EFFECT OF CHANGES IN ASPECT RATIO, SIDE AREA, FLIGHT-PATH ANGLE, AND NORMAL ACCELERATION ON LATERAL STABILITY

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Computations have been made to determine the effect of changes in wing aspect ratio, additional side area, flight-path angle, and normal acceleration on the relation between the fin area and the dihedral angle required for spiral and for oscillatory lateral stability for a hypothetical airplane of the pursuit or fighter categories. The calculations, however, are applicable to any type of airplane characterized by the parameters and the data employed. The results of the computations are presented in the form of diagrams of fin area plotted against dihedral angle showing the combinations of these variables for which the airplane has both spiral and oscillatory stability.

The diagrams indicate that the effect of wing aspect ratio on lateral stability is small. The increased crosswind force that accompanies an increase in side area is advantageous in that it makes less difficult the selection of a fin area and a dihedral angle that will give lateral stability at high speed as well as at low speed with flap extended. The effects of flight path and normal acceleration are such that airplanes, which are laterally stable in level flight, may become unstable in a glide or a climb or when subjected to normal acceleration.

INTRODUCTION

The results of investigations reported in references 1 and 2 show that the present-day trends in airplane design toward higher wing loadings and larger values of radii of gyration in roll and yaw make the attainment of lateral
stability difficult, if not impossible. It has become important, therefore, to study in detail all factors that affect the lateral stability of the airplane. The limits of fin area and dihedral angle required for lateral stability of an airplane as affected by variations in airplane density, radii of gyration in roll and yaw, wing chord, and tail length have previously been computed and presented in reference 1.

The present report is a continuation of the work of reference 1 and includes the effects on lateral stability of changes in aspect ratio, side area, flight-path angle, and normal acceleration. The results are given in the form of diagrams of dihedral angle against fin area for neutral spiral and oscillatory divergence.

AIRPLANE PARAMETERS

The airplane considered in the investigation was assumed to have the following characteristics: wing area $S_w$, 200 square feet; total weight $W$, 6000 pounds; wing loading $W/S$, 30 pounds per square foot; ratio of radius of gyration in roll to wing span $k_x/b$, 0.125; and ratio of radius of gyration in yaw to wing span $k_z/b$, 0.175.

The changes in the parameters studied are given in the following table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mu$</th>
<th>$A$</th>
<th>$b$ (ft)</th>
<th>$l_t/b$</th>
<th>$l_a/b$</th>
<th>$S_{l\theta}/S_w$</th>
<th>$\gamma$ (deg)</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value</td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>----</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>13.72</td>
<td>4</td>
<td>28.28</td>
<td>0.566</td>
<td>----</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>6.86</td>
<td>16</td>
<td>56.57</td>
<td>0.283</td>
<td>----</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Additional side area</td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>0.2</td>
<td>0.06</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>0.2</td>
<td>0.12</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Flight-path angle</td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>----</td>
<td>0</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>----</td>
<td>0</td>
<td>-20</td>
<td>1</td>
</tr>
<tr>
<td>Normal acceleration</td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>----</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>9.7</td>
<td>8</td>
<td>40</td>
<td>0.400</td>
<td>----</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>
where

\[ \mu = \text{ratio of airplane density to air density computed for standard air density at sea level (m/\rho S_w b)} \]

\[ m = \text{airplane mass} \]

\[ \rho = \text{air density} \]

\[ S_w = \text{wing area} \]

\[ b = \text{wing span} \]

\[ A = \text{aspect ratio} \]

\[ S_f = \text{fin area} \]

\[ S_{fa} = \text{additional side area} \]

\[ l_t/b = \text{ratio of distance from center of gravity of airplane to center of pressure of fin to b} \]

\[ l_a/b = \text{ratio of distance of } \Delta S_{fa} \text{ ahead of center of gravity to b} \]

\[ \gamma = \text{flight-path angle} \]

\[ g = \text{normal acceleration} \]

For the computations, one parameter was varied at a time while the others were kept at the mean value. These mean values were the same values that were used for like quantities in reference 1.

