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C O N F I D E N T I A L

Report No. E-S/F-MX-1016-1

VIBRATION AND FLUTTER ANALYSIS OF THE B-29 - F-84 TIP-TO-TIP CONFIGURATION

Prepared by: S. Pines
S. Pines

R. S. Levy
R. S. Levy

Approved by: F. McVay
F. McVay
Chief Structures Engr.

Assistance in Computation:
G. Miller

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REFERENCES

1. Ackerman, W. E. "Design Data for Flutter Analysis - Model XB-29," Boeing Report D-3755 dated 6-43.

2. Rosenbaum, R. "Vibration and Flutter Analysis - XP-84," R.A.C. Report ERS-58.

3. Smilg, B., Wasserman, L. "Application of Three Dimensions Flutter Theory to Aircraft Structure," AFTR 4798, dated 7-42.

NOTATION

SYMBOL	DEFINITION	UNITS
h_i	Vertical deflection of wing elastic axis at station i	in.
a_{ij}	Bending or torsion influence coefficient. Deflection at station i due to unit force at station j	in./# or rad./in.#
F_j	Acceleration force at station j	#
ω	Natural frequency of the wing; also flutter frequency	rad./sec.
(K)	Symmetric matrix of influence coefficients	
(M)	Diagonal matrix of "lumped" masses	slugs
M_{29}	Tip mass of B-29 wing tip	slugs
K_x	Radius of gyration of F-84 in roll	in.
B	F-84 wing span	in.
{ }	Brackets indicate a column vector	
β_i	Flapping of the F-84 about attachment hinge in the i th bending mode	rad.
M_o	Half the mass of the B-29 fuselage	slugs
m_i	Lumped mass at station i	slugs
θ_o	Roll of the B-29 fuselage	rad.
X	Station along the elastic axis	in. & ft.
I_y	Rolling moment inertia of half the B-29 fuselage	slug. ft.^{2}
(I)	Unit matrix	
T_j	Acceleration torque at station j	# in.
$\mathcal{J}_i$	Lumped moment of inertia at station i	slug. ft.^{2}

NOTATION

SYMBOL	DEFINITION	UNIT
α_i	Angle of attack at station i	rad.
$q(t)$	Generalized coordinate, assumed a harmonic function of time	
A_{ij}	Generalized mass due to inertia coupling of i th and j th degrees of freedom	slug. ft.^{2}
δ_i	Aileron deflection of the F-84 in the ith degree of freedom	rad.
ψ	Sweep-back angle of the skewed attachment hinge line	rad.
g	Percentage of critical structural damping	
V	True airspeed	ft./sec.

SUMMARY

The B-29 - F-84 tip-to-tip configuration, using a two point attachment, is shown to be free from flutter instability at an altitude of 20,000 ft. up to a speed of 300 mph (I.A.S.).

The aileron deflection, geared to the F-84 flapping at a ratio of $0.83\delta : \beta$, plus the $3°36'$ skewed axis of attachment are sufficient to supply flutter stability up to the design true airspeed of 300 mph at 20,000 ft. altitude.

INTRODUCTION

The purpose of the tip-to-tip project is to extend the action radius of protective fighters accompanying a B-29 bomber. At the suggestion of Republic Aviation and the Dynamics Branch of AMC it was agreed to determine the flutter characteristics of the airborne tip-to-tip configuration.

Due to the low bending, torsion and F-84 flapping frequencies anticipated for the attached configuration it was decided that any flutter instability would of necessity be of the dynamic instability, low frequency type, rather than the classical higher frequency variety involving second and higher elastic vibration modes. In addition, the damping influence of the tail surfaces was neglected, and it was agreed to include these effects only if the basic wing-fuselage flutter analysis showed instability.

The admissible degrees of freedom were taken to be:

a) Symmetric Case: B-29 bending (including the associated F-84 flap), B-29 torsion (including one attached F-84 wing panel), F-84 flapping.

b) Anti-symmetric Case: B-29 rigid roll (including F-84 flap), B-29 bending (including F-84 flap), B-29 torsion (including one elastic F-84 wing panel), F-84 flapping.

The aileron deflection, geared to the flapping of the F-84 relative to the B-29 wing tip was included in the flutter analysis only in so far as air loads are concerned. The inertia effects of this degree of freedom were neglected as an insignificant contribution to the net kinetic energy of the configuration in flapping. The aerodynamic effect of the induced F-84 angle of attack resulting from flapping about a skewed axis, sweptback from the line of flight, was taken into account, although this contribution to the net aerodynamic work done was found to be negligible in all cases.

The influence of rigid translation of the B-29 was neglected for the symmetric case on the argument that this is a stabilizing influence, and would be included only if flutter instability was evidenced in the analysis.

The vibration analysis was carried out using the B-29 mass and flexibility data supplied by Boeing in Reference 1. The F-84 mass and flexibility data were obtained from Reference 2. Uncoupled bending and torsion modes were computed and used in the flutter analysis.

The analysis is limited to the following condition of flight:

a. 97,500# B-29 bomber

b. Two 14,450# F-84 fighters

c. 20,000' altitude

VIBRATION ANALYSIS

a) Symmetric Modes (bending)

The vibration analysis employed in this report makes use of the usual matrix iteration methods.

The entire structural inertia is assumed to be concentrated along the elastic axis of the wing. For the purposes of this analysis, the e.a. was taken to be the 25% chord of the B-29 wing. Let a_{ij} be the cantilevered bending influence coefficient of the B-29 wing at station i, due to a unit (1 lb.) load at station j. Then, the symmetric mode deflection of station i, relative to the fuselage is given by

$$h_i - h_o = \sum_{j=1}^{n} a_{ij} F_j \tag{1}$$

Assuming zero structural damping, the acceleration force at station j is given by

$$F_j = \omega^2 m_j h_j \tag{2}$$

In matrix form, Eq.'s (1) and (2) become

$$\{h - h_o\} = \omega^2 (K)(M)\{h\} \tag{3}$$

For the B-29 wing tip mass, m_n, assuming that the attached F-84 does not experience elastic bending at these low frequencies, but merely flaps about the skewed axis, the tip force is given by

$$F_{tip} = \omega^2 [M_{29} + M_{eff}] h_n \tag{4}$$

The effective mass of the flapping F-84 is given by

$$M_{eff} = M_{84} \frac{k_x^2}{B^2 + k_x^2} \tag{5}$$

and the flap angle, β , resulting from a unit deflection of the B-29 wing tip is given by

$$\beta = - \frac{1}{B + \frac{k_x^2}{B}} \tag{6}$$

The B-29 fuselage deflection, h_o , is obtained by requiring that the vibrating configuration be in equilibrium.

$$h_o = - \frac{1}{M_o} \sum_{i=1}^{n} m_i h_i \tag{7}$$

The matrix equation (Eq. 3) may be written in its characteristic form by eliminating h_o , or

$$\{h\} = \frac{\omega^2}{\Sigma} \left({}^{\Sigma}I - (\bar{M}) \right) (K) (M) \{h\} \tag{8}$$

where,

$$\Sigma = \sum_{i=0}^{n} m_i$$

and

$$(\bar{M}) = \begin{pmatrix} m_1 & m_2 & \cdots & m_n \\ m_1 & m_2 & \cdots & m_n \\ \vdots & & & \\ m_1 & m_2 & \cdots & m_n \end{pmatrix} \tag{9}$$

The first bending mode is obtained by iteration of Eq. (8).

b) Anti-symmetric Modes (bending)

For the anti-symmetric case, the cantilever bending mode is given

$$h_i - \theta_0 x = \sum_{j=1}^{n} a_{ij} F_j \tag{10}$$

In matrix form this becomes

$$\{h - \theta_0 x\} = \omega^2 (K)(M)\{h\} \tag{11}$$

To eliminate the $\theta_0 x$'s to obtain the characteristic form of Eq. (11), we require that the acceleration moments about the fuselage center line be in equilibrium.

$$\theta_0 = -\frac{1}{I_y} \sum_{i=1}^{n} m_i x^2 h_i \tag{12}$$

Eq. (11) may then be written as

$$\{h\} = \omega^2 \left(I - \frac{1}{\Sigma'}(x)(\bar{M})(x)\right)(K)(M)\{h\} \tag{13}$$

where,

$$\Sigma' = I_y + \sum_{i=1}^{n} m_i x^2$$

and

$$(x) = \begin{pmatrix} x_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & x_2 & 0 & 0 & & 0 \\ 0 & 0 & x_3 & 0 & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \cdots & x_n \end{pmatrix} \tag{14}$$

The anti-symmetric mode is then obtained by iteration of Eq. (14).

Since no value was given for the B-29 fuselage rolling moment of inertia, an average value was obtained from similar airplanes using the formula

$$I_y = .03 \sum_{i=1}^{n} m_i x^2 \tag{15}$$

In Eq. (15) m_n was taken to be the true tip mass of the B-29 without the F-84 attachment.

c) Torsion Modes

For the torsional modes, the torsional influence coefficients were computed using the effective B-29 CTR data obtained from Reference 1. Since the F-84 is attached to the B-29 wing tip at two points, the flexibility of the F-84 wing panel in direct contact with the bomber wing was included in the torsional vibration analysis. The F-84 fuselage plus the outboard wing was taken as a rigid mass in torsion.

The symmetric wing torsion at station i, relative to the fuselage pitch is given by

$$\alpha_i - \alpha_0 = \sum_{j=1}^{n} a_{ij} T_j \tag{16}$$

where

$$T_j = \omega^2 \mathcal{I}_j \alpha_j \tag{17}$$

The index j runs from the first B-29 station to the F-84 fuselage.

In matrix form, Eq.'s (16) and (17) may be written as

$$\{\alpha - \alpha_0\} = \omega^2 (K)(\mathcal{I}) \{\alpha\} \tag{18}$$

The bomber fuselage pitch, α_0, may be eliminated by making use of the condition for rotational equilibrium.

$$\alpha_0 = -\frac{1}{\mathcal{I}_0} \sum_{i=1}^{n} \mathcal{I}_i \alpha_i \tag{19}$$

Eq. (18) may then be written as

$$\{\alpha\} = \frac{\omega^2}{2} \left({}^{2}I - (\bar{\mathcal{I}})\right)(K)(\mathcal{I}) \{\alpha\} \tag{20}$$

where $\Sigma = \mathcal{I}_0 + \sum_{i=1}^{n} \mathcal{I}_i$

and

$$(\bar{\mathcal{I}}) = \begin{pmatrix} \mathcal{I}_1 & \mathcal{I}_2 & \cdots & \mathcal{I}_n \\ \mathcal{I}_1 & \mathcal{I}_2 & \cdots & \mathcal{I}_n \\ \vdots & & & \\ \mathcal{I}_1 & \mathcal{I}_2 & \cdots & \mathcal{I}_n \end{pmatrix} \qquad (21)$$

The symmetric torsion mode is then obtained by iteration of Eq. (20).

For the anti-symmetric torsion mode there is no B-29 fuselage pitch, so that $\alpha_0 = 0$. The appropriate characteristic equation is then given by

$$\{\alpha\} = \omega^2 (K)(\mathcal{I})\{\alpha\} \qquad (22)$$

which can be solved as before by iteration.

The tables containing the computations for the elastic modes are listed below:

Mass data for B-29, F-84	TABLE 1
Bending and Torsional influence coefficients	TABLE 2
Matrix iteration for 1st sym. bending	TABLE 3
Matrix iteration for 1st anti-sym. bending	TABLE 4
Matrix iteration for 1st sym. torsion	TABLE 5
Matrix iteration for 1st anti-sym. torsion	TABLE 6

The plan form of the tip-to-tip configuration is shown in Figure 1. This figure also contains the location of the assumed elastic axes of both the B-29 and F-84, and the c.g. stations of the masses used in the vibration and flutter analysis.

FLUTTER ANALYSIS

a) Mechanical Terms

The flutter analysis is most conveniently carried out in terms of generalized coordinates. Let the admissible degrees of freedom be

Symmetric Case:

$$\left.\begin{aligned} h(x,t) &= h_h(x)\, q_h(t) + h_\beta(x)\, q_\beta(t) \\ \alpha(x,t) &= \alpha_1(x)\, q_\alpha(t) \end{aligned}\right\} \qquad (23a)$$

Anti-Symmetric Case

$$\left.\begin{aligned} h(x,t) &= h_h(x)\, q_h(t) + h_R(x)\, q_R(t) + h_\beta(x)\, q_\beta(t) \\ \alpha(x,t) &= \alpha_1(x)\, q_\alpha(t) \end{aligned}\right\} \qquad (23b)$$

The deflection modes $h_i(x)$, $\alpha_i(x)$ are defined over the entire coupled configuration as follows:

Symmetric Case

MODE	B-29	F-84
$h_h(x)$	1st sym. bending	$1 - \frac{1}{B + \frac{K_x}{B}}\left(x - \frac{650}{12}\right)$
$h_\beta(x)$	0	$\dfrac{x - \frac{650}{12}}{\frac{440}{12}}$
$\alpha_1(x)$	1st sym. torsion	1st sym. torsion

Anti-Symmetric Case:

MODE	B-29	F-84
$h_n(x)$	1st anti-sym. bending	$1 - \frac{1}{B + \frac{K_x^2}{B}}\left(x - \frac{850}{12}\right)$
$h_R(x)$	$\frac{x12}{850}$	$1 - \frac{1}{B + \frac{K_x^2}{B}}\left(x - \frac{850}{12}\right)$
$h_\beta(x)$	0	$\frac{x - \frac{850}{12}}{\frac{440}{12}}$
$\alpha_1(x)$	1st anti-sym. torsion	1st anti-sym. torsion

The kinetic energy for 1/2 of the entire configuration is given by

$$2T = \sum_{x=0}^{84\,t_{ip}} \left[m(x)\dot{h}^2(x,t) + 2\Delta(x)\dot{h}(x,t)\dot{\alpha}(x,t) + \mathcal{J}(x)\dot{\alpha}^2(x,t) \right] \tag{24}$$

In terms of the generalized coordinates Eq. (24) becomes

$$2T = \sum_i \sum_j A_{ij}(x,m)\, \dot{q}_i(t)\, \dot{q}_j(t) \tag{25}$$

The numerical tabulation of the kinetic energy integrals, A_{ij}, is listed in Tables 7 and 12 .

