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The Motion of an Underwater Projectile

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by

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INTRODUCTION

The motion of a projectile under water is a matter of considerable complexity about which there is comparatively little precise information although new evidence is steadily accumulating as a result of controlled tests at experimental establishments. In the present paper the approach to the problem is theoretical, an attempt being made to develop the dynamical equations sufficiently to reveal the salient features of the motion and check existing evidence. Approximate equations are formulated which, after reduction to non-dimensional form and simplification by the neglect of gravity forces, are solved in a particular case which has been studied experimentally. In the absence of precise data on hydrodynamic forces and moments, tentative estimates of these have had to be made. It is hoped that, with a greater body of fundamental evidence it will subsequently prove possible, using the theory, to make reasonably accurate forecasts of the performance of particular designs.

1. The Equations of Motion

Let P, Fig. 1, be the position of the centre of gravity of the projectile at any instant and EP the path traversed from entry. Refer the system to axes tangential and normal to the trajectory with origin P. Let

α = angle of entry relative to horizontal.

ϕ = slope of trajectory relative to horizontal.

θ = pitch upwards of projectile relative to trajectory.

V = velocity of projectile along path.

q = total angular velocity of projectile = $\dot{\theta} - \dot{\phi}$

m = mass of projectile.

g = acceleration of gravity

B = moment of inertia of projectile in pitch about centre of gravity.

k = radius of gyration in pitch about centre of gravity.

ℓ = overall length of projectile.

S = representative area, e.g. area of base of conical nose of projectile.

s = distance along trajectory from entry.

σ = s/ℓ

L = hydrodynamic lift normal to trajectory.

D = hydrodynamic drag along trajectory.

M = hydrodynamic nose-up pitching moment.

C_L = lift coefficient in rectilinear motion at fixed incidence and speed.

C_D = drag coefficient in rectilinear motion at fixed incidence and speed.

C_m = moment coefficient in rectilinear motion at fixed incidence and speed.

z_q = coefficient of lift deriving from angular velocity in pitch.

m_q = damping derivative in pitch.

ρ = density of water.

μ = $m/S\rho\ell$ = relative density coefficient.

K_1, K_2, K_3 = 'added mass' coefficients (acceleration derivatives)

F = $V^2/g\ell$ = Froude number.

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The equations of motion are

$$\left.\begin{aligned} mV\frac{dV}{ds} &= -D + mg\sin\phi \\ mV^2\frac{d\phi}{ds} &= -L + mg\cos\phi \\ B\dot{q} &= M \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

We may write approximately

$$\left.\begin{aligned} D &= K_1 mV\frac{dV}{ds} + \tfrac{1}{2}S\rho V^2 C_D \\ L &= K_2 mV^2\frac{d\phi}{ds} + \tfrac{1}{2}S\rho V\ell z_q q + \tfrac{1}{2}S\rho V^2 C_L \\ M &= -K_3 B\dot{q} + \tfrac{1}{2}S\rho V\ell^2 m_q q + \tfrac{1}{2}S\rho V^2\ell C_m \end{aligned}\right\} \quad \ldots\ldots (2)$$

The first term on the right hand side of each equation (2) is the part of the force or moment arising from acceleration, while the last term is the drag, lift or moment in rectilinear motion at constant speed and incidence. The terms containing z_q and m_q represent a force and a moment arising from the velocity of rotation q. As defined above, z_q and m_q are non-dimensional

The equations (1) may now be written

$$\left.\begin{aligned} (1+K_1)m\frac{1}{V}\frac{dV}{ds} &= -\tfrac{1}{2}S\rho C_D + \frac{mg}{V^2}\sin\phi \\ (1+K_2)m\frac{d\phi}{ds} &= -\tfrac{1}{2}S\rho C_L + \frac{mg}{V^2}\cos\phi - \tfrac{1}{2}S\rho z_q\frac{q\ell}{V} \\ (1+K_3)B\dot{q} &= \tfrac{1}{2}S\rho V^2\ell C_m + \tfrac{1}{2}S\rho V\ell^2 m_q q \end{aligned}\right\} \quad \ldots (3)$$

Write $s = \sigma\ell$

$\mu = m/S\rho\ell$

$F = V^2/g$

$B = mk^2$

so that σ expresses the path length in multiples of the projectile length, μ is a measure of the relative density of the projectile, and F is the Froude number.

The equations (3) then become

$$\left.\begin{aligned} (1+K_1)\frac{1}{V}\frac{dV}{d\sigma} &= -\frac{1}{2\mu}C_D + \frac{1}{F}\sin\phi \\ (1+K_2)\frac{d\phi}{d\sigma} &= -\frac{1}{2\mu}C_L + \frac{1}{F}\cos\phi - \frac{1}{2\mu}z_q\frac{q\ell}{V} \\ (1+K_3)\frac{\ell}{V}\frac{dq}{d\sigma} &= \frac{1}{2\mu}\left(\frac{\ell}{k}\right)^2 C_m + \frac{1}{2\mu}\left(\frac{\ell}{k}\right)^2 m_q\frac{q\ell}{V} \end{aligned}\right\} \quad \ldots\ldots (4)$$

