operating conditions are given in figure 3 as correction factors to be applied to the values obtained from the curves. These correction factors can ordinarily be neglected. The effect of changing from equivelocity to equilateral spacing is given in figure 4.

(3) Design charts are given relating the surface area, the tube length, and the number of tube banks of the reference charge-across-tube intercooler to the operating conditions (fig. 5). The remaining dimensional characteristics are determined by the following equations, which are general in that they apply for any core structure and for both types of tubular intercoolers:

The dimension of the tubular intercooler block in the direction of the flow across the tube banks

\[
L = m(s + d) \sqrt{(\frac{s_b}{s} + \frac{d}{s})^2 - \frac{1}{4}}
\]

where, for the equilateral arrangement, \( \frac{s_b}{s} = 1 \), and for the equivelocity arrangement, \( \frac{s_b}{s} = 1/2 \).
The width of the core

\[ w = \frac{(a + d)S}{\pi d \sin \theta} \]  

(2)

The face area perpendicular to the flow across the tubes

\[ A = wL \]  

(3)

and the face area perpendicular to the flow through the tubes

\[ a = wL \]  

(4)

The volume of the core

\[ v = AL = aL \]  

(5)

Figure 1 is a drawing showing the linear dimensions and core structure of a tubular intercooler.

(4) Curves showing the relation between the reference charge-through-tube intercooler and the reference charge-across-tube intercooler for the same operating condition are given in figure 6. Thus, for any set of operating conditions, the dimensions of the charge-across-tube intercooler with the reference core structure can be obtained from figure 5 and for the charge-through-tube intercooler with reference core structure from figures 5 and 6. The effect of change in core structure on the intercooler dimensions for both types of intercooler can be obtained by applying the curves of figures 2 and 3.

DISCUSSION

Relations between Dimensional Characteristics and Core Structure

Equivalence tube arrangement.— The variation in the dimensional characteristics of a tubular intercooler with
change in core structure when the operating conditions are kept constant can be considered as a product of a major variation, which is independent of the operating conditions, and a minor variation, which is dependent on the relative values of the weight flows and the pressure drops of the charge air and the cooling air. The major variation is the same whether the tubular intercooler is of the charge-across-tube or charge-through-tube type; whereas the minor variation is different for the two types of intercooler. Figure 2 is a plot of the major variation. The volume, the surface area, the number of tube banks, the face areas, and the linear dimensions of the tubular intercooler are shown to vary with tube diameter and transverse spacing for any given set of operating conditions and for the equivelocity tube arrangement ($s_b/s = 1/2$).

The effect of a change in transverse spacing when the tube diameter is at the reference value of 0.25 inch is shown by the solid curves. The long- and short-dashed curves show the effect of change in transverse spacing when the tube diameters are, respectively, 0.1875 and 0.125 inch. For any given transverse spacing, the effect of change in tube diameter from the reference value is given by the vertical displacement of the dashed curves from the corresponding solid curves. Curves for the 0.1875-inch tube diameter have been omitted in cases where their presence is inessential. In all cases the variations in dimensional characteristics are given as ratios based on the characteristics of the reference intercooler.

Figure 2 shows that a reduction in tube diameter and transverse spacing results in a marked decrease in intercooler volume at the expense of an increase in intercooler width. Accompanying these changes is a reduction in heat-transfer area and flow lengths and an increase in face area perpendicular to the flow across the tubes. For a given tube-wall thickness the heat-transfer area may be regarded as proportional to the intercooler weight. The number of tube banks and the face area perpendicular to the flow through the tubes decrease as the tube diameter is increased and as the transverse spacing is reduced. It should be noted that the face area $A$ perpendicular to the flow across the tubes is the frontal area of the charge-through-tube intercooler while the face area $A$ perpendicular to the flow through the tubes is the frontal area of the charge-across-tube intercooler. The tube length $l$ changes nearly linearly with tube diameter and varies only slightly with change in transverse spacing; whereas the dimension $L$ is very sensitive to change in transverse spacing and varies only slightly with tube diameter.
The minor variations are small and are plotted in figure 3 as corrections that should be multiplied by the major variation given in figure 2 to give the total variation. The corrections are too small to affect the basic trends shown in figure 2. In many practical cases the corrections are small enough to be neglected. Figure 3 shows that, when \( \theta \) is unity or when \( s \) is 0.25 inch, no correction is necessary. For any given value of \( \theta \) and \( s \), the correction for the charge-across-tube intercooler is very nearly the reciprocal of the correction for the charge-through-tube intercooler.

Table I gives the percentage change in tubular intercooler dimensional characteristics for any given set of operating conditions when the core structure is changed by a reduction in only tube-wall thickness from the reference value (0.01 in.) to 0.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>( d = 0.25 ) in.</th>
<th>( d = 0.125 ) in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>-4</td>
<td>-8</td>
</tr>
<tr>
<td>( l )</td>
<td>12</td>
<td>27.5</td>
</tr>
<tr>
<td>( L )</td>
<td>-2.5</td>
<td>-5</td>
</tr>
<tr>
<td>( m )</td>
<td>-2.5</td>
<td>-5</td>
</tr>
<tr>
<td>( w )</td>
<td>-12</td>
<td>-24</td>
</tr>
<tr>
<td>( A )</td>
<td>-1.5</td>
<td>-3</td>
</tr>
<tr>
<td>( a )</td>
<td>-14</td>
<td>-28</td>
</tr>
<tr>
<td>( r )</td>
<td>-4</td>
<td>-8</td>
</tr>
</tbody>
</table>

The percentage change depends on only the tube outside diameter. For tube-wall thicknesses between 0 and 0.01 inch linear interpolation is sufficiently accurate. The effect of change in tube wall thickness on the linear dimensions of the intercooler is seen, in general, to be small, the largest effect being on the intercooler width \( w \) and length \( l \). The width \( w \) decreases and the length \( l \) increases in nearly the same ratio when the wall thickness \( t \) is reduced.

Equilateral tube arrangement. The heat-transfer area \( S \), the tube length \( l \), the no-flow dimension \( w \), the number of tube banks \( m \), and the face area \( A \) are shown in this paper to be independent of the value of \( a_b/s \);
consequently, the curves of Figures 2 and 3 for these dimensional characteristics also apply when the equilateral arrangement is used. Equations (1), (4), and (5) show that the intercooler dimension in the direction of the flow across the tube banks $L$ and therefore the face area $a$ and volume $v$ are functions of $e_s$ and $e_s$. The ratios $L$, $a$, and $v$ obtained from the curves of figures 2 and 3 must therefore be modified by the factors plotted in figure 4 to obtain the variation in these dimensional characteristics when the equilateral arrangement is used.