With regard to the variation of aspect ratio, the changes in \( \mu \) and \( l_t/b \) result from the change in \( b \) that accompanies the change in \( A \). The wing loading and the value of \( l_t \) are constant for all values of \( A \). From elementary considerations the ratio of changes in radii of gyration to wing span in roll \( k_X/b \) and in yaw \( k_Z/b \) would be expected to change with \( A \); an examination of existing data from present-day types of airplane, however, showed no general tendency for these values to vary with \( A \).

The additional side area \( S_{fa} \) is used to represent an increase in the cross-wind force of an airplane over that of the streamline fuselage for which the original data were
obtained. For convenience, the additional side area $S_{fa}$ is divided into two parts: $\Delta S_{fa}$, which is added ahead of the center of gravity, and $\Delta S_f$, which is added to $S_f$. The value of $\Delta S_f$ is so chosen that it will exactly counteract the unstable yawing moment due to sideslip of $\Delta S_{fa}$; that is, the weathercock stability of the airplane is not affected by changes in $S_{fa}$.

Because few data are available for some of the aerodynamic characteristics required, the lateral-stability derivatives were computed from the dimensions and parameters given. The contributions of each component of the airplane are added to give the values for the complete airplane.

In the computation of the lateral-stability derivatives, the changes in the aerodynamic characteristics due to changes in the interference effects between the component part of the airplane and the effect of changes in power are assumed to be zero. Actually, for an airplane, some of these effects may be large, especially for conditions other than straight level flight. Some of the interference effects caused by wing location on the variation of the derivatives that depend upon sideslip are given in references 3 and 4. It follows that the derivatives used in the calculations may not represent the actual characteristics of the geometric arrangement assumed. The results, therefore, should be regarded as applicable to an airplane characterized by the stability derivatives used rather than by a given geometric arrangement.

The stability derivatives used and their variations with the airplane parameters were computed by the following relations, which are intended to include the effects of the parameters and particularly the changes in fin area, additional side area, and dihedral angle. These relations are the same as given in reference 1, modified for constant wing area and with a term added to include additional side area. The stability derivatives are the instantaneous rates of change of the aerodynamic coefficients with attitude or angular velocity when the attitude angle or the angular velocity is zero. For convenience, the partial derivatives are written in the form

\[
C_\beta, C_p, C_r, \text{ etc. instead of } \frac{\partial C_1}{\partial \beta}, \frac{\partial C_1}{\partial \theta}, \frac{\partial C_1}{\partial r}, \text{ etc.}
\]
where

\[ C_i \text{ force or moment coefficient (rolling-moment coefficient in this case)} \]
\[ \beta \text{ angle of sideslip} \]
\[ p \text{ rolling velocity} \]
\[ r \text{ yawing velocity} \]
\[ V \text{ velocity of flight} \]

The relation for the derivatives is

\[ C_{y\beta} = C_{y\beta(\text{fuselage})} + C_{y\beta(S_w)} + C_{y\beta(S_f+\Delta S_f)} + C_{y\beta(\Delta S_f a)} \]

\[ = -0.16 + 0 - 3.48 \frac{(S_f+\Delta S_f)}{S_w} - 3.48 \frac{\Delta S_f a}{S_w} \quad (1) \]

where

\[ C_y \text{ force coefficient along Y axis of airplane} \]
\[ \beta \text{ angle of sideslip, positive when the right wing is into the wind, radians} \]

and the subscripts indicate the contribution of the corresponding part of the airplane to \( C_{y\beta} \).

The constant 0.16 in \( C_{y\beta(\text{fuselage})} \) was computed from the relation given in reference 1 for an aspect ratio of 8 and represents the condition for a constant ratio of wing area to fuselage size.