Again, since $q = \dot{\theta} - \dot{\phi} = \frac{V}{\ell}\frac{d}{d\sigma}(\theta - \phi)$

$$\frac{\ell}{V}\frac{dq}{d\sigma} = \frac{1}{V}\frac{dV}{d\sigma}\frac{d}{d\sigma}(\theta - \phi) + \frac{d^2}{d\sigma^2}(\theta - \phi)$$

and $\frac{q\ell}{V} = \frac{d}{d\sigma}(\theta - \phi)$

and since $F = V^2/g\ell$

$$\frac{1}{F}\frac{dF}{d\sigma} = \frac{2}{V}\frac{dV}{d\sigma}$$

Making these substitutions in the equations (4) and using a dash to denote differentiation with respect to σ, we obtain finally

$$\left.\begin{aligned}
\tfrac{1}{2}(1 + K_1)\frac{F'}{F} &= -\frac{1}{2\mu}C_D + \frac{1}{F}\sin\phi \\
(1 + K_2)\phi' &= -\frac{1}{2\mu}C_L + \frac{1}{F}\cos\phi - \frac{1}{2\mu}z_q(\theta' - \phi') \\
(1 + K_3)(\theta'' - \phi'') + \left[\tfrac{1}{2}(1 + K_3)\frac{F'}{F} - \frac{1}{2\mu}\left(\frac{\ell}{K}\right)^2 m_q\right](\theta' - \phi') &= \frac{1}{2\mu}\left(\frac{\ell}{K}\right)^2 C_m
\end{aligned}\right\} \quad ..(5)$$

The equations (5) are in non-dimensional form.

In general, the coefficients C_L, C_D, C_m are functions of the incidence θ and of the dimensionless ratios R, F and Q where

R is the Reynolds Number $= \frac{V\ell}{\nu}$

Q is the Cavitation Number $= (p_H - p_C)/\tfrac{1}{2}\rho V^2$

ν is the Kinematic Viscosity of water (6)

p_H is the static pressure at the depth of the projectile.

p_C is the pressure in the cavity behind the projectile

2. Approximations to the Equations of Motion

In many cases of interest in underwater ballistics, the speed of the projectile, at any rate during the earlier stages of the motion, is sufficiently high for the gravitational forces to be negligible in comparison with the hydrodynamic ones. When this is so, the terms $\frac{1}{F}\sin\phi$ and $\frac{1}{F}\cos\phi$ in equations (5) are negligible compared with $\frac{1}{2\mu}C_D$ and $\frac{1}{2\mu}C_L$. In what follows we shall suppose this so.

Elimination of F and ϕ from the third equation (5) by means/

means of the first two gives

$$\theta'' + \frac{1}{2\mu}\left[\frac{1}{1+K_2}\frac{\partial C_L}{\partial\theta} - \frac{1}{1+K_1}C_D - \frac{1}{1+K_3}\left(\frac{\ell}{K}\right)^2 m_q\right]\theta'$$

$$-\left(\frac{1}{2\mu}\right)^2\left[\frac{1}{1+K_1}C_D + \frac{1}{1+K_3}\left(\frac{\ell}{K}\right)^2 m_q\right]\left(\frac{1}{1+K_2}C_L\right) = \frac{1}{2\mu}\left(1-\frac{1}{2\mu}\frac{1}{1+K_2}z_q\right)\left(\frac{\ell}{K}\right)^2\frac{1}{1+K_3}C_m \quad \ldots (7)$$

3. <u>Application to the motion of a symmetrical projectile in the phase prior to touch-down of the tail on the cavity wall.</u>

It is clear that in this case the lift and moment coefficients are odd functions of θ, while the drag coefficient is an even function. We may, therefore, write

$$\left.\begin{aligned} C_L &= a_o\theta + a_1\theta^3 + a_2\theta^5 + \ldots \\ C_D &= b_o + b_1\theta^2 + b_2\theta^4 + \ldots \\ C_m &= c_o + c_1\theta^3 + c_2\theta^5 + \ldots \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (8)$$

Where a_o, b_o, c_o etc. are functions of the parameters (6). We will suppose that, for the displacements occurring in the motion, the series (8) are sufficiently closely represented by their first terms, so that

$$\left.\begin{aligned} C_L &= a_o\theta \\ C_D &= b_o \\ C_m &= c_o\theta \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and furthermore that a_o, b_o, c_o may be assumed constant.

We will furthermore suppose, in the interests of formal simplicity, that the multipliers involving K_1, K_2 and K_3 are absorbed into the respective coefficients C_D, C_L or z_q, c_m or m_q, to which they may be regarded as corrections.

The equations (5) and (7) now become

$$\left.\begin{aligned} \frac{F'}{F} &= -\frac{b_o}{\mu} \\ \phi' &= -\frac{1}{2\mu}\left[a_o\theta + z_q(\theta' - \phi')\right] \\ \theta'' + \frac{1}{2\mu}\left[a_o - b_o - \left(\frac{\ell}{k}\right)^2 m_q\right]\theta' &- \left(\frac{1}{2\mu}\right)^2\left[a_o\left\{b_o + \left(\frac{\ell}{K}\right)^2 m_q\right\} + (2\mu - z_q)\left(\frac{\ell}{k}\right)^2 c_o\right]\theta = 0 \end{aligned}\right\} (10)$$

From the last equation it follows that stability of the motion requires

$$\left.\begin{aligned} &a_o - b_o - \left(\frac{\ell}{K}\right)^2 m_q > 0 \\ &a_o\left\{b_o + \left(\frac{\ell}{k}\right)^2 m_q\right\} + (2\mu - z_q)\left(\frac{\ell}{k}\right)^2 c_o < 0 \end{aligned}\right\} \ldots\ldots\ (11)$$

The equations (2) may be written

$$L = K_2 mV^2\frac{d\phi}{ds} + \tfrac{1}{2}S\rho V^2(a_o\theta + z_q\frac{q\ell}{V})$$

$$M = -K_3B\dot{q} + \tfrac{1}{2}S\rho V^2\ell(c_o\theta + m_q\frac{q\ell}{V})$$

$q\ell$ is the velocity, normal to the path, of a point on the projectile distant ℓ from the centre of gravity, and so $q\ell/V$ represents the angle of attack which the incident water makes with the surface element at distance ℓ from the centre of gravity by virtue of the rotational velocity q. Thus, if h_o be the distance of the centre of pressure on the nose from the centre of gravity, the nose is effectively at an incidence $\theta - \frac{qh_o}{V}$ to the stream and the lift on it is therefore,

$$\tfrac{1}{2}S\rho V^2 a_o(\theta - \frac{qh_o}{V})$$

It follows that

$$\left.\begin{aligned} z_q &= -\frac{h_o}{\ell}a_o \\ \text{and similarly, } m_q &= -\frac{h_o}{\ell}c_o \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