According to figure 4, for $e = 0.25$ inch, a change from the equivelocity to the equilateral arrangement when the operating conditions remain constant results in an increase in intercooler volume of 55 percent when $d = 0.25$ inch and of 96 percent when $d = 0.125$ inch. If the transverse spacing is reduced, the effect of the change from the equivelocity to the equilateral arrangement on the intercooler volume is decreased. For example, for $e = 0.05$ inch, the increase in volume is only 13 percent when $d = 0.25$ inch and 24 percent when $d = 0.125$ inch.

The effects of tube arrangement on heat transfer and pressure drop are discussed in appendix C. The only possible advantage of the equilateral arrangement over the equivelocity arrangement is that it may be easier to construct by some manufacturing procedures.

Dimensional Characteristics of the Reference Tubular Intercooler

Figure 5, together with equations (2) to (5), gives the dimensional characteristics of the reference charge-across-tube intercooler in terms of the operating conditions. Figure 5(a) shows the relation between heat-transfer area and operating conditions (from equation (31), appendix B). Figure 5(b) relates the number of tube banks to $a_v \Delta P_s$ and the heat-transfer area (from equation (32), appendix B). The ratios of the flow lengths to the number of tube banks are plotted in figure 5(c) as functions of $e$ and $e_s / e_s$ (from equation (33) of appendix B and equation (1)). The remaining characteristics can be easily calculated from equations (2) to (5).

In figure 6 the dimensional characteristics of the reference charge-through-tube intercooler relative to the
dimensional characteristics of the reference charge- across- tube intercooler are plotted in terms of the operating conditions. Figure 6 shows that the relative values of the flow lengths and the number of tube banks depend on the relative values of the weight flows and pressure drops of the charge air and the cooling air.

It is also seen in Figure 6 that, for the reference core structure, the required heat-transfer area, volume, and width of the charge- through- tube intercooler are not widely different from the corresponding dimensional characteristics of the charge- across- tube intercooler. For other than the reference core structure the same is true, because Figure 2 may be used for the two types of tubular intercooler subject to the minor corrections of Figure 3. Figure 2 shows that a reduction in tube spacing or in diameter permits a reduction in volume at the cost of increased width for both types of intercooler. When width is the limiting factor, as is often the case, both types of intercooler are limited to substantially the same minimum transverse spacing, heat- transfer area, and intercooler volume.

The frontal area of the reference charge- through- tube intercooler is seen from the curve for \( \frac{A_t}{A_m} \) in Figure 6 to be from 60 to 75 percent larger than that of the reference charge- across- tube intercooler for the equivelocity tube arrangement. Figure 2 shows that the frontal area decreases rapidly and area \( A \) increases as the tube spacing is reduced. Thus a reduction in tube spacing further widens the difference between the frontal areas of the two types of intercooler. A large reduction in tube diameter results in a comparatively small decrease in the frontal area \( A \). It is therefore concluded that, in general, the frontal area of the charge- through- tube intercooler is much greater than that of the charge- across- tube intercooler.

**Pressure Drops**

For the flow through the tubes, a close estimate of the magnitude of the velocity- profile loss, the entrance- exit loss, and the loss or gain due to momentum change of the air may be obtained from Figure 7. Figure 7(b) applies only for tubes with blunt entrance and exit sections. The pressure changes obtained from Figure 7 should be algebraically added to the skin- friction pressure drop (equation (8)) to obtain the total pressure drop through the tubes.
For flow across tubes the pressure loss or gain due to heat transfer should be obtained from figure 8 and algebraically added to the skin-friction and form-drag pressure drops given by equation (9) of appendix A. The use of figures 7 and 8 is explained in more detail in an example that is given in the following pages. When figures 3 to 6 and equations (1) to (5) are used in selecting an intercooler design, it is recommended that the pressure-drop values applied be a modification of specified design values to allow for pressure changes other than those accounted for by equations (8) and (9).

Illustration of Tubular Intercooler Design

It is assumed that the following operational requirements are to be satisfied in the design of a tubular intercooler:

(1) Altitude, ft .................................. 40,000
(2) \( \eta \), percent .................................. 70
(3) \( M_2 \), lb/sec .................................. 2
(4) \( \frac{H_1}{M_2} \) .................................. 2
(5) \( \sigma_{av} \Delta p f_1 \), in. of water ............ 1.5
(6) \( \sigma_{av} \Delta p f_2 \), in. of water ............ 6

The design procedure for a charge-across-tube intercooler is as follows:

(7) From items (4), (5), and (6)

\[ s = 2 \times \frac{1.6}{6} = 0.5 \]

(8) From figure 5(a) and items (2), (4), and (6)

\[ \frac{S_o}{M_{av}} = 19,000 \text{ square inches per pound per second} \]

(9) From figure 5(a) and item (7)

\[ y = 1.18 \]
Then, from item (8)

\[
\frac{S_o}{M_2} = 22,400 \text{ square inches per pound per second}
\]

(10) From figure 5(b) and items (6) and (9)

\[
m_o = 87 \text{ banks}
\]

(11) From figure 5(c) and items (4) and (7)

\[
\frac{l_o}{m_o} = 0.18
\]

and from item (10)

\[
l_o = 15.7 \text{ inches}
\]

(12) From figure 5(c) and item (10)

\[
L_o = 0.28 \times 87 = 24.4 \text{ inches}
\]

(13) From equation (2) and items (3), (9), (10), and (11), if \( d_o = 0.25 \)

\[
w_o = \frac{0.5 \times 22400 \times 2}{\pi \times 0.25 \times 15.7 \times 87} = 20.9 \text{ inches}
\]

Thus, the general dimensional characteristics of the intercooler that has the reference core and that meets the operational requirements are as follows:

(14) \( \dot{m}_o = 87 \text{ banks} \)

\[
l_o = 15.7 \text{ inches}
\]

\[
L_o = 24.4 \text{ inches}
\]

\[
w_o = 20.9 \text{ inches}
\]

\[
S_o = 44,800 \text{ square inches}
\]

\[
v_o = l_oL_ow_o = 8,000 \text{ cubic inches}
\]

\[
a_o = w_oL_o = 510 \text{ square inches}
\]

\[
N_o = S_o/\pi d_l_o = 3630 \text{ tubes}
\]
Figure 2 shows that the volume of this intercooler can be diminished by reducing the tube transverse spacing. The flow dimensions \( l \) and \( L \) are consequently reduced, but the no-flow dimension \( w \) is increased. In practice, a limit is often placed on the maximum value of \( w \) because of space limitations. If the maximum allowable width is 30 inches,

\[
(15) \quad \frac{w}{w_0} = \frac{30}{20.9} = 1.435
\]

(16) From figure 2 and item (15)

\[
s = 0.094 \text{ inch}
\]

also

\[
m/m_0 = 0.46 \quad \text{or} \quad m = 40 \text{ banks}
\]
\[
l/l_0 = 0.93 \quad \text{or} \quad l = 14.6 \text{ inches}
\]
\[
L/L_0 = 0.40 \quad \text{or} \quad L = 9.8 \text{ inches}
\]
\[
S/S_0 = 0.895 \quad \text{or} \quad S = 40,100 \text{ square inches}
\]
\[
v/v_0 = 0.53 \quad \text{or} \quad v = 4240 \text{ cubic inches}
\]
\[
a/a_0 = 0.57 \quad \text{or} \quad a = 291 \text{ square inches}
\]

\[
N = 3500 \text{ tubes}
\]

Thus, if the tube transverse spacing is changed from 0.250 inch to 0.094 inch, the value of \( v \) has been reduced to 53 percent and the value of \( a \) to 57 percent of the corresponding original values without change in performance. The large reduction in \( L \) and the small changes in \( l \) and \( N \) are noteworthy. Figure 2 shows that additional reduction in \( v, L, \) and \( l \) may be achieved by diminishing the tube diameter. Such change will be accompanied, however, by an increase in \( w \) and \( s \). Because \( l \) is quite sensitive to change in \( d \), a reduction in \( d \) permits large reductions in \( l \).