Since the elastic modes are the only modes which store potential energy the quadratic form for the potential energy may be written

$$2U = A_{hh}\,\omega_h^2\, q_h^2(t) + A_{\alpha\alpha}\,\omega_\alpha^2\, q_\alpha^2(t) \tag{26}$$

b) Aerodynamic Forces

For the purposes of computing the aerodynamic work done during the virtual displacement of any of the generalized coordinates it is necessary to include the F-84 change in angle of attack and aileron deflection resulting from flapping of the attached fighter.

Symmetric Case:

$$\left.\begin{aligned} h(x,t) &= h_h(x)\,q_h(t) + h_\beta(x)\,q_\beta(t) \\ \alpha(x,t) &= \alpha_1(x)\,q_\alpha(t) + \alpha_h(x)\,q_h(t) + \alpha_\beta(x)\,q_\beta(t) \\ \delta(x,t) &= \delta_h(x)\,q_h(t) + \delta_\beta(x)\,q_\beta(t) \end{aligned}\right\} \qquad (27a)$$

Anti-Symmetric Case:

$$\left.\begin{aligned} h(x,t) &= h_h(x)\,q_h(t) + h_R(x)\,q_R(t) + h_\beta(x)\,q_\beta(t) \\ \alpha(x,t) &= \alpha_1(x)\,q_\alpha(t) + \alpha_h(x)\,q_h(t) + \alpha_R(x)\,q_R(t) + \alpha_\beta(x)\,q_\beta(t) \\ \delta(x,t) &= \delta_h(x)\,q_h(t) + \delta_R(x)\,q_R(t) + \delta_\beta(x)\,q_\beta(t) \end{aligned}\right\} \qquad (27b)$$

The aileron deflection of the F-84 is a function of both the F-84 flapping and the relative angle between the attached wing tips due to the bending slope of the bomber wing.

$$\delta_i(x) = 0.83\left(-\beta_i + \frac{\partial h_i(x)}{\partial x}\Big|_{x=\frac{850}{12}}\right) \qquad (28)$$

The aileron deflection is defined only over the F-84 aileron intervals and is taken as a constant positive value inboard and constant negative value outboard.

The F-84 induced angle of attack due to fighter flapping is given by

$$\alpha_i(x) = \beta_i \tan \psi \tag{29}$$

This is taken as a constant angle of attack over the entire fighter span. Figure 2 illustrates the sign convention and kinematic relationships between δ_i, α_i and the flapping of the F-84 wing.

The oscillatory aerodynamic forces used in the analysis are the usual strip theory forces given in Ref. 3, for a tapered wing. The reference chord, b_r, is taken at 3/4 span of the bomber wing. The values of e and c; for the F-84 aileron, are taken from Ref. 2. The elastic axis locations are defined as

$$a_{29} = -\frac{1}{2}$$
$$a_{84} = -.20 \tag{30}$$

The work done by the aerodynamic forces is given by

$$W = \int \mathcal{L}\, h(x,t)\, dx + \int \mathcal{M}\, \alpha(x,t)\, dx + \int \mathcal{T}\, \delta(x,t)\, dx \tag{31}$$

The generalized forces are given by

$$Q_i = \frac{\partial W}{\partial q_i(t)} \tag{32}$$

which may be written in terms of the generalized coordinates as

$$Q_i = \pi \rho \omega^2 \sum_j B_{ij}\, q_j(t) \tag{33}$$

where B_{ij} is given by

$$B_{ij} = \left\{ \begin{array}{l} \int L_h b^2 h_i(x) h_j(x)\,dx + \int L_\alpha b^3 h_i(x)\alpha_j(x)\,dx + \int L_\delta b^3 h_i(x)\delta_j(x)\,dx \\ \int M_h b^3 \alpha_i(x) h_j(x)\,dx + \int M_\alpha b^4 \alpha_i(x)\alpha_j(x)\,dx + \int M_\delta b^4 \alpha_i(x)\delta_j(x)\,dx \\ \int \tilde{J}_h b^3 \delta_i(x) h_j(x)\,dx + \int \tilde{J}_\alpha b^4 \delta_i(x)\alpha_j(x)\,dx + \int \tilde{J}_\delta b^4 \delta_i(x)\delta_j(x)\,dx \end{array} \right\} \quad (34)$$

The aerodynamic coefficients L_i, M_i, $\tilde{J}_i$ are given by

$$\left.\begin{array}{l} L_h = 1 + \frac{b_r}{b} K_2(L_h) \\ L_\alpha = -a + \frac{b_r}{b} K_2(L_\alpha) + \left(\frac{b_r}{b}\right)^2 K_3(L_\alpha) - \left(\frac{1}{2}+a\right)\left(\frac{b_r}{b}\right) K_2(L_h) \\ L_\delta = L_\beta - (c-e) L_z \\ M_h = -a - \frac{b_r}{b}\left(\frac{1}{2}+a\right) K_2(L_h) \\ M_\alpha = \frac{1}{8}+a^2 + \frac{b_r}{b} K_2(M_\alpha) - \left(\frac{1}{2}+a\right)\frac{b_r}{b} K_2(L_\alpha) - \left(\frac{1}{2}+a\right)\left(\frac{b_r}{b}\right)^2 K_3(L_\alpha) \\ \qquad + \left(\frac{1}{2}+a\right)^2 \frac{b_r}{b} K_2(L_h) \\ M_\delta = M_\beta - \left(\frac{1}{2}+a\right) L_\beta - (c-e) M_z + (c-e)\left(\frac{1}{2}+a\right) L_z \\ \tilde{J}_h = T_h - (c-e) P_h \\ \tilde{J}_\alpha = T_\alpha - (c-e) P_\alpha - \left(\frac{1}{2}+a\right) T_h + (c-e)\left(\frac{1}{2}+a\right) P_h \\ \tilde{J}_\delta = T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z \end{array}\right\} \quad (35)$$

The integrals are evaluated for fixed values of the reduced frequency, $\frac{1}{k}$. Under this convention the complex values of $K_1(L_h)$, etc., are assumed constant

and are taken outside the integral sign, however, the varying chord remains under the integral sign.

c) Flutter Determinant

The Lagrangian equations of motion may be written for a fixed value of

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i(t)} + \frac{\partial U}{\partial q_i(t)} = Q_i \tag{36}$$

or, for the harmonic case where $\ddot{q}_i(t) = -\omega^2 q_i(t)$

$$-\omega^2 \sum_j A_{ij}\, q_j(t) + \omega_i^2 A_{ii}(1+\sqrt{-1}\,g)\, q_i(t) = \pi\rho\omega^2 \sum_j B_{ij}\, q_i(t) \tag{37}$$

The factor g is the structural damping which is taken proportional to the potential energy and in phase with the velocity for harmonic oscillations.

Eq. (37) represents as many equations as there are degrees of freedom. For a non-trival solution of these equations it is necessary and sufficient for the matrix of the coefficients of $q_i(t)$ to be singular. This places the condition on the flutter frequency, ω, to be a root of the polynomial of the determinant of the matrix.

Since the rigid body degrees of freedom are independent of ω, these coordinates may be solved for in terms of the elastic degrees of freedom. The equations may be reduced to a 2 x 2 determinant. The resulting quadratic polynomial is then solved for the critical flutter speed.

The flutter solution curves are plotted in the usual "$\frac{V}{\omega_\alpha}$ vs. g" coordinate system.

TABLE 1

MASS DATA FOR B-29 & F-84

INTERVAL	STA	MASS	STATIC MOMENT	MOMENT OF INERTIA	ROLLING MOMENT OF INERTIA			
IN.	IN.	SLUGS	SLUG IN.	SLUG IN.^{2}	SLUG IN.^{2}			
-45 - +45	0	31.53	2847	4736350	265630			B-29 TOTAL
45 - 130	90	12.06	-492	17100				WEIGHT = 97,500 LBS.
130 - 220	169	33.42	-1290	221530				
220 - 320	270	12.69	101.0	14510			REF. 1	
320 - 450	374	28.82	-1412	176450				
450 - 595	520	5.350	69.9	4920				
595 - 745	670	1.218	12.9	1040				
745 - 850	820	.363	2.6	260				
TIP WT.	850	.842	0	200				
850 - 877.5	850	.36	2.73	111				F-84 TOTAL
877.5 - 932.5	905	1.38	12.40	585				WEIGHT = 14,450 LBS
932.5 - 987.5	960	3.87	40.0	1640		REF. 2		
987.5 - 1042.5	1015	1.43	17.3	1070				
1042.5 - 1290	1070	30.2	985	350000		INCLUDES F-84		FUSELAGE + ENTIRE OUTBOARD WING

NOTES:

MASSES USED IN VIBRATION ANALYSIS ARE IN IN.-SLUG SYSTEM.

MASSES USED IN FLUTTER ANALYSIS ARE IN FT.-SLUG SYSTEM.

FUSELAGE MASSES FOR 1/2 FUSELAGE.

EFF. MASS OF F-84 IN FLAPPING - $M_{EFF} = \frac{K_x^2}{b^2+K_x^2} M_{84} = 3.31$

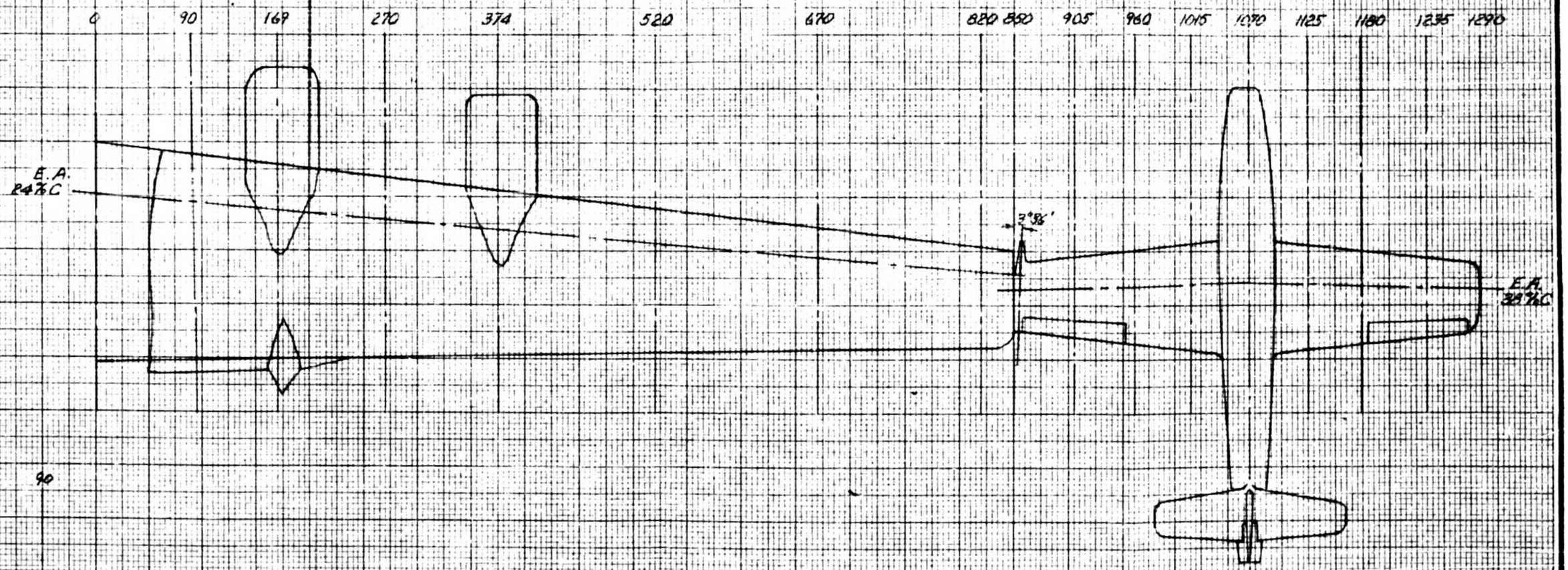
$b = 220$ IN.

$K_x = 68.52$ IN.

$M_{84} = 37.435$ SLUGS

FIGURE 1

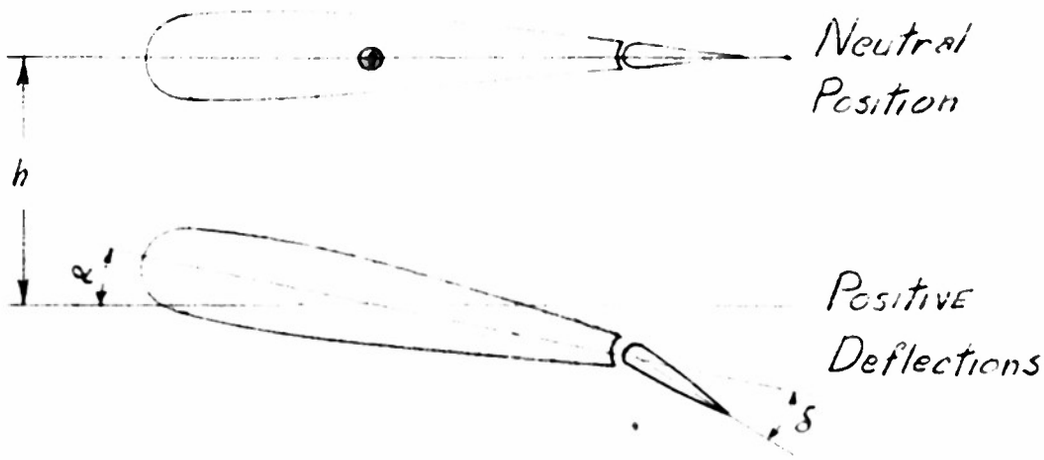
PLAN VIEW OF TIP-TO-TIP CONFIGURATION



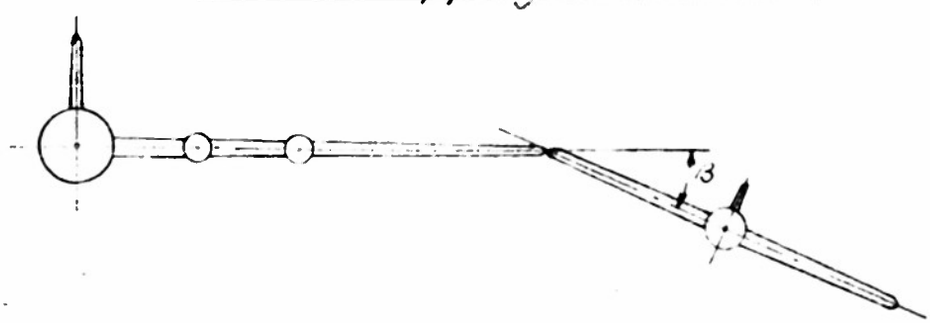
NOTE: C.G. LOCATION OF THE MASSES USED IN FLUTTER & VIBRATION

