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Pinkel, B.  
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Power Plants, Jet and Turbine (5)  
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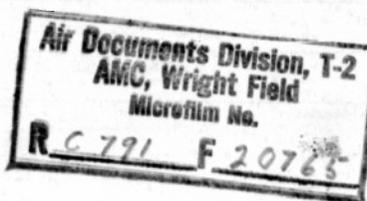
Performance charts for a jet-propulsion system consisting of a compressor, a combustion chamber, and a turbine

National Advisory Committee for Aeronautics, Washington, D. C.

U.S. Eng. Restr. Jun'48 56 diagr, graphs

Charts are presented for computing thrust, fuel consumption, and other performance values of a jet-propulsion system consisting of a compressor, combustion chamber, turbine, and exhaust nozzle. Conclusions of the analysis show that maximum thrust per unit mass rate of air flow occurs at a lower compressor pressure ratio than minimum specific fuel consumption, and that increase in combustion chamber discharge temperature causes an increase in thrust. However, an optimum chamber discharge temperature exists at which minimum specific fuel consumption is obtained and this temperature is sometimes less than the limits imposed by the strength-temperature characteristics of normally used materials.

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ADVANCE RESTRICTED REPORT

PERFORMANCE CHARTS FOR A JET-PROPULSION SYSTEM CONSISTING OF A  
COMPRESSOR, A COMBUSTION CHAMBER, AND A TURBINE

By Benjamin Pinkel and Irving M. Karp

Aircraft Engine Research Laboratory  
Cleveland, Ohio



Washington  
June 1946

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PERFORMANCE CHARTS FOR A JET-PROPULSION SYSTEM CONSISTING OF A  
COMPRESSOR, A COMBUSTION CHAMBER, AND A TURBINE

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SUMMARY

Convenient charts are presented for computing the thrust, fuel consumption, and other performance values of a jet-propulsion system consisting of a compressor, a combustion chamber, a turbine, and a discharge nozzle. These charts take into account the effects of ram pressure, compressor pressure ratio, ratio of combustion-chamber outlet temperature to atmospheric temperature, compressor efficiency, turbine efficiency, combustion efficiency, discharge-nozzle coefficient, losses in total pressure in the inlet to the jet-propulsion unit and in the combustion chamber, and variation in specific heats with temperature. The principal performance charts show clearly the effects of the primary variables and correction charts provide the effects of the secondary variables.

The performance of illustrative cases of jet-propulsion systems of this type is given. It is shown that maximum thrust per unit mass rate of air flow occurs at a lower compressor pressure ratio than minimum specific fuel consumption. The thrust per unit mass rate of air flow increases as the combustion-chamber discharge temperature increases. For minimum specific fuel consumption, however, an optimum combustion-chamber discharge temperature exists, which in some cases may be less than the limiting temperature imposed by the strength temperature characteristics of present materials.

INTRODUCTION

The jet-propulsion system consisting of a compressor, a combustion chamber, a turbine, and a discharge nozzle, which is generally known as the turbojet, is now under extensive development for the propulsion of high-speed airplanes. A simplified study of this jet-propulsion system in which general trends are shown has been presented in references 1 and 2.

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An analysis was made of the performance of such a system at the NACA Cleveland laboratory during 1944 for the purpose of providing convenient charts from which the performance of this system can be quickly and accurately obtained for any given set of operating conditions and system parameters. An attempt was made to predict accurate values of actual performance by the introduction of factors that account for the change in physical properties of the gas as it passes through the cycle and the effect of the change in mass by the addition of fuel. The charts take into account turbine efficiency, compressor efficiency, combustion efficiency, discharge-nozzle coefficient, losses in total pressure in the inlet duct and combustion chamber, ambient atmospheric conditions, flight velocity, compressor pressure ratio, and combustion-chamber outlet total temperature. These variables are grouped in a few simple charts from which their effects on performance can be readily obtained. The charts and the analysis are presented herein.

The performance of the subject jet-propulsion system is given for several interesting cases to illustrate some of the characteristics of the system.

#### ANALYSIS

A diagram of the jet-propulsion device under discussion is shown in figure 1. Air is inducted into the intake of the unit and delivered to the compressor inlet. Part of the dynamic pressure of the free air stream is converted into static pressure at the compressor inlet by the diffusing action of the inlet duct. The air is further compressed in passing through the compressor and is delivered to the combustion chamber where fuel is injected and burned. The products of combustion then pass through the turbine nozzles and buckets where an appreciable drop in pressure occurs and finally are discharged rearwardly through the discharge nozzle to provide thrust.

The variables affecting the performance are divided into a primary group and a secondary group. The variables of the primary group are shown on the principal charts for determining the performance of the jet-propulsion unit. The variables of the secondary group are shown on an auxiliary chart for determining a factor  $\epsilon$  usually close to unity, which also appears as a variable on the principal performance charts.

The primary group of variables includes:

- (a) Compressor efficiency  $\eta_c$
- (b) Compressor total-pressure ratio  $P_2/P_1$

- (c) Burner efficiency  $\eta_f$
- (d) Ratio of combustion-chamber outlet total temperature to free atmospheric temperature  $T_4/T_0$
- (e) Turbine efficiency  $\eta_t$
- (f) Airplane velocity  $V_0$
- (g) Atmospheric temperature  $T_0$
- (h) Discharge-nozzle velocity coefficient  $C_v$ , which includes losses in the tail pipe following the turbine

The secondary group includes:

- (a) Drop in total pressure across the inlet ducting caused by friction and turbulence  $\Delta p_d$
- (b) Drop in total pressure across the combustion chamber caused by both the mechanical obstruction of the burners and the momentum increase of the gases during combustion  $\Delta p_{(2-4)}$
- (c) Effect of the difference between the physical properties of hot exhaust gases during the expansion processes and cold air (The effect of change in specific heat of the gas during the other processes is included in the principal charts.)

A chart is given from which a factor  $\epsilon$  can be obtained corresponding to the values of the secondary group of variables. This factor  $\epsilon$  appears in the parameters on the principal performance charts.

The compressor efficiency  $\eta_c$  in this report is defined as the isentropic work done in the compressor, including the difference between the kinetic energy of the air at the compressor outlet and at the compressor inlet, divided by the compressor shaft work. The turbine efficiency  $\eta_t$  as defined in this report is the shaft work divided by the difference between the isentropic work available in expanding the gas from turbine inlet conditions to the static pressure at turbine discharge and the kinetic energy of the gas at the turbine discharge. It is emphasized that, in these definitions of compressor and turbine efficiencies, the kinetic energy of the gas leaving the compressor or turbine is not charged against the respective unit as an energy loss.

The symbols used solely in the derivations of performance equations are listed in appendix A. The significance of the symbols appearing in the charts and in the subsequent discussion are as follows:

A	ratio of compressor pressure ratio $P_2/P_1$ to reference pressure ratio $(P_2/P_1)_{ref}$
a, b, c	factors that measure effects produced by secondary variables
$C_v$	velocity coefficient of discharge nozzle
F	net jet thrust, (lb)
f	fuel-air ratio
h	lower heating value of fuel, (Btu/lb)
J	mechanical equivalent of heat, 778 (ft-lb/Btu)
M	mass rate of air flow, (slug/sec)
$P_0$	atmospheric free-air static pressure, (lb/sq ft absolute)
$P_1$	total pressure at compressor inlet, (lb/sq ft absolute)
$P_2$	total pressure at compressor outlet, (lb/sq ft absolute)
$\Delta p_d$	drop in total pressure across inlet duct, (lb/sq ft)
$\Delta P(2-4)$	over-all drop in total pressure across combustion chamber due to mechanical obstruction of the burners and momentum increase of gases during combustion, (lb/sq ft)
$P_c$	compressor-shaft horsepower input
$T_0$	atmospheric temperature, ( $^{\circ}R$ )
$T_1$	compressor-inlet total temperature, ( $^{\circ}R$ )
$T_2$	compressor-outlet total temperature, ( $^{\circ}R$ )
$T_4$	combustion-chamber outlet total temperature, ( $^{\circ}R$ )
$V_0$	airplane velocity, (ft/sec)

$V_5$	gas velocity at turbine discharge, (ft/sec)
$V_j$	jet velocity, (ft/sec)
$\Delta V_j$	increase in jet velocity due to effect of turbine-loss reheat, (ft/sec)
$W_f$	weight flow of fuel, (lb/hr)
$Y$	ratio of ram temperature rise to free-air atmospheric temperature, $V_0^2/2 J c_{pa} T_0$
$Z$	ratio of compressor power per unit mass rate of air flow to enthalpy of air at temperature $T_0$ , $550 P_c/J c_{pa} M T_0$
$\gamma_a$	ratio of specific heats of air
$\epsilon$	correction factor that accounts for over-all effects produced by secondary variables
$\eta_c$	compressor efficiency
$\eta_f$	efficiency of combustion of fuel in combustion chamber
$\eta_t$	turbine efficiency

$$(P_2/P_1)_{\text{ref}} = \left[ \left( \frac{1}{1+Y} \right)^2 \eta_c \eta_t \epsilon \frac{T_4}{T_0} \right]^{\frac{\gamma_a}{2(\gamma_a-1)}}$$

$(P_2/P_1)_{\text{ref}}$  also equal to the compressor pressure ratio for maximum thrust per unit mass rate of air flow when the rate of change of  $\epsilon$  with compressor pressure ratio is negligible.

All velocities are axial and all except  $V_0$  are relative to the unit.

The equations from which the charts are prepared are listed in appendix B and are derived in appendix C.

In some cases, when a large pressure drop occurs across the final jet-discharge nozzle, reheat associated with the energy losses in the turbine has an appreciable effect on the jet velocity. A chart is given whereby the effect of reheat on the jet velocity can be readily determined.

DISCUSSION OF CHARTS

Useful equations. - The net thrust of the jet-propulsion device, when the effect of the fuel weight is neglected, is given by the equation

$$F = M (V_j - V_o) \quad (1a)$$

When the effect of fuel weight is included, the thrust is given by

$$F = M (V_j - V_o) + f M V_j \quad (1b)$$

The net thrust horsepower thp is given by

$$\text{thp} = F V_o / 550 \quad (2)$$

The compressor-shaft horsepower per slug per second of air is expressed as

$$\left. \begin{aligned} P_c/M &= J c_{pa} T_o Z / 550 \\ &= 5675 Z (T_o/519) \end{aligned} \right\} \quad (3)$$

The compressor-inlet total temperature is obtained from

$$T_1/T_o = 1 + Y \quad (4)$$

The fuel consumption per unit mass rate of air flow is given in terms of the fuel-air ratio by the following relation

$$W_f/M = 115,920 f \quad (5)$$

By means of equations (1) to (5) and the curves of figures 2 to 7 the performance of the turbojet engine and some associated quantities of interest can be readily determined. The curves are given in a form which shows the effects of the important variables and enables either very accurate computations or rapid but less accurate computations to be made.

Curves for obtaining the flight Mach number, the value of  $Y$ , and the compressor-inlet total pressure for various values of the factor  $V_o \sqrt{519/T_o}$  are shown in figure 2. The compressor-inlet total temperature is obtained from the value of  $Y$  and equation (4).

The quantity  $\eta_c Z$  is plotted against the compressor total-pressure ratio and  $Y$  in figure 3. The compressor power (and hence the turbine power) is computed from equation (3) and the value of  $Z$ .

The effect of the variation in the specific heat of air during compression is neglected in this plot, the error introduced being less than 1 percent for the range of compressor pressure ratios shown in figure 3 and for compressor inlet temperatures up to 550° R.

The value of  $(p_2/p_1)_{ref}$  plotted against the factor  $\eta_c \eta_t \epsilon \frac{T_4}{T_0} \left( \frac{1}{1+\gamma} \right)^2$  is also given in figure 3. The actual compressor pressure ratio  $p_2/p_1$  divided by the quantity  $(p_2/p_1)_{ref}$  defines the value of the factor A used in figure 4(a). This quantity  $(p_2/p_1)_{ref}$  is useful in that it is equal to the compressor pressure ratio for maximum thrust per unit mass rate of air flow for any

given value of  $\eta_c \eta_t \epsilon \frac{T_4}{T_0} \left( \frac{1}{1+\gamma} \right)^2$ , if the rate of change of the factor  $\epsilon$  with respect to a change in pressure ratio is negligible. The factor  $\epsilon$  is one which accounts for the effects of pressure losses in the inlet duct to the system, pressure drop in the combustion chamber, and the deviation from the value of the specific heat of air at 519° R of the specific heats of the gases during the expansion through the turbine and the nozzle. In a well designed system the value of  $\epsilon$  is close to or slightly greater than unity and does not vary appreciably with  $p_2/p_1$ .

When the change in  $\epsilon$  with  $p_2/p_1$  is appreciable, then  $(p_2/p_1)_{ref}$  is less than the compressor pressure ratio giving maximum thrust per unit mass rate of air flow; however, even in this case the thrust per unit mass rate of air flow corresponding to  $(p_2/p_1)_{ref}$  is generally within 1 percent of the true maximum. Hence figure 3 permits a rapid approximation of the pressure ratio for maximum thrust per unit mass rate of air flow.

From the left-hand set of curves of figure 4(a), the jet-velocity factor  $V_j \sqrt{\frac{\eta_c \eta_t}{C_v^2}} \sqrt{\frac{519}{T_0}}$  can be determined as a function of  $\eta_c \eta_t \epsilon \frac{T_4}{T_0}$  and the parameter A or 1/A for zero flight speed. (When A is less than unity, the value of 1/A is used in reading values from fig. 4(a).) The jet-velocity factor can be obtained for airplane velocities other than zero by moving horizontally across the graph to the desired velocity curve on the right-hand set of curves and then reading the value on the lower abscissa. The thrust can then be computed from the value of  $V_j$  and equation (1a). As previously mentioned, the value of A is found by dividing the compressor

pressure ratio  $p_2/p_1$  by the value of  $(p_2/p_1)_{ref}$  obtained from figure 3 corresponding to the values of the parameters  $\eta_c$ ,  $\eta_t$ ,  $\epsilon$ ,  $T_4$ ,  $T_o$ , and  $Y$  being investigated.

It is noted in figure 4(a) that for given values of  $\eta_c$ ,  $\eta_t$ ,  $T_4$ , and  $T_o$ , if  $\epsilon$  remains constant as  $p_2/p_1$  or  $A$  varies, then the variation of jet velocity with pressure ratio occurs along the

constant  $\eta_c \eta_t \epsilon \frac{T_4}{T_o}$  line. In this case,  $V_j$  has a maximum value when  $A$  is equal to unity, which occurs at a pressure ratio equal to  $(p_2/p_1)_{ref}$ . Actually, however, for a given unit as  $p_2/p_1$  varies, the value of  $\epsilon$  changes slightly and hence  $\eta_c \eta_t \epsilon \frac{T_4}{T_o}$  changes, with the result that  $V_j$  has a maximum value for a value of  $p_2/p_1$  somewhat greater than  $(p_2/p_1)_{ref}$ . It should also be noted that  $(p_2/p_1)_{ref}$  is changed by the change in  $\epsilon$  and this new value must be used in computing the new value of  $A$  when  $p_2/p_1$  is varied. In any event, the value of  $V_j$  corresponding to  $A = 1$  is a close approximation to the jet velocity for maximum thrust per unit mass rate of air flow  $M$  for a given set of values of  $T_4$ ,  $T_o$ , and component efficiencies.

The losses in kinetic energy in the turbine passages appear as heat energy in the gas leaving the turbine. This energy will be termed "turbine-loss reheat." If there is further expansion of the gas in passing through the jet nozzle (caused by a reduction in static pressure in passing from the turbine exit to the jet-nozzle exit), a conversion of part of the turbine-loss reheat to kinetic energy occurs in the jet. If, however, the velocity at the turbine exit is substantially equal to the final jet velocity, no further expansion occurs and no kinetic energy is recovered from the turbine-loss reheat. The curves of figure 4(a) correspond to this case. The ratio of the increase in jet velocity to the final jet velocity  $\Delta V_j/V_j$  obtained when the velocity at the turbine discharge  $V_5$  is less than the final jet velocity is shown in figure 4(b).

Figure 4(b) shows that  $\Delta V_j/V_j = 0$  when  $C_v V_5/V_j = 1$  for all values of turbine efficiency. It is also noted that  $\Delta V_j/V_j$  approaches 0 as turbine efficiency approaches 1 for all values of  $C_v V_5/V_j$  because the turbine-loss reheat approaches 0 with increase in turbine efficiency.

It is evident from figure 4(b) that, for a given turbine efficiency, the smaller the ratio of  $C_v V_5/V_3$ , the greater is the recovery of turbine-loss reheat. Decrease in turbine-discharge velocity  $V_5$  is obtained by increase in annular area swept by the turbine buckets. Bucket stress is one of the principal limitations on bucket height and thus on bucket-annulus area.