The effect of changing the tube-wall thickness is given in table I.

The design procedure for a charge-through-tube inter-
cooler that satisfies the requirements of items (1) to (5) is as follows:

(17) From figure 6 and items (4), (7), and (14)

\[ \frac{m_t}{m_o} = 0.39 \text{ or } m = 87 \times 0.39 = 34 \text{ banks} \]
\[ \frac{l_t}{l_o} = 2.60 \text{ or } l = 15.7 \times 2.60 = 40.8 \text{ inches} \]
\[ \frac{L_t}{L_o} = 0.39 \text{ or } L = 24.4 \times 0.39 = 9.5 \text{ inches} \]
\[ \frac{w_t}{w_o} = 1.00 \text{ or } w = 20.9 \times 1.00 = 20.9 \text{ inches} \]
\[ \frac{S_t}{S_o} = 1.01 \text{ or } S = 44,800 \times 1.01 = 45,300 \text{ square inches} \]
\[ \frac{v_t}{v_o} = 1.01 \text{ or } v = 8000 \times 1.01 = 8100 \text{ cubic inches} \]
\[ \frac{A_t}{A_o} = 1.655 \text{ or } A = 510 \times 1.655 = 845 \text{ square inches} \]
\[ N = \frac{S_o}{\pi d_l o} = 1410 \text{ tubes} \]

Figure 2 may also be used for the charge-through-tube intercooler; it is again evident that if \( s \) is reduced, \( v \), \( l \), and \( L \) can be reduced at the cost of increasing \( w \). If \( w \) is limited to 30 inches,

(18) \[ \frac{w}{w_o} = \frac{30}{20.9} = 1.435 \]

(19) From figure 2 and item (18)

\[ s = 0.094 \text{ inch} \]

which is the same spacing as that required by the charge-across-tube intercooler (item (16)). This equality of transverse spacing is due to the equality of width-reduction ratios (items (15) and (18)) for the two intercoolers in this particular example.

(20) From figure 2 and items (17) and (19)

\[ \frac{m}{m_o} = 0.46 \text{ or } m = 34 \times 0.46 = 16 \text{ banks} \]
\[ \frac{l}{l_o} = 0.93 \text{ or } l = 40.8 \times 0.93 = 38 \text{ inches} \]
\[ \frac{L}{L_o} = 0.40 \text{ or } L = 9.5 \times 0.40 = 3.8 \text{ inches} \]
\[ \frac{S}{S_o} = 0.895 \text{ or } S = 45,300 \times 0.895 = 40,600 \text{ square inches} \]
\[
\begin{align*}
v/v_o &= 0.53 \quad \text{or} \quad v = 8100 \times 0.53 = 4300 \text{ cubic inches} \\
A/A_o &= 1.33 \quad \text{or} \quad A = 840 \times 1.33 = 1130 \text{ square inches}
\end{align*}
\]

\(N = 1360 \text{ tubes}\)

The volume and the heat-transfer area of this intercooler (item (20)) do not differ greatly from those of the charge-across-tube intercooler (item (16)). The frontal area of the charge-through-tube intercooler, however, is much the greater. This frontal area depends on the tube length and can be reduced by reducing the tube diameter. Figure 2 shows that, if the tube diameter were reduced to 0.125 inch and the transverse spacing kept at 0.25 inch, the following charge-through-tube intercooler is the result:

(21) From figure 2 and item (17)

\[
\begin{align*}
\frac{s}{s_o} &= 1.5 \quad \text{or} \quad s = 51 \text{ banks} \\
\frac{l}{l_o} &= 0.395 \quad \text{or} \quad l = 16.1 \text{ inches} \\
\frac{L}{L_o} &= 0.890 \quad \text{or} \quad L = 8.5 \text{ inches} \\
\frac{w}{w_o} &= 2.39 \quad \text{or} \quad w = 50.0 \text{ inches} \\
\frac{S}{S_o} &= 0.945 \quad \text{or} \quad S = 42,800 \text{ square inches} \\
\frac{v}{v_o} &= 0.840 \quad \text{or} \quad v = 6800 \text{ cubic inches} \\
\frac{A}{A_o} &= 0.945 \quad \text{or} \quad A = 800 \text{ square inches} \\
N &= 6760 \text{ tubes}
\end{align*}
\]

The large volume and frontal area of this intercooler as compared to those dimensions of the charge-across-tube intercooler (item (16)) cannot be further reduced by decreasing the tube diameter because of the consequent increase in the width, which already exceeds the specified limit. The effect on the intercooler dimensions or reducing the tube-wall thickness is indicated in table I.

The equivelocity tube arrangement was assumed in this example. The equilateral arrangement, though perhaps more convenient to construct, leads to an increase in over-all dimensions. (See fig. 4.)
The method of correcting the design pressure drops (items (5) and (6)) to account for pressure changes due to causes other than skin friction and form drag are made, for the charge-across-tube intercooler, as follows:

(22) From item (16) and for $d = 0.25$ inch and $t = 0.010$ inch

$$\frac{l}{d - 2t} = \frac{14.6}{0.25 - 0.02} = 63.5$$

(23) From figure 7(b) and items (3), (4), and (16), when $d = 0.25$ inch and $\mu_1 = 12.5 \times 10^{-6}$ pound per foot second

$$R = \frac{M_1 \times d}{\mu_1} = \frac{12 \times 4 \times 0.25}{0.59 \times 291 \times 12.5 \times 10^{-6}} = 5600$$

(24) From figure 7(a) and items (22) and (23)

$$\frac{\Delta P_{V_1}}{\Delta P_{f_1}} = 0.041$$

and from item (5)

$$\sigma_{av} \Delta P_{V_1} = 0.041 \times 1.5 = 0.06 \text{ inch of water}$$

(25) From figure 7(b) and item (16), for $d = 0.25$ inch,

$$\frac{\Delta P_{e_1}}{\Delta P_{V_1}} = 4.8$$

and from item (22)

$$\sigma_{av} \Delta P_{e_1} = 4.8 \times 0.06 = 0.3 \text{ inch of water}$$

The total pressure drop across the intercooler, excluding the loss due to the momentum change of the air across the intercooler is equal to the sum of the skin-friction pressure drop, entrance-exit pressure drop, and velocity profile pressure drop.
(26) From items (5), (24), and (25) this sum is

\[ 1.5 + 0.06 + 0.3 = 1.9 \text{ inches of water} \]

The momentum pressure drop of the cooling air is of special importance at high altitudes. This loss is approximated as follows:

(27) From item (1), if the ramming effects are neglected, the entrance temperature \( T_1 \), pressure \( P_1 \), and relative density \( \sigma_{1en} \) are, for NACA standard air, \(-67^\circ F\), 5.54 inches of mercury, and 0.24, respectively. The temperature rise across the supercharger \( \Delta T_s \) is assumed to be \( 250^\circ F \).