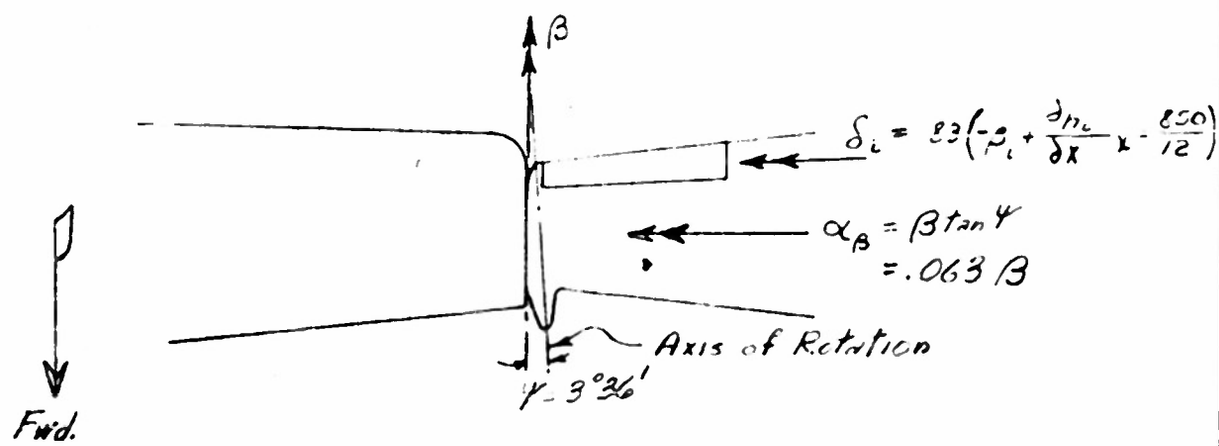
FIGURE 2

Wing Deflections



F-84 Flapping Deflection





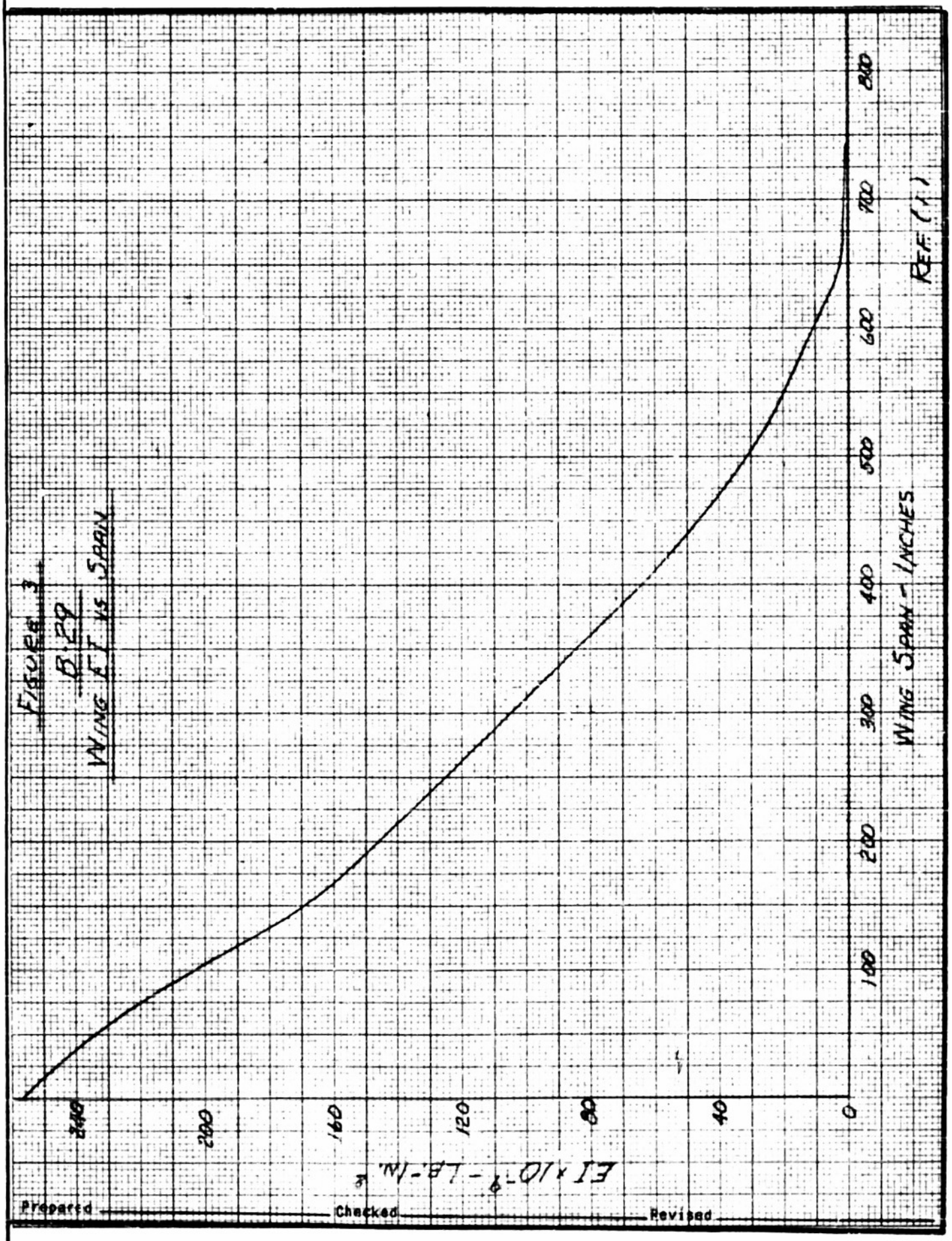


FIGURE 3
B-29
WING EI VS SPAN

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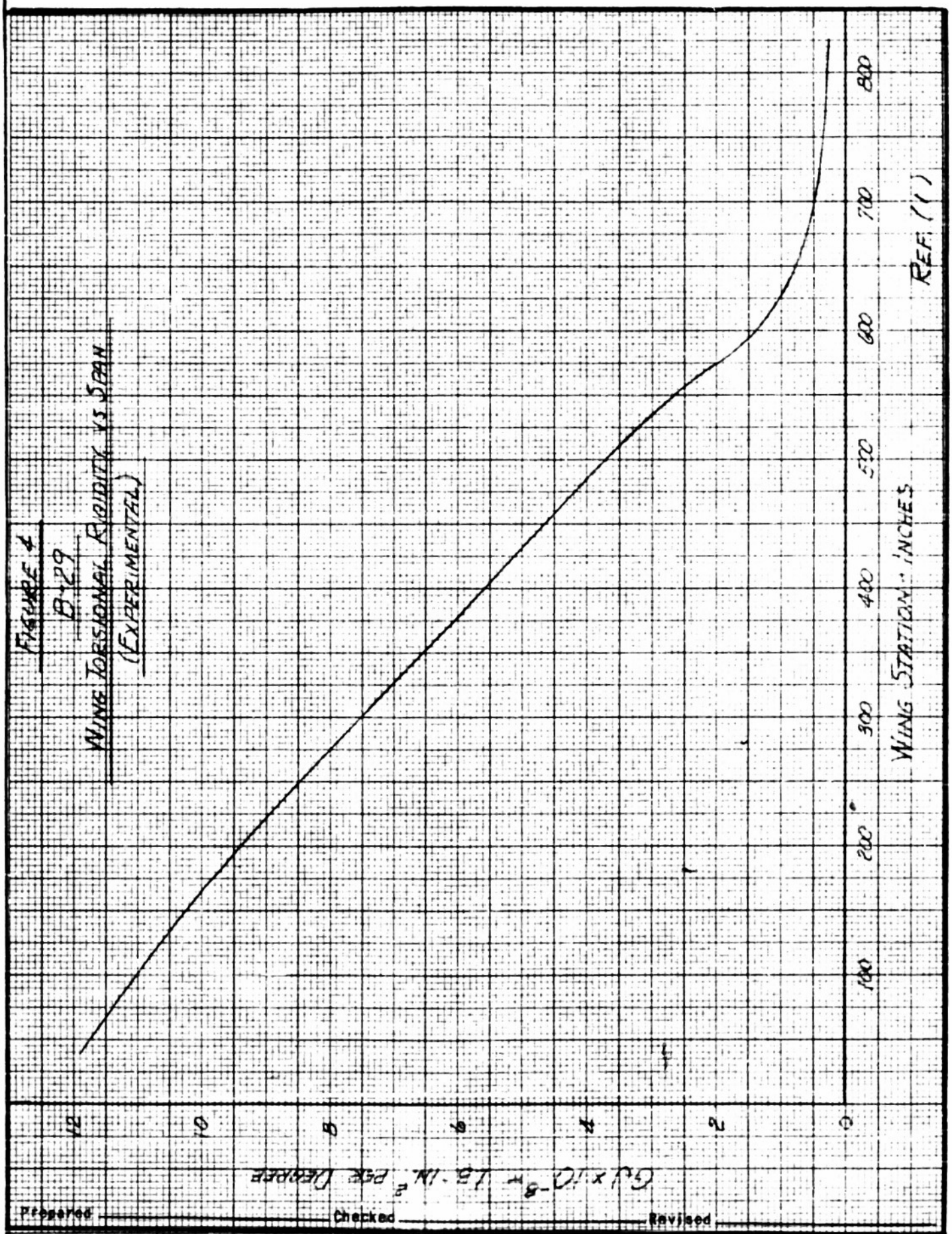


FIGURE 4
B-29
WING TORSIONAL RIGIDITY VS SPAN
(EXPERIMENTAL)
REF. (1)
WING STATION - INCHES
100
200
300
400
500
600
700
800
0
2
4
6
8
10
12
GJ x 10^-8 = LB-IN^2 PER DEGREE

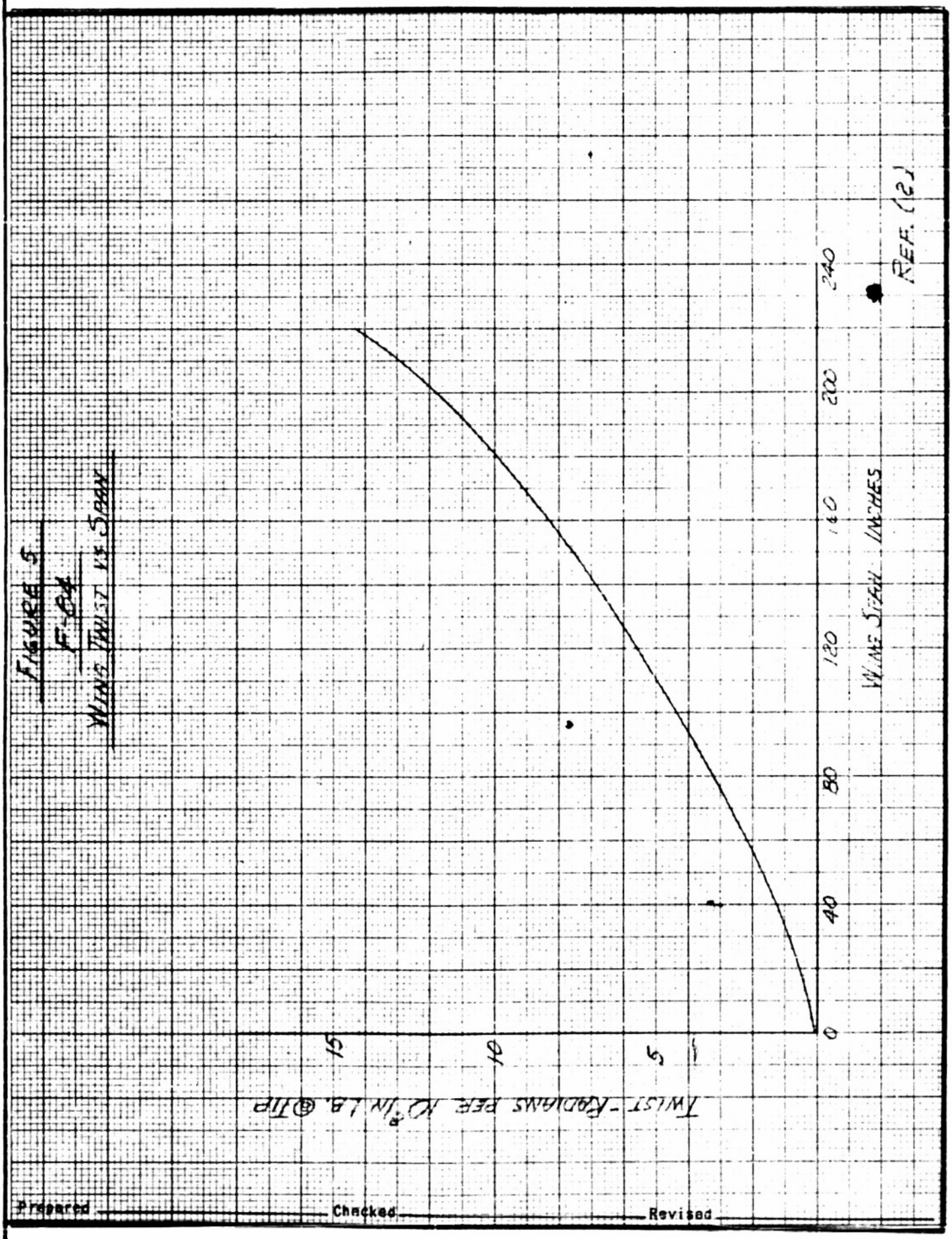


Figure 5
F-84
Wing Twist vs Span

TABLE 2

BENDING INFLUENCE COEFFICIENTS

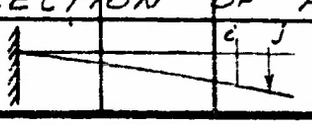
1	2	3	4	5	6	7	8				
	STA.										
	x	EI	$\frac{1}{EI}$	$\frac{\Delta x}{2}$	$\int_0^{850} \frac{dx}{EI}$	$\iint_0^{850} \frac{dx}{EI}$	$\iiint_0^{850} \frac{dx}{EI}$				
	IN.	$\times 10^{-9}$			$\times 10^{-9}$	$\times 10^{-9}$	$\times 10^{-9}$				
0	0	256	.003906		0	0	0				
	45	236	.004237	22.5	.1832	4.122	92.745				
1	90	209	.004785	22.5	.3862	16.933	566.48				
	130	182	.005495	20	.5918	36.493	1635.0				
2	169	159	.006289	19.5	.8216	64.055	3595.7				
	220	137	.007299	25.5	1.1681	114.792	8156.3				
3	270	117	.008547	25	1.5642	183.100	15604				
	320	96	.01042	25	2.0384	273.165	27010				
4	374	74	.01351	27	2.6845	400.683	45204				
	450	46	.02174	38	4.0240	655.606	85543				
5	520	26	.03846	35	6.1310	1011.03	143675				
	595	12	.08333	37.5	10.6772	1642.13	249169				
6	670	2	.5	37.5	32.5730	3264.80	427179				
	745	2	.5	37.5	70.0790	7114.02	816384				
7	820	2	.5	37.5	107.573	13775.7	1599749				
	820	10	.1	0							
8	850	10	.1	15	113.573	17092.9	2062778				
DEFLECTION OF POINT i											
				$j > i$		$a_{ij} = 2\iiint_0^i \frac{dx}{EI} + (x_j - x_i)\iint_0^i \frac{dx}{EI}$					

TABLE 2 (CONT.)