The third equation (10) is thus

$$\theta'' + \frac{1}{2\mu}\left[a_o - b_o + (\frac{\ell}{k})^2\frac{h_o}{\ell}c_o\right]\theta' - (\frac{1}{2\mu})^2\left[a_ob_o + 2\mu(\frac{\ell}{k})^2c_o\right]\theta = 0 \qquad \ldots. (13)$$

The equations (10) and (13) are linear equations with constant coefficients and may be directly integrated. The first equation gives

$$F = F_oe^{-b_o\sigma/\mu}$$

$$\text{or } V = V_oe^{-b_o\sigma/2\mu} = V_oe^{-b_os/2\mu\ell} \qquad \ldots\ldots\ldots\ldots\ldots (14)$$

The second and third equations are independent of V, from which it follows that the trajectory and attitude of the projectile are independent of the entry velocity except in so far as the drag, lift and moment coefficients depend on speed. That they do, in fact, vary with the latter, owing to the variation in the degree of cavitation with speed, is one reason why the actual trajectory depends on the entrance velocity. The other important influence is gravity.

4. <u>Estimation of the numerical values of the coefficients in particular cases</u>.

The calculation of the values of a_o, b_o and c_o in any particular case is difficult. Birkhoff and others (1), using a hypothesis originating from Newton, have suggested that the forces may be estimated by supposing the excess pressure at any point of the wetted surface to be $\frac{1}{2}\rho V_n^2$ per unit area, where V_n is the component of forward speed normal to the surface. This hypothesis is known to give values of the force coefficients of the right order in certain cases capable of exact solution, e.g. flat plate and sphere, but it results in zero pitching moment on the flat plate at any incidence, which is contrary to fact. Applied to a right circular cone of semi-angle β and base S with axis inclined at θ to the undisturbed flow/

flow, Birkhoff's hypothesis gives

$$\left.\begin{aligned} C_L &= \sin\theta\left[\cos 2\beta + \tfrac{1}{4}\sin^2\theta(1 + 5\cos 2\beta)\right] \\ C_D &= \cos^3\theta\sin^2\beta\left[1 + \tfrac{3}{2}\tan^2\theta\cot^2\beta\right] \\ C_m &= \frac{h_o}{2\ell}\sin 2\theta\left[\cos^2\beta - \frac{2}{3}\frac{h}{h_o}\sin^2\beta\right] \end{aligned}\right\} \quad \ldots\ldots\ldots (15)$$

where h_o is the distance of the centre of pressure from the centre of gravity and h is the height of the cone. The centre of pressure is distant $\frac{1}{3}h$ from the base of the cone.

Thus, neglecting K_1, K_2 and K_3, we have

$$\left.\begin{aligned} a_o &= \cos 2\beta \\ b_o &= \sin^2\beta \\ c_o &= \frac{h_o}{\ell}\left(\cos^2\beta - \frac{2}{3}\frac{h}{h_o}\sin^2\beta\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

In the above, the wetted area has been taken as the surface of the cone. It may be noted that with a sharp nosed cone, breakaway of the flow in pure cavitated flow cannot occur before the base edge, whatever the Cavitation Number, for the curvature of the streamlines away from the face, were breakaway to occur earlier, would imply a lower pressure within the fluid than at the surface of breakaway.

5. <u>Particular case of the theory</u>.

For a cone of semi-angle 30° the relations (16) give

$$\left.\begin{aligned} a_o &= 0.5 \\ b_o &= 0.25 \\ c_o &= \frac{h_o}{\ell}\left[\frac{3}{4} - \frac{1}{6}\frac{h}{h_o}\right] \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The following values apply to a model of a cylindrical rocket projectile with a 60° conical nose tested at Glen Fruin.

Weight 0.766 lb.

Dia. of base of cone 0.75 ins.

ℓ = 9.4 ins.

k = 2.58 ins.

Centre of gravity 3.93 ins from nose

Thus

$$\left.\begin{aligned} 2\mu &= 10.2 \\ (\ell/k)^2 &= 13.28 \end{aligned}\right\}$$

$$\left.\begin{aligned} h &= 0.65 \text{ ins.} \\ h_o &= 3.6 \text{ ins.} \\ c_o &= 0.27 \\ z_q &= -0.19 \\ m_q &= -0.1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

The equations (10) and (13) become

$$\left.\begin{aligned} V &= V_o e^{-0.0245\sigma} \\ \phi' &= -0.0481\theta + 0.0184\theta' \\ \theta'' + 0.159\theta' - 0.353\theta &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

The negative term in the last of these equations shows that the motion is unstable owing to the great preponderance of the term in c_o

The solution for θ has the form

$$\theta = Ae^{0.52\sigma} + Be^{-0.679\sigma} \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

where A and B are to be determined.

6. Initial Conditions.

Suppose that the entry conditions can be regarded as equivalent to a nose impulse I normal to the path. This will produce instantaneously an angular velocity θ_o and a transverse velocity w_o where

$$B\dot{\theta}_o = Ih_o$$

$$m w_o = I$$

The sideways velocity results in a negative pitch θ_o where

$$\theta_o = -\frac{w_o}{V_o} = -\frac{I}{mV_o}$$

accompanied by a reduction in ϕ equal to $-\theta_o$, $\phi - \theta$ remaining unchanged.