Although \( C_{y\beta(S_w)} \) varies with dihedral (references 3, 5, 6, and 7), this derivative has not been included because it is counteracted, at least partly, by the derivative of the side force due to rolling \( C_{y_p} \).
The rate of change of normal force on the fin and additional side area with angle of sideslip, \( C_{1\beta}(S_f + \Delta S_f) \) and \( C_{1\beta}(S_f, a) \), is equal to \(-3.48 \frac{(S_f + \Delta S_f)}{S_w}\) and \(-3.48 \frac{\Delta S_f, a}{S_w}\) in terms of the wing area. The term \( S_f/S_w \) is the ratio of fin area to wing area (a variable for this investigation) and 3.48 is the rate of change of normal-force coefficient on the fin with sideslip angle \( \beta \) for an effective aspect ratio of 3. The value obtained with the fin used on the model in reference 3 is about 3.48.

\[
C_{1\beta} = C_{1\beta}(S_w) + C_{1\beta}(S_f + \Delta S_f) + C_{1\beta}(\Delta S_f, a)
= K_1 \Gamma - \left( \frac{x}{b} - \frac{1}{b} \sin \theta \right) \left( 3.48 \frac{(S_f + \Delta S_f)}{S_w} \right) - \left( \frac{1}{b} \sin \phi \right) \left( 3.48 \frac{\Delta S_f, a}{S_w} \right)
\]

where

\( C_1 \)  
rolling-moment coefficient

\( C_{1\beta}(S_w) = K_1 \Gamma \)

\( K_1 \)  
varies with aspect ratio; it is equal to -0.0175 for \( A = 16 \), -0.0141 for \( A = 8 \), and -0.0114 for \( A = 4 \). The values of \( K_1 \) were obtained from data given in reference 6 for a wing of a 2:1 taper ratio.

\( \Gamma \)  
effective dihedral angle of wing, degrees

Effective dihedral angle is used throughout this report as a fictitious angle that would give the wing the value of \( C_{1\beta} \). Wing plan form and elevation, as well as large interference effects, contribute various amounts of \( C_{1\beta} \). (See references 3, 5, and 7.)

The contribution of the fin to the value of \( C_{1\beta} \) is

\[
\left( \frac{x}{b} - \frac{1}{b} \sin \phi \right) \left( 3.48 \frac{(S_f + \Delta S_f)}{S_w} \right)
\]
where \((\frac{\overline{X}}{b} - \frac{h}{b} \sin i)\) is the assumed ratio to the wing span of the vertical distance of the center of pressure on the fin above the center of gravity. The value of \(\overline{X}\) was assumed to be the distance from the fuselage center line to the center of pressure of the fin. Upon the assumption that the distance from the top of the fuselage to the center of pressure of the fin varied with the square root of the fin area and by the use of data from reference 3, the relation \(\frac{X}{b} = 0.025 + 0.23 \sqrt{(S_f + \Delta S_f)/S_w}\) was obtained and used for all variations of the fin. The value of \(i\), the angle of fuselage center line to the horizontal, was zero for all cases when the lift coefficient \(C_L\) was equal to 0.2. For other values of \(C_L\), the value of \(i\) depended upon both \(C_L\) and \(A\). The value of \(i\) for \(C_L = 2.8\) was the same as for \(C_L = 1.4\). The increase in \(C_L\) was obtained by merely deflecting the flaps with no change in the angle of attack. The term \(3.48 \frac{(S_f + \Delta S_f)}{S_w}\) was used in the expression for \(C \beta (S_f + \Delta S_f)\).

