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Note on longitudinal stability and trim changes at speeds near the speed of sound

- by -

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SUMMARY

By considering a typical aircraft design at both low subsonic and high supersonic speeds, where its aerodynamic characteristics can be roughly calculated by simple theory, some indications of the nature of the changes in longitudinal stability and trim of an aircraft to be expected on passing through the speed of sound are obtained. It is shown that:-

(1) The longitudinal stability will increase at supersonic speeds by as much as $0.46\bar{c}$ aft movement of the neutral point. Of this, $0.25\bar{c}$ is due to aft movement of the aerodynamic centre of the main wing, and the remainder to the disappearance of the downwash at the tailplane and the changed slopes of wing and tailplane lift curves.

(2) With present day aircraft and wing sections, a nose-down moment must be expected on passing from subsonic to supersonic speeds. It is estimated that this may require up to 7° of negative elevator to correct in a typical case. Methods of minimising this change by choice of tail-setting and of wing section are discussed.

(3) If this change in elevator angle is not applied by the pilot, the aircraft will trim at supersonic speeds at a small negative lift coefficient - roughly -0.1 if no change in elevator angle is made.

The majority of experiments have so far been made at high subsonic speeds, so that they examine only the beginning of these changes. They show, however, increases in longitudinal stability and in nose-down moment of the same sign and of the same order of magnitude as would be expected from the present estimates.

1. Introduction

The whole range of speeds of aircraft moving through air can be divided into three distinct régimes. The lowest speeds, up to the point where shock-waves first appear in the flow, have already been examined extensively both by experiments and calculation. The highest speeds, where the flow is everywhere supersonic, have also been studied a great deal, and although knowledge of the flow characteristics is by no means complete, a fair amount of consistent experimental and theoretical work has been done.

The intermediate region, near the speed of sound (say between Mach numbers of 0.7 and 1.3) is proving the most difficult to study, both experimentally and theoretically. By the use of high-speed wind tunnels and by dive tests on high speed aircraft, the lower part of this region is now being explored. Large changes in the aerodynamic characteristics of aircraft[1,2], occasionally dangerous in their effects, are being found.

Chief among these effects, on complete aircraft, is a large increase in the drag and an increasing nose-down change of trim of the aircraft as the Mach number increases.

The present report considers this change of trim, which is one of the most dangerous features of flight at high Mach number, by examining the difference between the longitudinal moments of typical aircraft at very low subsonic and very high supersonic speeds, i.e. at the two sides of the transition region near $M = 1$. It is shown, in fact, that with current designs of aircraft a nose-down trim change is inevitable in this transition region, and that appreciable negative elevator angles are required to counteract it. This conclusion is clearly of considerable importance to aircraft designed to fly at supersonic speeds, and methods of minimising the change are indicated. These may have some application to the diving of subsonic aircraft into the transition region, although of course the overall changes essential in reaching supersonic speeds may be marked, in this region, by other changes from the formation and movement of local shock waves.

2. Thin aerofoil theory at subsonic and supersonic speeds

A summary of thin aerofoil theory at speeds well above and well below the speeds of sound is given in Appendices I and II. With the notation used there, i.e. with the aerofoil extending from $x = -1$ to $x = +1$, and with $\cos\theta = -x$, the following aerodynamic characteristics at infinite aspect ratio are obtained:-

Quantity	Subsonic	Supersonic
Slope of lift coefficient curve, $dC_L/d\alpha$	$2\pi \times \frac{1}{\sqrt{1-M^2}}$	$4 \times \frac{1}{\sqrt{M^2-1}}$
Angle of zero lift, α_0	$\frac{1}{\pi}\int_0^{\pi} \frac{dy}{dx}(1-\cos\theta)\,d\theta$	0.
Aerodynamic centre position	Quarter chord, $x = -\frac{1}{2}$	Half chord, $x = 0$
Moment coefficient about aerodynamic centre	$\frac{1}{2\sqrt{1-M^2}}\int_0^{\pi} \frac{dy}{dx}(\cos 2\theta - \cos\theta)\,d\theta$	$-\frac{1}{2\sqrt{M^2-1}}\int_0^{\pi} \frac{dy}{dx}\sin 2\theta\,d\theta$

To apply these expressions to conventional aircraft, the effects of the vortex system produced by the lifting aerofoil have to be taken into account. At subsonic speeds the slope of the lift curve is reduced by the trailing vortices, and a downwash proportional to the wing lift is generated behind the aerofoil. At supersonic speeds it has been demonstrated that the trailing vortex system is confined to two 'Mach' cones issuing from the tips[7]. Further, the shock waves extend outwards from the aerofoil so far (according to the simple theory of Appendix II, they extend to infinity) that the downwash resulting from the bound vortex is extremely small. In the following estimates the effects of the vortex system at supersonic speeds have therefore been neglected.

It is first necessary to evaluate the expressions above for aerofoil characteristics, for the special cases of aerofoils in common use at present. This is done, for aerofoils of "conventional" and "low-drag" centre-lines, in the following sections.