From these data and from items (2) and (4)

\[ \beta_1 = \frac{M_2 \cdot \eta \cdot \Delta T_s}{M_1 \cdot 100 \cdot T_1 + 460} \]

\[ = \frac{1}{2} \times 0.7 \times \frac{250}{393} = 0.22 \]

(29) From items (26) and (27)

\[ \frac{\Delta P_1}{P_1} = \frac{1.9}{0.24 \times 5.54 \times 13.6} = 0.11 \text{ approximately} \]

(29) From figure 7(c) and items (27) and (28)

\[ \frac{\Delta P_H}{\Delta P_V} = 8 \]

and from item (24)

\[ \sigma_{1av} \Delta P_H = 0.5 \text{ inch of water} \]

(30) From items (26) and (29) a first approximation of

\[ \sigma_{1av} \Delta P_1 = 1.9 + 0.5 = 2.4 \text{ inches of water} \]
(31) The average relative density of the cooling-air can also be approximated as:

\[ \sigma_{\text{av}} \approx \sigma_{\text{en}} \left( \frac{2 + \beta_1 - \frac{\Delta P_1}{P_1}}{2 + \gamma_1} \right) \]

\[ = 0.24 \left( \frac{2 + 0.22 - 0.11}{2 + 0.44} \right) = 0.21 \]

(32) Thus, the cooling-air pressure drop is

\[ \Delta P_1 = \frac{2.4}{0.21} = 11.4 \text{ inches of water} \]

(33) A second approximation is given as:

\[ \frac{\Delta P_1}{P_1} = \frac{2.4}{0.21 \times 5.54 \times 13.6} = 0.15 \]

\[ \frac{\Delta P_H}{\Delta P_V} = 9.5 \]

and

\[ \sigma_{\text{av}} \Delta P_1 = 1.9 + (0.06 \times 9.5) = 2.5 \text{ inches of water} \]

\[ \sigma_{\text{av}} = 0.24 \left( \frac{2 + 0.22 - 0.15}{2 + 0.44} \right) = 0.204 \]

\[ \Delta P_1 = \frac{2.5}{0.204} = 12.3 \text{ inches of water} \]

No approximations after the second need be made.

(34) From Items (3) and (16), if \( \mu_2 \) is \( 14 \times 10^{-8} \) pound per foot-second,

\[ R_s = \frac{M_2 m}{M_1 \mu} = \frac{2 \times 40 \times 12}{3500 \times 14 \times 14 \times 10^{-8}} = 1340 \]
(35) From items (2) and (27)

\[ \beta_s = \frac{250 \times 0.7}{-67 + 250 + 460} = 0.272 \]

(36) The value of \( \frac{\Delta p_3}{p_3} \) will be nearly 0. Then, from figure 8(a) and item (35),

\[ \frac{\Delta p_H}{\Delta p_f} = 0.64 \]

and, from items (6) and (16) for \( R_s = 400 \),

\[ \sigma^2_{av} \Delta p_{H_2} = 0.64 \times \frac{6}{40} = 0.1 \text{ inch of water} \]

(37) From figure 8(b) and items (34) and (36)

\[ \sigma^2_{av} \Delta p_{H_2} = 1.28 \times 0.1 = 0.13 \text{ inch of water} \]

(38) From items (6) and (37)

\[ \sigma^2_{av} \Delta p_3 = 6.00 - 0.13 = 5.9 \text{ inches of water} \]

Correction of pressure drops for the charge-through-tube intercooler to account for pressure changes due to causes other than skin friction should be made by the method outlined in items (22) to (38). In the determination of pressure changes due to heat transfer, it should be kept in mind that temperature rise through the intercooler is accompanied by pressure drop and that temperature drop is accompanied by pressure rise. Corrections should be algebraically added to the pressure-drop values given in items (5) and (6). If the total pressure drops thus obtained do not satisfy design conditions, items (5) and (6) should be modified in the desired direction and the design procedure repeated.

CONCLUSIONS

From the foregoing analysis, it may be concluded that:
1. The volume of a tubular intercooler can be reduced without affecting the operating conditions by decreasing the flow lengths and the tube diameter and transverse spacing. This reduction in volume is accomplished, however, at the expense of increased intercooler width.

2. The charge-across-tube and the charge-through-tube intercoolers with identical core structures have nearly the same volume, amount of heat-transfer area, and width when operating at the same conditions.

3. For a core structure made up of 0.25-inch tubes spaced transversely 0.25 inch and diagonally to give the equivelocity arrangement, the frontal area of the charge-across-tube intercooler is about 60 percent of the frontal area of the charge-through-tube intercooler operating at the same conditions. The difference in frontal area increases rapidly with decrease in transverse spacing. A reduction in tube diameter increases the frontal area of the charge-across-tube intercooler and slightly decreases the frontal area of the charge-through-tube intercooler.

4. For constant operating conditions, the tube length changes nearly linearly with tube diameter and varies only slightly with transverse spacing; the dimension in the direction of the flow across the tube banks, however, is much more sensitive to changes in transverse spacing than to changes in tube diameter.

5. Curves showing the variation of dimensional characteristics with tube diameter and transverse spacing for constant operating conditions can be used to indicate the proper steps to be taken in adjusting the size and shape of an intercooler to fit the available space.

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APPENDIX A

DERIVATION OF FUNDAMENTAL EQUATIONS

Heat-Transfer and Pressure-Drop Equations

The heat-transfer and pressure-drop equations upon which the analysis of the tubular intercooler is based are obtained from reference 2. The equations for the flow across tube banks are given in this reference for the equilateral tube arrangement. These equations are shown in appendix C to apply equally as well for the equivelocity arrangement.