Again $\dot{\theta}_o = Ih_o/B = Ih_o/mk^2$

so that

$\theta_o' = (I/mV_o)(\ell/k)^2 h_o/\ell = 5.08I/mV_o$ in the particular example above

Then

$$A + B = -I/mV_o$$

$$0.52A - 0.679B = 5.08I/mV_o$$

$$\theta = \left[3.673e^{0.52\sigma} - 4.673\,e^{0.679\sigma}\right] I/mV_0$$

and .... (21)

$$\theta' = \left[1.909e^{0.52\sigma} + 3.172\,e^{-0.679\sigma}\right] I/mV_0$$

The following values are obtained

Table I

σ	$\theta / \frac{I}{mV_0}$	$\theta' / \frac{I}{mV_0}$	$R \frac{I}{mV_0}$
0	-1	5.080	-5.504
1	3.806	4.819	8.306
2	9.182	6.214	2.393
3	16.853	9.490	1.232
4	29.054	15.470	0.704

Suppose, for example, that $\theta = 5.5^0$ when $\sigma = 4$, i.e when the projectile has travelled four lengths from the instant of entry of the centre of gravity

Then $I/mV_0 = 0.003304$

The angular velocity $\frac{V}{\ell}\theta'$ is then 0.065V

The velocity has fallen to $0.907V_0$, so that for an entry velocity $V_0 = 300$ ft/sec, V = 272 and the angular velocity relative to the trajectory is 17.75 radians per second.

The corresponding initial path deflection and angular velocity are 0.19^0 and 6.43 radians per second.

The mean of these angular velocities in pitch is 12.09 radians per second, a value in good agreement with observation.

The radius of curvature, R, of the path is

$$\left|\frac{ds}{d\phi}\right| = \left|\ell / \phi^1\right| = \frac{(2\mu - z_q)\,\ell}{a_0\theta + z_q\theta'} \quad \text{............... (22)}$$

Instantaneous values of the radius of curvature in feet for the particular conditions above are quoted in Table II. The radius of curvature ranges from -1666 ft. at $\sigma = 0$ to 213 ft. at $\sigma = 4$ and changes sign at an infinite value when $\sigma = 0.58$ when the path is instantaneously straight.

7. <u>Motion subsequent to contact of the tail with the cavity wall</u>.

When the tail of the projectile touches down on the cavity wall, new hydrodynamic forces and moments are brought into play and the force and moment coefficients change their values.

Let/

Let the equation of pitching motion (10) prior to touch down of the tail be written

$$\theta'' + P\theta' - Q\theta = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Write $\psi = \theta - \theta_1$, where θ_1 is the incidence at which the tail first touches the cavity wall. The above equation is then equivalent to

$$\psi'' + P\psi' - Q\psi = Q\theta_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

It is plausible to assume that, for incidences greater than θ_1, the tail forces and moments may be expressed as functions of ψ. The equation (23) then assumes the form

$$\psi'' + (P + \xi)\psi' - (Q + \eta)\psi = (Q + \zeta)\theta_1 \quad \ldots\ldots\ldots (25)$$

where ξ, η and ζ are functions of ψ to be determined.

In this second phase of the motion write

$$\begin{aligned} C_L &= a_o\theta + C_L' \\ C_D &= b_o + C_D' \\ C_m &= c_o + C_m' \\ z_q &= -\frac{h_o}{l} a_o + z_q' \\ m_q &= -\frac{h_o}{l} c_o + m_q' \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

From equation (7) again simplified by the absorption of K_1, K_2 and K_3 into the coefficients, it follows that

$$\begin{aligned} \xi &= \frac{1}{2\mu}\left[\frac{\partial C_L'}{\partial\psi} - C_D' - \left(\frac{l}{k}\right)^2 m_q'\right] \\ \cdot\; &= \left(\frac{1}{2\mu}\right)^2 \left[\left\{C_D' + \left(\frac{l}{k}\right)^2 m_q'\right\}(a_o + C_L'/\psi) + \left\{b_o - \left(\frac{l}{k}\right)^2 \frac{h_o}{l} c_o\right\} C_L'/\psi \right. \\ &\quad \left. + \left(\frac{l}{k}\right)^2 \left\{-z_q' c_o + \left(2\mu + \frac{h_o}{l} a_o - z_q'\right) C_m'/\psi\right\}\right] \\ &= \left(\frac{1}{2\mu}\right)^2 \left[a_o\left\{C_D' + \left(\frac{l}{k}\right)^2 m_q'\right\} - \left(\frac{l}{k}\right)^2 z_q' c_o\right] \end{aligned} \quad \ldots\ldots (27)$$

Ref. 2 contains the results of experiments on the towing of cylinders in the seaplane tank at R.A.E. Lift, drag and moment were measured over a range of incidence and draught and the results plotted as functions of the draught at constant incidence and the incidence at constant draught. Neither of these plots is representative of the conditions when a projectile penetrates its cavity wall, for then the draught and incidence are varying simultaneously. Moreover, the cavity surface is not plane but curved, and is not stationary relative to the centre of gravity. For the existence of lift on the nose implies the creation of downwards momentum in the fluid. This will result in an asymmetric cavity and a tendency for the cavity to move down as the nose goes up i.e. the cavity wall will tend to move down with the tail.

Whatever the law of variation of the tail forces and moments with incidence, it is unlikely that they can be regarded as linear functions of the tail incidence over any but an infinitesimal range of angular displacement.

An application of Birkhoff's hypothesis leads to the conclusion that, for small degrees of immersion, and a fixed position of the cavity wall relative to the C.G., the lift at the tail should be proportional to $(\psi^{3/2})\epsilon$ where ϵ is the inclination of the axis of the projectile to the cavity wall. (See Fig. II).