2.1. Conventional wing sections

The general cubic centre-line considered by Glauert is representative of the older forms of aerofoil; in the notation of Appendix I it may be written:-

$$y = \frac{h}{4}(1 - x^2)(2 - \lambda - \lambda x)$$

where , h are arbitrary constants.

Special cases of this type of aerofoil are obtained with $\lambda = 0$ and $\lambda = 8/7$. The first of these gives a circular arc centre-line; the value $\lambda = 8/7$ gives an aerofoil with constant centre of pressure, i.e. $C_{M} = 0$, and is employed in the aerofoil R.A.F. 34. The aerodynamic characteristics of the general curve, and of these two special curves, are given in Table I. The value of h used in any particular application is dependent upon the lift coefficient at which the aerofoil is required to operate most efficiently; defining this lift coefficient, C_{LD}, as that at which the flow is tangential to the camber-line at the leading edge (see Appendix I), we have for the general cubic:-

$$C_{LD} = \pi h\left(1 - \frac{\lambda}{2}\right) \text{ at low Mach numbers.}$$

This has been used to express the no-lift angle and the subsonic and supersonic moment coefficients in Table I as a function of C_{LD}.

2.2. Low-drag sections

In the more modern low-drag sections the centre-line is chosen to give uniform loading over a fraction "a" of the chord from the leading edge, the loading then falling off to zero at the trailing edge. The simplest case of this type of loading, and the one most likely to be employed on high-speed aircraft (because it gives the lowest excess velocity on the upper surface for a given lift coefficient), is that with a = 1, i.e. uniform loading over the whole chord. This requires a constant velocity increment, $\frac{u}{V} = \mu$, on the upper surface at the design lift coefficient; and it is easily shown that the design lift coefficient, C_{LD}, is then 4μ (at low Mach number) and the equation of the aerofoil is given by:-

$$\frac{dy}{dx} = \frac{2\mu}{\pi} \log \cot \frac{\theta}{2} \quad \text{with the notation of Appendix I.}$$

Characteristics of this aerofoil are also given in Table I.

2.3. The aerofoil with flap

The characteristics of the thin aerofoil with hinged flap have already been calculated by Collar[6] for both subsonic and supersonic flow. They are included in Table I in two forms; relative to a chord-line joining leading and trailing edges, such as would be used in a supersonic double-wedge aerofoil having as camber-line two straight lines at an angle; and also relative to the chord of the unflapped portion of the wing, a form which would be applicable to control surfaces. From the second form it is seen that the effectiveness of a control is much reduced at supersonic speeds, if we compare the change in lift with changing control angle with the change with incidence. For typical aileron and elevator (or rudder), with E (control chord/total chord) = 0.2 and 0.4 respectively, Collar gives the following changes of lift-coefficient with α and β (control angle):-

Coefficient	Subsonic value $\times\sqrt{1-M^2}$ (infinite aspect ratio)	Supersonic $\times\sqrt{M^2-1}$	Aileron E = 0.2		Elevator E = 0.4	
			Subsonic $\times\sqrt{1-M^2}$	Supersonic $\times\sqrt{M^2-1}$	Subsonic $\times\sqrt{1-M^2}$	Supersonic $\times\sqrt{M^2-1}$
$a_1 = \frac{dC_L}{d\alpha}$	2π	4	6.28	4	6.28	4
$a_2 = \frac{dC_L}{d\alpha}$	$2\pi - 4\cos^{-1}\sqrt{E} + 4\sqrt{E(1-E)}$	4E	3.39	0.8	4.65	1.6

It will be seen that these figures present a strong case for the use of control surfaces occupying a much larger fraction of the main surface chord, and even of all moving surfaces, at supersonic and near-supersonic speeds.

3. Moment changes in passing through the speed of sound

3.1. General relations for longitudinal trim

Consider now an aircraft with its C.G. at k.c. behind the leading edge of the main chord c. It is assumed that the moments of the aerodynamic forces around the C.G. are balanced by the lift on an all-moving tailplane of tail volume $\left(= \frac{\text{tail area}}{\text{wing area}} \times \frac{\text{tail arm}}{\text{mean chord}} \right)$ V, at an angle η_T to the wing chord.

The change in η_T on passing through the speed of sound is obtained as follows, and the corresponding change in elevator angle η can be obtained by multiplying by $\frac{a_1}{a_2}$ as given in para. 2.3 above.