The contribution of the side area added ahead of the center of gravity is

\[
\frac{1}{b} \sin i \left( \frac{3.48 \Delta S_f a}{S_w} \right)
\]

which is similar to that of \(S_f + \Delta S_f\) except that the center of pressure of \(\Delta S_f a\) was assumed to be on the fuselage center line.

\[
C_{n\beta} = C_{n\beta} \text{(fuselage)} + C_{n\beta}(S_w) + C_{n\beta}(S_f + \Delta S_f) + C_{n\beta}(\Delta S_f a)
\]

\[
= \frac{-0.204}{\sqrt{A}} + K_a + \frac{h}{b} \left( 3.48 \frac{S_f + \Delta S_f}{S_w} \right) - \frac{1}{b} \left( 3.48 \frac{\Delta S_f a}{S_w} \right)
\]

where \(C_n\) is the yawing-moment coefficient.

A value of \(K_a\) of 0.009 was used for plain wings and of 0.030 when the flaps were deflected. Values for \(K_a\) may vary considerably with wing forms. (See references 5 and 7.)
The value \( C_{n\beta}^{(\text{fuselage})} = -0.204/\sqrt{A} \) was obtained from data in reference 3. The change from the expression given in reference 1 was necessary because for this investigation the fuselage and the wing area are constant.

The contribution of the fin and the side area to the value of \( C_{n\beta} \) is the computed variation of the normal force with \( \beta \) on the fins \( C_{Y\beta}(S_f + \Delta S_f) \) and \( C_{Y\beta}(\Delta S_{fa}) \) times the nondimensional lever arms \( l_t/b \) and \( l_a/b \), respectively.

\[
C_{l_p} = C_{l_p}(S_w) = K_3 \tag{4}
\]

where the values of \( K_3 \) (from reference 6) are -0.6 for \( A = 16 \), -0.5 for \( A = 8 \), and -0.4 for \( A = 4 \). The values of \( C_{l_p}(S_f + \Delta S_f) \) and \( C_{l_p}(\text{fuselage}) \) are probably very small as compared with \( C_{l_p}(\text{wing}) \) and therefore have not been used.

\[
C_{n_p} = C_{n_p}(S_w) + C_{n_p}(S_f + \Delta S_f) + C_{n_p}(\Delta S_{fa})
\]

\[
= K_4 C_L + 2\frac{l_t}{b} \left( \frac{\bar{X}}{b} - \frac{l_t}{b} \sin i \right) \left( 3.48 \frac{S_f + \Delta S_f}{S_w} \right) - 2\frac{l_a}{b} \left( \frac{l_a}{b} \sin i \right) \left( 3.48 \frac{\Delta S_{fa}}{S_w} \right) \tag{5}
\]

where \( K_4 \) varies with the aspect ratio and is (from reference 6) -0.089 for \( A = 16 \), -0.065 for \( A = 8 \), and -0.040 for \( A = 4 \).

The expression for \( C_{n_p}(S_f + \Delta S_f) \), that is,

\[
2\frac{l_t}{b} \left( \frac{\bar{X}}{b} - \frac{l_t}{b} \sin i \right) \left( 3.48 \frac{S_f + \Delta S_f}{S_w} \right)
\]

is the rate of change of the yawing moment with the rolling
velocity, produced by assuming that the normal force on the fin is proportional to the angle induced at the center of pressure of the fin by the rolling velocity; actually, the induced angle is a variable along the fin.

The expression for $C_{n_p}(\Delta S_{f_a})$ is the same as for $C_{n_p}(S_f+\Delta S_f)$ with the center of $\Delta S_{f_a}$ on the fuselage center line.

$$C_{n_r} = C_{n_r}(S_w) + C_{n_r}(S_f+\Delta S_f) + C_{n_r}(\Delta S_{f_a})$$

$$= 0.25 C_L + 2 \frac{l_t}{b} \left( \frac{x}{b} - \frac{l_t}{b} \sin i \right) \left( 3.48 \frac{S_f+\Delta S_f}{S_w} \right)$$

$$- 2 \frac{l_a}{b} \left( \frac{l_a}{b} \sin i \right) \left( 3.48 \frac{\Delta S_{f_a}}{S_w} \right)$$