The compressor-outlet total temperature  $T_2$  plotted against the factor  $T_0(1 + Y + Z)$  is shown in figure 5. This curve includes the variation in the specific heat of the air during compression and was computed using reference 3.

The fuel-air ratio factor  $\eta_p f$  is plotted in figure 6 against  $T_4 - T_2$  (the rise in total temperature in the combustion chamber) for various values of  $T_4$ . These curves were constructed using unpublished data based on the latest available information on specific heats of air and exhaust-gas mixtures and are for a fuel having a lower heating value of 18,900 Btu per pound and a hydrogen-carbon ratio of 0.185. For fuels having other values of  $h$ , the value of  $f$  given in figure 6 is corrected accurately by multiplying it by the factor  $18,900/h$ . The effect of the hydrogen-carbon ratio of the fuel on  $f$  is generally small and for a range of hydrogen-carbon ratios from 0.16 to 0.21 the error due to the deviation from the value of 0.185 is less than one-half of 1 percent. The fuel consumption per unit mass rate of air flow is obtained from the value of  $f$  and equation (5).

The value of  $\epsilon$ , which takes care of the effect of the secondary group of variables, is obtained from figure 7. The quantity  $\epsilon$  is given by the relation  $\epsilon = 1 - a - b + c$ , where  $a$ ,  $b$ , and  $c$  are given in figure 7. The effect of the drop in total pressure across the inlet duct  $\Delta p_d$  is shown in figure 7(a). The effect of the over-all drop in total pressure across the combustion chamber  $\Delta p(2-4)$  is introduced in figure 7(b). Reference 4, which discusses combustion in a chamber of constant flow area, is useful in evaluating the momentum-pressure drop in the combustion chamber. A correction for the difference between the physical properties of the hot gases and the cold air, involved in the computation of the expansion processes through the turbine and the jet nozzle is given in figure 7(c). Although  $\epsilon$  does not differ appreciably from unity, a change in  $\epsilon$  of 1 percent in some cases may introduce a change of several percent in the thrust.

In the discussion of the charts, the effect of the weight of injected fuel was not mentioned. It is shown in appendix C that the effect of the weight of fuel on the jet velocity can be taken

into account by using for the value of  $\eta_t$  in the charts the product of the actual turbine efficiency and  $(1 + f)$ . This term appears in the factor  $\eta_c \eta_t \epsilon \frac{T_4}{T_0} \left( \frac{1}{1 + Y} \right)^2$  in figure 3 used in finding  $(p_2/p_1)_{ref}$  and in the factors  $\eta_c \eta_t \epsilon \frac{T_4}{T_0}$  and  $V_j \sqrt{\eta_c \eta_t / C_v} \sqrt{519/T_0}$  of figure 4(a). The value of  $V_j$  determined is then used in equation (1b) which takes into account the additional weight of fuel introduced.

As an example of the use of these figures, consider a system having the following performance and operating parameters:

- |   |        |
|---|--------|
| 1. Compressor efficiency $\eta_c$ . . . . .   | 0.80   |
| 2. Turbine efficiency $\eta_t$ . . . . .  | 0.90   |
| 3. Combustion efficiency $\eta_f$ . . . . .   | 0.97   |
| 4. Discharge-nozzle velocity coefficient $C_v$ . . . . .  | 0.96   |
| 5. Airplane velocity $V_0$ , (ft/sec) . . . . .   | 733    |
| 6. Compressor total-pressure ratio $p_2/p_1$ . . . . .  | 6      |
| 7. Atmospheric free-air static pressure $p_0$ , (in. Hg) . . . . .                              | 29.9   |
| 8. Atmospheric temperature $T_0$ , ( $^{\circ}$ R) . . . . .                                    | 519    |
| 9. Combustion-chamber outlet total temperature $T_4$ , ( $^{\circ}$ R) . . . . .                | 1960   |
| 10. Drop in total pressure across inlet duct $\Delta p_{d1}$ , (in. Hg) . . . . .               | 0.5    |
| 11. Drop in total pressure across combustion chamber<br>$\Delta p_{(2-4)}$ , (in. Hg) . . . . . | 3      |
| 12. h, (Btu/lb) . . . . .   | 18,500 |

(a) Determination of Y and flight Mach number

From items 5 and 8

- |   |     |
|---|-----|
| 13. $V_0 \sqrt{\frac{519}{T_0}}$ , (ft/sec) . . . . . | 733 |
|---|-----|

From item 13 and figure 2

- |                                  |        |
|----------------------------------|--------|
| 14. Y . . . . .                  | 0.0861 |
| 15. Flight Mach number . . . . . | 0.656  |

(b) Determination of Z and compressor power

Using items 6 and 14, read on figure 3

- |                          |       |
|--------------------------|-------|
| 16. $\eta_c Z$ . . . . . | 0.726 |
|--------------------------|-------|

From items 16 and 1

17.  $Z$  . . . . . 0.908

Using items 17 and 8 in equation (3) the compressor power per unit mass rate of air flow is

18.  $P_c/M$ , (hp)/(slug/sec) . . . . . 5153

(c) Determination of fuel-air ratio and fuel consumption

From items 8, 14, and 17

19.  $T_o (1 + Y + Z)$ , ( $^{\circ}R$ ) . . . . . 1035

Using item 19 and figure 5

20.  $T_2$ , ( $^{\circ}R$ ) . . . . . 1025

From items 20 and 9

21.  $T_4 - T_2$ , ( $^{\circ}F$ ) . . . . . 935

From items 21 and 9 and figure 6

22.  $\eta_{pf}$  . . . . . 0.01372

Using items 22 and 3

23.  $f$  . . . . . 0.01414

Since the lower heating value of the fuel is equal to 18,500 Btu per pound (item 12), item 23 has to be multiplied by the factor  $\frac{18,900}{18,500}$  and the adjusted value is

24.  $f$  . . . . . 0.01445

From item 24 and equation (5)

25.  $W_f/M$ , (lb/hr)/(slug/sec) . . . . . 1675

(d) Determination of the factor  $\epsilon$

From items 7, 10, and 11

26.  $\Delta p_d/p_o$  . . . . . 0.017

27.  $\Delta p_{(2-4)}/p_o$  . . . . . 0.10

while from items 14 and 16

28.  $Y + \eta_c Z$  . . . . . 0.812

Using items 26, 28, and figure 7(a)

29.  $u$  . . . . . 0.006

Using items 27, 29, and figure 7(b)

30.  $b$  . . . . . 0.004

From figure 2 and item 13

31.  $\frac{P_1 + \Delta P_d}{P_0}$  . . . . . 1.335

From items 31, 26, and 6

32.  $\frac{F_2}{P_0}$  . . . . . 7.91

which when used with item 9 in figure 7(c) gives

33.  $c$  . . . . . 0.035

From items 29, 30, and 33

34.  $\epsilon = 1 - .006 - .004 + .035$  . . . . . 1.025

(e) Determination of  $(p_2/p_1)_{ref}$  and  $A$

Using items 1, 2, 34, 9, 8, and 14

35.  $\eta_c \eta_t \epsilon \frac{T_4}{T_0} \left( \frac{1}{1+Y} \right)^2$  . . . . . 2.363

From item 35 and figure 3

36.  $(P_2/P_1)_{ref}$  . . . . . 4.50

From items 6 and 36

37.  $A$  . . . . . 1.333

(f) Determination of jet velocity, net thrust per unit mass rate of air flow, and other performance quantities

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Using items 1, 2, 34, 9, and 8

38.  $\eta_c \eta_t \epsilon \frac{T_4}{T_0}$  . . . . . 2.787

From items 38, 37, 13, and figure 4(a) the jet-velocity factor is

39.  $V_j \sqrt{\frac{\eta_c \eta_t}{C_v}} \sqrt{\frac{519}{T_0}}$ , (ft/sec) . . . . . 1806

and from items 39, 1, 2, 4, and 8

40.  $V_j$ , (ft/sec) . . . . . 2044

The net thrust per unit mass rate of air flow is obtained from items 40, 5, and equation (1a)

41.  $F/M$ , (lb)/(slug/sec) . . . . . 1311

The thrust horsepower per unit mass rate of air flow is calculated from items 41, 5, and equation (2)

42.  $\text{thp}/M$ , (thp)/(slug/sec) . . . . . 1747

From items 25 and 41

43.  $W_f/F$ , (lb/hr)/(lb thrust) . . . . . 1.278

and from items 25 and 42

44.  $W_f/\text{thp}$ , (lb)/(thp-hr) . . . . . 0.959

(g) Effect of the weight of injected fuel and turbine-loss reheat on jet velocity and thrust

Where more accurate results are desired, the calculations are made taking into account the effect of the weight of fuel introduced and the effect of turbine-loss reheat. The effect of the fuel on jet velocity is handled by using for the value of  $\eta_t$  the product of the turbine efficiency and  $(1 + f)$ . This will now be done for the case just considered.

From items 24 and 35

45.  $\eta_c \eta_t \epsilon \frac{T_4}{T_0} \left( \frac{1}{1 + f} \right)^2$  . . . . . 2.596

From figure 3 the corresponding

46.  $(P_2/P_1)_{ref}$  . . . . . 4.61

From items 6 and 46

47. A . . . . . 1.30

Similarly accounting for fuel flow, item 38 becomes

48.  $\eta_c \eta_t \frac{T_4}{T_0}$  . . . . . 2.827

so that from items 47, 48, and 13, and figure 4(a)

49.  $V_j \sqrt{\frac{\eta_c \eta_t}{C_v}} \sqrt{\frac{519}{T_0}}$ , (ft/sec) . . . . . 1838

Again taking into account the effect of fuel by adjusting the  $\eta_t$  term

50.  $V_j$ , (ft/sec) . . . . . 2065

which differs from item 40 by 1 percent

The effect of reheat may be important when  $\eta_t$  is considerably less than unity and the velocity at turbine discharge is appreciably less than the final jet velocity. Let it be assumed in the example being discussed that the turbine is designed to have a discharge velocity of

51.  $V_5$ , (ft/sec) . . . . . 700

Then from items 4, 50, and 51

52.  $C_v V_5/V_j$  . . . . . 0.33

From items 8, 9, and 17

53.  $\frac{T_4}{T_0 Z}$  . . . . . 4.16

From figure 4(b) corresponding to items 2, 52, and 53

54.  $\Delta V_j/V_j$  . . . . . 0.012

and from items 50 and 54

55.  $\Delta V_j$ , (ft/sec) . . . . . 25

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Using items 55 and 50

56. Corrected  $V_j$ , (ft/sec) . . . . . 2090

Thus in this case, reheat provides an additional 1 percent increase in the value of  $V_j$ .

The thrust per unit mass rate of air flow is obtained from items 56, 5, and equation (1b)

57.  $F/M$ , (lb)/(slug/sec) . . . . . 1357

compared with 1311 where the effects of fuel and reheat were neglected.

From equation (2) and items 57 and 5

58. thp/M, (thp)/(slug/sec) . . . . . 1806

and using items 25 and 57

59.  $W_F/F$ , (lb/hr)/(lb) . . . . . 1.234

and items 25 and 58 give

60.  $W_F/\text{thp}$ , (lb/thp-hr) . . . . . 0.926

(h) Optimum thrust per unit mass flow of air

As previously pointed out, the value of  $V_j$  corresponding to  $(P_2/P_1)_{\text{ref}}$  is very close to the value of  $V_j$  giving maximum thrust per unit mass rate of air flow. The compressor pressure ratio  $P_2/P_1$  for maximum  $F/M$  is slightly greater than  $(P_2/P_1)_{\text{ref}}$  because of the increase in  $\epsilon$  with pressure ratio. The value of the maximum  $F/M$  and the corresponding value of  $P_2/P_1$  can be obtained by computing  $V_j$  for a range of values of  $P_2/P_1$  in the vicinity of and greater than  $(P_2/P_1)_{\text{ref}}$  by the method previously illustrated for a compressor pressure ratio of 6. From a plot of  $V_j$  against  $P_2/P_1$  the maximum value of  $V_j$  (and hence  $F/M$ ) and the corresponding value of  $P_2/P_1$  can be read. This computation for the previously illustrated case was made and the results are presented in the following table.

The effect of the weight of fuel and the turbine-loss reheat were neglected in calculating the values given in the table. Since

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item 36 gave a value for  $(p_2/p_1)_{ref}$  of 4.5, the range of compressor pressure ratios chosen started at this value. In the calculation of  $\epsilon$  the values of  $\Delta p_d$  and  $\Delta p(2-4)$  were assumed to remain constant at the values given in items 10 and 11, as  $p_2/p_1$  varied.

$\frac{p_2}{p_1}$	$\epsilon$	$\eta_c \eta_t \epsilon \frac{T_4}{T_0}$	$\left(\frac{p_2}{p_1}\right)_{ref}$	A	$V_j$ (ft/sec)	F/M (lb slug/sec)	$W_f/M$ (lb/hr slug/sec)	$W_f/F$ (lb/hr lb)
4.5	1.021	2.776	4.47	1.007	2057	1324	1820	1.375
4.6	1.021	2.776	4.47	1.029	2057	1324	1810	1.367
4.8	1.022	2.779	4.48	1.071	2058	1325	1791	1.352
5.0	1.022	2.779	4.48	1.116	2056	1323	1770	1.338
6.0	1.025	2.787	4.50	1.333	2044	1311	1675	1.278

The table shows the increase in  $\epsilon$  with increase in  $p_2/p_1$ .

This causes an increase in the value of  $\eta_c \eta_t \epsilon \frac{T_4}{T_0}$  and the corresponding value of  $(p_2/p_1)_{ref}$ . The percentage increase in A is slightly less than the percentage increase in  $p_2/p_1$  because of the increase in  $(p_2/p_1)_{ref}$ . The maximum value of F/M is 1325 as compared with a value of F/M of 1324 obtained at a compressor pressure ratio of 4.5 which was the  $(p_2/p_1)_{ref}$  for the previous example (see item 36). The values of  $V_j$  and F/M varied so slightly over the range of compressor pressure ratios from 4.5 to 5.0 that they were calculated using the formulas given in the appendixes rather than using the charts in order to detect the variation. It is noted that the true optimum occurs at a  $p_2/p_1$  of about 4.8 which is about 7 percent greater than the  $p_2/p_1$  of 4.5. If a maximum value of F/M is the main design consideration, it is doubtful that the additional complication to obtain the higher compressor pressure ratio is warranted by the small increase in F/M obtained. When fuel consumption is also an important consideration, the increase in compressor pressure ratio may be desirable as indicated by the values of  $W_f/F$  in the table.

JET-PROPULSION-UNIT PERFORMANCE

For the illustration of the performance and some of the characteristics of the subject jet-propulsion system, several cases of interest will be discussed.

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The following parameters are assumed:

Compressor efficiency $\eta_c$ . . . . .	0.85
Turbine efficiency $\eta_t$ . . . . .	0.90
Discharge-nozzle velocity coefficient $C_v$ . . . . .	0.97
Combustion efficiency $\eta_f$ . . . . .	0.96
Heating value of fuel $h$ , (Btu/lb) . . . . .	18,900
$\epsilon$ . . . . .	1.00

These compressor and turbine efficiencies are not unreasonably high when it is considered that in the definition of efficiency in this report the compressor and the turbine are credited with the kinetic energy of the gases at the compressor and turbine exits, respectively.

The computed turbojet performance in this illustrative case includes the contribution of the fuel weight.

The values of component efficiencies and  $\epsilon$  for any given turbojet engine vary with altitude and flight speed. In the present computations, the component efficiencies and  $\epsilon$  were assumed constant at the values listed; hence, the illustrative curves represent the performance of a series of turbojet engines having the listed characteristics. One curve is also given for a case in which the variation of  $\epsilon$  with compressor pressure ratio is considered.

When  $V_0 = 0$ ,  $T_0 = 519^\circ R$ , figure 3 shows the rate of fuel consumption per unit thrust and the static thrust per unit mass rate of air flow plotted against the compressor pressure ratio for various values of the gas total temperature at the combustion-chamber exit. It is noted that minimum specific fuel consumption occurs at a higher compressor pressure ratio than maximum thrust. A curve for  $T_4 = 1960^\circ R$  where the variation in  $\epsilon$  with  $p_2/p_1$  is considered is also shown in figure 8. For this curve, values of  $\Delta p_3/p_0 = 0.04$  and  $\Delta p_{(2-4)}/p_0 = 0.10$  were chosen and assumed to remain constant. (For a given unit, however,  $\Delta p_{(2-4)}$  will also vary with  $p_2/p_1$  so that the determination of the actual variation in  $\epsilon$  with compressor pressure ratio becomes quite complex.) It is seen from figure 8 that the value of compressor pressure ratio for a maximum value of  $F/M$  is greater for the case where  $\epsilon$  varies with pressure ratio than for the case where  $\epsilon$  is assumed constant; and that the peak value of  $F/M$  for the first case is slightly higher than that for the second case.