At subsonic speeds, the pitching moment coefficient due to air forces on the main wing is given by :-

$$C_M = C_{M_0} + (k - \tfrac{1}{4})C_L \qquad C_L = \text{main wing lift coefficient.}$$

This must be balanced by the moment due to the tailplane, which is at an incidence

$$\frac{C_L}{a} + \alpha_0 + \eta_T - \frac{d\epsilon}{dC_L} \cdot C_L$$

where a is the slope of the $C_L - \alpha$ curve for the main wing, and ϵ is the downwash at the tailplane due to the main wing. If a' is the slope of the tailplane C_L curve, the tail load coefficient is thus given by:

$$C_{M_0} + (k - \tfrac{1}{4})C_L = Va' \left[C_L\left(\frac{1}{a} - \frac{d\epsilon}{dC_L}\right) + \alpha_0 + \eta_T \right] \quad \ldots\ldots\ldots\ldots\ldots (1)$$

whence

$$\eta_T = -\alpha_0 + \frac{C_{M_0}}{Va'} + C_L \left[\frac{k - \frac{1}{4}}{Va'} + \frac{d\epsilon}{dC_L} - \frac{1}{a} \right] \quad \ldots\ldots (2)$$

Similarly at supersonic speeds, the tail load coefficient is given by:-

$$C_{M_0} + (k - \tfrac{1}{2})C_L = Va' \left[C_L/a + \eta_T\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and

$$\eta_T = \frac{C_{M_0}}{Va'} + C_L\left[\frac{k - \frac{1}{2}}{Va'} - \frac{1}{a}\right] \ldots\ldots\ldots\ldots (4)$$

Here the downwash at the tailplane has been neglected, for the reason given in para. 2; and the aerodynamic centre is assumed to be at 0.5 c.

3.2. Numerical values for the trim change in typical aircraft

In equations (1) to (4), the values of α_0 and C_{M_0} are those given in Table I. To obtain numerical values for the trim change, it is necessary to substitute typical values for the remaining quantities. The following values are representative of aircraft of to-day:-

tail volume $V = 0.4$
tail aspect ratio $= 3$, so that $a' = 3.8$ per radian ($M = 0$)
wing aspect ratio $= 6$, so that $a = 4.8$ per radian ($M = 0$)
downwash at tail $\epsilon = \frac{1.7}{\pi A} C_L$, so that $\frac{d\epsilon}{dC_L} = 0.090$ ($M = 0$).

At supersonic speeds $a = a' = \frac{4}{\sqrt{M^2-1}}$, so that it can be assumed, with sufficient accuracy for the present rough estimates, that.

$$Va'\sqrt{1 - M^2} \text{ (subsonic)} = Va'\sqrt{M^2 - 1} \text{ (supersonic)} = 1.6.$$

With these values of the constants, equations (2) and (4) become:-

$$\left.\begin{array}{ll} \underline{\text{Subsonic}}: & 1.6\eta_T = -1.6\alpha_0 + C_{M_0}\sqrt{1 - M^2} + C_L\sqrt{1-M^2}\left[k - 0.44\right] \\ \underline{\text{Supersonic}}: & 1.6\eta_T = C_{M_0}\sqrt{M^2 - 1} + C_L\sqrt{M^2 - 1}\left[k - 0.90\right] \end{array}\right\} (5)$$

A significant fact in these equations is that at a fixed value of $C_L\sqrt{1 - M^2}$ (subsonic) or $C_L\sqrt{M^2 - 1}$ (supersonic) the values of η_T are constant in each régime, regardless of the actual Mach number.

Now substituting from Table I, the change in η_T is obtained in terms of the design C_L (C_{LD}) of the section, the lift coefficient C_{L1} at subsonic speeds, and the lift coefficient C_{L2} at supersonic speeds. To simplify the result, a further approximation can be made, by using the fact that the margin of longitudinal stability at subsonic speeds is normally small. A sufficiently representative result is obtained if k is assumed to be 0.4, so that the stability margin is 0.04c at subsonic speeds and 0.5c. at supersonic speeds. (The very large margin at supersonic speeds may be a serious difficulty in providing good control; but may be offset in practice by the fact that the range of lift coefficient required is small). With this assumption, the change in tailsetting is:-

(1) Circular arc centre-line

$$\Delta\eta_T \text{(radians)} = -0.135\,C_{LD} - 0.5\,C_{L2}\sqrt{M^2-1} + 0.04\,C\ \sqrt{1-M^2}$$

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(2) Constant C.P. centre-line (R.A.F. 34)

$$\Delta\eta_T \text{ (radians)} = -0.186\, C_{LD} - 0.5\, C_{L_2}\sqrt{M^2 - 1} + 0.04\, C_{L_1}\sqrt{1-M^2}$$

(3) Low-drag section centre-line, a = 1.0

$$\Delta\eta_T \text{ (radians)} = -0.102\, C_{LD} - 0.5\, C_{L_2}\sqrt{M^2-1} + 0.04\, C_{L_1}\sqrt{1-M^2}$$

(4) Centre-line two straight lines, meeting at half-chord ($\bar{x}$ = 0.5)

$$\Delta\eta_T \text{ (radians)} = -0.159\, C_{LD} - 0.5\, C_{L_2}\sqrt{M^2-1} + 0.04\, C_{L_1}\sqrt{1 - M^2}.$$

It is clear from these figures that on passing through the speed of sound a change of trim is always to be expected on existing designs of aircraft, and this will always be in such a direction (at normal lift coefficients, i.e. positive values of C_{LD}, C_{L_1}, C_{L_2}) that a more negative tailsetting or elevator angle will be required. Although the low-drag section with constant chordwise loading shows some advantage over the remaining designs of aerofoil, there is not a great deal of difference between them.