TORSIONAL INFLUENCE COEFFICIENTS

1	2	3	4	5	6	7	8	9	10	11	12
	STA.										
	x	$\frac{\Delta x}{2}$	GJ	$\frac{1}{GJ}$	$\int_{45}^{x} \frac{dx}{GJ}$	$\frac{1}{GJ}/57.3$					
	IN.	IN.	$\times 10^{-7}$	$\times 10^{-7}$	PER 0						
B-29	0										
	45	22.5	118.0	.00847	.1906	.003326					
	90	22.5	111.6	.00896	.5827	.01017					
	130	20	105.5	.00948	.9515	.01661					
	169	19.5	99.0	.01010	1.3334	.02327					
	220	25.5	90.0	.01111	1.8742	.03271					
	270	25	80.5	.01242	2.4625	.04298					
	320	25	71.0	.01408	3.1250	.05454					
	374	27	60.3	.01658	3.9528	.06898					
	425	25.5	51.0	.01961	4.8756	.08509					
	475	25	41.5	.02410	5.9684	.1042					
	520	22.5	32.8	.03049	7.1967	.1256					
	570	25	21.0	.04762	9.1494	.1597					
	620	25	10.3	.09709	12.7672	.2228					
	670	25	6.0	.1667	19.3619	.3379					
	720	25	3.7	.2703	30.2869	.5286					
	770	25	2.8	.3571	45.9719	.8023					
	820	25	2.6	.3846	64.5144	1.1259					
B-29	850	15	10	.10	71.7834	1.2528					
				$\int_{x}^{850} \frac{dx}{GJ}$							
F-84	850			1.4311							
	905			.8726	72.3419	1.2625					
	960			.4991	72.7154	1.2690					
	1015			.1920	73.0225	1.2744					
F-84	1070			.0	73.2145	1.2777					

TABLE 2 (CONT.) BENDING & TORSIONAL INFLUENCE COEFFICIENTS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	BENDING	INFLUENCE		COEFFICIENTS			~	B-29												
				$K \times 10^{-7}$																
i \ j	90	169	270	374	520	670	820	850												
90	11.330	24.707	41.809	59.419	84.141	109.54	134.94	140.02												
169	24.707	71.914	136.61	203.23	296.25	392.83	488.91	508.13												
270	41.809	136.61	302.08	502.50	769.83	1044.5	1319.1	1374.1												
374	59.419	203.23	502.50	904.08	1487.1	2070.1	2691.1	2811.3												
520	84.141	296.25	769.83	1487.1	2833.5	4390.0	5906.6	6209.9												
670	109.54	392.83	1044.5	2070.1	4390.0	8543.6	13441	14420												
820	134.94	488.91	1319.1	2691.1	5906.6	13441	31995	36128												
850	140.02	508.13	1374.1	2811.3	6209.9	14420	36128	41256												

	TORSIONAL	INFLUENCE		COEFFICIENTS			~	B-29	& F-84		F-84	STA.	
		90	169	270	374	520	670	820	850	905	960	1015	1070
90		.01017											
169			.02327										
270				.04298									
374					.06898								
520						.1256							
670							.3379						
820								1.1259					
850									1.2528				
905	F-84 STA.									1.2625			
960											1.2670		
1015												1.2744	
1070													1.2777

TABLE 3 — SYMMETRIC BENDING

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA.																				
x	$M(x)$		h_1	Mh_1	KMh_1	$\frac{\Sigma}{\omega^2}(h_2)$	h_2	Mh_2	KMh_2	$\frac{\Sigma}{\omega^2}(h_3)$	h_3									
IN.	SLUG																			
90	12.06		-.0674	-.813	638.5	-1565545	-.06839	-.8248	640.8	-1571387	-.06837									
169	33.42		-.0586	-1.96	2364.3	-1344876	-.05864	-1.9597	2372.5	-1346953	-.05861									
270	12.67		-.03595	-.456	6524.1	-805 7539	-.03513	-.4458	6544.6	-806 2564	-.03508									
374	28.82		.00333	.0960	13503.4	98 7843 5	.004307	.1241	13540.7	100505.1	.004373									
520	5.350		.0974	.521	29954.9	2250 948	.09727	.5204	30020.9	2256 363	.09750									
670	1.218		.316	.385	68505.3	7227196	.3151	.3838	68616.4	7238455	.3149									
820	.363		.865	.314	166817	19968946	.8706	.3160	167023	19992246	.8697									
850	4.152		1.0	4.15	189877	22957382	1.0	4.1520	190102	22983353	1.0									
						$.436 \times 10^{-7}$				$.4351 \times 10^{-7}$										
FUS.	31.53																			
Σ =	129.603																			

EFF. MASS @ B-29 TIP $= M_{29} + \frac{K_x^2}{b^2 + K_x^2} M_{84}$

$= .842 + (.0884)(37.495)$

$= 4.152$

$B = 220$

$K_x = 571(12) = 68.52$

$M_{84} = \frac{14450}{386} = 37.495$

$h_0 = -\frac{1}{M_0} \Sigma M_i h_i = -.031715 \times 2.269$

$= -.07196$

$\omega^2 = \frac{10^7 \times 129.603}{22\,983\,353} = 56.39$ RAD/SEC^2

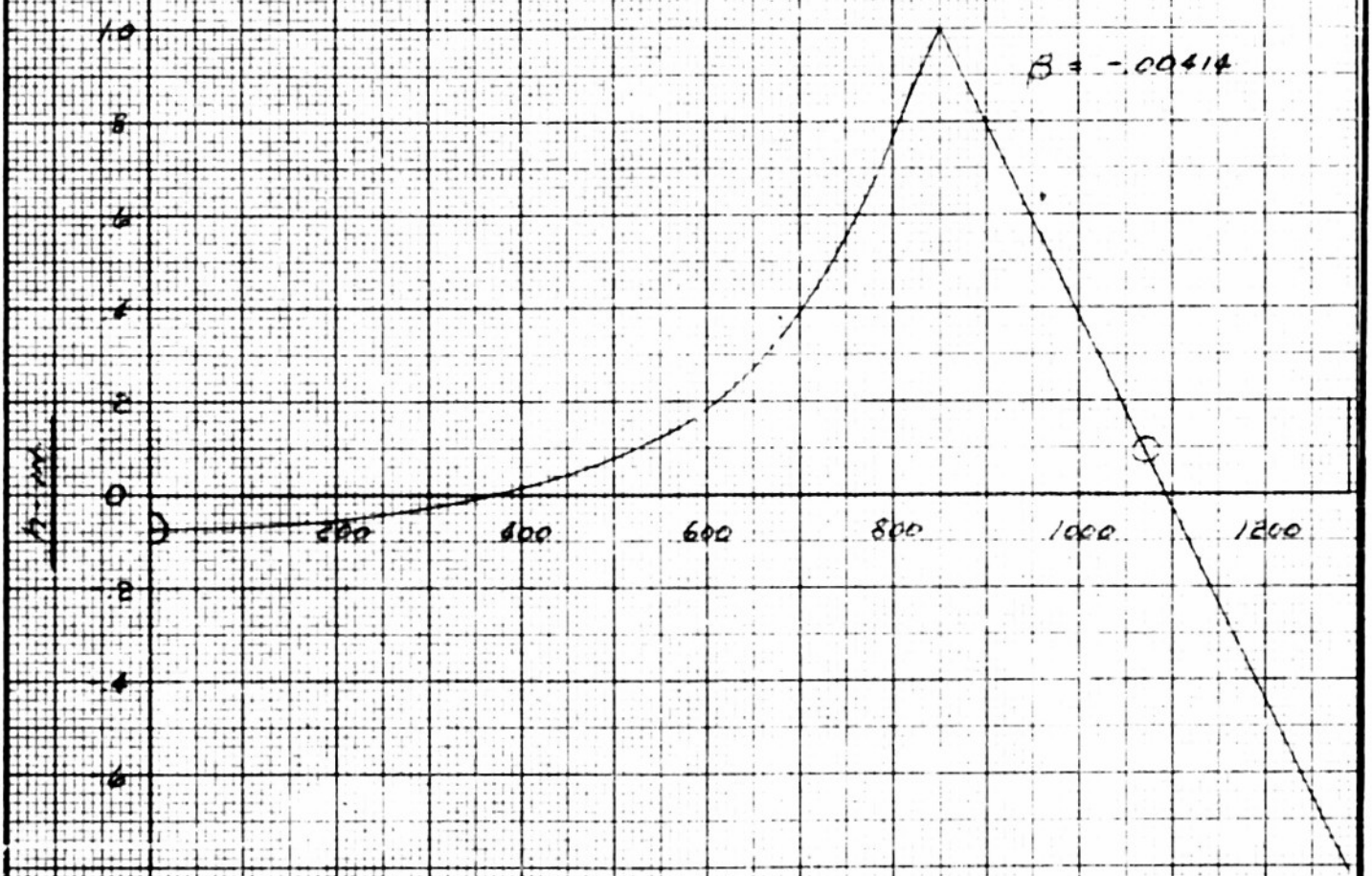
$\omega = (9.55)(7.51) = 72$ CPM

$\beta = -\frac{1}{B + \frac{K_x^2}{M}} = -\frac{1}{241} = -.00414$

$\beta_{FT} = -.04968$

FIGURE 6
1st BENDING - SYMMETRIC

$h'(\ell_1) = \frac{1 - .8699}{-.30} = -.00436$



1st FOOT SYSTEM:

$\beta_n = 12(-.00414 - .00424) = -.10176$

STA. - IN.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
		$I - \frac{1}{\Sigma'}(x)(\bar{M})(x) =$			T					h_1	Kh_1	TKh_1	h_2	Kh_2	TKh_2	h_3	Kh_3	TKh_3	h_4	
.99151	-.044158	-.026788	-.054271	-.021751	-.006380	-.002327	-.005596			-.03	280	-1300	-.021	281	-1335	-.02223	286	-1357	-.02229	
-.015935	.91708	-.050302	-.15824	-.040843	-.011981	-.004370	-.010507			-.05	1000	-1900	-.030	1019	-2015	-.03363	1037	-2049	-.03365	
-.025458	-.13247	.91964	-.25281	-.065252	-.019141	-.006982	-.016787			-.06	2700	-2100	-.034	2752	-2095	-.03497	2801	-2129	-.03497	
-.035264	-.18350	-.11132	.64981	-.090385	-.026513	-.009671	-.023253			-.05	5500	-1100	-.018	5618	-1096	-.01829	5717	-1112	-.01826	
-.049030	-.25513	-.15477	-.48670	.87433	.036863	-.013446	-.032330			.02	12000	2900	.047	12325	2970	.04991	12541	3046	.05003	
-.063173	-.32873	-.19942	-.62735	-.16192	.95250	-.017325	-.041656			.3	28000	16000	.260	27916	15888	.2652	28397	16162	.2655	
-.077317	-.40232	-.24407	-.76780	-.19817	-.058131	.97880	-.050982			.8	66000	52000	.83	66345	51624	.8617	67443	52470	.8618	
-.080145	-.41704	-.25300	-.79587	-.20542	-.060257	-.021979	.94715			1.0	74000	59000	1.0	75168	59708	1.0	76402	60880	1.0	