Figure 9(a) is a replot of figure 8 and shows compressor pressure ratio and fuel consumption per unit thrust plotted against thrust per unit mass rate of air flow. Similar curves are presented

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in figures 9(b) and 9(c) for other combinations of atmospheric temperature and airplane velocity. A scale of specific fuel consumption in pounds per thrust horsepower-hour is added on figures 9(b) and 9(c).

The amount of air handled by a unit is limited by the diameter of the unit. When high thrust per unit mass rate of air flow rather than low specific fuel consumption is the primary consideration, it is apparent from figure 9 that high combustion-chamber discharge temperatures should be used. High thrust is the more important consideration in take-off, climb, and maximum-speed operation.

The curves of figure 9 show that, with no limitation on compressor pressure ratio, higher thrust per unit mass rate of air flow and lower specific fuel consumption can be obtained by increasing the combustion-chamber outlet temperature until the value giving minimum specific fuel consumption is reached. For figures 9(a), 9(b), and 9(c), this temperature is less than  $1460^{\circ}$  R, about  $2210^{\circ}$  R, and  $1710^{\circ}$  R, respectively. Further increase in temperature permits an increase in thrust at the cost of increase in specific fuel consumption. As the gas temperature at the combustion-chamber outlet is increased, a large increase in compressor pressure ratio is required to maintain nearly minimum specific fuel consumption.

If the available compressor pressure ratio is limited, the combustion-chamber outlet temperature for minimum specific fuel consumption is very sensitive to the other operating conditions. For example, at a limiting compressor pressure ratio of 4, minimum specific fuel consumption occurs at a temperature below the lowest values shown in figure 9. If the limiting compressor pressure ratio is 8, the combustion-chamber discharge temperature for minimum specific fuel consumption is still less than the lowest temperature shown in figure 9(c) for an atmospheric temperature of  $412^{\circ}$  R but approaches an intermediate value of approximately  $1710^{\circ}$  R for an atmospheric temperature of  $519^{\circ}$  R (fig. 9(b)). The optimum combustion-gas temperature is also very sensitive to the efficiencies of the components of the jet-propulsion units.

In figure 10(a) the specific fuel consumption and the thrust per unit mass rate of air flow are plotted against airplane velocity for the conditions listed in the figure for the following cases:

- (a) Compressor pressure ratio chosen to give values of  $A = 1$
- (b) Compressor pressure ratio chosen to give minimum specific fuel consumption

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It is noted that the specific fuel consumption for case (a) is between 15 and 23 percent higher than for case (b) for airplane velocities between 300 and 800 feet per second; the percentage difference in specific fuel consumption is greater at the lower airplane velocities and at the lower atmospheric temperatures.

The thrust per unit mass rate of air flow is between 21 and 31 percent higher for case (a) than for case (b) for airplane velocities between 300 and 800 feet per second; the greater percentage difference in thrust per unit mass rate of air flow occurs at the lower airplane velocities and the lower atmospheric temperature.

Figure 10(b) shows the compressor pressure ratios and the values of  $A$  that are associated with the performance values given in figure 10(a). The large increase in required pressure ratio from the condition of  $A = 1$  to the condition of minimum specific fuel consumption is noted.

CONCLUSIONS

The following conclusions are based on an analysis of a jet-propulsion system comprising a compressor, a combustion chamber, a turbine, and a discharge nozzle:

1. Maximum thrust per unit mass rate of air flow occurs at a lower compressor pressure ratio than minimum specific fuel consumption.
2. Increase in combustion-chamber discharge temperature causes an increase in thrust. An optimum temperature, however, exists at which minimum specific fuel consumption is obtained. This temperature for minimum specific fuel consumption is at some conditions less than the temperature limit imposed by the strength-temperature characteristics of the materials of present jet-propulsion units.

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APPENDIX A

ADDITIONAL SYMBOLS USED IN THE DERIVATIONS OF PERFORMANCE EQUATIONS

Symbols used in the derivations of performance equations, in addition to those given in the report, are:

- $A'$  factor defined as equal to  $\left[ \frac{P_2/P_1}{(P_2/P_1)_{ref}} \right]^{\frac{\gamma_a-1}{\gamma_a}}$  or  $A^{\frac{\gamma_a-1}{\gamma_a}}$
- $c_p$  average specific heat at constant pressure of the exhaust gases during the expansion process. This term, when used with the temperature change accompanying the expansion, gives the change in enthalpy per unit mass. (Btu)/(slug)(°F)
- $\bar{c}_p$  average specific heat at constant pressure of the gases during the combustion process. This term, when used with the temperature change during combustion, is used to determine the fuel consumption. (Btu)/(slug)(°F)
- $c_{pa}$  specific heat of air at constant pressure at a  $T_0 = 519^\circ R$ , 7.73 (Btu)/(slug)(°F)
- $c_{pa2}$  specific heat of air at constant pressure at compressor outlet total temperature. It is equal to the enthalpy per unit mass (zero enthalpy arbitrarily fixed at absolute zero temperature) divided by the total temperature. (Btu)/(slug)(°F)
- $K, K'$  ratios of functions expressed in terms of physical properties of exhaust gas to same functions expressed in terms of physical properties of cold air. These functions are described in appendix C.
- $P_4$  total pressure at turbine inlet, (lb/sq ft absolute)
- $P_{5s}$  static pressure at turbine discharge, (lb/sq ft absolute)
- $P_t$  turbine-shaft horsepower output
- $R$  gas constant of exhaust gas, (ft-lb)/(slug)(°F)
- $R_a$  gas constant of air, (ft-lb)/(slug)(°F)
- $T_{5s}$  gas temperature at turbine discharge, (°R)

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- $\Delta T_{es}$  reheat due to net turbine loss, ( $^{\circ}F$ )
- $W_{th}$  work obtainable from isentropic expansion of exhaust gas,  
(ft-lb)/(slug)
- $\gamma$  ratio of specific heats of exhaust gas
- $\rho_0$  density of atmospheric air, (slug/cu ft)

The subscript 1 refers to the hypothetical case of no burning, no turbine in system, compressor-shaft power input  $\eta_c P_c$ , compressor efficiency 100 percent, and no losses in system beyond compressor.

APPENDIX B

EQUATIONS FOR THE PERFORMANCE FIGURES

The equation numbers correspond to those in the derivation given in appendix C.

Figure 2:

$$Y = \frac{v_o^2}{2 J c_{pa} T_o} = \frac{1}{2 J c_{pa} 519} \left( v_o \sqrt{\frac{519}{T_o}} \right)^2 \quad (C6)$$

$$\frac{P_1 + \Delta P_d}{P_o} = \left[ 1 + \frac{1}{2 J c_{pa} 519} \left( v_o \sqrt{\frac{519}{T_o}} \right)^2 \right]^{\frac{\gamma_a}{\gamma_a - 1}} \quad (C70)$$

$$\text{Flight Mach number} = \sqrt{\frac{1}{(\gamma_a - 1) J c_{pa} 519} \left( v_o \sqrt{\frac{519}{T_o}} \right)^2} \quad (C72)$$

Figure 3:

$$\frac{P_2}{P_1} = \left( 1 + \frac{\eta_c Z}{1 + Y} \right)^{\frac{\gamma_a}{\gamma_a - 1}} \quad (C67)$$

$$\left( \frac{P_2}{P_1} \right)_{\text{ref}} = \left[ \left( \frac{1}{1 + Y} \right)^2 \eta_c \eta_t \epsilon \frac{T_4}{T_o} \right]^{\frac{\gamma_a}{2(\gamma_a - 1)}} \quad (C68)$$

Figure 4(a):

$$v \sqrt{\frac{\eta_c \eta_t 519}{C_v^2 T_o}} = \sqrt{\frac{519}{T_o} v_o^2 + 2 J c_{pa} 519 \left[ \eta_c \eta_t \epsilon \frac{T_4}{T_o} - \left( A' + \frac{1}{A'} \right) \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} + 1} \right]} \quad (C37)$$

where  $A' = A \frac{\gamma_a - 1}{\gamma_a}$

Figure 4(b):

$$\frac{\Delta V_1}{V_1} = \frac{\frac{1}{2} \left[ 1 - \left( \frac{C_v V_5}{V_1} \right)^2 \right] \left( \frac{1}{\eta_t} - 1 \right)}{\frac{T_4}{T_0} \frac{c_p}{Z c_{pa}} - 1} \quad (C62)$$

Figure 5:

$$T_2 = \frac{c_{pa}}{c_{pa2}} T_0 (1 + Y + Z) \quad (C69)$$

Figure 6:

$$\eta_r f = \frac{\bar{c}_p (T_4 - T_2)}{32.2 h} \quad (C67)$$

where  $\bar{c}_p$  is determined from unpublished data

Figure 7(a):

$$a = \frac{\Delta P_a}{P_0} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{Y + \eta_c Z} \quad (C43)$$

Figure 7(b):

$$b = \frac{\Delta P(2-4)}{P_0} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{Y + \eta_c Z} \left( \frac{1}{1 + Y + \eta_c Z} \right)^{\frac{\gamma_a}{\gamma_a - 1}} \quad (C45)$$

Figure 7(c):

$$c = \frac{W_{th}/R T_4}{\left( \frac{\gamma_a}{\gamma_a - 1} \right) \left[ 1 - \left( \frac{P_0}{P_2} \right)^{\frac{\gamma_a - 1}{\gamma_a}} \right]} \frac{R}{R_a} - 1 \quad (C50)$$

APPENDIX C

DERIVATION OF EQUATIONS FOR JET VELOCITY, THRUST, THRUST  
HORSEPOWER, FUEL CONSUMPTION, SPECIFIC FUEL  
CONSUMPTION, AND MISCELLANEOUS EXPRESSIONS

From the momentum equation the net jet thrust, when the effect of the mass of fuel is neglected, is

$$F = M (V_j - V_o) \quad (C1a)$$

and when the mass of fuel is included

$$F = M (V_j - V_o) + f M V_j \quad (C1b)$$

The thrust horsepower developed by the jet is

$$\text{thp} = F V_o / 550 \quad (C2)$$

Jet Velocity and Thrust

Consider the hypothetical case of a unit running with a compressor efficiency of 100 percent but with a compressor-shaft power input equal to  $\eta_c P_c$  (that is, the product of the actual compressor efficiency by the actual shaft power input). Also assume no turbine in the system and no burning (that is, the compressor is considered to be driven by an engine). The available jet kinetic energy, assuming no losses after the compressor but accounting for the losses in the intake system leading to the compressor, is

$$\frac{1}{2} M V_{j1}^2 = \frac{1}{2} M V_o^2 + 550 \eta_c P_c - \frac{\Delta P_d}{\rho_o} M \quad (C3)$$

The following approximation is accurate for a wide range of  $T_{21}$  and  $P_2/P_o$ .

$$\frac{1}{2} M V_{j1}^2 = M J c_{pa2} T_{21} \left[ 1 - \left( \frac{P_o}{P_2} \right)^{\frac{\gamma_a - 1}{\gamma_a}} \right] \quad (C4)$$

From the conservation of energy,

$$\frac{c_{pa2}}{c_{pa}} T_{21} - T_0 = \frac{550 \eta_c P_c}{M J c_{pa}} + \frac{V_0^2}{2 J c_{pa}} \quad (C5)$$

By definition

$$Y = V_0^2 / 2 J c_{pa} T_0 \quad (C6)$$

$$Z = 550 P_c / J c_{pa} M T_0 \quad (C7)$$

then

$$\frac{c_{pa2}}{c_{pa}} T_{21} = T_0 (1 + Y + \eta_c Z) \quad (C8)$$

and

$$V_{j1}^2 = V_0^2 \left( 1 + \eta_c \frac{Z}{Y} - \frac{\Delta P_d}{\frac{1}{2} \rho_0 V_0^2} \right) \quad (C9)$$

Now

$$\frac{\Delta P_d}{\frac{1}{2} \rho_0 V_0^2} = \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{2 J c_{pa} T_0}{V_0^2} = \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{Y} \quad (C10)$$

and equation (C9) becomes

$$V_{j1}^2 = V_0^2 \left[ 1 + \eta_c \frac{Z}{Y} - \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{Y} \right] \quad (C11)$$

The compressor energy transferred to the gas in this hypothetical case is equal to the useful energy transferred to the gas in the actual case where the shaft power input is  $P_c$  and the compressor efficiency is  $\eta_c$ . Thus, the compressor-discharge pressure  $p_2$  is the same in both cases. The compressor-discharge temperature for the hypothetical case  $T_{21}$  differs from the true compressor-discharge temperature. When  $V_{j1}$  and  $T_{21}$  are eliminated from equations (C4), (C8), and (C11), the following relation is obtained:

$$2 J c_{pa} T_0 (1 + Y + \eta_c Z) \left[ 1 - \left( \frac{P_0}{P_2} \right)^{\frac{\gamma_a - 1}{\gamma_a}} \right] = V_0^2 \left[ 1 + \eta_c \frac{Z}{Y} - \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{Y} \right] \quad (C12)$$

which is used later to evaluate the compressor outlet pressure  $P_2$ .  
By definition

$$\eta_t = \frac{550 P_t}{(1 + f) M J c_p T_4 \left[ 1 - \left( \frac{P_{58}}{P_4} \right)^{\frac{\gamma - 1}{\gamma}} \right] - \frac{(1 + f) M V_5^2}{2}} \quad (C13)$$

Now consider the actual system with burning taking place and turbine power being removed to drive the compressor. The jet velocity (when the effect of reheat due to the turbine loss, which occurs in the further expansion of the gases from turbine-discharge static pressure to atmospheric pressure, is neglected) is given by

$$V_j = c_v \sqrt{2 J c_p T_4 \left[ 1 - \left( \frac{P_0}{P_4} \right)^{\frac{\gamma - 1}{\gamma}} \right] - \frac{550 P_t}{\frac{1}{2} M \eta_t (1 + f)}} \quad (C14)$$

For simplification, the effect of the weight of the fuel injected will be neglected by dropping the term  $f$  in equation (C14). The effect of the presence of the fuel on the jet velocity  $V_j$  can be taken into account in the subsequent equations and charts for  $V_j$  by using, for the value of  $\eta_t$ , the product of the turbine efficiency and  $1 + f$ , as the quantities  $\eta_t$  and  $f$  appear only as the product  $\eta_t (1 + f)$  in equation (C14). Now

$$\left[ 1 - \left( \frac{P_0}{P_4} \right)^{\frac{\gamma - 1}{\gamma}} \right] = 1 - \left( \frac{P_0}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} \left( 1 - \frac{\Delta P(2 - 4)}{P_2} \right)^{-\frac{(\gamma - 1)}{\gamma}} \quad (C15)$$

When the last term of equation (C15) is expanded into a series,

$$\left(1 - \frac{\Delta P(2-4)}{P_2}\right)^{-\left(\frac{\gamma-1}{\gamma}\right)} = 1 + \frac{\gamma-1}{\gamma} \frac{\Delta P(2-4)}{P_2} \quad (C16)$$

for small  $\Delta P(2-4)/P_2$ . Since only enough turbine power is removed to drive the compressor

$$P_t = P_c \quad (C17)$$

When equations (C15), (C16), and (C17) are substituted into equation (C14),

$$V_j = C_v \sqrt{2Jc_p T_4 \left[1 - \left(\frac{P_0}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right] - 2Jc_p T_4 \left(\frac{P_0}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{\gamma-1}{\gamma}\right) \frac{\Delta P(2-4)}{P_2} - \frac{550 P_c}{\frac{1}{2} M \eta_t}} \quad (C18)$$

Let

$$K = \frac{\left[1 - \left(\frac{P_0}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right] c_p}{\left[1 - \left(\frac{P_0}{P_2}\right)^{\frac{\gamma_a-1}{\gamma_a}}\right] c_{pa}} \quad (C19)$$

and

$$K' = \frac{\left(\frac{P_0}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{\gamma-1}{\gamma}\right) c_p}{\frac{\gamma_a-1}{\gamma_a} \left(\frac{P_0}{P_2}\right) \left(\frac{\gamma_a-1}{\gamma_a}\right) c_{pa}} \quad (C20)$$