A typical present-day fighter such as Spitfire has a wing loading of about 30 lb./sq.ft. and reaches its highest Mach number at about 30,000 ft. It is usually fitted with wings of conventional section very like (2) above, with a maximum camber of about 2% chord, giving C_{LD} = 0.2. The lift coefficient in level flight is $0.67 \times 10^{-3}. \frac{W_S}{M^2 p}$, where W_S is the wing loading and p the relative pressure (0.3 at 30,000 ft). If such an aircraft is accelerated through the speed of sound, the change in tailsetting to be expected between the two extremes of the transition region (say $M_1 = 0.71$ to $M_2 = 1.22$, so that $\sqrt{1 - M_1^2} = \sqrt{M_2^2 - 1} = 0.71$) will be as follows:-

$$\Delta\eta_T \text{(degrees)} = -2.13 \text{(from } C_{LD} \text{ term)} - 0.71 \text{(from } C_{L_1}, C_{L_2}) = -2.84 \quad \ldots\ldots(6)$$

This applies to level flight; the second term will be somewhat reduced in a dive. If the elevator angle to trim is zero at high subsonic speeds, as is usual in aircraft designs, then a negative elevator angle of about -7^o will be required for trim at supersonic speeds (using the value of $\frac{a_1}{a_2}$ given in para. 2.3). This change in elevator angle can be reduced to about -4^o by choosing the tailsetting so that the elevator angle to trim is zero at supersonic speeds.

In fact, however, an aircraft designed to achieve supersonic speeds will have a large wing loading (to satisfy the requirements of large power and low drag coefficient) and for strength reasons will operate at high altitude. An aircraft with a wing loading of 50 lb./sq.ft., passing through the speed of sound at 40,000 ft., will have C_{L_1} = 0.36 at M_1 = 0.71, C_{L_2} = 0.12 at M_2 = 1.22. With a wing section of type (4) above, the change in tailsetting required will be

$$\Delta\eta_T = -9.1\, C_{LD} - 1.9 \text{ degrees.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Thus with a design C_{LD} of only 0.1, elevator angles of -7^o (supersonic)

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or +4° (subsonic) will be required as above. Even with symmetrical wing sections, a change equivalent to 4.8 degrees of elevator at supersonic speeds, or 2.6 degrees at subsonic speeds, is necessary.

The above rough estimates have neglected the influence of the fuselage on pitching moments, to simplify the picture. In fact, however, the fuselage is not expected to modify greatly the values of $\Delta\eta_T$ estimated in (6) and (7). It will be seen from Table I that in all the unreflexed aerofoils, i.e. all aerofoils except the constant C.P. type, the value of $C_{M_0}\sqrt{1-M^2}$ in subsonic flow differs only slightly from the value of $C_{M_0}\sqrt{M^2-1}$ in supersonic flow. Thus in equations (5), the change in η_T due to change in C_{M_0} is usually negligible compared with that due to changes in no-lift angle α_0 and in aerodynamic centre. The same is likely to hold true in the case of the fuselage.

4. Reduction of the change in trim

Some methods of reducing the change in elevator angle to trim in passing through the speed of sound are obvious from the results of para. 3. It is clear, for instance, that a smaller change in elevator angle is required if the tailsetting is chosen to give zero elevator at supersonic speeds, rather than at high subsonic speeds. This implies that the elevator angle should be about +3 or +4 degrees at speeds just below the shock-stall.

Modification of the wing section to reduce the trim change is also possible; but here care is necessary to ensure that such modification does not conflict with other requirements. From the strength point of view, for instance, it is clear that the tail load should be as small as possible throughout, so that each side of equations (1) and (3) should be reduced to the minimum possible value. Further, as will be discussed in more detail in para. 5, the beginning of the transition region is marked by the formation of shock waves on both upper and lower surfaces, and these probably have the smallest effect on trim when the wing incidence is very near the design angle α_1 (Appendix I), i.e. when the lift coefficient is $\dfrac{C_{LD}}{\sqrt{1-M_1^2}}$.

For this reason, although equations (5) - (7) show that the change $\Delta\eta_T$ can be reduced to zero by designing the main wing section with negative camber, so that C_{LD} = -0.07 (equation (6)) to -0.21 (equation (7)), yet this is not recommended. It would conceivably lead to a strong initial change of trim on reaching shockstalling speeds, disappearing later as the aircraft accelerated to higher Mach numbers.