$\Sigma' = I_{y(0)} + \Sigma M x^2 = 265,630 + 11,245,820 = 11,511,450$

$\frac{1}{\Sigma'} = .08687 \times 10^{-6}$

$B = 220$

$K_1 = 5.71(12) = 68.52$

$M_{84} = \frac{14450}{386} = 37.435$

$\omega^2 = \frac{10^7}{60880} = 164.3$

$\omega = (9.55)(12.82) = 122$ cpm

$\beta = -\frac{1}{B + \frac{K_1^2}{B}} = -\frac{1}{241} = -.00414$

$\beta_{FT} = -.04968$

FIGURE 8
1ST BENDING - ANTI-SYMMETRIC

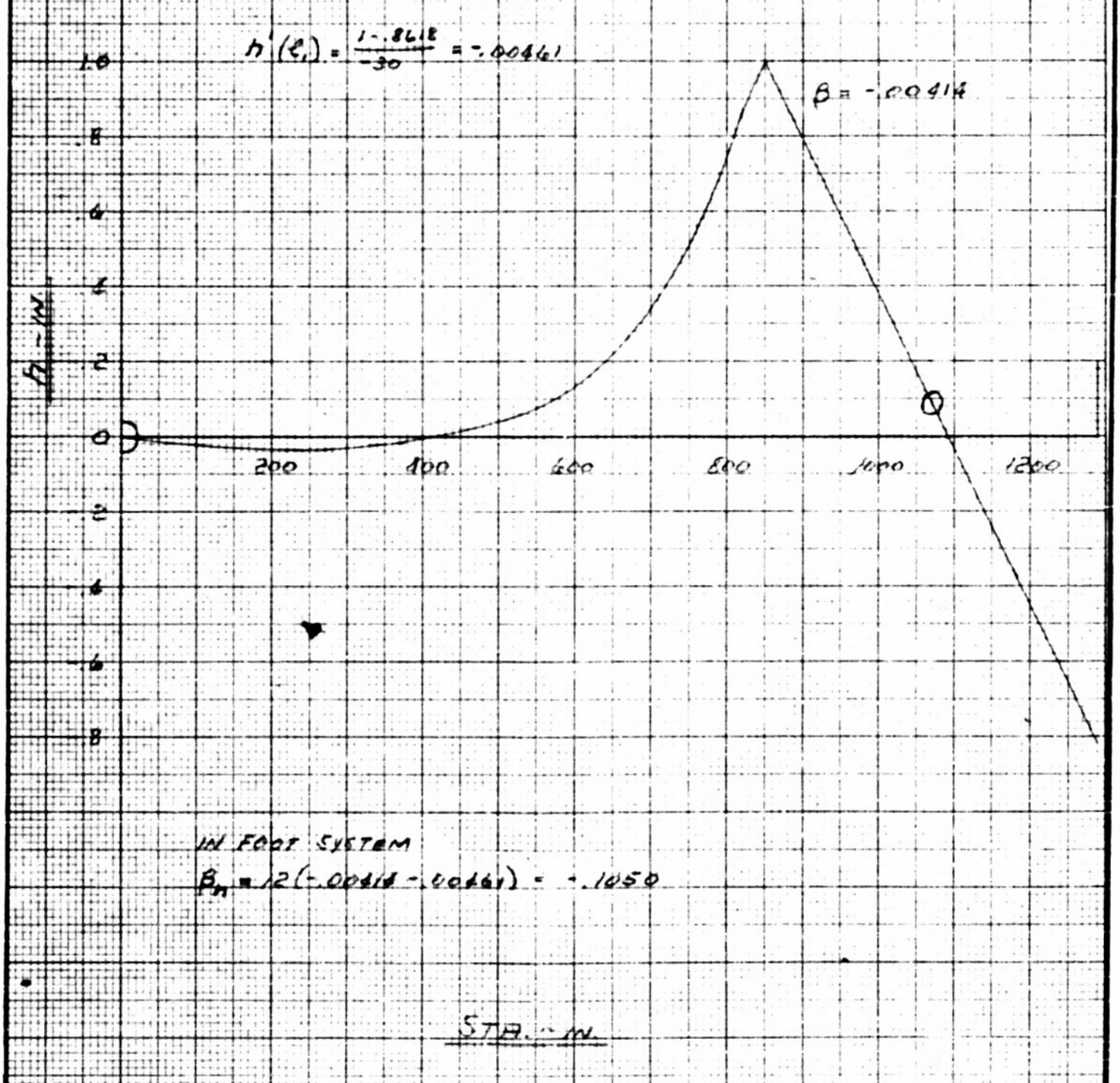


TABLE 5 SYMMETRIC TORSION

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA.																				
x	$\vartheta(x)$		α_1	$\vartheta\alpha_1$	$K\vartheta\alpha_1$	$\frac{\Sigma}{\omega^2}\alpha_2$	α_2	$\vartheta\alpha_2$	$K\vartheta\alpha_2$	$\frac{\Sigma}{\omega^2}\alpha_3$	α_3									
	$\times 10^{-3}$			$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-6}$		$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-6}$										
90	17.10		-.059	-1.01	3.73	-145 998	-.0626	-1.070	3.443	-147 246	-.06324									
169	221.53		-.047	-10.4	8.24	-121 077	-.0519	-11.50	7.892	-122 662	-.05268									
270	14.51		-.025	-.363	15.0	-83 723	-.0359	-.5209	14.81	-84 436	-.03627									
374	176.45		.0095	1.68	24.3	-32 335	-.0139	-2.453	23.95	-33 931	-.01457									
520	4.92		.057	.280	44.3	78 178	.0335	.1648	44.00	76 857	.03301									
670	1.04		.22	.229	119	490 945	.211	.2194	119.1	491 832	.2112									
820	.26		.83	.216	398	2 032 602	.872	.2267	397.9	2 032 376	.8729									
850	.20		1.0	.20	443	2 281 257	.979	.1958	442.7	2 279 924	.9792									
905	.585		1.0	.585	446	2 297 834	.986	.5768	446.1	2 298 711	.9873									
960	1.640		1.0	1.64	449	2 314 411	.993	1.629	448.4	2 311 420	.9928									
1015	1.070		1.0	1.07	451	2 325 462	.998	1.068	450.3	2 321 919	.9973									
1070	350.0		1.0	350.0	452	2 330 988	1.0	350.0	451.5	2 328 550	1.0									
						$.429 \times 10^{-6}$				$.4295 \times 10^{-6}$										
FUS.	4736.35																			
$\Sigma =$	5525.655																			

$$\omega^2 = (10^7)(.4295 \times 10^{-12})(5525.655 \times 10^3) = 23.73$$

$$\omega = (9.55)(4.87) = 47 \text{ cpm}$$

$$\alpha_0 = -\frac{1}{I_0}\Sigma I_i \alpha_i = (-.00021113)(338.23)$$

$$= -.07141$$

FIGURE 7
1st TORSION - SYMMETRIC

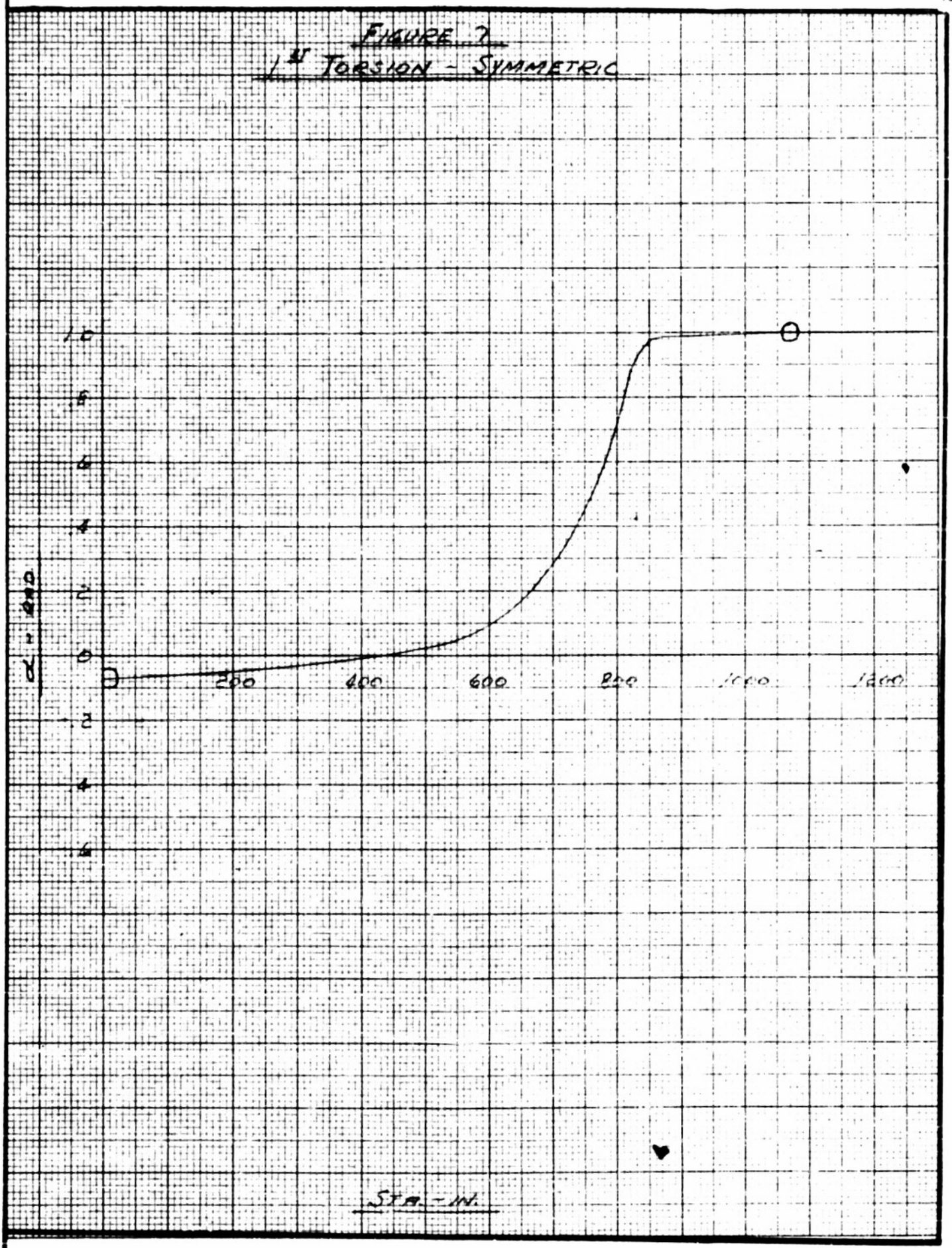


TABLE 6 — ANTI-SYMMETRIC TORSION

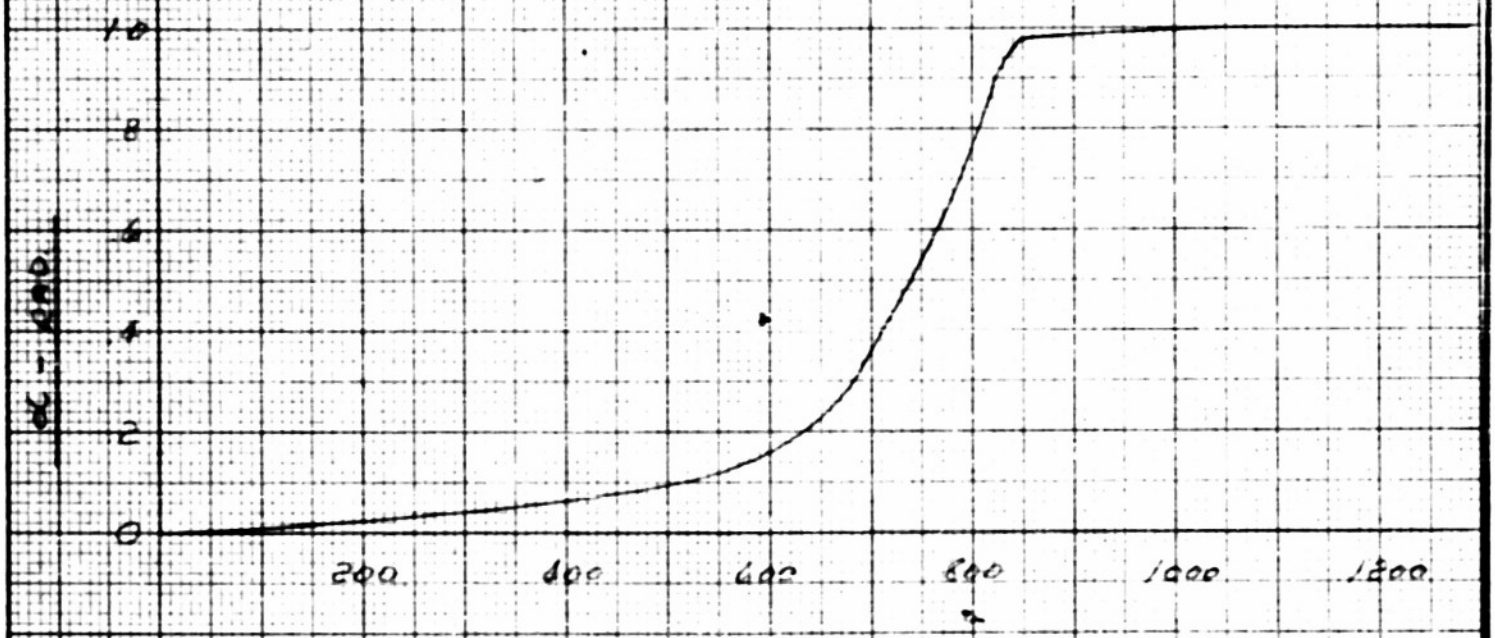
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA.																				
x	$\vartheta_{(x)}$		α_1	$\vartheta\alpha_1$	$K\vartheta\alpha_1$	α_2	$\vartheta\alpha_2$	$K\vartheta\alpha_2$	α_3											
	$\times 10^{-3}$			$\times 10^{-3}$	$\times 10^{-3}$		$\times 10^{-3}$	$\times 10^{-3}$												
90	17.10		.010	.171	3.783	.00835	.1428	3.755	.008293											
169	221.53		.023	5.10	8.654	.0191	4.231	8.591	.01897											
270	14.51		.041	.595	15.88	.0351	.5093	15.78	.03485											
374	176.45		.066	11.6	25.40	.0561	9.899	25.26	.05579											
520	4.92		.11	.541	45.47	.100	.4920	45.33	.1001											
670	1.04		.28	.291	120.6	.266	.2766	120.5	.2661											
820	.26		.88	.229	399.3	.881	.2291	399.2	.8816											
850	.20		.98	.196	444.2	.981	.1962	444.0	.9806											
905	.585		.99	.579	447.6	.988	.5780	447.5	.9883											
960	1.640		.99	1.62	449.9	.993	1.629	449.8	.9934											
1015	1.070		1.0	1.07	451.8	.997	1.067	451.7	.9976											
1070	350.0		1.0	350	452.0	1.0	350	452.8	1.0											