The heat-transfer coefficient for the flow of air through tubes may be expressed as

\[ \frac{h_d}{k} = 0.0198 \left( \frac{\rho V_d}{\mu} \right)^{0.8} \quad (6) \]

and for the flow of air across tube banks as

\[ \frac{h_d}{k} = 0.131 \left( \frac{\rho V_d}{\mu} \right)^{0.89} \quad (7) \]

The surface-friction pressure drop for flow through tubes may be expressed as

\[ \sigma_{av} \Delta p_f = 0.049 \left( \frac{\mu}{\rho V_d} \right)^{0.2} \left( \frac{\rho V}{\mu} \right)^2 \frac{4l}{10.4 \rho_0 g d_1} \quad (8) \]

The surface-friction and form-drag losses for flow across tube banks may be expressed as

\[ \sigma_{av} \Delta p_f = 0.8 \left( \frac{\mu}{\rho V_s} \right)^{0.22} \left( \frac{\rho V}{\rho V} \right)^2 \frac{4m}{10.4 \rho_0 g} \quad (9) \]

The Charge-Across-Tube Intercooler

If the thermal resistance of the tube wall is neglected, the over-all heat-transfer coefficient is given by
The charge-air weight flow is given as

$$ M_2 = \rho_2V_2 \frac{N}{m} \text{ s}^1 \quad (11) $$

and the cooling-air weight flow as

$$ M_1 = \rho_1V_1N \frac{\pi d_1^2}{4} \quad (12) $$

The outside tube surface

$$ S = \pi Nd_1 \quad (13) $$

and the inside tube surface

$$ S_1 = \pi Nd_1^1 \quad (14) $$

and

$$ \frac{S}{S_1} = \frac{d}{d_1} \quad (15) $$

From equations (11) and (13)

$$ \frac{md}{s} = \frac{S}{M_2} \frac{\rho_2V_2d}{\mu_2} \frac{\mu_2}{\pi d} \quad (16) $$

The substitution of equations (9) and (16) in equation (7) gives

$$ h_2 = 0.292 \frac{k_2}{\mu_2^{0.75} \beta^{0.19} \phi^{0.068}} \left( \frac{M_2\sigma_{2xy}\Delta Pf}{s} \right)^{0.25} \quad (17) $$

From equations (12) and (14),

$$ \frac{1}{d_1} = \frac{1}{4} \frac{S_1}{M_1} \frac{\rho_1V_1d_1}{\mu_1} \frac{\mu_1}{d_1} \quad (18) $$
The substitution of equations (8) and (18) in equation (6) gives

\[ h_1 = 0.119 \frac{k_1}{\mu_1^{0.88} d_1^{0.14}} \left( \frac{M_1 \sigma_{av} \Delta p_f}{S_1} \right)^{0.88} \]  

(19)

Then, from equations (15), (17), and (19) and from the relations for air, \( \frac{c_p \mu}{k} = \) a constant and \( c_p = \) a constant

\[ \frac{h_2 S}{h_1 S_1} = 2.45 \mu_2^{0.14} d_1^{0.06} \left( \frac{S_1}{M_1 \sigma_{av} \Delta p_f} \right)^{0.03} \]  

(20)

From equations (10), (15), (17), and (20)

\[ \frac{S}{M_2} = 3.38 \left( \frac{UA_r}{M_2 c_p} \right)^{1.33} \frac{u_2^{0.36} \sigma_{av}^{0.03}}{\mu_2^{0.33}} \left( \frac{1}{\sigma_{av} \Delta p_f} \right)^{0.33} \]

\[ \left[ 1 + 2.45 \mu_2^{0.35} d_1^{0.06} \left( \frac{S}{M_2} \right)^{0.03} \right]^{1.33} \]

(21)

It is necessary that equation (21) be approximated in such a manner that \( S/M_2 \) can be expressed as a product of factors each of which is a function either of internal dimensions only or of operating conditions only. This approximation is possible only if a small correction factor \( \phi \), which is a function both of internal dimensions and operating conditions, is introduced. It should be noted that the heat-transfer factor \( UA_r/M_2 c_p \) can be considered as an operating condition because it is a function of \( \eta \) and \( M_1/M_2 \). (See reference 1.)

A close approximation of equation (21) is
\[
\frac{S}{M_2} = 3.38 \left( \frac{UA_r}{M_2 c_p} \right)^{1.33} \left( \frac{a_0.86d_0.08}{\mu_2^{0.33}} \right)^{0.33} \left( \frac{1}{\sigma_{av} \Delta p f_2} \right)^{0.33} \\
\left[ 1 + \frac{1}{1.223} \left( \frac{S}{M_2} \right)^{0.038} \right]^{1.33} \\
\left( \frac{1 + 2.78 \frac{\mu_2^{0.95}}{\mu_1^{0.14}} \frac{d_0.65}{d_1^{0.57}d_0.19}}{1.33} \left[ \frac{1 + \frac{1}{1.223} \left( \frac{S}{M_2} \right)^{0.038}}{2} \right] \right)^{1.33} \rho_m^{1.33} \tag{22}
\]

where
\[
\rho_m = 1 + \frac{1}{\frac{1 + 2.78 \frac{\mu_2^{0.95}}{\mu_1^{0.14}} \frac{d_0.65}{d_1^{0.57}d_0.19}}{1.33} \left[ \frac{1 + \frac{1}{1.223} \left( \frac{S}{M_2} \right)^{0.038}}{2} \right] \left( \frac{1 + 2.78 \frac{\mu_2^{0.95}}{\mu_1^{0.14}} \frac{d_0.65}{d_1^{0.57}d_0.19}}{1.33} \right) \left( \frac{1 + \frac{1}{1.223} \left( \frac{S}{M_2} \right)^{0.038}}{2} \right) \right)^{1.33} \tag{23}
\]

For the range of values of \( S/M_2 \) from 35 to 700 square feet per pound per second, the factor
\[
\left[ 1 + \frac{1}{1.223} \left( \frac{S}{M_2} \right)^{0.038} \right]^{1.33}
\]

in equation (22) can be shown to be nearly equal to 0.87 \( \left( \frac{S}{M_2} \right)^{0.026} \). Thus, equation (22) can be written
\[
\frac{S}{M_2} = 3.025 \left( \frac{UA_r}{M_2 c_p} \right)^{1.365} \left( \frac{a_0.267d_0.082}{\mu_2^{0.34}} \right)^{0.34} \left( \frac{1}{\sigma_{av} \Delta p f_2} \right)^{0.34} \\
\left( 1 + 2.78 \frac{\mu_2^{0.95}}{\mu_1^{0.14}} \frac{d_0.65}{d_1^{0.57}d_0.19} \right)^{1.365} \\
\left( \frac{1 + 2.78 \frac{\mu_2^{0.95}}{\mu_1^{0.14}} \frac{d_0.65}{d_1^{0.57}d_0.19}}{1.365} \right)^{1.365} \rho_m^{1.365} \tag{24}
\]
The error due to the foregoing approximations depends on the values of \( S/M_2 \), \( s \), \( \theta \), and \( \sigma_{av}\Delta p_f \). For values of 