Of the three requirements for which the wing section must be designed, viz:-
(a) zero trim change, $\Delta\eta_T = 0$,
(b) smallest possible tail load,
(c) design lift coefficient C_{LD} appropriate to beginning of shock-stall,

the best solution of (a) and (b) only is that giving negative C_{LD}; for this will result in positive values of C_{M_0}, which will offset the usual negative moment from the fuselage. A solution of (a) and (c) only can be obtained, for any family of aerofoils, by choosing the parameters of the family to satisfy $\Delta\eta_T = 0$ in equations (5) with the appropriate values of C_{L_1}, C_{L_2} in terms of C_{LD}.

For example, if the trim change is to be zero between $M_1 = 0.71$ and $M_2 = 1.22$, and C_{L_1} at $M_1 = 0.71$ is to be equal to $\frac{C_{LD}}{\sqrt{1-M_1^2}}$, then equations (5) give, with $k = 0.4$:-

$$1.6\Delta\eta_T = 0 = \left[C_{M_0}\sqrt{M_2^2-1}\right]_{\text{supersonic}} - \left[C_{M_0}\sqrt{1-M_1^2} - 1.6\alpha_0\right]_{\text{subsonic}}$$

$$- 0.127\, C_{LD}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

As C_{M_0} and α_0 are proportional to C_{LD}, this equation gives a centre-line shape, the ordinates of which are directly proportional to C_{LD}. If a cubic form is chosen, this equation becomes, with the notation of Table I:-

$$-\frac{2}{3}\pi - \frac{1}{16}\left(\frac{3\lambda - 4}{2 - \lambda}\right) + \frac{1.6}{4\pi}\left(\frac{3\lambda - 4}{2 - \lambda}\right) - 0.127 = 0$$

which gives $\lambda = 2.43$. The corresponding section is quite unusual, with a reflexed leading edge instead of the more usual reflexed trailing edge. It is undesirable because of a very large tail load at subsonic speeds ($C_{M_0}\sqrt{1 - M_1^2} = -1.3 \times C_{LD}$) although at supersonic speeds this reduces to a normal value - Table I shows that at supersonic speeds C_{M_0} is the same for all members of the cubic family.

The above discussion is sufficient to indicate that there is considerable scope for the design of aerofoils which will permit of smooth transition from subsonic to supersonic speeds in flight; i.e. which will permit the requirements of small tail-loads, zero trim change from subsonic to supersonic speeds, and smallest possible disturbance of trim during the transition period, to be mutually satisfied.

5. <u>Comparison with experimental results</u>

As stated in para.1, experiments up to the present have been largely concentrated on the region of subsonic speeds where shock-waves are just forming, i.e. on the lower boundaries of the transition region. It is of interest to observe, however, that the changes which the above simple theory would indicate, through the transition region, are already being found in practice. Thus it has been demonstrated on several aircraft[2] that at Mach numbers well above the critical Mach number at which shock installing first occurs, there is a pronounced nose-down change of pitching moment requiring several degrees of negative elevator to correct. Fig.1 compares the change in elevator actually found in the initial shock-stalling period with the predicted change over the whole transition region. The indications are that the change may actually be greater, at some stage near $M = 1$ than the final supersonic value.

Occasionally dives have been made on present-day aircraft in which the pilot's strength was insufficient to counter the trim change. Although very few records are available of such flights, for obvious reasons, the pilots describe the aircraft as apparently adopting an attitude giving a negative lift coefficient - this is deduced from the negative "g" and from the fact that loose objects fly up above the pilots' heads. This tendency to trim at negative lift is in accord with the deductions above; in fact the aircraft considered in equation (6) would trim itself, if no change in elevator angle were made, at a lift coefficient of -0.13 at supersonic speeds.

A final point shown by the experimental evidence from high-speed tunnels is the very pronounced increase in longitudinal stability which accompanies the shock-stall. According to para.3.2, an increase in stability equivalent to 0.46 $\bar{c}$ movement of the neutral point is to be expected at supersonic speeds; 0.25 $\bar{c}$ due to aft movement of the aerodynamic centre, and the remainder from the disappearance of the downwash at the tailplane and the changed slopes of the wing and tailplane lift curves. Increases in stability equivalent to 0.26 c̄ aft movement of the neutral point have already been observed on a Typhoon model in the R.A.E., high speed tunnel at a Mach number of 0.75, and even greater movements have been found on a Spitfire[8] model. 7

These facts suggest that in spite of the extremely complex nature of the flow in the transition region near M = 1, longitudinal stability and trim changes on the aircraft are generally of the same sign, and of the same order of magnitude, as might be expected if steady transition from subsonic to supersonic flow were taking place.