When equations (C7), (C12), (C19), and (C20) are used in equation (C16),

$$V_j = C_v \sqrt{\frac{T_4}{T_0} \left[ \frac{V_0^2 \left( 1 + \eta_c \frac{Z}{Y} - \frac{\Delta P_d}{P_0} \right) \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{1}{Y}}{(1 + Y + \eta_c Z)} \right] \left[ \frac{\frac{\Delta P(2-4)}{P_2} \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{2J_0 \eta_c T_0}{V_0^2} \left( \frac{P_0}{P_2} \right) \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{1}{(1 + Y + \eta_c Z)}}{1 + \eta_c \frac{Z}{Y} - \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{1}{Y}} \right] - \frac{V_0^2 Z}{\eta_c Y}} \quad (C21)$$

or

$$V_j = C_v \sqrt{\frac{T_4}{T_c} \frac{V_0^2 \left( 1 + \eta_c \frac{Z}{Y} \right)}{1 + Y + \eta_c Z} \epsilon - \frac{V_0^2 Z}{\eta_c Y}} \quad (C22)$$

where  $\epsilon$  is defined by the relation

$$\epsilon = \left[ 1 - \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{1}{(Y + \eta_c Z)} \right] \left[ \frac{\frac{\Delta P(2-4)}{P_2} \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \left( \frac{2J_0 \eta_c T_0}{V_0^2} \right) \left( \frac{P_0}{P_2} \right) \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{1}{(1 + Y + \eta_c Z)}}{1 + \eta_c \frac{Z}{Y} - \frac{\Delta P_d}{P_0} \left( \frac{\gamma_a^{-1}}{\gamma_a} \right) \frac{1}{Y}} \right] \quad (C23)$$

Equation (C22) can be written

$$V_j = V_o \sqrt{C_v^2 \epsilon \frac{T_4}{T_o} \frac{[1 + \eta_c (Z/Y)]}{(1 + Y + \eta_c Z)} - \frac{C_v^2 Z}{\eta_t Y}} \quad (C24)$$

When equation (C24) is substituted in equation (C1a)

$$F = M (V_j - V_o) = M V_o \left[ \sqrt{C_v^2 \epsilon \frac{T_4}{T_o} \frac{[1 + \eta_c (Z/Y)]}{(1 + Y + \eta_c Z)} - \frac{C_v^2 Z}{\eta_t Y}} - 1 \right] \quad (C25)$$

and equation (C6) is used in equation (C25)

$$\frac{F}{M} \sqrt{\frac{519}{T_o}} = \sqrt{2 J c_{pa} 519} \left[ \sqrt{C_v^2 \epsilon \frac{T_4}{T_o} \frac{(Y + \eta_c Z)}{(1 + Y + \eta_c Z)} - \frac{\eta_c Z}{\left( \frac{\eta_c \eta_t}{C_v^2} \right)}} - \sqrt{Y} \right] \quad (C26)$$

Fuel Consumption

$$\eta_f f = \frac{\bar{c}_p (T_4 - T_2)}{32.2 h} \quad (C27)$$

From the conservation of energy

$$c_{pa2} T_2 = c_{pa} T_o + \frac{V_o^2}{2J} + \frac{550 P_c}{M J} \quad (C28)$$

so that

$$T_2 = \frac{c_{pa}}{c_{pa2}} T_o (1 + Y + Z) \quad (C29)$$

Pressure Ratio for Optimum Thrust

For a given  $V_o$ ,  $T_o$ ,  $T_4$ ,  $\eta_c$ ,  $\eta_t$ , and  $C_v$ , neglecting the change in  $\epsilon$  due to a change in  $\eta_c Z$ , the maximum thrust per unit mass rate of air flow with respect to compressor power input (or pressure ratio) is obtained when

$$\frac{\partial \left( \frac{F}{M} \sqrt{\frac{519}{T_0}} \right)}{\partial (\eta_c Z)} = 0 = C_v^2 \epsilon \frac{T_4}{T_0} \left( \frac{1}{1 + Y + \eta_c Z} \right)^2 - \frac{C_v^2}{\eta_c \eta_t} \quad (C30)$$

from which

$$1 + Y + (\eta_c Z)_{\text{ref}} = \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_0}} \quad (C31)$$

Define A' by the relation

$$A' = \frac{1 + Y + \eta_c Z}{1 + Y + (\eta_c Z)_{\text{ref}}} \quad (C32)$$

$$1 + Y + \eta_c Z = A' \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_0}} \quad (C33)$$

Jet Velocity, Thrust, and Specific Fuel Consumption

in Terms of the Factor A'

Equation (C24) can be written

$$V_J = \frac{V_0}{\sqrt{Y}} \sqrt{C_v^2 \epsilon \frac{T_4}{T_0} \frac{(Y + \eta_c Z)}{(1 + Y + \eta_c Z)} - \frac{C_v^2 \eta_c Z}{\eta_c \eta_t}} \quad (C34)$$

When equation (C33) is used in equation (C34),

$$V_J = \frac{V_0}{\sqrt{Y}} \sqrt{\left( \frac{C_v^2}{\eta_c \eta_t} \right) \left( \eta_c \eta_t \epsilon \frac{T_4}{T_0} \right) \frac{\left( A' \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_0}} - 1 \right)}{A' \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_0}}} - \frac{C_v^2}{\eta_c \eta_t} \left( A' \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_0}} - 1 - Y \right)} \quad (C35)$$

and

$$V_j \sqrt{\frac{\eta_o \eta_t}{C_v^2}} = \frac{V_o}{\sqrt{Y}} \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} - \left(A' + \frac{1}{A'}\right) \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} + 1 + Y}} \quad (C36)$$

When equation (C6) is substituted in equation (C36),

$$V_j \sqrt{\frac{\eta_c \eta_t}{C_v^2}} \sqrt{\frac{519}{T_o}} = \sqrt{\frac{519}{T_o}} V_o \sqrt{2Jc_{pa} 519 \left[ \eta_c \eta_t \epsilon \frac{T_4}{T_o} - \left(A' + \frac{1}{A'}\right) \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} + 1} \right]} \quad (C37)$$

Equation (C26) becomes in terms of  $A'$

$$\frac{F}{M} \sqrt{\frac{519}{T_o}} = \sqrt{2Jc_{pa} 519} \left[ \sqrt{\frac{C_v^2}{\eta_c \eta_t}} \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} - \left(A' + \frac{1}{A'}\right) \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} + 1 + Y - \sqrt{Y}}} \right] \quad (C38)$$

The fuel consumption per unit thrust is obtained from equations (C27) and (C38) and is

$$\eta_f \frac{W_f}{F} \sqrt{\frac{T_o}{519}} = \frac{3600 \bar{c}_p}{h \sqrt{2Jc_{pa} 519}} \left[ \frac{T_4 - T_2}{\sqrt{\frac{C_v^2}{\eta_c \eta_t}} \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} - \left(A' + \frac{1}{A'}\right) \sqrt{\eta_c \eta_t \epsilon \frac{T_4}{T_o} + 1 + Y - \sqrt{Y}}} \right] \quad (C39)$$

Evaluation of the Correction Factor  $\epsilon$

From equations (C12) and (C6)

$$\left(\frac{P_o}{P_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}} = 1 - \frac{Y + \eta_c Z - \frac{\Delta P_d}{P_o} \left(\frac{\gamma_a - 1}{\gamma_a}\right)}{1 + Y + \eta_c Z} \quad (C40)$$

When equation (C40) is used in equation (C23),

$$c = \left[ 1 - \frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{1}{(Y + \eta_c Z)} \right] \left\{ K - K' \frac{\frac{\Delta P(2-4)}{P_2} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \left[ 1 + \frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \right]}{Y + \eta_c Z - \frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right)} \right\} \quad (C41)$$

or

$$c = K \left[ 1 - \frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \left( \frac{1}{Y + \eta_c Z} \right) \right] - K' \frac{\Delta P(2-4)}{P_2} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{\left[ 1 + \left( \frac{\Delta P_d}{P_o} \right) \left( \frac{\gamma_a - 1}{\gamma_a} \right) \right]}{(Y + \eta_c Z)} \quad (C42)$$

let

$$a = \frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \left( \frac{1}{Y + \eta_c Z} \right) \quad (C43)$$

and

$$b = \frac{\Delta P(2-4)}{P_2} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \frac{\left[ 1 + \frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \right]}{Y + \eta_c Z} \quad (C44)$$

When equation (C40) is used in equation (C44) and the  $\frac{\Delta P_d}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right)$  term in the numerator is neglected because it is small in comparison with unity,

$$b = \frac{\Delta P(2-4)}{P_o} \left( \frac{\gamma_a - 1}{\gamma_a} \right) \left( \frac{1}{1 + Y + \eta_c Z} \right)^{\frac{\gamma_a}{\gamma_a - 1}} \quad (C45)$$

When equations (C43) and (C45) are substituted into equation (C42),

$$c = K(1 - a) - K'b$$

The terms  $K$  and  $K'$  are close to unity in value whereas the values of  $a$  and  $b$  are small in comparison with unity; therefore, only a very small error is introduced by letting

$$c = K - a - b \quad (C46)$$

Defining the quantity  $c$  as

$$c = K - 1 \quad (C47)$$

then

$$g = 1 - a - b + c \quad (C48)$$

Now

$$K = \frac{W_{th}/R T_4}{\left(\frac{\gamma_a}{\gamma_a - 1}\right) \left[1 - \left(\frac{P_0}{P_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]} \frac{R}{R_a} \quad (C49)$$

where the values of  $W_{th}/R T_4$  are obtained from reference 5. These values correspond to the required temperature  $T_4$  and pressure ratio  $P_2/P_0$ . Therefore,

$$c = \frac{W_{th}/R T_4}{\left(\frac{\gamma_a}{\gamma_a - 1}\right) \left[1 - \left(\frac{P_0}{P_2}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right]} \frac{R}{R_a} - 1 \quad (C50)$$

#### Correction for Reheat Accompanying Irreversibility in the Turbine

The actual jet velocity including the reheat in the turbine is given by the equation

$$\frac{v_j^2}{C_v^2} - v_5^2 = 2 J c_p T_{5s} \left[1 - \left(\frac{P_0}{P_{5s}}\right)^{\frac{\gamma-1}{\gamma}}\right] \quad (C51)$$

from which the following equation in terms of differentials is obtained:

$$2 \frac{V_j}{C_v} dV_j = 2 J c_p \left[ 1 - \left( \frac{P_0}{P_{5s}} \right)^{\frac{\gamma-1}{\gamma}} \right] dT_{5s} \quad (C52)$$

When equation (C51) is used in equation (C52),

$$2 \frac{V_j}{C_v} dV_j = \left( \frac{V_j^2}{C_v} - V_5^2 \right) \frac{dT_{5s}}{T_{5s}} \quad (C53)$$

$T_{5s}$  is the independent variable, therefore

$$\Delta T_{5s} \equiv dT_{5s}$$

For small values of  $\Delta T_{5s}$  the following equation is very nearly true:

$$\Delta V_j = dV_j \quad (C54)$$

If these expressions for  $dT_{5s}$  and  $dV_j$  are used in equation (C53)

$$\frac{\Delta V_j}{V_j} = \frac{1}{2} \left[ 1 - \left( \frac{C_v V_5}{V_j} \right)^2 \right] \frac{\Delta T_{5s}}{T_{5s}} \quad (C55)$$

$\Delta T_{5s}$  is the amount of reheat and is equal to

$$\Delta T_{5s} = \frac{550 P_t}{M J c_p} \left( \frac{1}{\eta_t} - 1 \right) \quad (C56)$$

whereas the gas temperature at the turbine discharge

$$T_{5s} = T_4 - \frac{550 P_t}{M J c_p} - \frac{V_5^2}{2 J c_p} \quad (C57)$$

When equations (C6), (C7), and (C17) are used in equations (C56) and (C57),

$$\Delta T_{5s} = Z T_0 \left( \frac{c_{p0}}{c_p} \right) \left( \frac{1}{\eta_t} - 1 \right) \quad (C58)$$

$$T_{5s} = T_4 - Z T_0 \left( \frac{c_{pa}}{c_p} \right) - \frac{V_5^2}{V_0^2} Y T_0 \frac{c_{pa}}{c_p} \quad (59)$$

and, when equations (C58) and (C59) are substituted into equation (C55),

$$\frac{\Delta V_J}{V_J} = \frac{\frac{1}{2} \left[ 1 - \left( \frac{C_v V_5}{V_J} \right)^2 \right] Z T_0 \frac{c_{pa}}{c_p} \left( \frac{1}{\eta_t} - 1 \right)}{T_4 - Z T_0 \frac{c_{pa}}{c_p} - \frac{V_5^2}{V_0^2} Y T_0 \frac{c_{pa}}{c_p}} \quad (C60)$$

or

$$\frac{\Delta V_J}{V_J} = \frac{\frac{1}{2} \left[ 1 - \left( \frac{C_v V_5}{V_J} \right)^2 \right] \left( \frac{1}{\eta_t} - 1 \right)}{\frac{T_4}{T_0 Z} \frac{c_p}{c_{pa}} - 1 - \frac{V_5^2 Y}{V_0^2 Z}} \quad (C61)$$

The  $V_5^2 Y/V_0^2 Z$  term in the denominator is small in comparison with  $(T_4/T_0 Z) (c_p/c_{pa}) - 1$  and can be neglected, resulting in

$$\frac{\Delta V_J}{V_J} = \frac{\frac{1}{2} \left[ 1 - \left( \frac{C_v V_5}{V_J} \right)^2 \right] \left( \frac{1}{\eta_t} - 1 \right)}{\frac{T_4}{T_0 Z} \frac{c_p}{c_{pa}} - 1} \quad (C62)$$

#### Derivation of Miscellaneous Expressions

$$(a) \quad \eta_c Z = \eta_c \frac{550 P_c}{M J c_{pa} T_0} \quad (C63)$$

where the compressor power is accurately given for a wide range of compressor pressure ratios and compressor inlet temperatures by the relation

$$P_c = \frac{M J c_{pa} T_1}{550 \eta_c} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{\gamma_a-1}{\gamma_a}} - 1 \right] \quad (C64)$$

and

$$T_1 = T_o + \frac{V_o^2}{2 J c_{pa}} = T_o (1 + Y) \quad (C65)$$

When equations (C64) and (C65) are used in equation (C63),

$$\eta_c Z = (1 + Y) \left[ \left( \frac{P_2}{P_1} \right)^{\frac{\gamma_a-1}{\gamma_a}} - 1 \right] \quad (C66)$$

or

$$\frac{P_2}{P_1} = \left( 1 + \frac{\eta_c Z}{1 + Y} \right)^{\frac{\gamma_a}{\gamma_a-1}} \quad (C67)$$

(b) When equation (C31) is substituted into equation (C67),

$$\left( \frac{P_2}{P_1} \right)_{\text{ref}} = \left[ \left( \frac{1}{1 + Y} \right)^2 \eta_c \eta_t \epsilon \frac{T_4}{T_o} \right]^{\frac{\gamma_a}{2(\gamma_a-1)}} \quad (C68)$$

(c) The ideal ram pressure ratio is

$$\frac{P_1 + \Delta p_d}{P_o} = \left( \frac{T_1}{T_o} \right)^{\frac{\gamma_a}{\gamma_a-1}} \quad (C69)$$

and when equations (C65) and (C6) are used in equation (C69)

$$\frac{P_1 + \Delta p_d}{P_o} = (1 + Y)^{\frac{\gamma_a}{\gamma_a-1}} = \left[ 1 + \frac{1}{2 J c_{pa} 519} \left( V_o \sqrt{\frac{519}{T_o}} \right)^2 \right]^{\frac{\gamma_a}{\gamma_a-1}} \quad (C70)$$

(d) The Mach number at the inlet to the unit (or the flight Mach number) is

$$\text{Flight Mach number} = V_o / \sqrt{\gamma_a R_a T_o} \quad (C71)$$

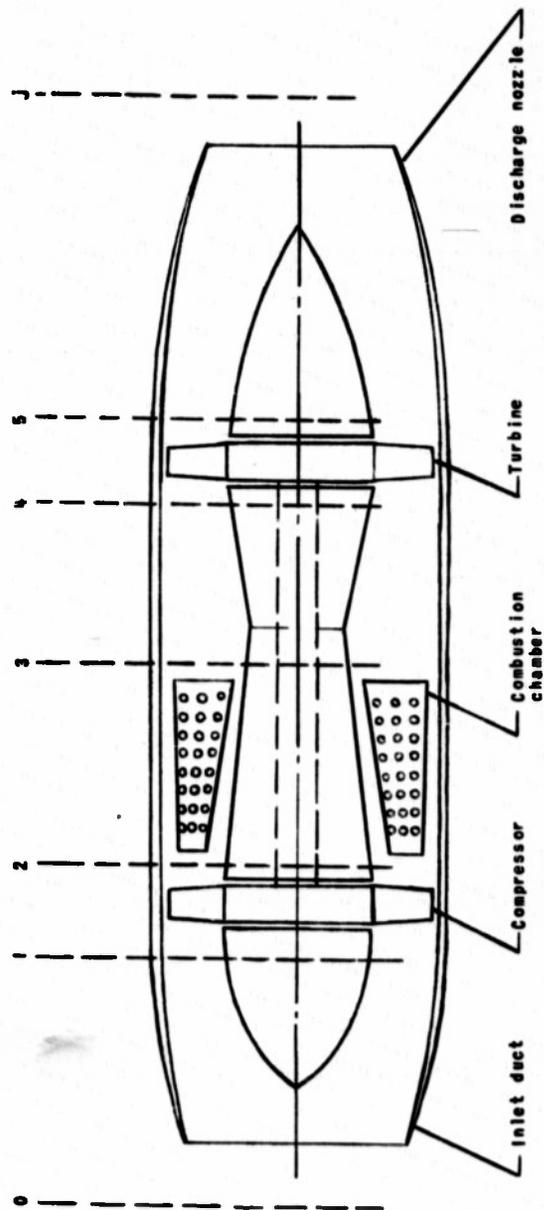
$$= \sqrt{\frac{1}{(\gamma_a - 1) J c_{pa} 519}} \left( V_o \sqrt{\frac{519}{T_o}} \right) \quad (C72)$$

or, when equation (C6) is used in equation (C71),

$$\text{Flight Mach number} = \frac{\sqrt{2 J c_{pa} T_o Y}}{\sqrt{\gamma_a R_a T_o}} = \sqrt{\left( \frac{2}{\gamma_a - 1} \right) Y} \quad (C73)$$

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Figure 1. - Schematic diagram of the jet-propulsion system.