\( S/M_2 \) from 35 to 7.00 square feet per pound per second, of 
\( s \) from 0.012 to 0.25 inch, of \( \theta \) from 1/4 to 4, and of 
\( \sigma_{av}\Delta p_f \) from 4 to 16 inches of water the maximum error 
is 2.5 percent. For usual values of these quantities the error 
is less than 1 percent. From equations (6), (15), 
(18), and (19):

\[
\frac{l}{d_1} = 2.35 \frac{d_1^{0.71}}{\mu_1^{0.07} d_1^{0.64}} \left( \frac{S}{M_2} \right)^{0.64} \left( \frac{M_2}{M_1} \right)^{0.64} \left( \sigma_{av}\Delta p_f \right)^{0.36} \quad (25)
\]

From equations (7), (16), and (17):

\[
\frac{md}{s} = \frac{3.2}{\pi} \frac{d_1^{0.36}}{\mu_2^{0.08} d_1^{0.36}} \left( \frac{S}{M_2} \right)^{0.64} \left( \sigma_{av}\Delta p_f \right)^{0.36} \quad (26)
\]

Equations (23) to (26), together with equations (1) to (5) presented in the section on Method, give the dimensional characteristics of a charge-across-tube intercooler in terms of the internal dimensions and operating conditions.

The Charge-through-Tube Intercooler

The equations for the charge-through-tube intercooler are derived in the same manner as for the charge-across-tube intercooler. Only the final results are therefore given.

\[
\frac{S}{M_2} = 3.025 \left( \frac{UA}{M_2 c_p} \right)^{1.365} \left( \frac{M_2}{M_1} \right)^{0.34} \frac{0.367 d_1^{0.06} \mu_1^{0.34}}{} \left( \frac{1}{\sigma_{av}\Delta p_f} \right)^{0.34}
\]

\[
\left( 1 + 2.78 \frac{\mu_1^{0.025}}{\mu_2^{0.11}} d_1^{0.65} \right)^{1.365}
\]

\[
\left( 1 + 1.08 \frac{\theta^{0.25}}{\sigma_{av}\Delta p_f^{0.036}} \right)^{1.365} \theta^{1.365} \quad (27)
\]
where
\[
\beta_t = 1 + \frac{(2.78 \frac{\mu_1^{0.25} d^{0.65}}{\mu_2^{0.14} d_2^{0.57} s^{0.19}} - 1)(1.08 \frac{\mu_2^{0.25} \sigma_{av} \Delta P_{f_2}^{0.038}}{\mu_1^{0.14} d_1^{0.57} s^{0.19}} - 1)}{\frac{2.78}{\mu_1^{0.25} d_1^{0.57} s^{0.19}} + 1)(1.08 \frac{\mu_2^{0.25} \sigma_{av} \Delta P_{f_2}^{0.038}}{\mu_1^{0.14} d_1^{0.57} s^{0.19}} + 1)}
\]  (28)

also
\[
\frac{1}{d_1} = 2.35 \frac{d_1^{0.71}}{\mu_2^{0.07} d^{0.64}} \left( \frac{S}{M_2} \right)^{0.64} \left( \sigma_{av} \Delta P_{f_2} \right)^{0.36}
\]  (29)

and
\[
\frac{md}{s} = 3.2 \frac{d^{0.36}}{\mu_1^{0.08} s^{0.88}} \left( \frac{S}{M_2} \right)^{0.64} \left( \frac{M_2}{M_1} \right)^{0.64} \left( \sigma_{av} \Delta P_{f_1} \right)^{0.36}
\]  (30)

Equations (27) to (30), together with equations (1) to (5) presented in the section on Method, give the dimensional characteristics of a charge-through-tube intercooler in terms of the internal dimensions and operating conditions.

APPENDIX B

EQUATIONS USED IN PREPARATION OF DESIGN CHARTS

Equations Showing the Relations between the Dimensional Characteristics of the Reference Intercooler and the Operating Conditions

The equations presented herein are obtained from equations derived in appendix A and from those given in the section on Method. The presentation is outlined in the sections on Method and Discussion. The viscosities are evaluated at 200°F for the charge air and at 100°F for the cooling air. The terms \( \frac{\mu_2^{0.25}}{\mu_1^{0.14}} \)
\( \frac{\mu_1^{0.35}}{\mu_2^{0.14}} \) (equations (24) and (27)) are given the approximate value of 0.30. The term \((\sigma_{av}\Delta p_{f_2})^{0.038}\) in the same equations is assigned the constant value of 1.08 because its variation is small in the range of values of \(\sigma_{av}\Delta p_{f_2}\) occurring in intercooler practice. Except when otherwise stated, the equations presented in this appendix apply to both the equilateral and the equivalentity tube arrangement. When substitution is made in equation (24) of these values and of the values assigned to \(s, d\) and \(d_1\) for the reference charge-across-tube intercooler,

\[
\frac{S_{m_0}}{M_2 y} = 110.7 \left( \frac{U_{Ar}}{M_3c_p} \right)^{1.365} \left( \frac{1}{\sigma_{av}\Delta p_{f_2}} \right)^{0.34} \tag{31}
\]

where \(U_{Ar}/M_3c_p\) is a function of \(\eta\) and \(M_1/M_2\) and where

\[
y = \left( \frac{1 + \theta^{-0.39}}{2} \right)^{1.365} \phi_{m_0}
\]

By a similar substitution, equation (26) becomes

\[
m_{m_0} = 1.822 \left( \frac{S_{m_0}}{M_2} \right)^{0.64} \left( \sigma_{av}\Delta p_{f_2} \right)^{0.36} \tag{32}
\]

and, from equation (25), it follows that

\[
\frac{l_{m_0}}{m_{m_0}} = 0.039 \frac{M_2}{M_1} \theta^{0.38} \tag{33}
\]

Equations (1) to (5) can be used with equations (31) to (33) to express other dimensional characteristics in terms of operating conditions.

Similar relations for the reference charge-through-tube intercooler are not expressed directly but by means of ratios obtained by dividing the equations for the charge-through-tube intercooler by the corresponding equations for the charge-across-tube intercooler. These relations follow:
\[
\frac{S_{t,0}}{S_{m,0}} = \frac{1.07}{0.34} \left( \frac{1 + \theta^{0.85}}{1 + \theta^{-0.89}} \right) \left( \frac{\rho_{t,0}}{\rho_{m,0}} \right)^{1.365} (34)
\]

\[
\frac{L_{t,0}}{L_{m,0}} = \frac{M_1}{M_2} \left( \frac{1}{\theta} \right)^{0.36} \left( \frac{S_{t,0}}{S_{m,0}} \right)^{0.64} (35)
\]

\[
\frac{m_{t,0}}{m_{m,0}} = \frac{M_1}{M_2} \theta^{0.36} \left( \frac{S_{t,0}}{S_{m,0}} \right)^{0.64} (36)
\]

\[
\frac{L_{t,0}}{L_{m,0}} = \frac{M_2}{M_1} \theta^{0.36} \left( \frac{S_{t,0}}{S_{m,0}} \right)^{0.64} (37)
\]

\[
\frac{w_{t,0}}{w_{m,0}} = \frac{1}{\left( \frac{S_{t,0}}{S_{m,0}} \right)^{0.236}} (38)
\]

\[
\frac{A_{t,0}}{A_{m,0}} = \frac{0.922}{\sqrt{\left( \frac{d + d}{s + d} \right)^2 - \frac{1}{4}} (39)}
\]

\[
\frac{v_{t,0}}{v_{m,0}} = \frac{S_{t,0}}{S_{m,0}} (40)
\]