References

No.	Author	Title, etc.
1	Young	Survey of compressibility effects in aeronautics. R.A.E. Report No. Aero 1725.
2	Smelt, Charnley, and Rose	Drag and trim changes on Spitfire, Mustang and Thunderbolt in flight at high Mach numbers. R.A.E. Report No. Aero 1906. January, 1944.
3	Birnbaum	Die tragende Wirbelfläche als Hilfsmittel zur Behandlung des ebenen Problems der Tragflügeltheorie - ZAMM.1923.
4	Taylor	Applications to aeronautics of Ackeret's theory of aerofoils moving at speeds greater than that of sound. R. & M. 1467, April, 1932.
5	Schlichting	Aerofoil theory at supersonic velocities. Luftfahrtforschung. Vol.13, No.10.
6	Collar	Theoretical forces and moments on a thin aerofoil with hinged flap at supersonic speeds. R.A.E. Report No. SME.3273.
7	Caldwell, Evans	High speed wind tunnel tests on a model of Typhoon I. R.A.E. Report No. Aero 1815.
8	Mair, Hutton and Gamble	High speed wind tunnel tests on the Spitfire I. R.A.E. Report No. Aero 1810.

Attached

Appendix I and II
Drg. No. 14014 3 Fig. 1.

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APPENDIX I

Thin aerofoil theory - subsonic speeds

Following Birnbaum[3], we represent the aerofoil by a curve in the (x, y) plane extending from (-1, 0) to (+1, 0). Assuming that y is small, and that the flow around the aerofoil is that produced by a distribution of vorticity along the aerofoil

$$\frac{\gamma(x)}{V} = 2 \sum_{1}^{\infty} A_n \sin n\theta \quad \text{.............................} \quad (1)$$

where V is the speed of flight
and $\cos\theta = -x$,
then the disturbance velocities produced by the aerofoil at its boundary are given by

along x axis: $\pm \frac{u}{V} = \frac{1}{2}\frac{\gamma(x)}{V} = \sum_{1}^{\infty} A_n \sin n\theta$ (+ for upper surface, - for lower)

perpendicular to x axis: $\frac{v}{V} = \sum_{1}^{\infty} A_n \cos n\theta$.

The circulation distribution (1) is only possible if the flow is tangential to the aerofoil surface at the leading edge; this requires the stream velocity V to have an initial angle α_1 to the x axis, given by:-

$$\alpha_1 = \frac{1}{\pi} \int_0^{\pi} \frac{dy}{dx}\, d\theta \quad \text{.............................} \quad (2)$$

This angle is usually called the optimum angle of incidence for the aerofoil. The flow in this condition can be found from the boundary condition at the aerofoil surface, viz:-

$$\frac{v}{V} + \alpha_1 - \frac{dy}{dx} = 0$$

whence
$$A_n = \frac{2}{\pi} \int_0^{\pi} \frac{dy}{dx} \cos n\theta\, d\theta \quad \text{......................} \quad (3)$$

The velocity over the surface is $(V \pm u)$ to the first order, so that the normal force per unit length $\div \frac{1}{2}\rho V^2 C$ (where C, the chord = 2) is given by

$$\text{normal force}/\tfrac{1}{2}\rho V^2 C = \frac{\frac{1}{2}\rho \times 4uV}{\rho V^2} = \frac{2u}{V} = \frac{\gamma(x)}{V} .$$

Thus the left coefficient at the optimum incidence α_1 is given by

$$C_L = \int_{-1}^{1} \frac{\gamma(x)}{V}\, dx$$

$$= \int_0^{\pi} 2 \sum_{1}^{\infty} A_n \sin n\theta \sin\theta\, d\theta$$

$$= \pi A_1 .$$

The no-lift angle, α_0, is given by

$$\alpha_0 = \alpha_1 - \frac{C_L}{2\pi}$$
$$= \frac{1}{\pi}\int_0^{\pi} \frac{dy}{dx}(1 - \cos\theta)\, d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Again, by integration of the normal force, the pitching moment about mid-chord at the optimum incidence α_1 is given by

$$C_{M\ \text{half-chord}} = -\frac{1}{2}\int_{-1}^{1} \frac{\gamma(x)}{V}\, x\, dx$$
$$= \frac{1}{2}\int_0^{\pi} 2\sum_{1}^{\infty} A_n \sin n\theta \cos\theta \sin\theta\, d\theta$$
$$= \frac{\pi}{4} A_2.$$

Thus the pitching moment coefficient about the quarter-chord, which is independent of incidence, is given by

$$C_{M\ \text{quarter chord}} = \frac{\pi}{4} A_2 - \frac{1}{4} C_L$$
$$= \frac{1}{2}\int_0 \frac{dy}{dx}(\cos 2\theta - \cos\theta)\, d\theta \quad \ldots\ldots\ldots \quad (5)$$

The solution above applies to uncompressible flow ($M = 0$). For compressible flow without shock, Glauert has shown small pressure changes (such as are considered above) to be increased by a factor $\frac{1}{\sqrt{1 - M^2}}$. This results in the values of C_L, C_M found above being multiplied by the same factor, whilst the incidence α_0 and α_1 remain unchanged.

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6. ~~[illegible]~~ American Embassy.