Furthermore, since $\epsilon = \epsilon_0 + \psi$, where ϵ_0 is the incidence at first contact, the lift should be proportional to $\epsilon_0\psi^{3/2} + \psi^{5/2}$

In the case of the pitching moment, there is the further complication that a leverage about the centre of gravity, varying with the degree of immersion, is involved. It is clear, however, that to a first approximation, the moment, like the lift, should be proportional to $\psi^{3/2}\epsilon$. The drag, on the same hypothesis, is proportional to ϵ^2 and may, therefore, be neglected compared with the lift.

With forces and moments proportional to $\psi^{3/2}$ the equation (24) is solvable only by successive approximation. Since the object of the present analysis is merely to illustrate the salient features of the motion, we propose, in the absence of definite evidence to the contrary, to assume that the tail lift and moment are proportional to ψ over the small range of this variable occurring in the motion, and write

$$\left.\begin{aligned} C_L' &= a_1\psi \\ C_D' &= 0 \\ C_m' &= -c_1\psi \end{aligned}\right\} \qquad \text{.............................} \quad (28)$$

where the coefficients are, for convenience, based on the same area as that used in defining the nose coefficients.

Since the tail is effectually at the incidence $\epsilon + qh_1/V$ where h_1 is the distance of the centre of pressure of the tail from the centre of gravity, and since $\frac{\partial C_L'}{\partial \epsilon} = \frac{\partial C_L'}{\partial \psi}$, etc., it follows that

$$\left.\begin{aligned} z_q' &= \frac{h_1}{l} a_1 \\ m_q' &= -\frac{h_1}{l} c_1 \end{aligned}\right\} \qquad \text{........................} \quad (29)$$

The results (27) now become

$$\left.\begin{aligned} \xi &= \frac{1}{2\mu}\left[a_1 + \frac{h_1}{l}\left(\frac{l}{k}\right)^2 c_1\right] \\ \eta &= -\left(\frac{1}{2\mu}\right)^2\left[\left(\frac{l}{k}\right)^2\left\{\frac{h_0+h_1}{l}(a_0c_1 + a_1c_0) + 2\mu c_1\right\} - a_1b_0\right] \\ \zeta &= -\left(\frac{1}{2\mu}\right)^2\left(\frac{l}{k}\right)^2\frac{h_1}{l}(a_0c_1 + a_1c_0) \end{aligned}\right\} \;\ldots\; (30)$$

With/

With an average value for h_1, ξ, η, ζ reduce to constants and the equation (24) becomes integrable. When the lift and moment are assumed proportional to a higher power of ψ than the first, ξ, η, ζ are zero when $\psi = 0$ and increase steadily in numerical value with ψ.

Stability of the system is assured if $P + \xi > 0$ and $Q + \eta < 0$. These replace the conditions $P > 0$, $Q < 0$ relevant before touch down of the tail. The equilibrium incidence is determined by $\psi'' = \psi' = 0$. If the equilibrium values be denoted $\bar{\psi}$ and $\bar{\theta}$.

$$\frac{\bar{\psi}}{\theta} = -\frac{Q+\zeta}{Q+\eta}, \quad \frac{\bar{\theta}}{\theta} = \frac{\eta-\zeta}{Q+\eta} \quad \ldots\ldots\ldots\ldots\ldots (31)$$

8. <u>Particular Example</u>

There are at present no reliable experimental results from which values of a_1 and c_1 can be estimated. By way of example therefore a typical value will be selected for the equilibrium radius of curvature of the trajectory as observed in tests on models.

We make the following assumptions

$$\left.\begin{array}{lr} \text{(1) Angle at touch down of tail on cavity wall} & \theta_1 = 5.5^\circ \\ \text{(2) Equilibrium radius of curvature} & R = 65 \text{ ft} \\ \text{(3) } c_1 = \frac{h_1}{\ell} a_1 & \\ \text{(4) } \frac{h_1}{\ell} = 0.42 & \end{array}\right\} (32)$$

The last value corresponds with a mean centre of pressure of the tail forces one sixth of the total length from the trailing edge. The third assumption results in ξ, η and ζ being proportional to a_1. With the numerical values previously used (see (18))

$$\left.\begin{array}{l} \xi = 0.328a_1 \\ \eta = -0.594a_1 \\ \zeta = -0.568a_1 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

The radius of curvature is determined from the equation connecting ϕ and θ. When the tail forces are present this becomes

$$\phi' = -\frac{1}{2\mu}\left[a_0\theta + a_1\psi + z_q(\theta' - \phi')\right] \quad \ldots\ldots\ldots (34)$$

where z_q is the total derivative as given in (26)

$$\text{Then } R = \left|\frac{ds}{d\phi}\right| = \left|\frac{\ell}{\phi'}\right| = \frac{(2\mu - z_q)\ell}{a_0\theta + a_1\psi + z_q\theta'} \quad \ldots\ldots (35)$$

Equation (35) replaces the previous equation (22)

The/

The equilibrium radius of curvature is, therefore

$$R = \frac{(2\mu - z_q)\ell}{a_0\bar{\theta} + a_1\bar{\psi}} \qquad \text{.......................... (36)}$$

Insertion of the values of $\bar{\theta}$ and $\bar{\psi}$ as determined by (31) and (33) results in a quadratic equation for a_1 in terms of $R\theta$, With the numerical values of (32) we obtain

$$\left.\begin{array}{l} a_1 = 3.36 \\ \text{and by (31)} \\ \bar{\theta} = 6.39^\circ \end{array}\right\} \qquad \text{.......................... (37)}$$

The following results may now be deduced.

$$\left.\begin{array}{lcl} C_L' & = & 3.36 \\ C_m' & = & -1.41 \\ z_q' & = & 1.41 \\ m_q' & = & -1.99 \\ \xi & = & 1.1 \\ \eta & = & -1.99 \\ \zeta & = & -0.086 \end{array}\right\} \qquad \text{.......................... (38)}$$

The equation (25) becomes

$$\psi'' + 1.26\psi' + 1.64\psi = 0.0217 \qquad \text{............... (39)}$$

The motion is, therefore, stable.