$$\omega^2 = \frac{10^7}{452.8 \times 10^3} = 22.085$$

$$\omega = (9.55)(4.7) = 45 \text{ CPM}$$

FIGURE 9

1ST TORSION - ANTI-SYMMETRIC



PREPARED G.M.
CHECKED
REVISED

TABLE 7

GENERALIZED MASSES – SYMMETRIC CASE

MODES: h, α, β

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
ELEMENT	INTERVAL	STA.	x_1 IN.	x_1 FT	$M_{(x)}$	$S_{(x)}$	$I_{(x)}$	h_h	α_1	h_β	A_{hh}	$A_{\beta\beta}$	$A_{\alpha\alpha}$	$A_{h\beta}$	$A_{h\alpha}$	$A_{\beta\alpha}$				
			(3) - 850	(4) ÷ 12	SLUGS	SLUG-FT.	SLUG-FT.^{2}	1 - .04968(5)		(4) ÷ 440	(6)(9)^2	(6)(11)^2	(8)(10)^2	(6)(9)(11)	(7)(9)(10)	(7)(11)(10)				
1	FUS. -45	0	0		378.36	7847	394.696	-.07196	-.07141	0	1.959	0	2013	0	40.33	0				
2	45-130	90	↑		144.72	-49.2	1425	-.06837	-.06324	↑	.6765	↑	5.699	↑	-.2127	↑				
3	130-220	169			401.04	-1790	18461	-.05861	-.05268		1.378		51.23		-5.527					
4	220-320	270			152.28	101.0	1209	-.03508	-.03627		1874		1.590		.1285					
5	320-450	374			345.84	-1412	14704	.004373	-.01457		.00661		3.121		.08997					
6	450-575	520			64.20	69.9	410	.09730	.03301		.6078		.4468		.2245					
7	575-745	670			14.62	12.9	86.6	.3149	.2112		1.450		3.863		.8579					
8	745-850	820	↓		4.36	2.6	21.7	.8699	.8729	↓	3.274	↓	16.53	↓	1.914	↓				
9	TIP WT.	850	0	0	14.4	2.7	25.9	1.0	.9792	0	14.40	0	24.83	0	2.644	0				
10	877.5-932.5	905	55	4.6	16.56	12.4	48.7	.7215	.9873	.1250	9.856	.2587	47.47	1.597	9.445	1.530				
11	932.5-987.5	960	110	9.2	46.44	40	137	.5429	.9928	.250	13.69	2.903	135.0	6.303	21.56	9.928				
12	987.5-1042.5	1015	165	13.7	17.16	17.3	89.2	.3194	.9973	.375	1.251	2.413	88.72	2.055	5.511	6.469				
13	1042.5-1097.5	1070	220	18.3	277.92	913	28883	.09086	1.0	.500	2.294	69.48	28883	12.63	82.96	456.5				
14	1097.5-1152.5	1125	275	22.9	17.16	17.3	89.2	-.1377	↑	.625	.3254	6.703	89.2	-1.477	-2.382	10.81				
15	1152.5-1207.5	1180	330	27.5	46.44	40	137	-.3662		.75	6.628	26.12	137	-12.75	-14.65	30.0				
16	1207.5-1262.5	1235	385	32.1	16.56	12.4	48.7	-.5947	↓	.875	5.857	12.68	48.7	-8.617	-7.374	10.85				
17	1262.5-1290	1290	440	36.7	4.32	2.7	9.2	-.8233	1.0	1.0	2.928	4.32	9.2	-3.557	-2.223	2.7				
										Σ =	66.896	124.88	31559	-3.816	133.36	528.79				

TABLE 8

AERODYNAMIC INTEGRALS - SYMMETRIC CASE

MODES: h, α, β

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA.									B-29											
x	b	b^2	b^3	b^4	h_h FT.	h_β	α_1	h_h^2	bh_h^2	$b^2h_h^2$	$\alpha_1 h_h b^3$	$\alpha_1 h_h b^2$	$\alpha_1 h_h b$	$b^4\alpha_1^2$	$b^3\alpha_1^2$					
IN.	FT	(2)²	(2)(3)	(2)(4)	1-(.04968) x (X, FT)	X, ÷440		(6)²	(2)(9)	(3)(9)	(8)(6)(4)	(8)(6)(3)	(8)(6)(2)	(5)(8)²	(4)(8)²					
90	7.84	61.466	481.89	3778.0	-.07	0	-.06	.0049	.038416	.30118	2.0239	.25816	.032928	13.601	1.7348					
170	7.41	54.908	406.87	3014.9	-.06	↓	-.055	.0036	.026676	.19767	1.3427	.18120	.024453	9.1201	1.2308					
250	6.95	48.303	335.71	2333.2	-.045		-.04	.002025	.014074	.097814	.60428	.086945	.012510	3.7331	.53714					
330	6.50	42.250	274.63	1785.1	-.015		-.025	.000225	.001463	.009506	.10299	.015844	.002437	1.1157	.17164					
410	6.09	37.088	225.87	1375.5	.02		-.05	.0004	.002436	.014835	-.22587	-.037088	-.006090	3.4387	.56467					
490	5.62	31.584	177.50	997.55	.07		.02	.0049	.027538	.15476	.24850	.044218	.007868	.39902	.0710					
570	5.21	27.144	141.42	736.80	.145		.065	.021025	.10954	.57070	1.9329	.25583	.049104	3.1130	.59750					
650	4.75	22.563	107.17	509.06	.27		.17	.0729	.34627	1.6448	4.9191	1.0356	.21803	14.712	3.0972					
730	4.33	18.749	81.183	351.52	.48	↓	.375	.2304	.99763	4.3198	14.613	3.3748	.77940	49.433	11.416					
810	3.88	15.054	58.410	226.63	.815	0	.80	.66423	2.5772	9.9992	38.083	9.8152	2.5298	145.04	37.382					
B-29	Σ=								4.14124	17.3103	63.0445	15.0307	3.65044	243.706	56.8027					
B-29	ΣΔX =								27.610	115.408	420.318	100.310	24.337	1624.8	378.704					
																$b^2\alpha_1^2$	$b^3 h_h$	$b^2 h_h$	$b\, h_h$	$\alpha_1 b^4$
									F-84							(3)(8)²	(4)(6)	(3)(6)	(2)(6)	(8)(5)
877.5	2.83	8.0089	22.665	64.142	.88615	.0625	.985	.78526	2.2223	6.2891	19.783	6.9906	2.4702	62.232	21.990	7.7704	20.065	7.0971	2.5078	63.180
932.5	3.33	11.089	36.926	122.96	.65845	.1875	.99	.43356	1.4438	4.8077	24.071	7.2286	2.1707	120.51	36.191	10.868	26.314	7.3016	2.1926	121.73
987.5	3.84	14.746	56.625	217.44	.43075	.3125	.995	.18555	.71251	2.7361	24.269	6.3201	1.6458	215.27	56.060	14.577	24.371	6.3518	1.6541	216.35
1042.5	4.33	18.749	81.183	351.52	.20305	.4375	.999	.041229	.17852	.77300	16.468	3.8032	.87834	350.82	81.021	18.712	16.484	3.8070	.87921	351.17
1097.5	4.33	18.749	81.183	351.52	-.02465	.5625	1.0	.0006076	.002631	.011392	-2.0012	-.46216	-.10673	351.52	81.183	18.749	-2.0012	-.46216	-.10673	351.52
1152.5	3.84	14.746	56.625	217.44	-.25234	.6875	↑	.063675	.24451	.93895	-14.289	-3.7210	-.96899	217.44	56.625	14.746	-14.289	-3.7210	-.96899	217.44
1207.5	3.33	11.089	36.926	122.96	-.48004	.8125		.23044	.76737	2.5553	-17.726	-5.3232	-1.5985	122.96	36.926	11.089	-17.726	-5.3232	-1.5985	122.96
1262.5	2.83	8.0089	22.665	64.142	-.70775	.9375	1.0	.50091	1.4176	4.0117	-16.041	-5.6683	-2.0029	64.142	22.665	8.0089	-16.041	-5.6683	-2.0029	64.142
F-84	Σ=	105.186	394.798	1512.12					6.98924	22.1232	34.5338	9.16784	2.48792	1504.89	392.661	104.542	35.2168	9.36284	2.55659	1508.47
F-84	ΣΔX =	482.067	1809.36	6930.05					32.032	101.391	158.268	42.016	11.402	6896.9	1799.6	479.116	161.379	42.001	11.717	6913.4

B-29 ~ ΔX = 6.667

F-84 ~ ΔX = 4.583

TABLE 8 (CONT.)

F-84

AERODYNAMIC INTEGRALS ~ SYMMETRIC CASE

MODES: h, α, β

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA.	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)								
χ	$\alpha_1 b^3$	$\alpha_1 b^2$	$b^2 h_h h_\beta$	$b h_h h_\beta$	$b^3 h_\beta$	$b^2 h_\beta$	$\alpha_1 h_\beta b^3$	$\alpha_1 h_\beta b^2$	$\alpha_1 h_\beta b$	$b h_\beta$	$b h_\beta^2$	$b^2 h_\beta^2$								
	(8)(4)	(8)(3)	(3)(6)(7)	(2)(6)(7)	(4)(7)	(3)(7)	(8)(7)(4)	(8)(7)(3)	(8)(7)(2)	(2)(7)	$(2)(7)^2$	$(3)(7)^2$								
832.5	22.325	7.8888	.44357	.15673	1.4166	.50056	1.3954	.49305	.17422	.17687	.011055	.031285								
932.5	36.557	10.928	1.3690	.41112	6.9236	2.0792	6.8544	2.0584	.61813	.62437	.11707	.38985								
987.5	56.342	14.672	1.9849	.51690	17.675	4.6081	17.607	4.5851	1.194	1.2	.375	1.440								
1042.5	81.102	18.730	1.6656	.38466	35.518	8.2027	35.482	8.1945	1.8925	1.8944	.82879	3.3973								
1097.5	81.183	18.749	-.25996	-.060038	45.665	10.546	45.665	10.546	2.4356	2.4356	1.3700	5.9323								
1152.5	56.625	14.746	-2.5582	-.66618	38.930	10.138	38.930	10.138	2.64	2.64	1.815	6.9698								
1207.5	36.926	11.089	-4.3251	-1.2988	30.002	9.0098	30.002	9.0098	2.7056	2.7056	2.1983	7.3205								
1262.5	22.665	8.0089	-5.3140	-1.8777	21.248	7.5083	21.248	7.5083	2.6531	2.6531	2.4873	7.0391								
$\Sigma =$	393.725	104.862	-6.99419	-2.43351	197.398	52.5927	197.184	52.5927	14.3131	14.3299	9.20251	32.5201								
$\Sigma^* \Delta\chi =$	1804.4	480.583	-32.054	-11.152	904.67	241.032	903.694	241.032	65.597	65.674	42.175	149.040								

AILERON COEFFICIENTS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	χ	b	b^2	b^3	b^4	h_h	h_β	α_1	$b^3 h_h$	$\alpha_1 b^4$	$b^3 h_\beta$
	IN.	FT.	$(2)^2$	(2)(3)	(2)(4)	$1-(.04968)\times(\chi_1 \div 440)$	$\chi_1 \div 440$		(4)(6)	(8)(5)	(4)(7)
INB'D.	876	2.82	7.9524	22.426	63.241	.89236	.059091	.985	20.012	62.292	1.3252
"	909	3.12	9.7344	30.371	94.758	.75574	.13409	.988	22.953	93.621	4.0724
"	942	3.42	11.696	40.0	136.80	.61912	.20909	.992	24.765	135.71	8.3636
OUTB'D.	1198	3.42	11.696	40.0	136.80	-.44072	.79091	1.0	-17.629	136.80	31.636
"	1231	3.12	9.7344	30.371	94.758	-.57734	.86591	1.0	-17.534	94.758	26.299
"	1264	2.82	7.9524	22.426	63.241	-.71396	.94091	1.0	-16.011	63.241	21.101
$\Sigma =$				$\Sigma \delta_i \alpha_j =$	0				118.904	-3.176	-65.2748
$\Sigma \Delta\chi =$				$\Sigma \delta_i \delta_j =$	1621.4				326.99	-8.734	-179.506

These summations actually include ail. defl. which is taken positive for inb'd. and negative for outb'd.

F-84 ~ $^*\Delta\chi$ = 4.583

F-84 ~ AIL. $\Delta\chi$ = 2.750

TABLE 9

AERODYNAMIC FORCES - SYMMETRIC

$$
\begin{aligned}
B_{hh} = {} & \sum_{\ell 4} b^2 h_h^2 + b_r K_2(L_h) \sum_{\ell 4} b h_h^2 + \sum_{\ell 4} b^2 h_h^2 + b_r K_2(L_h) \sum_{\ell 4} b h_h^2 - a \alpha_h \sum_{\ell 4} b^3 h_h \\
& + b_r K_2(L_\alpha) \alpha_h \sum_{\ell 4} b^2 h_h + b_r^2 K_3(L_\alpha) \alpha_h \sum_{\ell 4} b h_h - \left(\tfrac{1}{2}+a\right) b_r K_2(L_h) \alpha_h \sum_{\ell 4} b^2 h_h \\
& + \left[L_\beta - (c-e) L_z\right] \delta_h \sum_{a} h_h b^3 - a \alpha_h \sum_{\ell 4} b^3 h_h - b_r \left(\tfrac{1}{2}+a\right) K_2(L_h) \alpha_h \sum_{\ell 4} b^2 h_h \\
& + \left(\tfrac{1}{8}+a^2\right) \alpha_h^2 \sum_{\ell 4} b^4 + b_r K_2(M_\alpha) \alpha_h^2 \sum_{\ell 4} b^3 - \left(\tfrac{1}{2}+a\right) b_r K_2(L_\alpha) \alpha_h^2 \sum_{\ell 4} b^3 \\
& - \left(\tfrac{1}{2}+a\right) b_r^2 K_3(L_\alpha) \alpha_h^2 \sum_{\ell 4} b^2 + \left(\tfrac{1}{2}+a\right)^2 b_r K_2(L_h) \alpha_h^2 \sum_{\ell 4} b^3 \\
& + \left[T_h - (c-e) P_h\right] \delta_h \sum_{a} b^3 h_h + \left[T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z\right] \delta_h^2 \sum_{a} b^4
\end{aligned}
$$

$$
\begin{aligned}
B_{h\alpha} = {} & \tfrac{1}{2} \sum_{\ell 4} \alpha_1 h_h b^2 + b_r K_2(L_\alpha) \sum_{\ell 4} \alpha_1 h_h b^2 + b_r^2 K_3(L_\alpha) \sum_{\ell 4} \alpha_1 h_h b - a \sum_{\ell 4} \alpha_1 h_h b^3 \\
& + b_r K_2(L_\alpha) \sum_{\ell 4} \alpha_1 h_h b^2 + b_r^2 K_3(L_\alpha) \sum_{\ell 4} \alpha_1 h_h b - \left(\tfrac{1}{2}+a\right) b_r K_2(L_h) \sum_{\ell 4} \alpha_1 h_h b^2 \\
& + \left(\tfrac{1}{8}+a^2\right) \alpha_h \sum_{\ell 4} b^4 \alpha_1 + b_r K_2(M_\alpha) \alpha_h \sum_{\ell 4} b^3 \alpha_1 - \left(\tfrac{1}{2}+a\right) b_r K_2(L_\alpha) \alpha_h \sum_{\ell 4} b^3 \alpha_1 \\
& - \left(\tfrac{1}{2}+a\right) b_r^2 K_3(L_\alpha) \alpha_h \sum_{\ell 4} b^2 \alpha_1 + \left(\tfrac{1}{2}+a\right)^2 b_r K_2(L_h) \alpha_h \sum_{\ell 4} b^3 \alpha_1 \\
& + \left[T_\alpha - (c-e) P_\alpha - \left(\tfrac{1}{2}+a\right) T_h + \left(\tfrac{1}{2}+a\right)(c-e) P_h\right] \delta_h \sum_{a} b^4 \alpha_1
\end{aligned}
$$