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APPENDIX II

Thin aerofoil theory - supersonic speeds

At supersonic speeds the simple theory of Ackeret, as outlined by Taylor[4], can be employed. By this theory, the pressure at any point of the surface of the thin aerofoil (subject to the conditions detailed in Ref.4) is given by:-

$$\text{Pressure} = \rho V^2\beta \Big/ \sqrt{M^2 - 1}$$

where β is the angle which the surface makes at the point, with the initial V direction. For the line aerofoil extending from (-1, 0) to (1, 0) as in Appendix I, we have:

on upper surface $\beta = -\alpha + \frac{dy}{dx}$

on lower surface $\beta = \alpha - \frac{dy}{dx}$ where α is the incidence of the x-axis

Hence the local lift at the point x is given by

$$L = 2\frac{\rho V^2}{\sqrt{M^2 - 1}}\left(\alpha - \frac{dy}{dx}\right) \text{ per unit length}$$

and the lift coefficient is

$$C_L = \int_{-1}^{1}\frac{L}{\frac{1}{2}\rho V^2}\frac{dx}{2} = \frac{4\alpha}{\sqrt{M^2-1}} - 2\int_{-1}^{1}\frac{dy}{dx}\,dx.$$

$$= \frac{4\alpha}{\sqrt{M^2-1}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Thus the slope of the lift curve is $\frac{4}{\sqrt{M^2-1}}$, and the angle of zero lift is $\alpha = 0$.

The moment coefficient about mid-chord is:-

$$C_{M\,\text{mid-chord}} = -\frac{1}{\frac{1}{2}\rho V^2.4.}\int_{-1}^{1}\frac{2\rho V^2}{\sqrt{M^2-1}}\left(\alpha - \frac{dy}{dx}\right)x\,dx$$

$$= \frac{1}{\sqrt{M^2-1}}\int_{-1}^{1}x\frac{dy}{dx}\,dx. \quad \ldots\ldots\ldots\ldots \quad (7)$$

This is independent of α_1, i.e. the mid-chord is the aerodynamic centre. Equation (7) may be written in the two alternative forms:-

$$C_{M\,\text{mid-chord}} \times \sqrt{M^2-1} = -\int_{-1}^{1}y\,dx = -\frac{1}{2}\int_{0}^{\pi}\frac{dy}{dx}\sin 2\theta\,d\theta.$$

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TABLE I

Thin aerofoil characteristics at subsonic and supersonic speeds (infinite aspect ratio)

Aerofoil	Design C_{LD} ($\alpha = 0$)	No-lift angle α_o		Pitching moment at zero lift C_{m_o}		In terms of C_{LD}:-			
		Subsonic	Supersonic	Subsonic $\times\sqrt{1-M^2}$	Supersonic $\times\sqrt{M^2-1}$	α_o Subsonic	α_o Supersonic	$C_{m_o}\sqrt{1-M^2}$ Subsonic	$C_{m_o}\sqrt{M^2-1}$ Supersonic
Conventional (cubic)*: (1) General	$\pi h(1-\frac{\lambda}{2})$	$\frac{h}{8}(3\lambda-4)$	0	$\frac{\pi h}{32}(7\lambda-8)$	$\frac{h}{3}(2-\lambda)$	$\frac{C_{LD}}{4\pi}\left(\frac{3\lambda-4}{2-\lambda}\right)$	0	$\frac{C_{LD}}{16}\left(\frac{7\lambda-8}{2-\lambda}\right)$	$-\frac{2\,C_{LD}}{3\pi}$
(2) Circular arc, $\lambda = 0$	πh	$-\frac{1}{2}h$	0	$-\frac{1}{4}\pi h$	$-{}^{2}/_{3}h$	$-\frac{C_{LD}}{2\pi}$	0	$-\frac{1}{4}C_{LD}$	$-\frac{2\,C_{LD}}{3\pi}$
(3) Constant C.P., $\lambda = 8/7$	$^{3}/_{7}\pi h$	$-{}^{1}/_{14}h$	0	0	$-{}^{2}/_{7}h$	$-\frac{C_{LD}}{6\pi}$	0	0	$-\frac{2\,C_{LD}}{3\pi}$
[illegible]** $a = 1.0$	4μ	$-\frac{2\mu}{\pi}$	0	$-\mu$	$-\frac{2\mu}{\pi}$	$-\frac{C_{LD}}{2\pi}$	0	$-\frac{1}{4}C_{LD}$	$-\frac{C_{LD}}{2\pi}$

.... contd.