With the boundary conditions

$$\left.\begin{array}{l} \psi = 0 \\ \psi' = 0.051 \end{array}\right\} \qquad \text{.......................... (40)}$$

at $\sigma = 4$, which correspond with the terminal conditions in the first phase (see Table I), the solution is

$$\left.\begin{array}{rl} \psi & = 0.0156 + \{0.037 \sin 1.115(\sigma-4) - 0.0156 \cos 1.115(\sigma-4)\} e^{-0.63(\sigma-4)} \\ & = \{0.051 \cos 1.115(\sigma-4) - 0.006 \sin 1.115(\sigma-4)\} e^{-0.63(\sigma-4)} \end{array}\right\} (41)$$

The incidence, therefore, oscillates about the steady value 6.39° with steadily decreasing amplitude, each oscillation being completed in 5.64 lengths travel.

The angular velocity is zero when

$$\psi = n\pi + \text{arc tan } 8.572 \qquad \text{.................... (42)}$$

The radius of curvature at any instant may be determined from/

from equation (35) and the inclination of the path to the horizontal by integration of (34) with the appropriate functional forms of θ and θ' inserted.

Typical values are given in Table II, in which σ denotes the number of lengths travelled from the instant of entry of the centre of gravity, θ the incidence of the projectile to its path in degrees, ϕ the inclination of the trajectory to the horizontal in degrees, R the radius of curvature of the path in feet and V/V_0 the ratio of the instantaneous and initial forward speeds.

TABLE II

σ	θ	$\phi-\alpha$	R	V/V_0	Position
0	-0.19	-0.19	-1666	1	Entry
1.0	0.72	-0.20	2514	0.976	
2.0	1.74	-0.23	724	0.953	
3.0	3.19	-0.29	373	0.929	
4.0	5.50	-0.48	213	0.907	Touch down of tail
4.1	5.78	-0.56	103.5		
4.2	6.04	-0.73	82.3		
4.3	6.27	-0.83	69.7		
4.5	6.65	-0.95	59.2	0.90	
5.0	7.20	-1.46	43.2		
5.3	7.28	-1.80	41.8	0.88	Max. Incidence Min. R.
6.0	7.02	-2.48	46.7		
7.0	6.40	-3.25	64.7		
8.12	6.25	-3.95	71.2	0.84	Min. Incidence Max. R.
10.94	6.41	-5.86	64.2	0.78	2nd Max. Incidence
∞	6.39		65.0		Steady condition

There is thus an initial small refraction of the path, resulting from the assumption of an entry impulse, but the trajectory remains practically straight till touch down of the tail. For $\sigma > 10$, ϕ is sufficiently accurately obtained by assuming that the radius of curvature has the constant value 65 ft. The slope of the trajectory is then given very closely by

$$\phi = \alpha + 1.69 - 0.69\sigma \text{ degrees} \quad \ldots\ldots\ldots\ldots\ldots \text{(43)}$$

With/

With this assumption the trajectory would be horizontal at $\sigma = 24.2$ for an entry angle $= 15^\circ$, and the speed ratio would then be 0.55

The variations of θ and R with σ are shown graphically in Fig. III.

9. <u>Equations of Motion in more general case</u>.

As was mentioned earlier, the trajectory in any actual case is modified by the action of gravity and by variations of the hydrodynamic force and moment coefficients with speed. If these effects are included, the equations of motion (5) are not directly integrable but are nevertheless suitable for mechanical integration on a machine such as the Differential Analyser. The same is true of equation (25) when ξ, η and ζ are not constants.

The principal part of the effect of gravity may, however, be taken into account by including a term 1/F in the second equation (5) in place of the term $^1/F \cos\phi$. Since $1/F$ may be expressed as a function of σ by means of the first equation, in which the term $^1/F \sin\phi$ is neglected, the equations for θ and ϕ remain directly integrable.

The equations (23) and (25) are modified by the addition of a term

$$\frac{1}{2\mu}\left[b_o - \left(\frac{l}{k}\right)^2 m_q\right] \frac{1}{F_o} e^{b_o\sigma/\mu} \qquad \ldots\ldots\ldots\ldots\ldots\ (44)$$

to the right hand side, where F_o is the Froude Number at entry, and m_q has its appropriate value. The solutions for θ and ψ now contain the above term as a particular integral multiplied by the factors.

$$\left.\begin{array}{ll} & \dfrac{\mu^2}{b_o^2 + b_o\mu P - \mu^2 Q} \\[2ex] \text{and} & \dfrac{\mu^2}{b_o^2 + b_o\mu(P + \xi) - \mu^2(Q + \eta)} \end{array}\right\} \qquad \ldots\ldots\ (45)$$

respectively.

There is no longer an equilibrium radius of curvature in the second phase, but a fictitious equilibrium radius may be defined as that which would be attained were gravity absent.

The solutions (21) and (41) have been recalculated assuming the entry impulse to result in a pitch of 5.5° at $\sigma = 4$ as before, and the fictitious equilibrium radius to be 65 ft.