$$
\begin{aligned}
B_{h\beta} = {} & \sum_{\ell 4} b^2 h_h h_\beta + b_r K_2(L_h) \sum_{\ell 4} b h_h h_\beta - a \alpha_\beta \sum_{\ell 4} b^3 h_h + b_r K_2(L_\alpha) \alpha_\beta \sum_{\ell 4} b^2 h_h \\
& + b_r^2 K_3(L_\alpha) \alpha_\beta \sum_{\ell 4} b h_h - \left(\tfrac{1}{2}+a\right) b_r K_2(L_h) \alpha_\beta \sum_{\ell 4} b^2 h_h + \left[L_\beta - (c-e) L_z\right] \delta_\beta \sum_{a} b^3 h_h \\
& - a \alpha_h \sum_{\ell 4} b^3 h_\beta - b_r \left(\tfrac{1}{2}+a\right) K_2(L_h) \alpha_h \sum_{\ell 4} b^2 h_\beta + \left(\tfrac{1}{8}+a^2\right) \alpha_h \alpha_\beta \sum_{\ell 4} b^4 \\
& + b_r K_2(M_\alpha) \alpha_h \alpha_\beta \sum_{\ell 4} b^3 - \left(\tfrac{1}{2}+a\right) b_r K_2(L_\alpha) \alpha_h \alpha_\beta \sum_{\ell 4} b^3 \\
& - \left(\tfrac{1}{2}+a\right) b_r^2 K_3(L_\alpha) \alpha_h \alpha_\beta \sum_{\ell 4} b^2 + \left(\tfrac{1}{2}+a\right)^2 b_r K_2(L_h) \alpha_h \alpha_\beta \sum_{\ell 4} b^3 \\
& + \left[T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z\right] \delta_h \delta_\beta \sum_{a} b^4 + \left[T_h - (c-e) P_h\right] \delta_h \sum_{a} h_\beta b^3
\end{aligned}
$$

TABLE 9 (CONT.)

$$B_{\alpha h} = \frac{1}{2}\sum_{29} b^3\alpha_1 h_h - a\sum_{84} b^3\alpha_1 h_h - b_r\left(\frac{1}{2}+a\right)K_2(l_h)\sum_{84} b^2\alpha_1 h_h + \left(\frac{1}{8}+a^2\right)\alpha_h\sum_{84} b^4\alpha_1$$
$$+ b_r K_2(M_\alpha)\alpha_h\sum_{84} b^3\alpha_h - \left(\frac{1}{2}+a\right)b_r K_2(l_\alpha)\alpha_h\sum_{84} b^3\alpha_1$$
$$-\left(\frac{1}{2}+a\right)b_r^2 K_3(l_\alpha)\alpha_h\sum_{84} b^2\alpha_1 + \left(\frac{1}{2}+a\right)^2 b_r K_2(l_h)\alpha_h\sum_{84} b^3\alpha_1$$
$$+\left[M_\beta - \left(\frac{1}{2}+a\right)L_\beta - (c-e)M_z + (c-e)\left(\frac{1}{2}+a\right)L_z\right]\delta_h\sum_a b^4\alpha_1$$

$$B_{\alpha\alpha} = \frac{3}{8}\sum_{29}\alpha_1^2 b^4 + b_r K_2(M_\alpha)\sum_{29}\alpha_1^2 b^3 + \left(\frac{1}{8}+a^2\right)\sum_{84}\alpha_1^2 b^4 + b_r K_2(M_\alpha)\sum_{84}\alpha_1^2 b^3$$
$$-\left(\frac{1}{2}+a\right)b_r K_2(l_\alpha)\sum_{84}\alpha_1^2 b^3 - \left(\frac{1}{2}+a\right)b_r^2 K_3(l_\alpha)\sum_{84}\alpha_1^2 b^2$$
$$+\left(\frac{1}{2}+a\right)^2 b_r K_2(l_h)\sum_{84}\alpha_1^2 b^3$$

$$B_{\alpha\beta} = -a\sum_{84}\alpha_\alpha h_\beta b^3 - b_r\left(\frac{1}{2}+a\right)K_2(l_h)\sum_{84}\alpha_\alpha h_\beta b^2 + \left(\frac{1}{8}+a^2\right)\alpha_\beta\sum_{84}\alpha_\alpha b^4$$
$$+ b_r K_2(M_\alpha)\alpha_\beta\sum_{84} b^3\alpha_\alpha - \left(\frac{1}{2}+a\right)b_r K_2(l_\alpha)\alpha_\beta\sum_{84}\alpha_\alpha b^3$$
$$-\left(\frac{1}{2}+a\right)b_r^2 K_3(l_\alpha)\alpha_\beta\sum_{84}\alpha_\alpha b^2 + \left(\frac{1}{2}+a\right)^2 b_r K_2(l_h)\alpha_\beta\sum_{84}\alpha_\alpha b^3$$
$$+\left[M_\beta - \left(\frac{1}{2}+a\right)L_\beta - (c-e)M_z + (c-e)\left(\frac{1}{2}+a\right)L_z\right]\delta_\beta\sum_a\alpha_\alpha b^4$$

$$B_{\beta h} = \sum_{84} b^2 h_h h_\beta + b_r K_2(l_h)\sum_{84} b h_h h_\beta - a\alpha_h\sum_{84} h_\beta b^3 + b_r K_2(l_\alpha)\alpha_h\sum_{84} h_\beta b^2$$
$$+ b_r^2 K_3(l_\alpha)\alpha_h\sum_{84} h_\beta b - \left(\frac{1}{2}+a\right)b_r K_2(l_h)\alpha_h\sum_{84} h_\beta b^2$$
$$+\left[L_\beta - (c-e)L_z\right]\delta_h\sum_a b^3 h_\beta - a\alpha_\beta\sum_{84} h_h b^3 - b_r\left(\frac{1}{2}+a\right)K_2(l_h)\alpha_\beta\sum_{84} h_h b^2$$
$$+\left(\frac{1}{8}+a^2\right)\alpha_\beta\alpha_h\sum_{84} b^4 + b_r K_2(M_\alpha)\alpha_\beta\alpha_h\sum_{84} b^3 - \left(\frac{1}{2}+a\right)b_r K_2(l_\alpha)\alpha_\beta\alpha_h\sum_{84} b^3$$
$$-\left(\frac{1}{2}+a\right)b_r^2 K_3(l_\alpha)\alpha_\beta\alpha_h\sum_{84} b^2 + \left(\frac{1}{2}+a\right)^2 b_r K_2(l_h)\alpha_\beta\alpha_h\sum_{84} b^3$$
$$+\left[T_\beta - (c-e)P_\beta - (c-e)T_z + (c-e)^2 P_z\right]\delta_\beta\delta_h\sum_a b^4 + \left[T_h - (c-e)P_h\right]\delta_\beta\sum_a b^3 h_h$$

TABLE 9 (CONT.)

$$
\begin{aligned}
B_{\beta\alpha} = {} & -a \sum_{84} \alpha_1 h_\beta b^3 + b_r K_2(l_\alpha) \sum_{84} \alpha_1 h_\beta b^2 + b_r^2 K_3(l_\alpha) \sum_{84} \alpha_1 h_\beta b \\
& - \left(\tfrac{1}{2} + a\right) b_r K_2(l_h) \sum_{84} \alpha_1 h_\beta b^2 + \left(\tfrac{1}{8} + a^2\right) \alpha_\beta \sum_{84} \alpha_1 b^4 + b_r K_2(M_\alpha) \alpha_\beta \sum_{84} \alpha_1 b^3 \\
& - \left(\tfrac{1}{2} + a\right) b_r K_2(l_\alpha) \alpha_\beta \sum_{84} \alpha_1 b^3 - \left(\tfrac{1}{2} + a\right) b_r^2 K_3(l_\alpha) \alpha_\beta \sum_{84} \alpha_1 b^2 \\
& + \left(\tfrac{1}{2} + a\right)^2 b_r K_2(l_h) \alpha_\beta \sum_{84} \alpha_1 b^3 + \left[T_\alpha - (c-e) P_\alpha - \left(\tfrac{1}{2} + a\right) T_h + \left(\tfrac{1}{2} + a\right)(c-e) P_h\right] \times \\
& \delta_\beta \sum_a \alpha_1 b^4
\end{aligned}
$$

$$
\begin{aligned}
E_{\beta\beta} = {} & \sum_{84} h_\beta^2 b^2 + b_r K_2(l_h) \sum_{84} h_\beta^2 b - a \alpha_\beta \sum_{84} h_\beta b^3 + b_r K_2(l_\alpha) \alpha_\beta \sum_{84} h_\beta b^2 \\
& + b_r^2 K_3(l_\alpha) \alpha_\beta \sum_{84} h_\beta b - \left(\tfrac{1}{2} + a\right) b_r K_2(l_h) \alpha_\beta \sum_{84} h_\beta b^2 \\
& + \left[l_\beta - (c-e) l_z\right] \delta_\beta \sum_a h_\beta b^3 - a \alpha_\beta \sum_{84} h_\beta b^3 - b_r \left(\tfrac{1}{2} + a\right) K_2(l_h) \alpha_\beta \sum_{84} h_\beta b^2 \\
& + \left(\tfrac{1}{8} + a^2\right) \alpha_\beta^2 \sum_{84} b^4 + b_r K_2(M_\alpha) \alpha_\beta^2 \sum_{84} b^3 - \left(\tfrac{1}{2} + a\right) b_r K_2(l_\alpha) \alpha_\beta^2 \sum_{84} b^2 \\
& - \left(\tfrac{1}{2} + a\right) b_r^2 K_3(l_\alpha) \alpha_\beta^2 \sum_{84} b^2 + \left(\tfrac{1}{2} + a\right)^2 b_r K_2(l_h) \alpha_\beta^2 \sum_{84} b^3 \\
& + \left[T_h - (c-e) P_h\right] \delta_\beta \sum_a b^3 h_\beta + \left[T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z\right] \delta_\beta^2 \sum_a b^4
\end{aligned}
$$

$e = .4$

$b_r = 5.21$

$(c-e) = .05$

$a_{B\text{-}29} = -.5$

$a_{F\text{-}84} = -.2$

TABLE 10

EVALUATION OF TABLE 9

$$B_{hh} = 216.433 + 310.616\,K_2(L_h) - 1.54977\,K_2(L_\alpha) - 2.19645\,K_3(L_\alpha) + .386499\,K_2(M_\alpha) + 27.5653\,L_\beta - 1.37826\,L_z + 27.5653\,T_h - 1.37826\,P_h + 11.5217\,T_\beta - .576083\,P_\beta - .576083\,T_z + .028804\,P_z$$

$$B_{h\alpha} = 234.512 - 71.0859\,K_2(L_h) + 759.047\,K_2(L_\alpha) + 995.146\,K_3(L_\alpha) - 60.1659\,K_2(M_\alpha) - .736276\,T_\alpha + .036814\,P_\alpha + .220883\,T_h - .011044\,P_h$$

$$B_{h\beta} = -33.1693 - 56.5150\,K_2(L_h) + .41405\,K_2(L_\alpha) + .58682\,K_3(L_\alpha) - .10817\,K_2(M_\alpha) - 7.3867\,L_\beta + .369335\,L_z - 15.1324\,T_h + .756618\,P_h - 3.08769\,T_\beta + .15438\,P_\beta + .15438\,T_z - .007192\,P_z$$

$$B_{\alpha h} = 228.53 - 71.0859\,K_2(L_h) + 18.0498\,K_2(L_\alpha) + 25.0461\,K_3(L_\alpha) - 60.1659\,K_2(M_\alpha) - .73628\,M_\beta + .22088\,L_\beta + .036814\,M_z - .011044\,L_z$$

$$B_{\alpha\alpha} = 1747.29 + 843.83\,K_2(L_h) - 2812.77\,K_2(L_\alpha) - 3701.54\,K_3(L_\alpha) + 11,348.97\,K_2(M_\alpha)$$

$$B_{\alpha\beta} = 182.69 - 374.859\,K_2(L_h) - 6.88267\,K_2(L_\alpha) - 6.69208\,K_3(L_\alpha) + 16.0756\,K_2(M_\alpha) + .197301\,M_\beta - .05919\,L_\beta - .009865\,M_z + .00296\,L_z$$

$$B_{\beta h} = -33.2842 - 55.7001\,K_2(L_h) - 8.00602\,K_2(L_\alpha) - 11.8660\,K_3(L_\alpha) - .103166\,K_2(M_\alpha) - 15.1324\,L_\beta + .756618\,L_z - 7.38670\,T_h + .369335\,P_h - 3.08769\,T_\beta + .154385\,P_\beta + .154385\,T_z - .007719\,P_z$$

$$B_{\beta\alpha} = 182.69 - 374.859\,K_2(L_h) + 1249.17\,K_2(L_\alpha) + 1773.87\,K_3(L_\alpha) + 16.0756\,K_2(M_\alpha) + .197301\,T_\alpha - .05919\,T_h - .009865\,P_\alpha + .002960\,P_h$$

TABLE 10 (CONT.)

$$B_{\beta\beta} = 149662 + 218.446\, K_2(L_h) + 2.1559\, K_2(L_\alpha) + 3.03656\, K_3(L_\alpha)$$
$$+ .02828\, K_2(M_\alpha) + 4.05504\, T_h - .20275\, P_h + 4.05504\, L_\beta$$
$$- .20275\, L_z + .82691\, T_\beta - .041345\, P_\beta - .041345\, T_z + .0020673\, P_z$$

TABLE II — FLUTTER DETERMINANT — ALT. = 30,000'