TABLE I - continued

Aerofoil	Design C_{LD} ($a = 0$)	No-lift angle α_0 Subsonic	No-lift angle α_0 Supersonic	Pitching moment at zero lift C_{M_0} Subsonic $\times\sqrt{1 - M^2}$	Pitching moment at zero lift C_{M_0} Supersonic $\times\sqrt{M^2 - 1}$	In terms of C_{LD}: α_0 Subsonic	In terms of C_{LD}: α Supersonic	In terms of C_{LD}: $C_{M_0}\sqrt{1 - M^2}$ Subsonic	In terms of C_{LD}: $C_{M_0}\sqrt{M^2 - 1}$ Supersonic
Flat plate with flap over fraction E of chord at t.e; flap angle β (1) Relative to chord joining L.E and T.E.	$4\beta\sqrt{E(1-E)}$	$\frac{\beta}{\pi}\left[2\cos^{-1}\sqrt{E} - \pi(1-E) - 2\sqrt{E(1-E)}\right]$	0	$-2\beta\frac{(1-E)}{\sqrt{E(1-E)}}$	$-2\beta E(1-E)$	$\frac{C_{LD}}{4\pi}\left[\frac{2\cos^{-1}E - \pi(1-E)}{\sqrt{E(1-E)}} - 2\right]$	0	$-\frac{C_{LD}}{2}(1-E)$	$-\frac{C_{LD}}{2}\sqrt{E(1-E)}$
(2) Relative to unflapped chord-line	-do-	$+\frac{\beta}{\pi}\left[2\cos^{-1}E - \pi - 2\sqrt{E(1-E)}\right]$	$-E\beta$	-do-	-do-	-	-	-	-

* Conventional section has equation $y = \frac{h}{4}(1 - x^2)(2 - \lambda - \lambda x)$.

** Low-drag section ($a = 1.0$) has equation $\frac{dy}{dx} = \frac{2\mu}{\pi}\log\cot\frac{\theta}{2}$.

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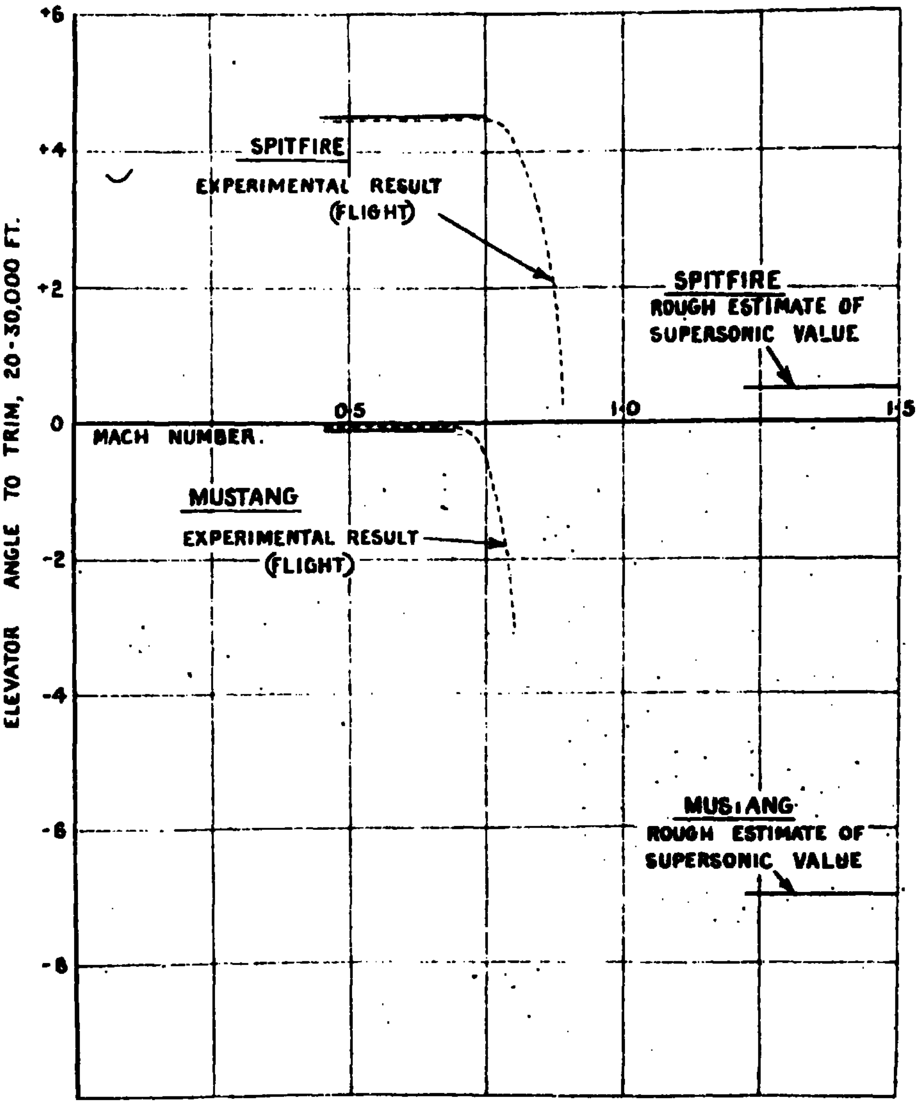
FIG. I



CHANGE OF ELEVATOR ANGLE TO TRIM AT HIGH MACH NUMBERS. FLIGHT RESULTS COMPARED WITH ESTIMATED SUPERSONIC VALUE.

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