(6)

where the value of 0.25 was obtained from data given in reference 6, and the terms for $C_{n_r}(S_f+\Delta S_f)$ and $C_{n_r}(\Delta S_{f_a})$ are of the same form as for $C_{n_p}(S_f+\Delta S_f)$ and $C_{n_p}(\Delta S_{f_a})$; but, in this case, the angle is induced by the yawing velocity and the values are probably more nearly representative of the actual values of the derivative than the values given for $C_{n_p}(S_f+\Delta S_f)$ and $C_{n_p}(\Delta S_{f_a})$.

$$C_{n_r} = C_{n_r}(\text{fuselage}) + C_{n_r}(S_w) + C_{n_r}(S_f+\Delta S_f) + C_{n_r}(\Delta S_{f_a})$$

$$= -0.0242 \frac{K_s C_L^2 - K_s}{A} - 2 \left( \frac{l_t}{b} \right)^2 \left( 3.48 \frac{S_f+\Delta S_f}{S_w} \right)$$

$$- 2 \left( \frac{l_a}{b} \right)^2 \left( 3.48 \frac{\Delta S_{f_a}}{S_w} \right)$$

(7)

where $K_s$ (from reference 6) is -0.0113 for $A = 16$, -0.02105 for $A = 8$, and -0.03838 for $A = 4$. The constant $K_s$ depends on the profile-drag coefficient of the
wing (reference 6) and is assumed to be 0.003 for the plain wing and 0.030 for the wing with flaps deflected. The value $-0.0242/\sqrt{A}$ was computed from the relations given in reference 1 for the fuselage. This form is used in the present investigation because the ratio of wing area to fuselage size is constant and $C_{n_f}(\text{fuselage})$ varies inversely with span.

The expressions for $C_{n_r}(S_f+\Delta S_f)$ and $C_{n_r}(\Delta S_{fa})$ are:

$$2\left(\frac{l_t}{b}\right)^2 \left(3.48 \frac{S_f+\Delta S_f}{S_w}\right)$$ and $$2\left(\frac{l_a}{b}\right)^3 \left(3.48 \frac{\Delta S_{fa}}{S_w}\right)$$

are the nondimensional forms of the rate of change of yawing momcnt due to the fin with yawing velocity.

**CALCULATIONS**

The boundaries of neutral spiral and oscillatory stability were computed by use of the lateral-stability equations from the theory of small oscillations as given in reference 8. Lateral stability depends upon the values and the algebraic sign of the term $E$ of the stability equation and Routh's discriminant. When the value of $E$ becomes negative, the airplane becomes spirally unstable. The lateral oscillations increase in amplitude when the value of Routh's discriminant becomes negative. The limits of the stable region are therefore defined by the values of $\Gamma$ and $S_f/S_w$ when $E = 0$ and Routh's discriminant = 0.

**RESULTS AND DISCUSSION**

The results of this investigation are presented in the form of diagrams, figures 1 to 4, showing the variations in the computed boundaries of spiral and oscillatory stability with $S_f/S_w$, the ratio of fin area to wing area, and with $\Gamma$, the effective dihedral angle. The values of $S_f/S_w$ where $C_{n_B}$ is zero and 0.05 and the values of $C_{r_B}$ when $\Gamma$ is $10^6$ are indicated on the figures. The weathercock stability is neutral where $C_{n_B}$ is zero.
The results, in general, indicate that the value of $\Gamma$ required for spiral stability increases with $C_L$ and $S_f/S_w$. The value of $S_f/S_w$ required for oscillatory stability decreases with $\Gamma$ for several degrees and then increases with continued increase in $\Gamma$. For some cases the diagrams do not cover a sufficient range of $\Gamma$ to show the increase in $S_f/S_w$. The rate of change of $S_f/S_w$ with $\Gamma$ required for oscillatory stability increases with $C_L$.