Fig. 2

NACA ARR NO. E6E14

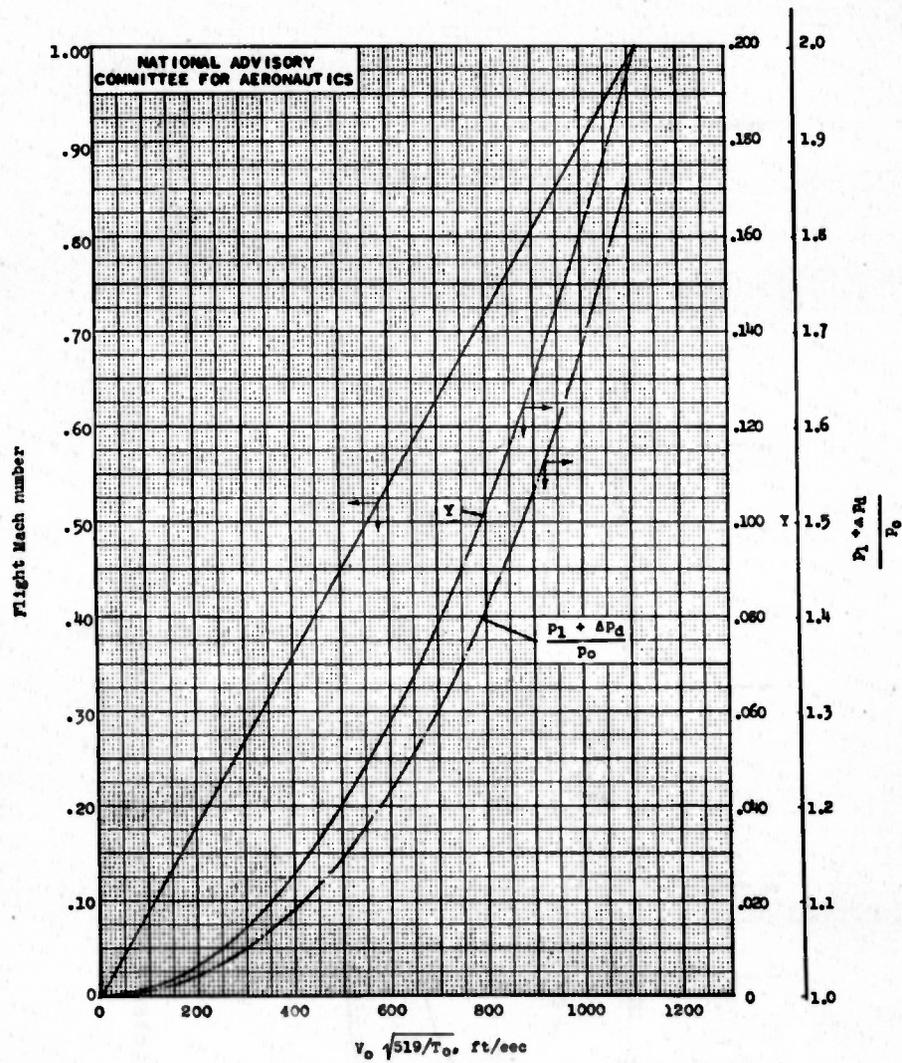


Figure 2.- Chart for determining  $Y$ , flight Mach number, and compressor inlet total pressure for various airplane velocities and atmospheric temperatures.

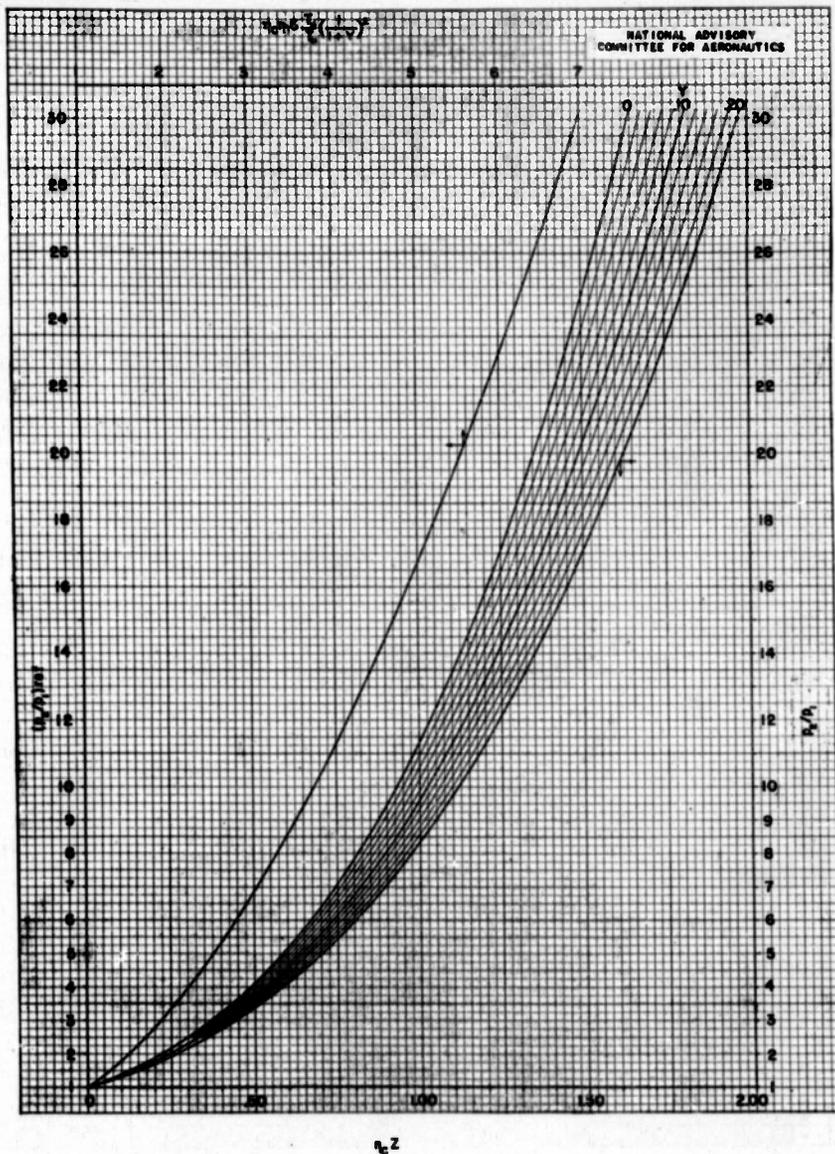


Figure 3. - Chart for determining the reference compressor pressure ratio for various values of  $\eta_c \eta_t \frac{E T_4}{T_0} \frac{1}{1+\gamma}$  and the compressor pressure ratio for various values of  $\gamma$  and  $\eta_c Z$ . (A  $19\frac{1}{2}$ -in. by 28-in. print of this chart is enclosed with the report.)

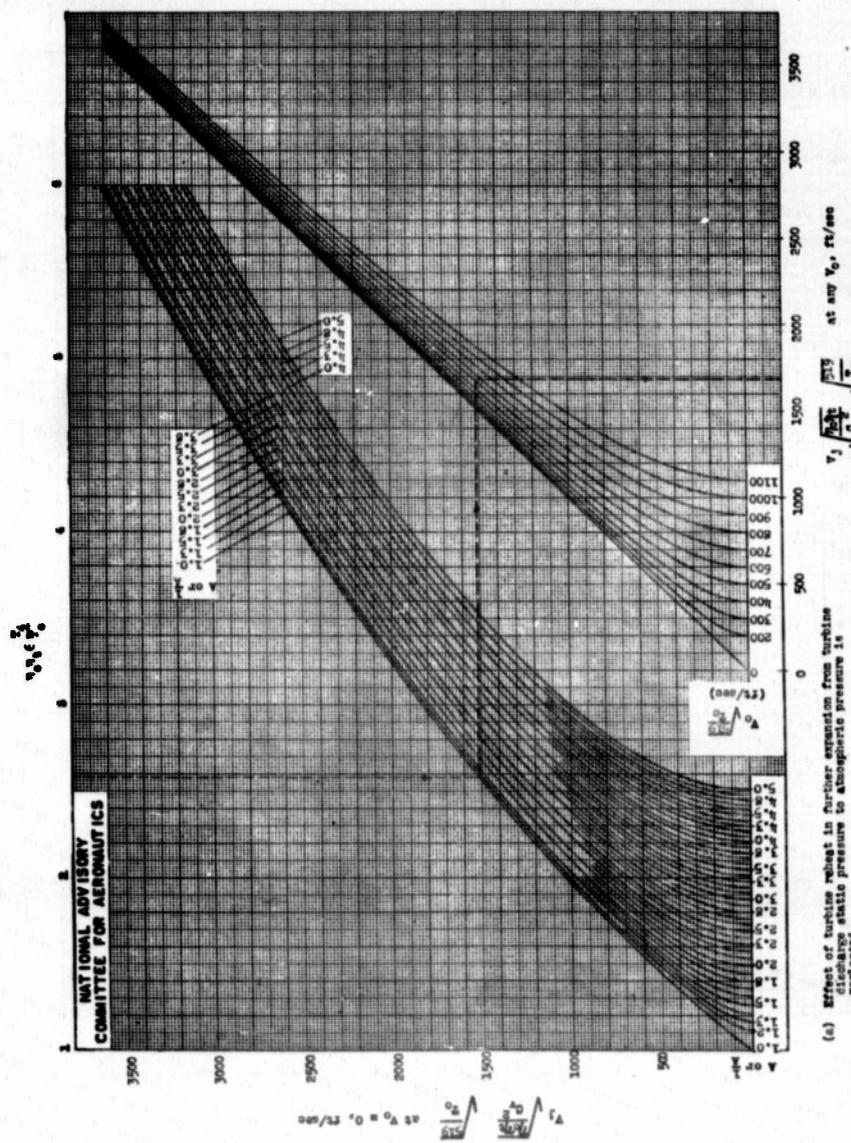
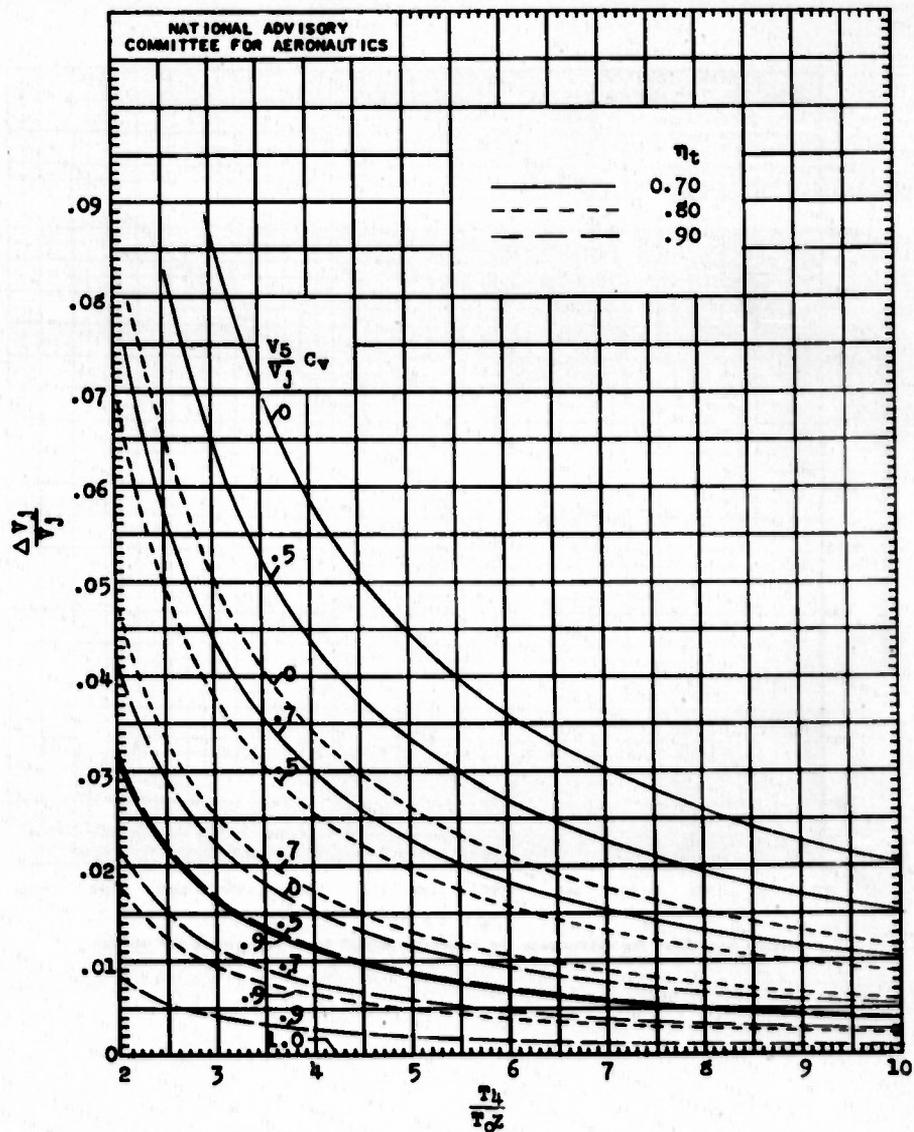


Figure 4. - Chart for determining jet velocity. (This chart has been divided into two sections, an 18-in. by 36 $\frac{1}{2}$ -in. section and a 32 $\frac{1}{2}$ -in. by 36-in. section, both of which are enclosed with the report.)



(b) Correction to jet velocity due to reheat in turbine.

Figure 4. - Concluded.

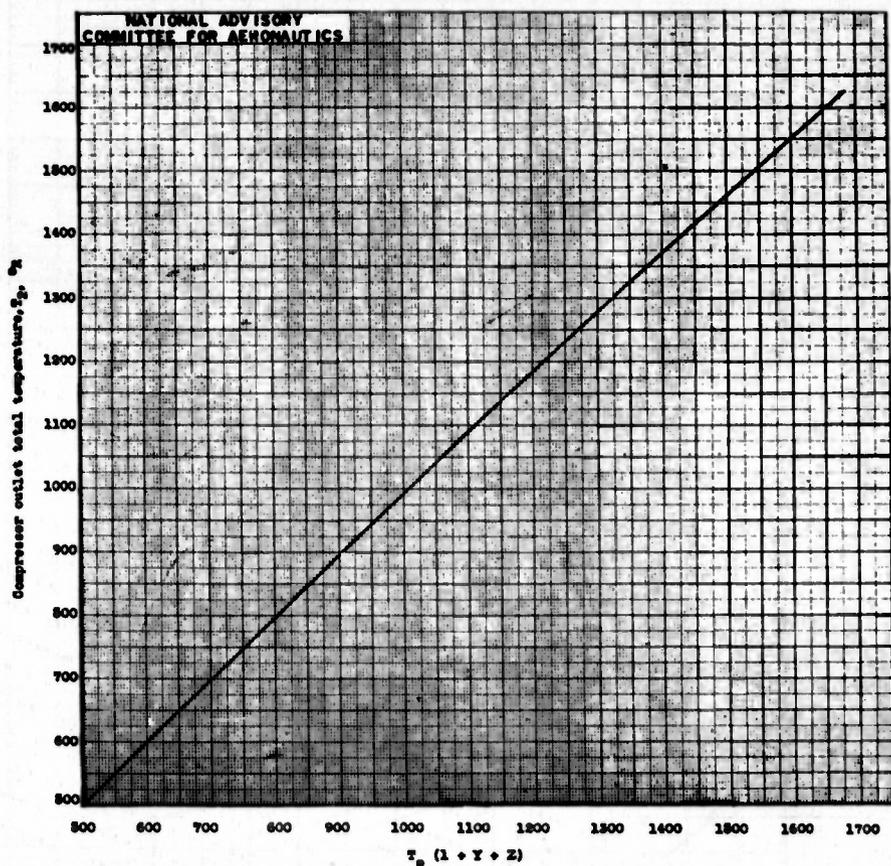


Figure 5. - Chart for determining the compressor outlet total temperature for various values of the factor  $T_0 (1 + Y + Z)$ .