**Variation of Tubular Intercooler Dimensional Characteristics**

**with Tube Transverse Spacing for Constant Operating Conditions**

Tube outside diameter of 0.25 inch. — If the assigned values of \( \mu_1, \mu_2, (\sigma_{av} \Delta P_{f,2})^{0.038}, d = 0.25 \text{ in.} \) and
t (= 0.010 in.) are substituted in equations (24) and (27), and if the resulting expressions are divided, respectively, by the corresponding equations for the reference intercooler,

\[
\frac{S}{S_0} = \left(\frac{s}{s_0}\right)^{0.267} \left(\frac{1 + \frac{0.64}{s_0^{0.19}}}{1 + \frac{0.64}{s_0^{0.19}}}\right)^{1.385} Z
\]  

(41)

for both the charge-across-tube and the charge-through-tube intercoolers when the operating conditions are constant. By definition,

\[
Z = \left(\frac{\rho_t}{\rho_{t_0}}\right)^{1.365}
\]

for the charge-through-tube intercooler and

\[
Z = \left(\frac{\rho_m}{\rho_{m_0}}\right)^{1.365}
\]

for the charge-across-tube intercooler. The term \((\rho_t / \rho_{t_0})^{1.365}\) can be shown to be very nearly equal to \((\rho_m / \rho_{m_0})^{1.365}\) when \(s\) for the one type of intercooler is taken as \(1/s\) for the other. The factor \(Z\) is plotted against \(s\) in Fig. 3 as a minor correction to be applied to \(S/S_0\). The same figure shows a plot of other powers of \(Z\) that occur implicitly in the equations (42) to (48), which follow.

If the procedure used in deriving equation (41) is applied to equations (25) and (29), it follows that

\[
\frac{l}{l_0} = \left(\frac{S}{S_0}\right)^{0.64}
\]

(42)

In similar manner,

\[
\frac{m}{m_0} = \left(\frac{s}{s_0}\right)^{0.75} \left(\frac{s}{s_0}\right)^{0.64}
\]

(43)
\[
\frac{L}{L_0} = \left( \frac{\frac{1}{48} + s}{\frac{1}{48} + s_0} \right)^m \\
\frac{L}{L_0} = \left( \frac{\frac{24s + 1}{48s + 1} - \frac{1}{4}} \left( \frac{\frac{1}{48} + s}{\frac{1}{48} + s_0} \right)^m \right) (\text{equilateral arrangement}) \]

\[
\frac{w}{w_0} = \left( \frac{\frac{1}{48} + s}{\frac{1}{48} + s_0} \right) \left( \frac{S}{S_0} \right)^{0.36} \frac{m_0}{m} \quad (45)
\]

\[
\frac{A}{a_0} = \frac{w_1}{w_0} \quad (46)
\]

\[
\frac{a}{a_0} = \frac{wL}{w_0L_0} \quad (47)
\]

\[
\frac{v}{v_0} = \frac{wL}{w_0L_0} \quad (48)
\]

It should be noted that equations (42) to (48) hold for both types of tubular intercooler as in the case of equation (41).

**Outside tube diameter of 0.125.**—If the assigned values of \( \mu_1, \mu_2, (\sigma_{av} \Delta P_{f_2})^{0.038}, d = 0.125 \) inch, and \( t = 0.010 \) inch are substituted in equations (24) and (27) and if the resulting expressions are divided, respectively, by the corresponding equations for the reference intercooler,

\[
\frac{S}{S_0} = 0.945 \left( \frac{s}{s_0} \right)^{0.267} \left( \frac{1 + \frac{0.64}{0.19}}{1 + \frac{0.64}{s_0^{0.19}}} \right)^{1.385} \quad (49)
\]

**Note:** The symbols used in the equations are as follows:
- \( L, L_0 \): Length of the intercooler.
- \( m, m_0 \): Constants.
- \( w, w_0 \): Flow rate.
- \( A, a, a_0 \): Cross-sectional area.
- \( v, v_0 \): Velocity.
- \( S, S_0 \): Cross-sectional area.
- \( \mu \): Friction factor.
- \( d, t \): Diameter and thickness.
- \( \sigma_{av} \Delta P_{f_2} \): Average pressure drop.

These equations are used to calculate the performance characteristics of tubular intercoolers under various conditions.
Similarly,
\[
\frac{l}{l_0} = 0.41 \left(\frac{s}{s_0}\right)^{0.84}
\]  
(50)

\[
\frac{m}{m_0} = 1.56 \left(\frac{s}{s_0}\right)^{0.72} \left(\frac{S}{S_0}\right)^{0.84}
\]  
(51)

\[
\frac{L}{L_0} = \left(\frac{\frac{1}{96} + \frac{s}{s_0}}{\frac{1}{48} + \frac{s}{s_0}}\right) \frac{m}{m_0}
\]  
(equilateral arrangement)

\[
\frac{L}{L_0} = \sqrt{\frac{\left(\frac{48s + 1}{96s + 1}\right)^2 - 1}{\left(\frac{48s_0 + 1}{48s_0 + 1}\right)^2 - 1}} \frac{\left(\frac{\frac{1}{96} + \frac{s}{s_0}}{\frac{1}{48} + \frac{s}{s_0}}\right) m}{m_0}
\]  
(equivalence arrangement)

(52)

\[
\frac{w}{w_0} = 4.88 \left(\frac{\frac{1}{96} + \frac{s}{s_0}}{\frac{1}{48} + \frac{s}{s_0}}\right) \left(\frac{S}{S_0}\right)^{0.38} \frac{m_0}{m}
\]  
(53)

Equations (46) to (48) apply for both diameters (0.25 and 0.125 in.).