The stability boundaries for $C_L = 2.8$ may appear to be inconsistent with the other lift coefficients. The differences, however, result from the changes in $C_L$, $C_{n\alpha}$, and $C_{nT}$ produced by the flaps and from the assumption that the angle of attack is the same as for the condition with no flaps, that is, $C_L = 1.4$.

Effect of changes in aspect ratio on stability boundaries. – For spiral stability, increasing $A$ increases the minimum permissible value of $\Gamma$ and decreases the allowable value of $S_f/S_w$. (See fig. 1.) For oscillatory stability, increasing $A$ decreases the value of $S_f/S_w$ required for the normal range of values of $\Gamma$, but for very large values of $\Gamma$ at the high lift coefficients, this effect may be reversed. The resulting effect of increasing aspect ratio on lateral stability is to reduce slightly the required value of $S_f/S_w$ and to increase the required value of $\Gamma$.

Effect of changes in side area on stability boundaries. – Adding side area $S_{fa}$ as assumed in this investigation increases the damping coefficients $C_{Y\beta}$ and $C_{n_T}$ and has no effect on the weathercock-stability coefficient $C_{n\alpha}$. Increasing $S_{fa}/S_w$ decreases the minimum permissible value of $\Gamma$ for spiral stability and allows larger values of $S_f/S_w$ to be used. (See fig. 2.) For oscillatory stability, increasing $S_{fa}/S_w$ reduces the required value of $S_f/S_w$.

The resulting effect of increasing $S_{fa}/S_w$ is to allow much greater variations in the value of $S_f/S_w$ that can be used. Another variation of side area has been re-
ported in reference 1, that is, the case of changes in tail length. Increasing the tail length through the normal range and decreasing $S_f/S_w$ so that $C_{nB}$ remains constant produces only minor changes in the stability boundaries. Decreasing $S_f/S_w$ and increasing the tail length increases $C_{nT}$ but decreases the side area and therefore $C_{yB}$. Adding the side area increases both $C_{yB}$ and $C_{nT}$. It appears that, in critical cases, it would be best to add side area as far from the center of gravity as possible.

**Effect of changes in flight path on the stability boundaries.**—A change in the flight path from a dive to a climb greatly reduces the allowable range of $S_f/S_w$ required for spiral stability. (See fig. 3.)

For the range of values of $\Gamma$ normally used, flight path has very little effect on the oscillatory stability boundaries. At the high lift coefficients and large values of $\Gamma$, changing the flight path from a dive to a climb slightly reduces the possibility of oscillatory instability.

The resulting effect of changing the flight path is that an airplane will have much less spiral stability and may have slightly more oscillatory stability in a climb than it will have in a dive. This result indicates that, for satisfactory stability throughout the flight range, an airplane should have more lateral stability than is indicated by results for level flight.

As stated before, the computations do not include the direct effects of the propeller nor of the slipstream. The large change in power required between a glide and a climbing condition is known to have a large effect on the stability derivatives of some airplanes.

**Effect of changes in normal acceleration on lateral stability.**—Normal acceleration has no effect on the spiral stability boundary, because it enters into the $E$ term of the equations of motion as a multiplying factor. (See reference 8.) It increases the magnitude of the value of $E$ with respect to the other terms, however, and the spiral motion will therefore be affected.
Increasing the normal acceleration increases the value of \( S_f/S_w \) required and decreases the maximum value of \( \Gamma \) allowable for oscillatory stability. (See fig. 4.)

These results indicate that an airplane which is normally stable may be unstable when subjected to normal accelerations. The computations were based on the assumption that the airplane is in straight level flight and is subjected to a continuous normal acceleration. This flight condition is a physical impossibility. It is believed, however, that these flight conditions may be approached for short intervals during pull-ups, in banked turns, and in gusts.