The value of I/mV_o becomes 0.003,288 and equation (21) contains the additional term

$$(76.8e^{0.52\sigma} + 49.6e^{-0.679\sigma} - 126.4e^{0.049\sigma})10^{-6} \qquad \ldots\ (46)$$

The nett effect in the phase prior to touch-down of the tail is therefore a modification of the previous solution for by/

by the addition of the term

$$(18.032e^{0.52\sigma} + 124.368e^{-0.679\sigma} - 126.4e^{0.049\sigma})10^{-6} \quad \ldots (47)$$

The effect is insignificant.

The solution for ψ, equation (41), becomes, to the same order of accuracy.

$$\psi = 0.0156 + \{0.0368 \sin 1.115(\sigma - 4) - 0.0161 \cos 1.115(\sigma - 4)\} e^{-0.63(\sigma - 4)}$$
$$+ 0.000{,}551 e^{0.049(\sigma - 4)} \quad \ldots\ldots\ldots\ldots (48)$$

Once again, the change is insignificant in the important range of values of σ.

The radius of curvature and inclination of the trajectory at any point may be obtained as before.

10. Application to scale models.

Suppose that the characteristics of an underwater projectile are studied by testing a geometrically similar model of correct inertial scaling. The values of μ and k are then the same for the model as for the full scale and, if the test be made at the same value of F as applies to the full scale, the gravity forces will bear the same ratio to the hydrodynamic forces in both cases. If the effects of the Reynolds and Cavitation Numbers on the hydrodynamic forces can be neglected, the equations (5) determining the motion in the two cases will be identical and the trajectories geometrically similar. Furthermore the attitudes at corresponding points and their space rates of change in terms of σ will likewise be identical.

If the velocities on full scale and in the model test are both sufficiently high for gravity forces to be negligible in the part of the motion under study, equality of Froude Number is of no significance and attention may be directed to the other parameters. By what has been said earlier, § 4, it appears unlikely that Cavitation Number is an important variable on sharp-nosed cone-headed projectiles in the phase prior to touch down of the tail on the cavity wall. But although the edge of breakaway on the nose may, in this case, be supposed independent of Cavitation Number, the extent of the cavity, and hence the conditions at the tail, will vary considerably with the latter. So long as the cavity remains open to the air, or approximately at atmospheric pressure, Froude scaling will result in equality of Cavitation Number since the pressure difference $p_H - p_C$ (See (6)) will then be directly proportional to the depth. In general, however, unless the atmospheric pressure be reduced during the model test, the Cavitation Number will be higher for the model than for the full scale projectile and the cavity smaller relative to the model in consequence.

Better agreement between model and full scale Cavitation Numbers is likely to be obtained if the model test is made at a higher speed than Froude scaling would indicate, and this has the beneficial effect of increasing the Reynolds Number, although the discrepancy between model and full scale must remain very large./

large. Reynolds Number is, however, unlikely to be of much significance.

11. Conclusions.

The theory developed in this paper can be regarded as only approximate, nevertheless the conclusions are in accordance with observation so far as the salient features of the motion of an underwater projectile in cavitated flow are concerned. These are a preliminary phase, prior to touch down of the tail on the cavity wall, during which the path remains practically straight, followed by a second phase, after touch down, in which the path rapidly assumes constant curvature while the projectile executes a heavily damped oscillation about a stable position of equilibrium. In certain cases, the tail of the projectile has been observed to leave the cavity wall for a short space during the initial stages of this second phase which continues till the velocity has fallen sufficiently for the cavity to change radically or for gravity forces to become important.

In the particular example of the paper, which related to an underwater rocket projectile having a 60° conical nose, the forces on the nose of the projectile were calculated on an approximate theory and the tail forces were then deduced from assumptions based on the observation of actual trajectories. The computed motion was found to be in good agreement with estimates from observation, and this encourages the hope that with more detailed and systematic information obtained from further experiments, it may be possible, at a later stage, to use the present theory to estimate the characteristics of an underwater projectile with good accuracy.

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G.F.4.

FIG. 1.