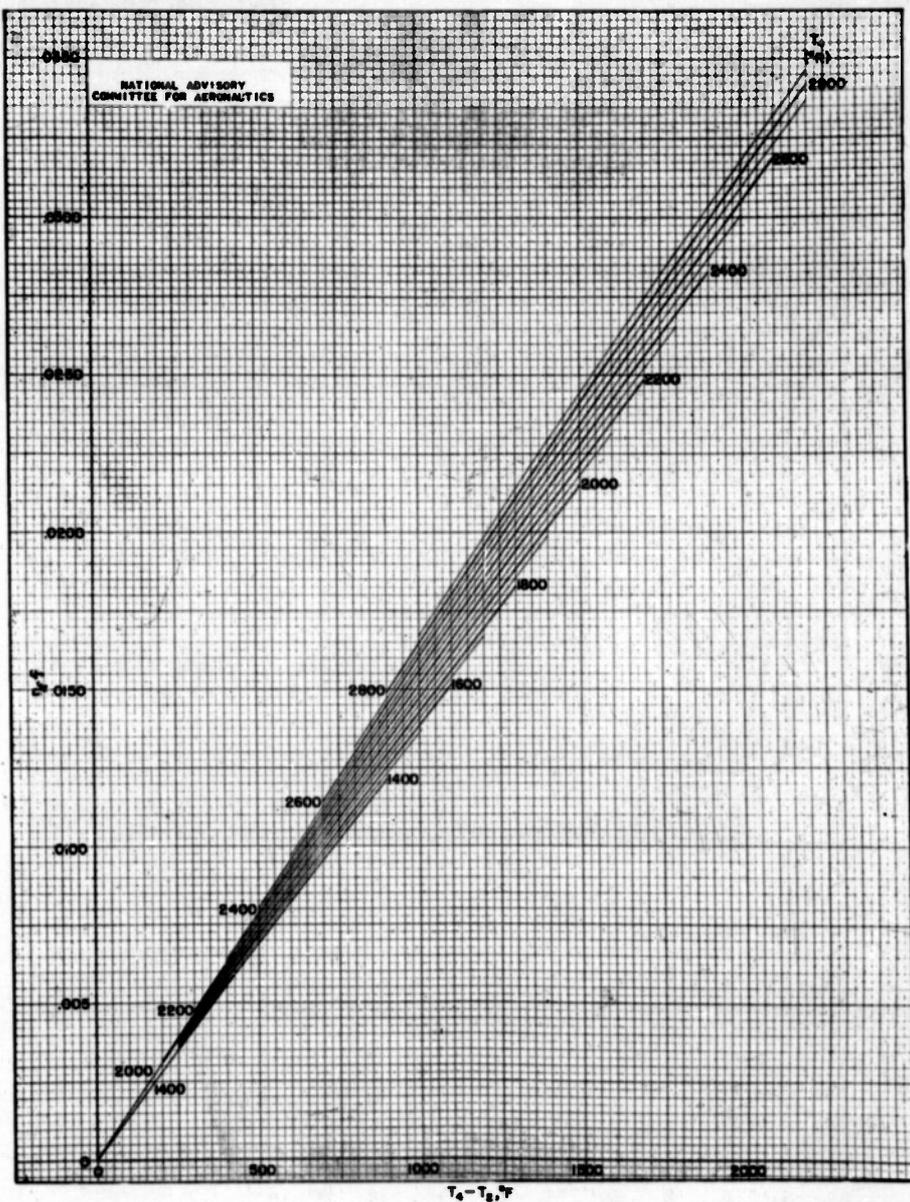


Figure 6. - Chart for determining the fuel-air ratio for various values of rise in total temperature across the combustion chamber and combustion chamber outlet total temperature. ( $h = 18,900$  Btu/lb) (A  $2\frac{3}{4}$ -in. by  $3\frac{1}{2}$ -in. print of this chart is enclosed with the report.)

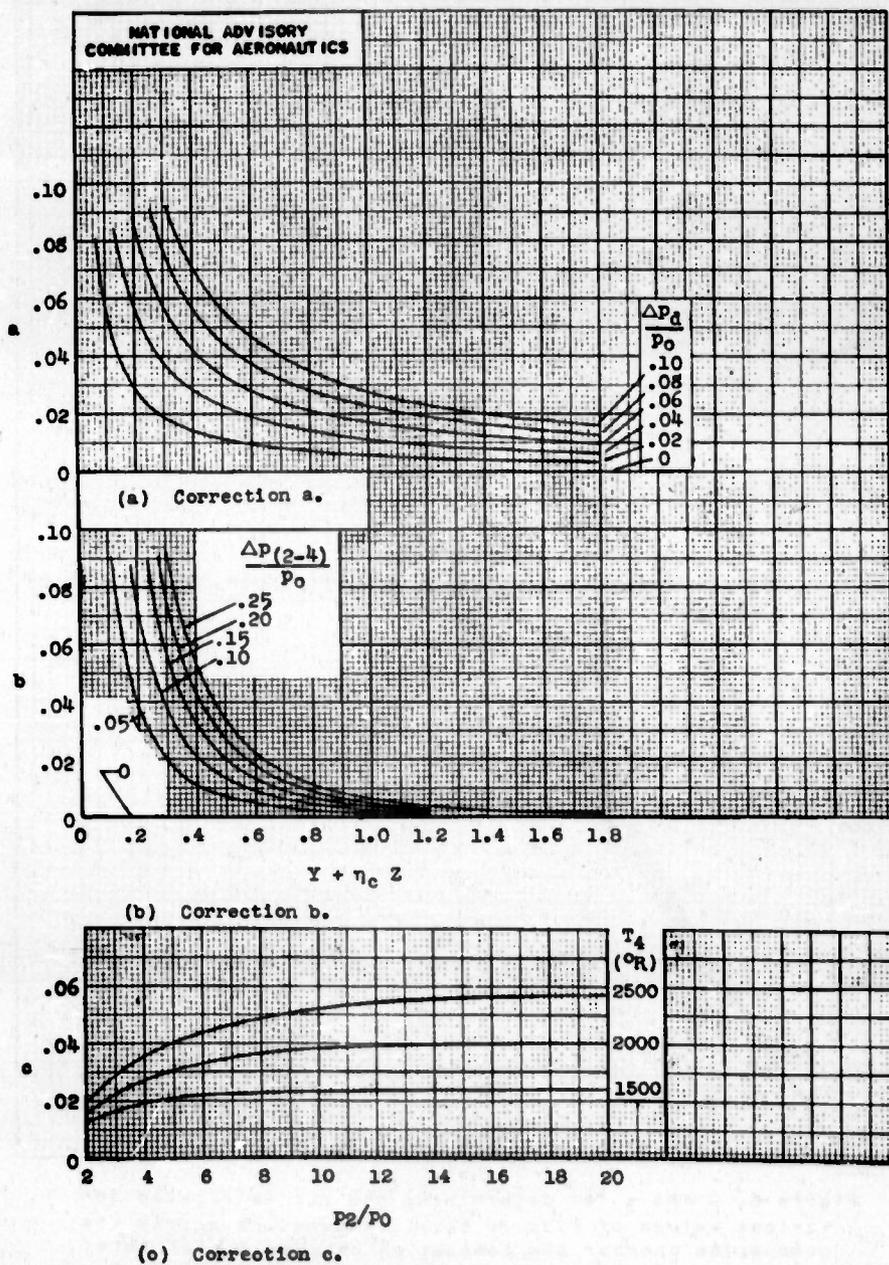


Figure 7.- Chart for determining the factor  $\epsilon$ . ( $\epsilon = 1 - a - b + c$ )

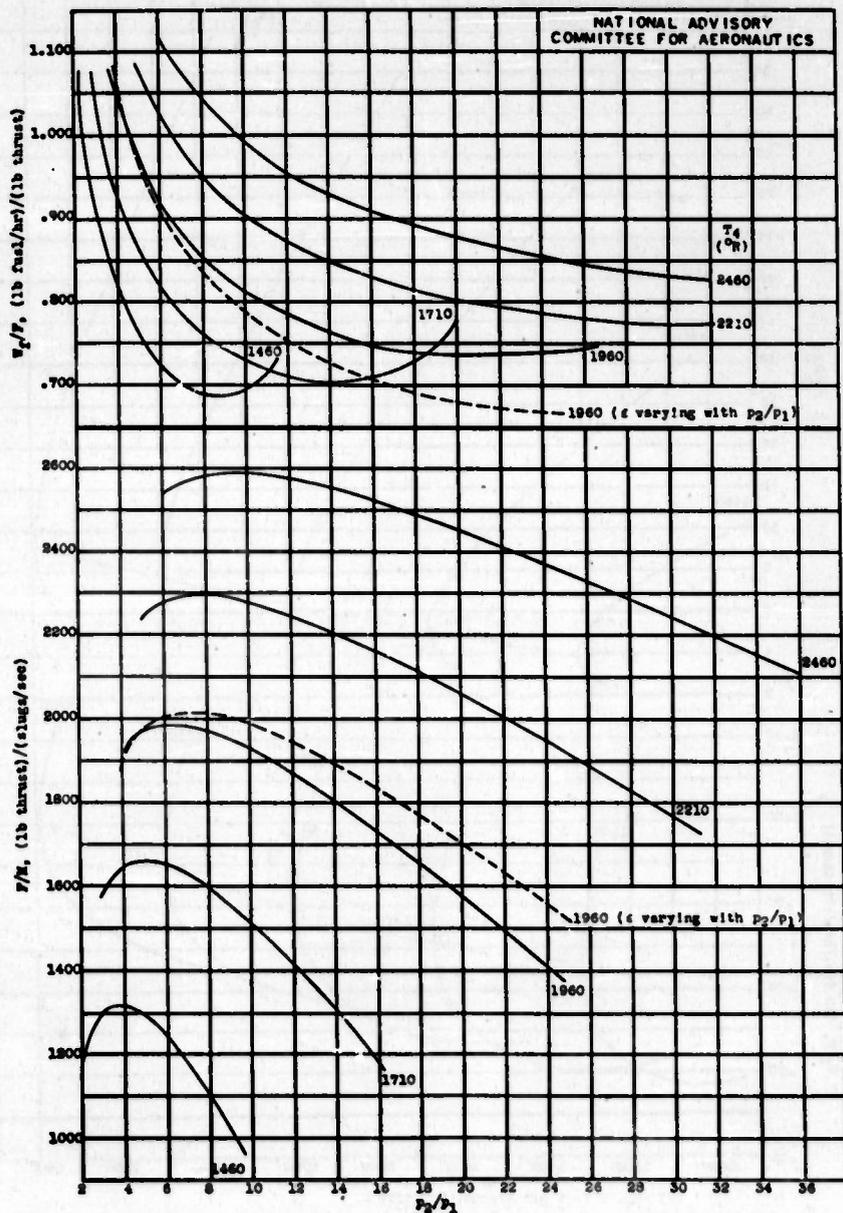


Figure 8. - Fuel rate per unit thrust and thrust per unit mass rate of air flow for various compressor pressure ratios and combustion-chamber discharge temperatures for illustrative case. ( $V_0$ , 0 ft/sec;  $T_0$ , 519° R;  $\eta_c$ , 0.85;  $\eta_{gs}$ , 0.95;  $\eta_p$ , 0.99;  $\mu$ , 18,900 Stn/lb;  $C_v$ , 0.97;  $\epsilon$ , 1.00.)

Fig. 9a

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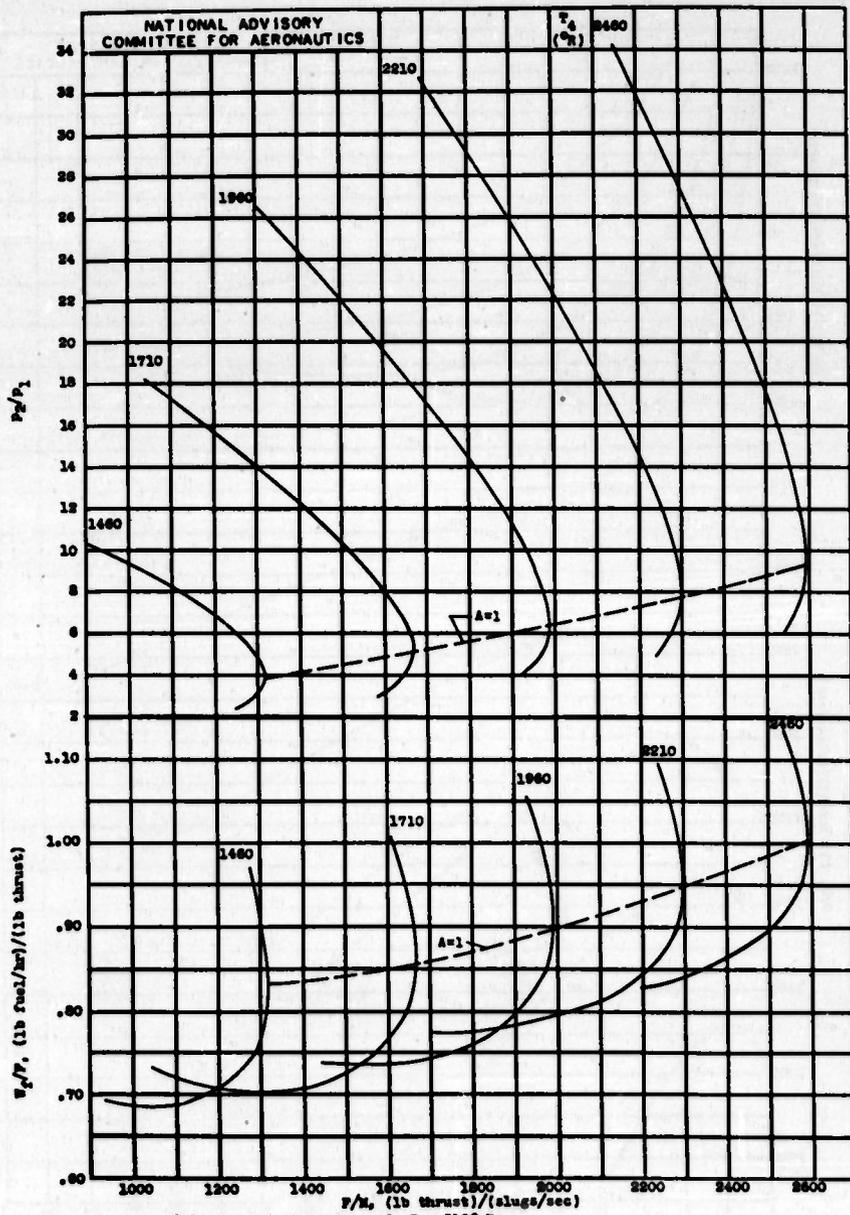
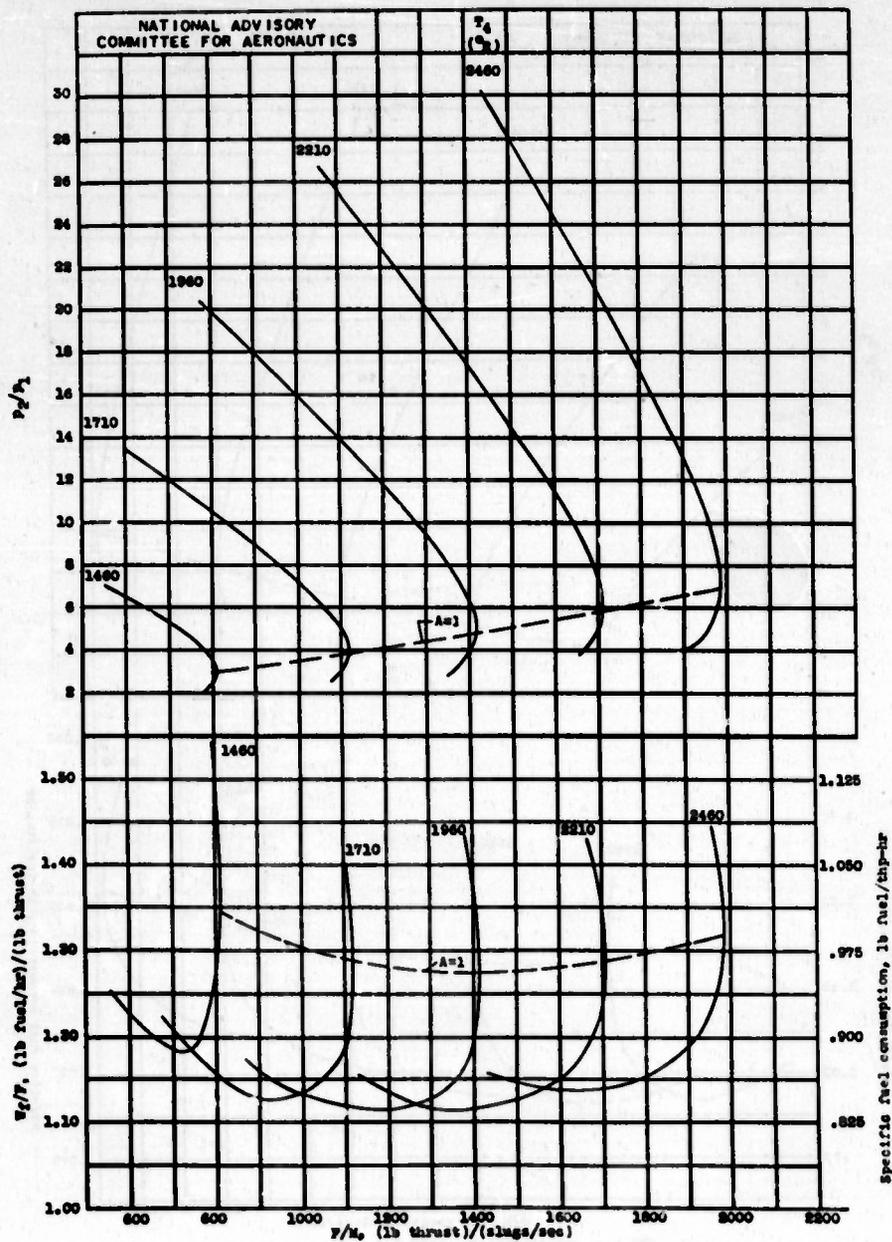