It was thought that possibly, in practice, the period of the unstable motion of the airplane might be too long to be of any particular importance because of the short time that the acceleration would exist. Accordingly, computations were made for the motions in roll \( \phi \), in sideslip \( \beta \), and in heading \( \Psi \) with unit rolling moment and with unit yawing moment, each applied independently with normal accelerations of \( 1g \) and \( 6g \). The motions were obtained by solving the equations of motion. The assumed characteristics of the airplane are the same as were used in obtaining the stability boundaries. A value for \( S_f/S_w \) of 0.05 and a value for \( \Gamma \) of \( 6^\circ \) was chosen for the computations in order that the oscillatory motion would be stable for \( 1g \) acceleration and unstable for \( 6g \) acceleration. (See fig. 4(c).)

The resulting motions of the airplane are given in figures 5 to 7. In figures 5 to 7 the motions are given as time in seconds to obtain the angle of bank \( \phi \), the angle of sideslip \( \beta \), and the angular heading \( \Psi \) in radians for unit applied rolling moment or unit applied yawing moment.

The motions for \( 6g \) acceleration were computed for a value of \( C_L \) of 1.4. The motions were also computed for \( 1g \) acceleration at values of \( C_L \) of 1.4 and 0.233 for comparisons at the same attitude and at the same airspeed. In order to obtain an acceleration of \( 6g \), it is necessary for this assumed airplane to travel at a speed of 328.7 feet per second as compared with 134.2 feet per second for \( 1g \) at a value of \( C_L \) of 1.4.

Figures 5 and 6 show large oscillations in roll and sideslip within 3 seconds for the condition with \( 6g \) acceleration although, in this same time interval, there is
no evidence of oscillation with $l_g$ acceleration for the same disturbance. These oscillations are likely to confuse the pilot as well as to allow the airplane to attain attitudes in which the aerodynamic characteristics and the stalling characteristics are greatly different from those in the normal flight range.

**General comments.** The results of this investigation are intended to indicate general effects of changes in certain airplane parameters and flight conditions and are not intended to indicate the stability characteristics of any particular airplane. The general results of the investigation may be summed up by pointing out the factors that make the attainment of lateral stability more critical and difficult, and the conditions of flight in which lateral instability is most likely to be encountered:

1. Small amounts of side area for equal values of $C_{n\phi}$
2. A climbing attitude for spiral instability and a dive for oscillatory instability
3. Any maneuver in which the normal acceleration is increased

Other factors, such as the type and degree of stability desired and the choice of aerodynamic characteristics used, have a large effect on the interpretation of these results.

Although the type and the degree of stability that affects the control and the riding qualities of the airplane are outside the scope of this report, some of the following factors should be mentioned: It is normally considered that, for satisfactory stability characteristics, the oscillatory motion should be highly damped, that is, the airplane should have a large amount of oscillatory stability. This stability can be obtained by proportioning the effective fin area and the effective dihedral angle in order that their values, when plotted on a diagram such as figure 1 for a particular airplane, will be in a stable area and well away from the zero oscillatory stability boundary of the airplane. The distance from the boundary, although an indication, is not a quantitative measure of the amount of damping. For good riding qualities in rough air, the amount of spiral stability should be small; some stability is considered desirable, particularly for flying conditions of poor visibility, although spiral instability is generally considered to be preferable to a
poorly damped oscillatory motion. The value of $C_{n\beta}$ should be positive, giving weathercock stability; the value, however, is dependent upon the spiral and oscillatory stability requirements. These concepts are further substantiated by the experimental data of reference 9 in which are reported the results of a free-flight-tunnel investigation in which the directional stability and dihedral were varied over a wide range. In another free-flight-tunnel investigation (reference 10) it was found that although lateral stability was easier to obtain with larger values of the side-force derivative $C_{Y\beta}$, the lateral control characteristics were considerably impaired and in certain configurations increasing $C_{Y\beta}$ gave poorer over-all lateral flying characteristics.