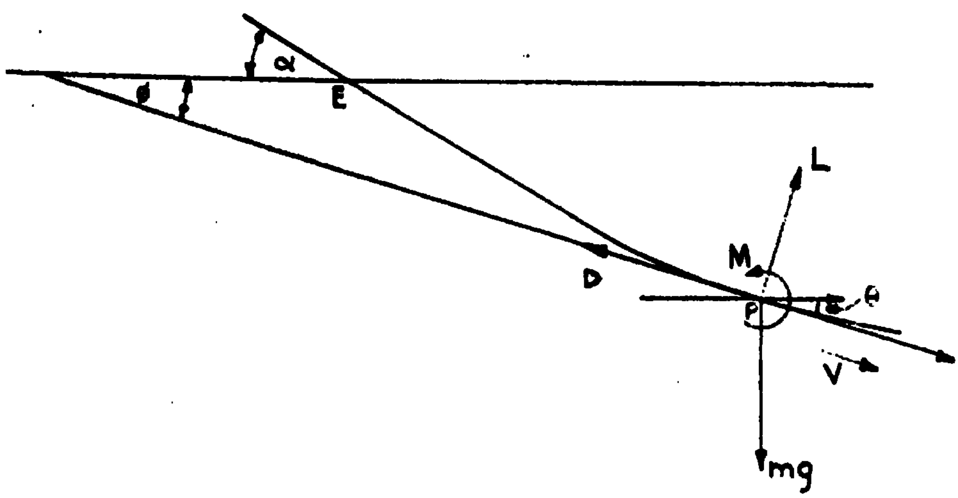
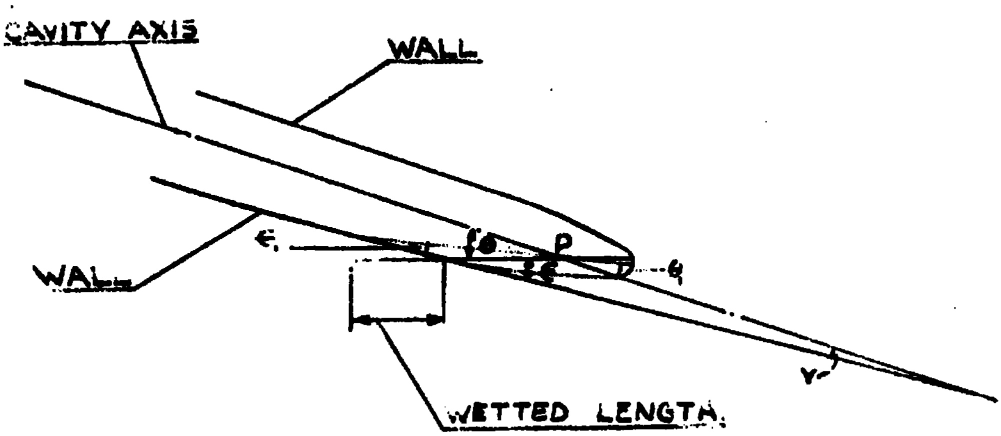
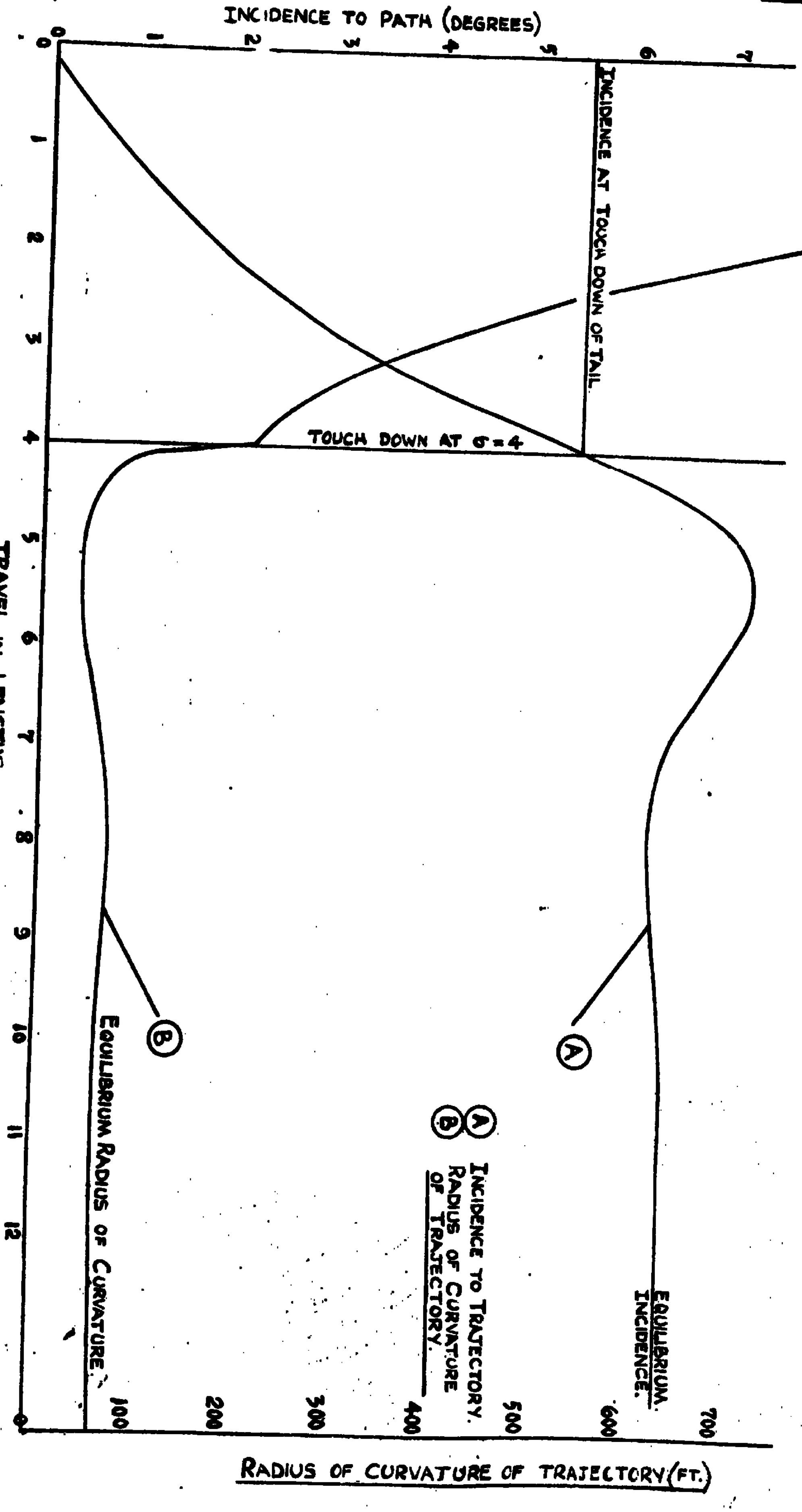


FIG. 2



G.F. 4

FIG. III



INCIDENCE TO PATH (DEGREES)
0
1
2
3
4
5
6
7
INCIDENCE AT TOUCH DOWN OF TAIL.
TOUCH DOWN AT σ = 4
TRAVEL IN LENGTHS:
0
1
2
3
4
5
6
7
8
9
10
11
12
A
B
A INCIDENCE TO TRAJECTORY.
B RADIUS OF CURVATURE OF TRAJECTORY.
EQUILIBRIUM INCIDENCE.
EQUILIBRIUM RADIUS OF CURVATURE.
100
200
300
400
500
600
700
RADIUS OF CURVATURE OF TRAJECTORY (FT.)

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