The Effect on Intercooler Dimensional Characteristics of Changing the Tube–Wall Thickness for Any Transverse Spacing and for Constant Operating Conditions

By the general procedure used in obtaining equations (41) to (53), the effect of changing the tube–wall thickness from the reference intercooler value of 0.010 inch to 0 is shown in the following equations. The primed symbols refer to the dimensional characteristics when the tube–wall thickness is 0. When \( d = 0.25 \) in.,
The ratio $\frac{\rho_m'}{\rho_m}$ being very nearly equal to unity

$$\frac{l'}{l} = 1.153 \left( \frac{S'}{S} \right)^{0.34}$$ \hspace{1cm} (55)

$$\frac{m'}{m} = \left( \frac{S'}{S} \right)^{0.64}$$ \hspace{1cm} (56)

$$\frac{L'}{L} = \frac{m'}{m}$$ \hspace{1cm} (57)

$$\frac{w'}{w} = 0.868 \left( \frac{S}{S'} \right)^{0.28}$$ \hspace{1cm} (58)

$$\frac{\dot{A}'}{\dot{A}} = \frac{w'l'}{wl}$$ \hspace{1cm} (59)

$$\frac{a'}{a} = \frac{w'LL'}{wl}$$ \hspace{1cm} (60)

$$\frac{v'}{v} = \frac{w'l'L'}{wlL}$$ \hspace{1cm} (61)

When $d = 0.125$ in.,

$$\frac{S'}{S} = \left( \frac{1 + \frac{0.59}{g^{0.19}}}{1 + \frac{0.64}{g^{0.19}}} \right)^{1.365}$$ \hspace{1cm} (62)
The ratio \( \frac{\rho_m}{\rho_m} \) does not appear in equation (62) because its value is very-nearly equal to unity.

\[
\frac{1}{l} = 1.346 \left( \frac{S'}{S} \right)^{0.64} \quad (63)
\]

\[
\frac{m'}{m} = \left( \frac{S'}{S} \right)^{0.64} \quad (64)
\]

\[
\frac{L'}{L} = \frac{m'}{m} \quad (65)
\]

\[
\frac{w'}{w} = 0.743 \left( \frac{S}{S'} \right)^{0.28} \quad (66)
\]

Equations (59) to (61) apply for both diameters (0.25 and 0.125 in.).

Equations for Pressure Drop Due to Sources Other than Surface Friction and Form Drag

Figures 7 and 8 are based on the following relations derived from pressure drop equations given in reference 3:

For flow through tube—(fig. 7),

\[
\frac{\Delta p_V}{\Delta p_f} = \frac{0.09}{4\lambda \frac{l}{d_1}}
\]

\[
\frac{\Delta p_g}{\Delta p_V} = (1 - f)^3 + \epsilon
\]

where \( \epsilon \) is a function of \( f \) as shown in reference 2.
For flow across tubes (fig. 8),

\[
\frac{\Delta P_h}{\Delta P_v} = 22.2 \left( \frac{1 + \beta}{1 - \frac{\Delta P}{P}} - 1 \right)
\]

APPENDIX C

EFFECT OF STAGGERED-TUBE ARRANGEMENT ON HEAT TRANSFER

AND PRESSURE DROP FOR FLOW ACROSS TUBE BANKS

References 4 and 5 give the results of heat-transfer and pressure-drop tests on a large number of tube arrangements. The test data from these references for the staggered-tube arrangements are plotted herein in figures 9 and 10 as Nusselt number \( \frac{h d}{k} \) and friction factor \( \lambda \) against \( \frac{s_b}{s} \) for various values of \( s/d \) and Reynolds number. The staggered-tube arrangement is defined by the value of \( s_b/s \). (See fig. 1.) When \( s_b/s = 1 \), the arrangement is equilateral. When \( s_b/s = 1/2 \), the arrangement is such that the velocity changes of the air flowing across the tube banks is minimum and the arrangement is called the equivelocity arrangement.

Figure 9 shows that for \( s/d = 1 \) and 2 a decrease in \( s_b/s \) results in a slight increase in heat-transfer coefficient until \( s_b/s = 1/2 \). Decrease in \( s_b/s \) below this value apparently results in an abrupt reduction in the heat-transfer coefficient. The range of data for \( s/d = 0.25 \) and 0.50 is limited in that no arrangements below \( s_b/s = 1/2 \) were tested. The available data, however, fall in line with the values plotted for \( s/d = 1 \) and 2, the scatter of data being random.

Although quite scattered and limited, the friction data of figure 10 seem to indicate a slight increase in friction factor with decrease in \( s/d \). Also, for values of \( s_b/s \) above 1/2 the friction factor seems to be only
slightly dependent on $s_b/s$. For $s_b/s$ below $1/2$ and 
for $s/s_d = 2$, the only value of $s/s_d$ for which enough 
data below $s_b/s = 1/2$ is available, the friction fac-
tor drops rapidly with decrease in $s_b/s$.

The evidence in figures 9 and 10 of a drop in heat-
transfer and friction as $s_b/s$ is reduced below $1/2$ is 
somewhat meager and additional data is required before 
definite conclusions can be drawn concerning operation at 
a value of $s_b/s$ of less than $1/2$. Operation at $s_b/s$ 
of $1/2$ is definitely better than at higher values of $s_b/s$ 
from the standpoint of space economy for equal performance. 
The heat-transfer coefficients and friction factors as 
determined by the equations used in this paper (obtained 
from reference 2) are also indicated in figures 9 and 10. 
It is noted that the use of the equations should yield 
conservative estimates of intercooler performance for both 
the equivelocity and the equilateral arrangements. Test 
results reported in reference 3 for the equilateral ar-
rangement and for $s/s_d = 0.38$ were in good agreement 
with the performance predicted by the heat-transfer and 
pressure-drop equations used in this paper.

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Figure 1. Drawing of a tubular intercooler
Subscript $O$ refers to a reference tubular intercooler, which is defined as one having $s = d = 0.25\text{ in.}$, $t = 0.01\text{ in.}$, and $s/s = \frac{1}{2}$.

---

**Figure 2**—Variation of tubular intercooler dimensional characteristics with tube transverse spacing and tube diameter for constant operating conditions and for the equivelocity staggered tube arrangement. Charge-across-tube and charge-through-tube types.
(a) Corrections for $s/s_o$ and $v/v_o$.

Figure 3: Corrections for variation of dimensional characteristics shown in figure 2.
Figure 4.— Effect on L, a, and V of change from equivelocity to equilateral tube arrangement.
Figure 5.- Variation of dimensional characteristics of reference charge-across-tube intercooler with operating conditions for the equivelocity arrangement.
Figure 6.-Comparison of dimensional characteristics of the reference charge-through-tube intercooler with those of the reference charge-across-tube intercooler.
Figure 7.- Pressure drops for flow through tubes.
Figure 8.—Pressure change due to density variation of air flowing across tube banks.

(a) $R_s = 400$
(b) Correction of $\Delta p_H$ for Reynolds numbers above 400.

Figure 8b
Figure 9.- Effect of tube arrangement on heat transfer for flow across staggered tube banks.
Figure 10 - Effect of tube arrangement on friction factor for flow across staggered tube banks.
ABSTRACT:

An analysis of cross-flow tubular intercooler which has a staggered tube arrangement was made for turbulent-flow conditions. Charts are presented showing how over-all dimensions of intercooler vary with tube diameter, transverse spacing, and wall thickness, without affecting operation of intercooler. The charts are applicable to charge-across-tube and charge-through-tube intercoolers. Charts and equations relating the intercooler dimensional characteristics and the operating conditions are given.