Figure 9. - Compressor pressure ratio and fuel rate per unit thrust for various thrusts per unit mass rate of air flow and combustion-chamber discharge temperatures for illustrative case. ( $\eta_c = 0.88$ ;  $\eta_p = 0.80$ ;  $\eta_{pc} = 0.86$ ;  $h = 18,500$  Btu/lb;  $C_p = 0.97$ ;  $\epsilon = 1.00$ .)



(b)  $V_0$ , 733 feet per second;  $T_0$ , 519° R.

Figure 9. - Continued.

Fig. 9c

NACA ARR No. E6E14

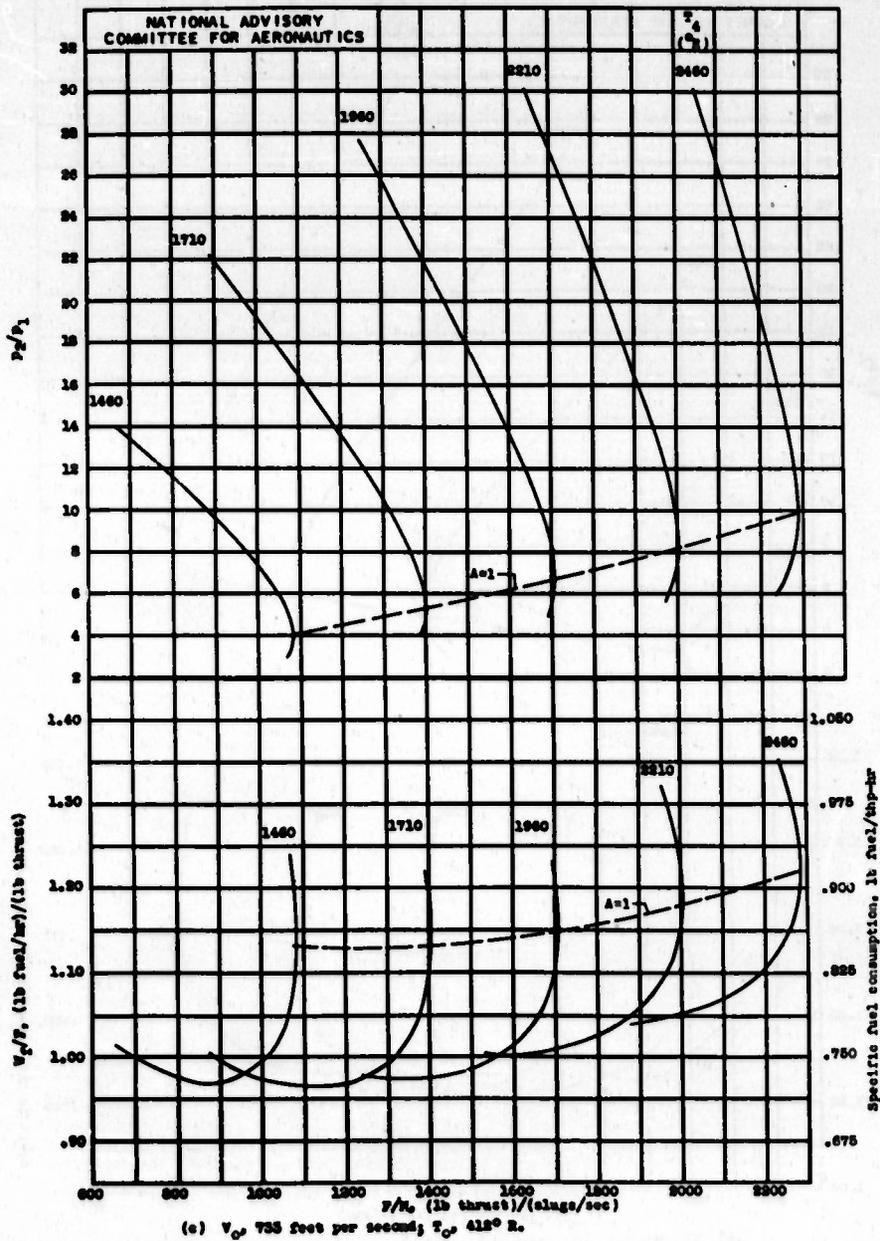
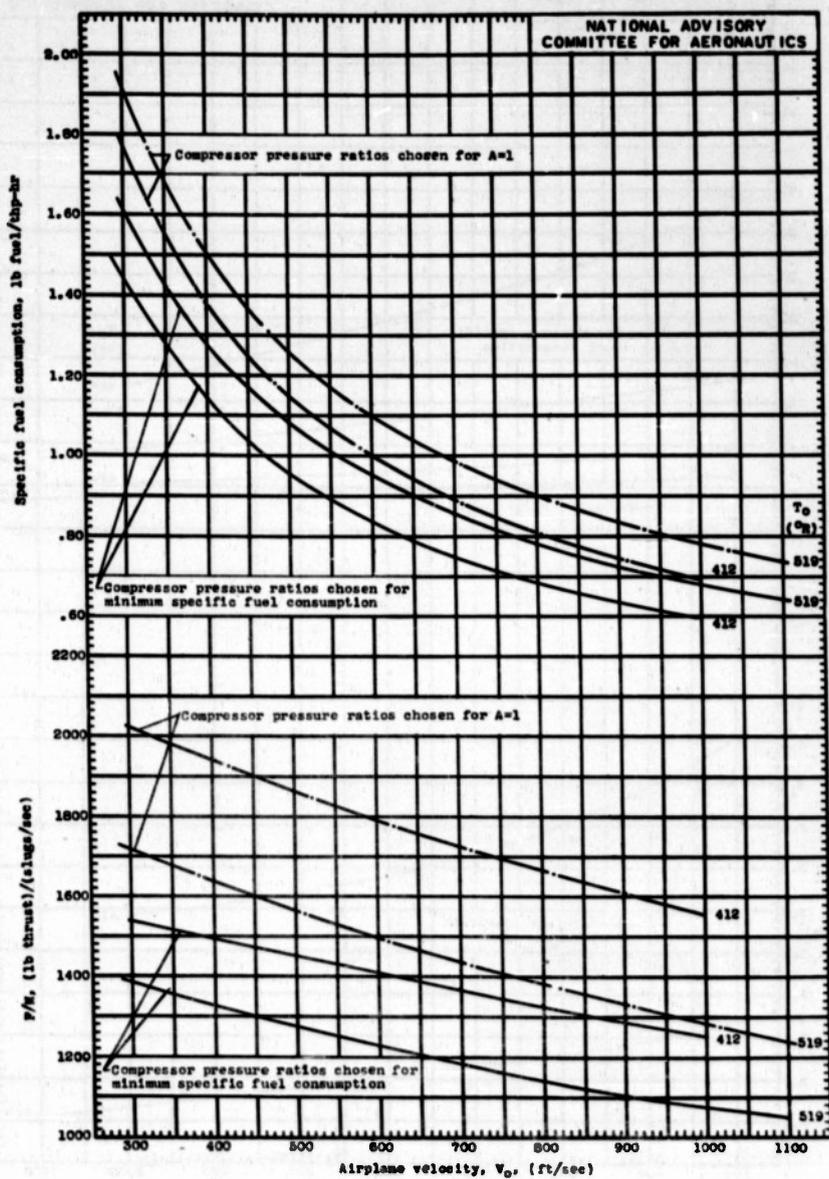


Figure 9. - Concluded.



(a) Specific fuel consumption and thrust per unit air flow at various airplane velocities.

Figure 10.- Performance of jet-propulsion unit at conditions for minimum specific fuel consumption and for pressure ratios giving  $A=1$  for illustrative cases. ( $T_4$ , 1960° R;  $\eta_c$ , 0.85;  $\eta_h$ , 0.90;  $\eta_p$ , 0.98;  $C_v$ , 0.97;  $h$ , 18,900 Btu/lb;  $\epsilon$ , 1.00.)

Fig. 10b

NACA ARR No. E6E14

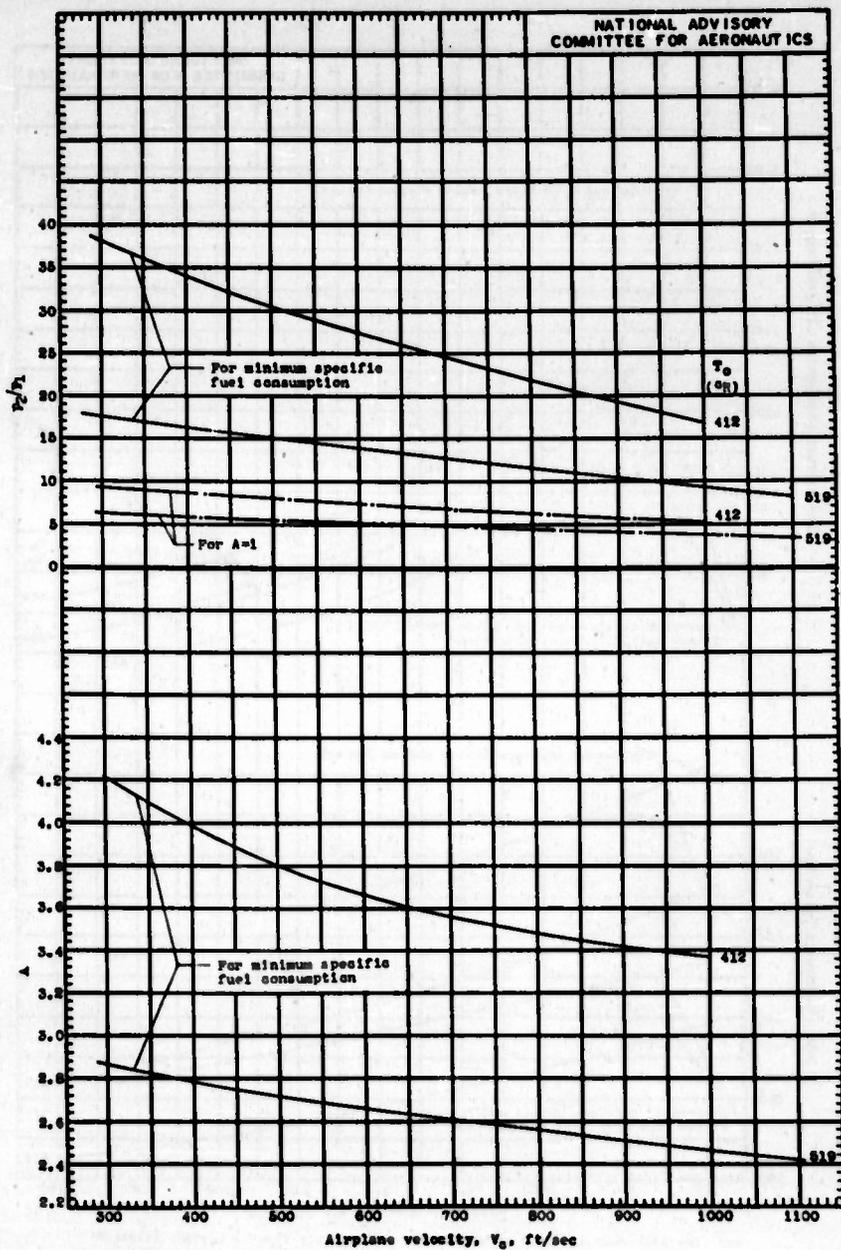


Figure 10.- Concluded.

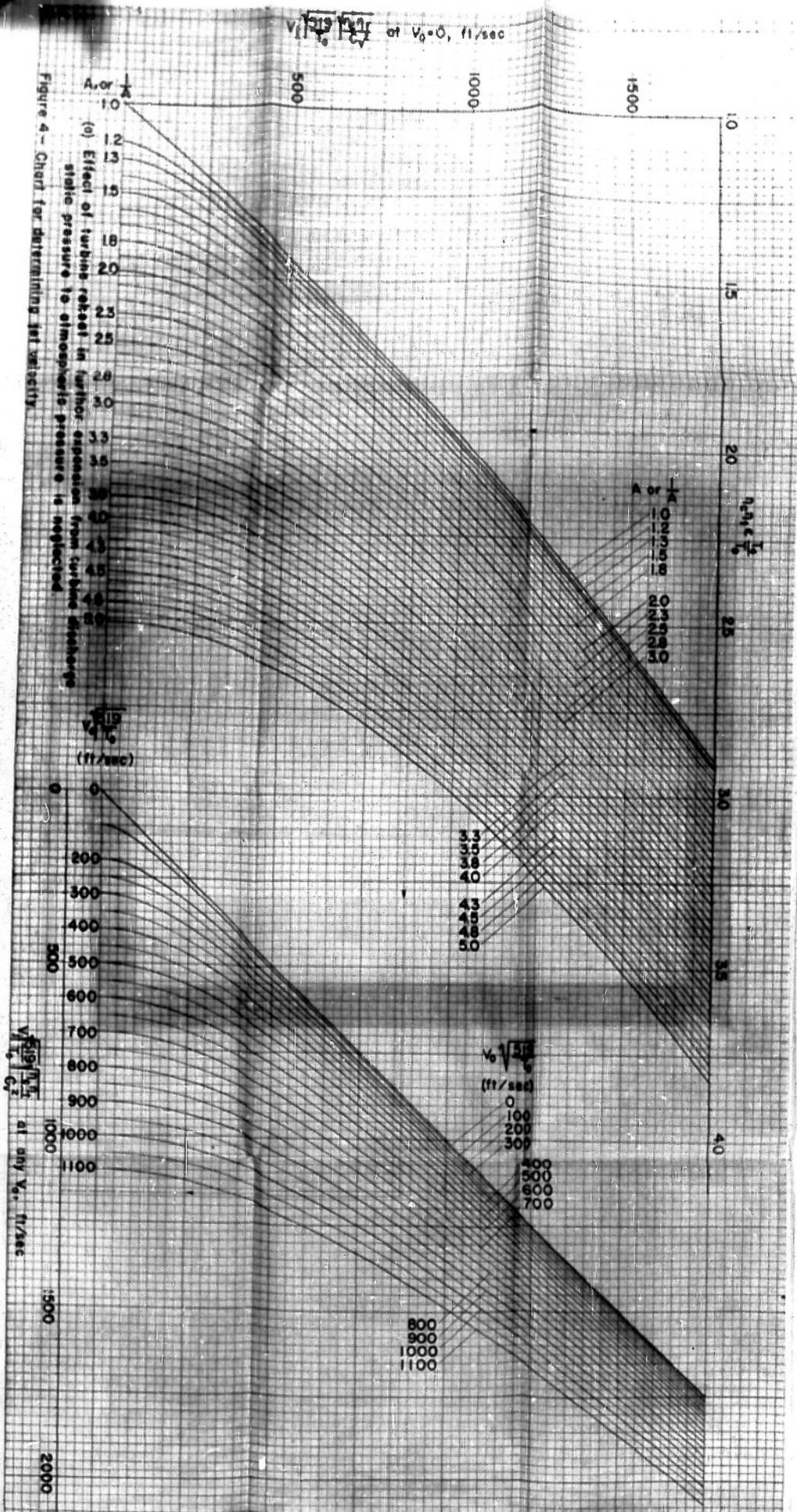


Figure 4 - Chart for determining jet velocity

(a) Effect of turbine reheat in further expansion from turbine discharge static pressure to atmospheric pressure is neglected