CONCLUSIONS

From the analysis for spiral and oscillatory lateral stability in which assumed data are used for changes in certain airplane parameters, the following conclusions may be drawn:

1. An increase in the aspect ratio of the wing of an airplane requires a smaller fin area but makes little difference in the attainment of lateral stability.

2. Adding side area makes the attainment of lateral stability considerably less difficult.

3. An airplane is more likely to be spirally unstable and slightly less likely to have oscillatory instability in a climbing attitude than in a gliding attitude.

4. Normal accelerations are likely to cause oscillatory instability and the resulting notions may be confusing to the pilot.

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REFERENCES


Effective dihedral angle, $T$, deg
(a) $C_l = 0.2$.

Figure 1. — Continued.

Effect of aspect ratio $A$ on boundaries of oscillatory and spiral stability. $k_x/b_0 = 0.125; k_y/b_0 = 0.175; W/5 = 30$.

Figure 2. — Effect of aspect ratio $A$ on oscillatory instability. $T = 0$. $C_l = 0$.
Figure 1. — Continued.

(c) $C_L 1.4$.

(d) $C_L 2.8$.

Figure 1. — Concluded.
Figure 2.- Effect of additional side area \( S_{fw} \) on boundaries of oscillatory and spiral stability. \( A, B \); \( k_x/b, 0.125; k_y/b, 0.175; \mu, 9.70; l_1/b, 0.4 \).

Figure 2.- Continued.
Figure 2.— Continued.

Figure 2.— Concluded.
Figure 3. - Effect of flight path angle $\gamma$ on boundaries of oscillatory and spiral stability. $A_0, B_0, x/b, 0.125; k_2/b_0, 0.125; n_2, 0.40; y/b, \pm 0.2; W/s, 30.$

Figure 3. - Continued.
Effective dihedral angle, $\Gamma$, deg

Figure 3. - Continued.
Figure 4. Effect of normal accelerations on boundaries of oscillatory and spiral stability. $C_L, K_x, \beta_1, 0.125; k_2, \beta_2, 0.175; K_y, x, 70; \gamma, \beta, 0.04$.

(b) $C_L, 0.8$.

Figure 4—Continued.
Effective dihedral angle, $\Gamma$, deg

(C) $C_{L}=1.4$.

Figure 4.- Continued.

Effective dihedral angle, $\Gamma$, deg

(D) $C_{L}=2.8$.

Figure 4.- Concluded.
Figure 5.— Effect of normal acceleration on change in angle of bank $\phi$ in radians with unit rolling moment $C_r=1$ and unit yawing moment $C_n=1$.

$C_x/b_0.125$ $C_z/b_0.175$; $\mu_9.70$;

$\text{wing}, 40 \text{ ft}$; $\Gamma^\circ$; $S_f/S_W0.05$.

Figure 6.— Effect of normal acceleration on change in angle of sideslip $\beta$ in radians with unit rolling moment $C_r=1$ and with unit yawing moment $C_n=1$.

$C_x/b_0.125$ $C_z/b_0.175$; $\mu_9.70$; $\text{span}, 40 \text{ ft}$; $\Gamma^\circ$; $S_f/S_W0.05$.
Figure 7.— Effect of normal acceleration on change in angle of heading $\psi$ in radians with unit rolling moment $C_l = 1$ and unit yawing moment $C_n = 1$, $k_x/b = 0.125$; $k_y/b = 0.175$; $\mu = 9.70$; span, 40 feet; $\Gamma$, $S^3$; $S_f/S_w$, 0.05.
ABSTRACT:

Lateral stability calculations were made for a hypothetical fighter which are applicable to any type of fighter characterized by parameters and data employed. Diagrams indicate that effect of wing aspect ratio is small and that increase in side area is advantageous to extent of easier selection of fin area and dihedral angle. Results are given graphically with dihedral angle plotted against fin area for neutral spiral and oscillatory divergence.
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Effect of Change in Aspect Ratio, Side Area, Flight Path Angle, and Normal Acceleration on Lateral Stability

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