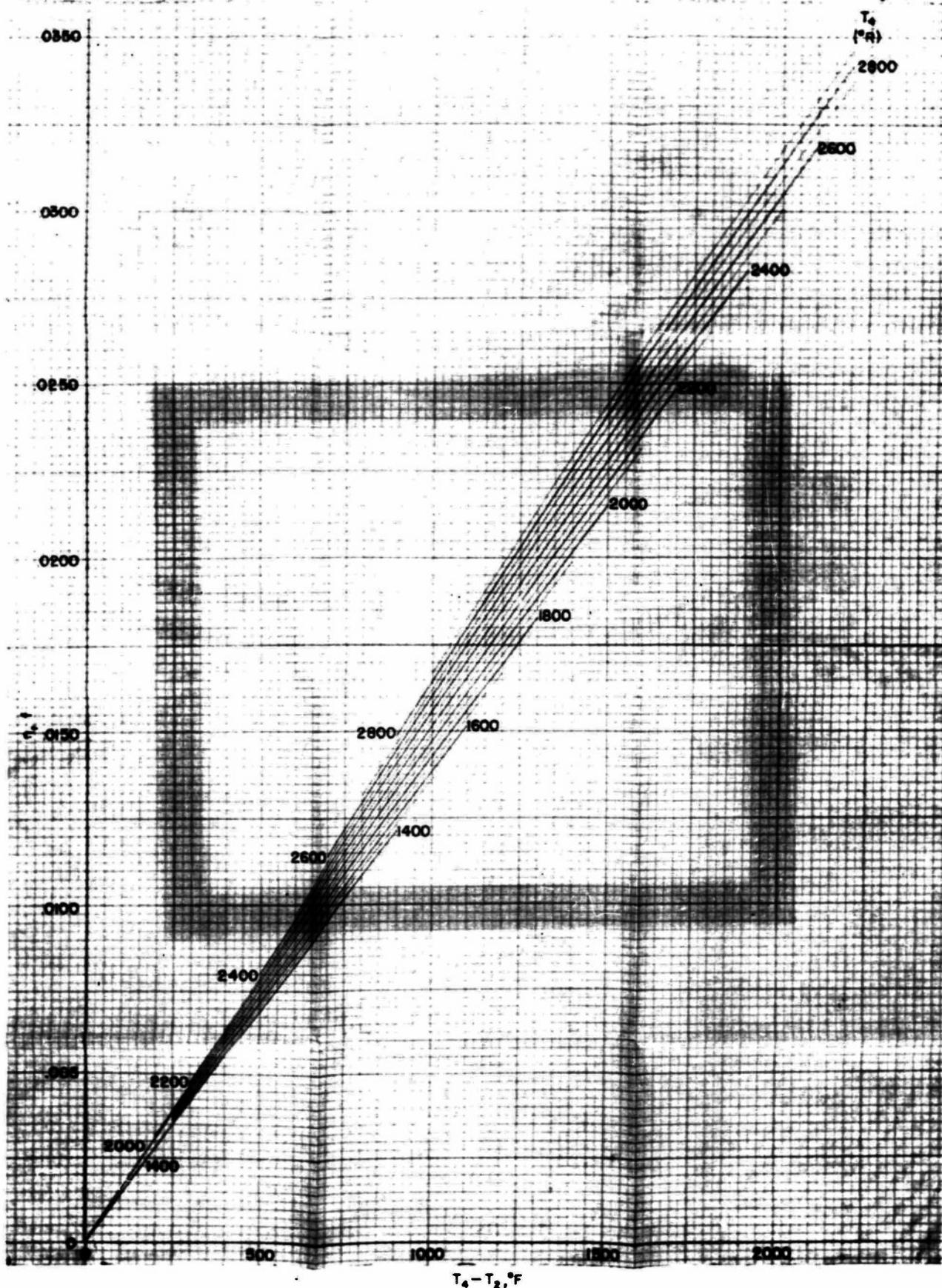


Figure 6.- Chart for determining the fuel-air ratio for various values of rise in total temperature across the combustion chamber and combustion chamber outlet total temperature. ( $h=18,900$  Btu/lb)

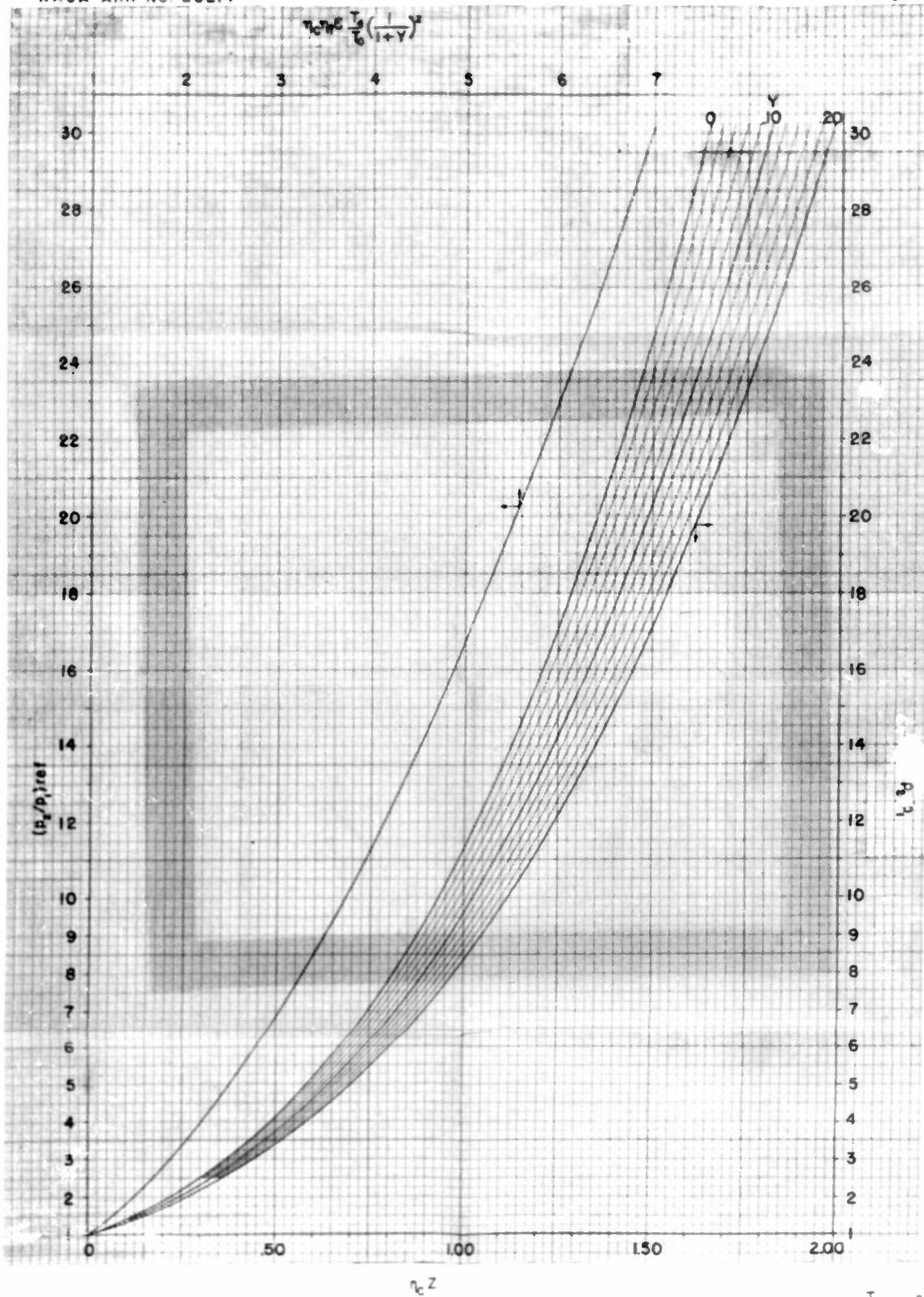


Figure 3 - Chart for determining the reference compressor pressure ratio for various values of  $\eta_c \eta_1 \epsilon \frac{T_4}{T_0} \left(\frac{1}{\gamma}\right)^2$  and the compressor pressure ratio for various values of  $\gamma$  and  $\eta_c Z$ .

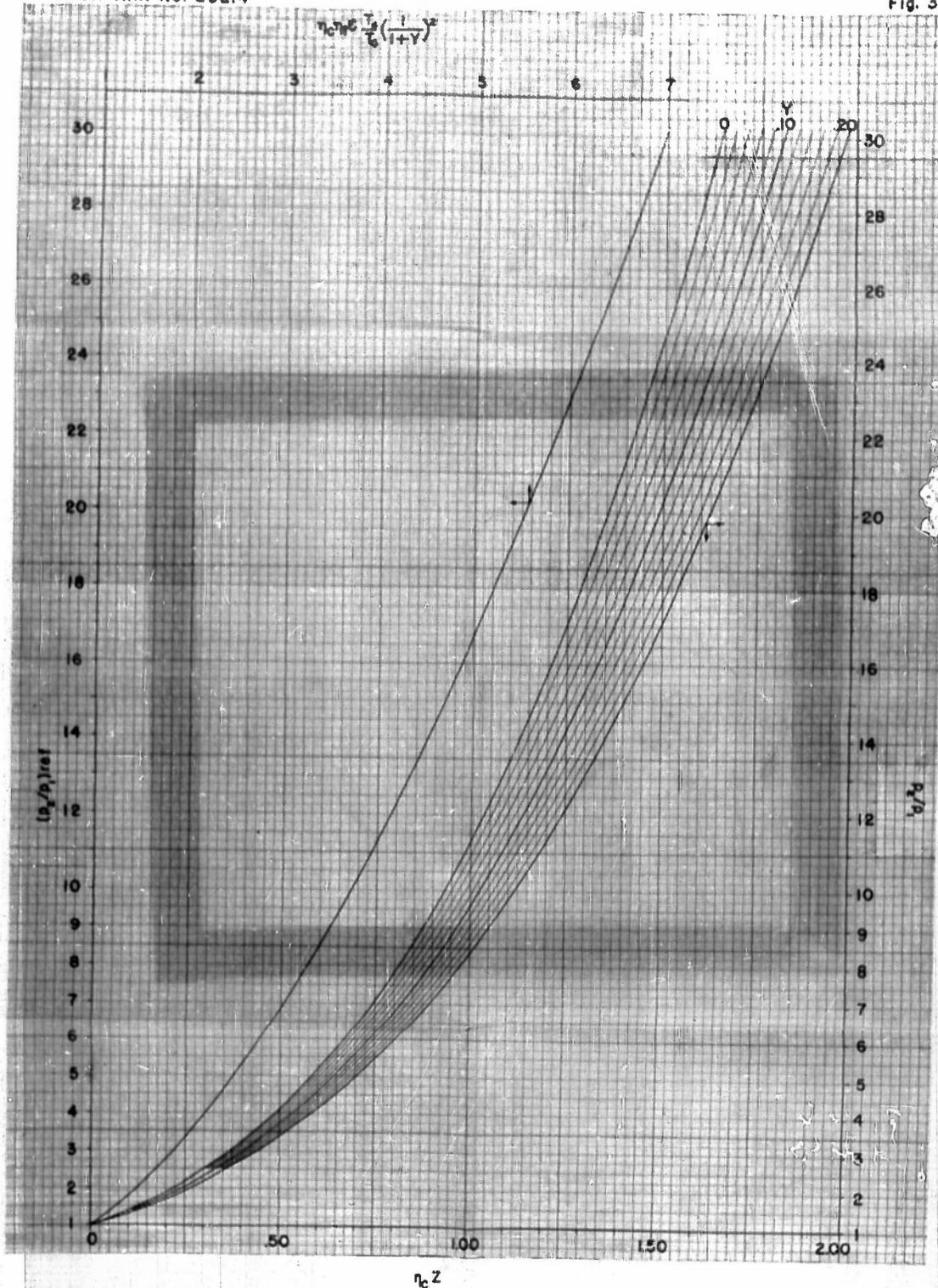


Figure 3.— Chart for determining the reference compressor pressure ratio for various values of  $\eta_c \eta_c \left( \frac{1}{1+\gamma} \right)^2$  and the compressor pressure ratio, for various values of  $\gamma$  and  $\eta_c Z$ .

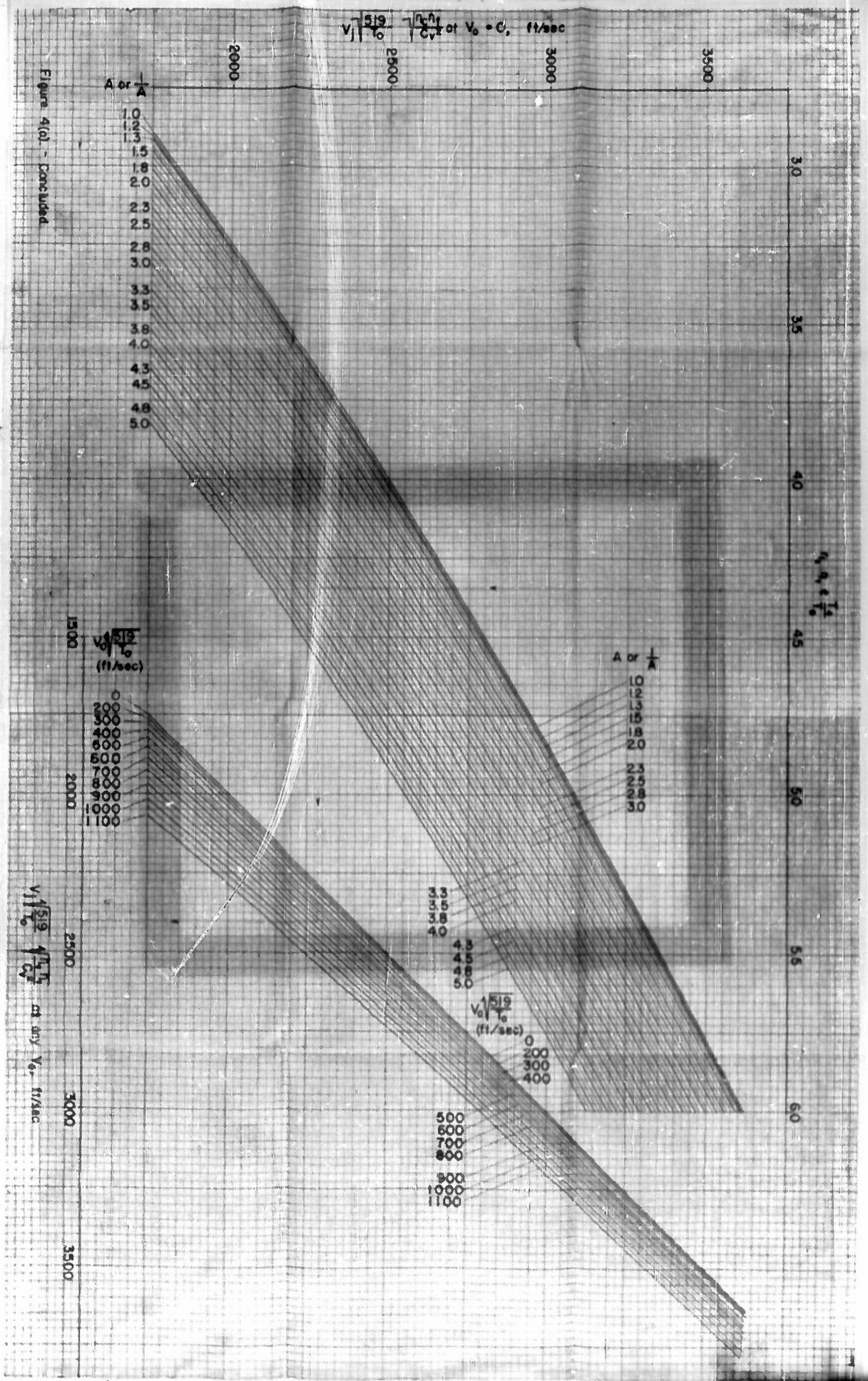


Figure 4(a) - Concluded

Fig. 4a concl.

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## ABSTRACT

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