MODES: h, α, β

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1/K	→	1.25		2.50		5.00		10.00		16.67									
			R	I	R	I	R	I	R	I	R	I								
(1)	1 +	$.1894\times10^{-4}$	B_{hh}	1.00185	-.00515	.99676	-.01836	.98161	-.04121	.93213	-.09030	.82294	-.15581							
(2)	1.9936 +	"	$B_{h\alpha}$	1.96161	-.027132	1.84004	-.034934	1.28728	.016752	-1.18514	.30032	-2.40407	.88516							
(3)	-.05705 +	"	$B_{h\beta}$	-.05723	.00154	-.05614	.00337	-.05260	.00743	-.04013	.01590	-.01149	.02693							
(4)	.004226 +	$.4014\times10^{-7}$	$B_{\alpha h}$	.00423	0	.00423	.00001	.00420	.00003	.00407	.00009	.00375	.00017							
(5)	1 +	"	$B_{\alpha\alpha}$	1.00036	-.00037	1.00136	-.00093	1.00592	-.00261	1.02640	-.00751	1.07805	-.01577							
(6)	.01675 +	"	$B_{\alpha\beta}$	.01676	.00002	.01677	.00005	.01680	.00011	.01685	.00024	.01696	.00042							
(7)	-.03056 +	$.1014\times10^{-4}$	$B_{\beta h}$	-.03033	.00102	-.02865	.00199	-.02121	.00354	.00762	.00513	.03996	.00518							
(8)	4.2344 +	"	$B_{\beta\alpha}$	4.20252	-.02279	4.08641	-.02270	3.56516	.04093	.19864	.31249	-4.31951	.92740							
(9)	1 +	"	$B_{\beta\beta}$	1.00077	-.00313	.99721	-.00698	.99517	-.01597	.98410	-.03573	.96202	-.06277							
(10)	(9) ×	$(\bar{9})$	1.00155	0	.99847	0	.99062	0	.96973	0	.92442	0								
(11)	$(\bar{9})$ ÷	(10)	.99922	.00313	1.00074	.00699	1.00459	.01612	1.01482	.03685	1.03508	.06754								
(12)	(7) ×	(11)	-.030310	.000924	-.028685	.001791	-.021867	.003206	.007543	.005487	.039966	.011638								
(13)	(8) ×	(11)	4.19931	-.004618	4.09159	.005861	3.58086	.098588	.19357	.22797	-4.94906	.66188								
(14)	(12) ×	(3)	.001733	-.000100	.001604	-.000197	.001126	-.000331	-.000390	-.000100	-.001324	.002367								
(15)	(13) ×	(3)	-.24031	.002017	-.22972	.013460	-.18909	.021420	-.011393	-.006071	.039040	-.14058								
(16)	(1) -	$(14) = a_{11}$	1.00012	-.008250	.99516	-.018163	.98048	-.040879	.93252	-.09020	.82431	-.15818								
(17)	(2) -	$(15) = a_{12}$	2.20192	-.034269	2.06976	-.048396	1.47687	-.004668	-1.17175	.30639	-2.44311	1.02604								
(18)	(12) ×	(6)	-.000508	.000015	-.000321	.000029	-.000368	.000051	.000126	.000094	.001569	.000233								
(19)	(13) ×	(6)	.070381	-.000077	.068616	.000303	.060148	.002050	.003207	.003888	-.084214	.009147								
(20)	(4) -	$(18) = a_{21}$	.004738	-.000015	.004711	-.000019	.004568	-.000021	.003944	-.000004	.002181	-.000063								
(21)	(5) -	$(19) = a_{22}$	.92998	-.000293	.93274	-.001233	.94577	-.004660	1.02319	-.01378	1.16236	-.03717								
(22)	$a_{11} \times$	a_{22}	.92009	-.002745	.92820	-.018166	.92712	-.043231	.95316	-.10292	.95412	-.20437								
(23)	$a_{21} \times$	a_{12}	.010432	-.000198	.009750	-.000267	.006946	-.000052	-.004620	.001213	-.016169	.002307								
(24)	(22) -	(23)	.91966	-.002547	.91845	-.017899	.92037	-.043179	.95724	-.10413	.97629	-.20210								

$\frac{\pi\rho}{A_{hh}} = .1894\times10^{-4}$ $\frac{\pi\rho}{A_{\alpha\alpha}} = .4014\times10^{-7}$ $\frac{\pi\rho}{A_{\beta\beta}} = .1014\times10^{-4}$

$\frac{A_{h\alpha}}{A_{hh}} = 1.9936$ $\frac{A_{\alpha h}}{A_{\alpha\alpha}} = .004226$ $\frac{A_{\beta h}}{A_{\beta\beta}} = -.03056$

$\frac{A_{h\beta}}{A_{hh}} = -.05705$ $\frac{A_{\alpha\beta}}{A_{\alpha\alpha}} = .01675$ $\frac{A_{\beta\alpha}}{A_{\beta\beta}} = 4.2344$

TABLE 11 (CONT.)

P = 2.376

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1/K	→	1.25		2.50		5.00		10.00		16.67									
			R	I	R	I	R	I	R	I	R	I								
(25)	(24) ÷ P	= η	.38708	-.003269	.38658	-.007534	.38738	-.018174	.40311	-.043828	.40840	-.082168								
(26)	$a_{11} \div 2P + a_{22} \div 2 = \lambda$		.67552	-.001883	.67585	-.004440	.67928	-.010935	.70789	-.024686	.75465	-.045755								
(27)	λ^2		.45632	-.002544	.45675	-.006002	.46130	-.014856	.50050	-.034950	.56740	-.069058								
(28)	$\lambda^2 - \eta$		.069240	.000725	.070170	.001532	.073920	.003318	.09739	.008878	.15900	.018110								
(29)	$\sqrt{(28)}$		.26314	.001378	.26491	.002892	.27195	.006102	.31240	.014224	.39940	.022708								
(30)	$\lambda + (29) =$	$x_1 + iy_1$	.93866	-.000505	.94076	-.001548	.95123	-.004833	1.02029	-.010462	1.15405	-.023047								
(31)	.373 $(1/K)/\sqrt{x_1}$;	iy_1/x_1	.4812	-.00054	.9614	-.00165	1.912	-.00508	3.693	-.01025	5.788	-.01997								
(32)	$\lambda - (29) =$	$x_2 + iy_2$	.41238	-.003261	.41094	-.007332	.40733	-.017037	.39549	-.038910	.35525	-.068463								
(33)	.373 $(1/K)/\sqrt{x_2}$;	iy_2/x_2	.7261	-.00791	1.455	-.01784	2.922	-.04183	5.931	-.09838	10.43	-.1927								
	P = 1.5																			
(25)	(24) ÷ P	= η	.61311	-.0051780	.61230	-.011933	.61358	-.028786	.63849	-.069420	.64686	-.13807								
(26)	$a_{11} \div 2P + a_{22} \div 2 = \lambda$		.77836	-.0028965	.77809	-.0066708	.77971	-.015956	.82243	-.035766	.85570	-.065185								
(27)	λ^2		.63737	-.0046249	.63690	-.010648	.63928	-.025520	.67511	-.058830	.72832	-.11158								
(28)	$\lambda^2 - \eta$		.02426	.0005531	.0246	.001285	.0257	.003266	.03662	.01059	.08146	.02649								
(29)	$\sqrt{(28)}$		.15577	.0017755	.15690	.0040964	.16063	.010166	.19331	.027371	.28907	.045820								
(30)	$\lambda + (29) =$	$x_1 + iy_1$	.95413	-.0011210	.95499	-.0025244	.96034	-.00579	1.01574	-.008375	1.14497	-.019365								
(31)	.373 $(1/K)/\sqrt{x_1}$;	iy_1/x_1	.4773	-.001175	.9542	-.002696	1.903	-.006029	3.701	-.008245	5.811	-.01691								
(32)	$\lambda - (29) =$	$x_2 + iy_2$	.64259	-.0046720	.64119	-.010767	.63908	-.026122	.62912	-.063157	.56683	-.11101								
(33)	.373 $(1/K)/\sqrt{x_2}$;	iy_2/x_2	.5816	-.007271	1.165	-.01679	2.333	-.04087	4.703	-.1004	8.259	-.1958								

$$\frac{V}{\omega_\alpha} = \frac{b_r}{14}\,\frac{1/K}{\sqrt{x}}$$

$$P = \left(\frac{\omega_n}{\omega_\alpha}\right)^2$$

$b_r = 5.21$

FIGURE 10
SYMMETRIC BENDING-TORSION-FLAPPING FLUTTER

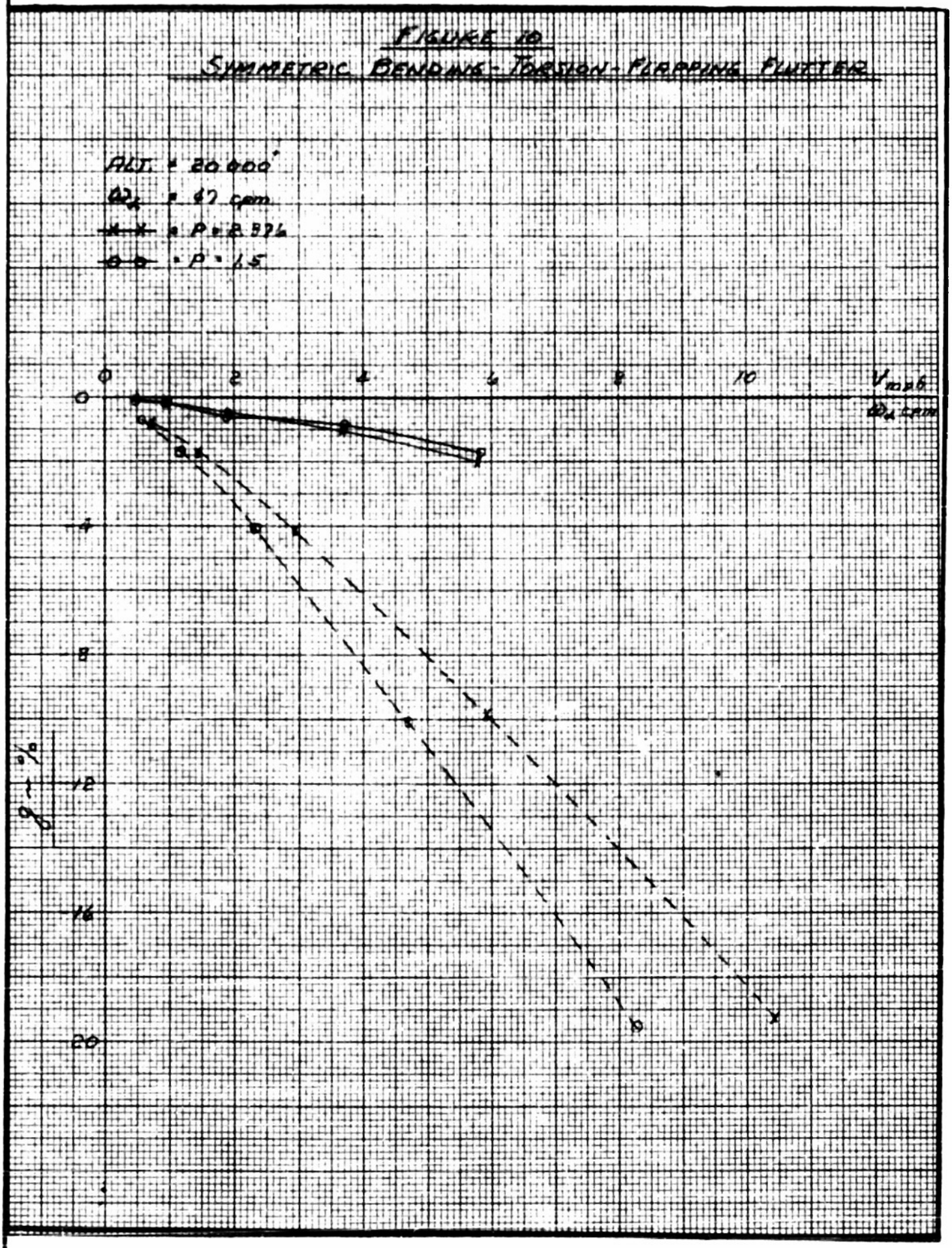


TABLE 12

GENERALIZED MASSES - ANTI-SYMMETRIC CASE

MODES: h, α, β, R

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
ELEMENT	STA.	$M_{(x)}$	$S_{(x)}$	$I_{(x)}$	h_R	h_h	h_β	α_1	A_{RR}	A_{hh}	$A_{\beta\beta}$	$A_{\alpha\alpha}$	$A_{R\beta}$	$A_{h\beta}$	$A_{R\alpha}$	$A_{h\alpha}$	$A_{\beta\alpha}$			
		SLUGS	SLUG-FT.	SLUG-FT.^{2}	(2) ÷ 850				$(3)(6)^2$	$(3)(7)^2$	$(3)(8)^2$	$(5)(9)^2$	(3)(6)(8)	(3)(7)(8)	(4)(6)(9)	(4)(7)(9)	(4)(8)(9)			
1	0	378.36	7847	394.696	0	0	0	0	0	0	0	0	0	0	0	0	0			
2	90	144.72	-49.2	1425	.08894	-.02229	↑	.008293	.96687	.0719	↑	.098003	↑	↑	-.036287	.009095	↑			
3	169	401.04	-1790	18461	.16701	-.03365	↓	.01897	11.186	.4541	↓	6.6434	↓	↓	-5.6710	1.1426	↓			
4	270	152.28	101.0	1209	.26682	-.03497	↓	.03485	10.841	.1862	↓	1.4684	↓	↓	.93917	-.12309	↓			
5	374	345.84	-1412	14704	.36960	-.01826	↓	.05579	47.242	.11531	↓	45.767	↓	↓	-29.115	1.4384	↓			
6	520	64.20	69.9	410	.51388	.05003	↓	.1001	16.953	1.6646	↓	4.1082	↓	↓	3.5956	.35006	↓			
7	670	14.62	12.9	86.6	.66212	.2655	↓	.2661	6.4094	1.0306	↓	6.1321	↓	↓	2.2729	.91138	↓			
8	820	4.36	2.6	21.7	.81036	.8618	↓	.8816	2.86314	3.2381	↓	16.866	↓	↓	1.8575	1.9754	↓			
9	850	14.40	2.7	25.9	1.0	1.0	0	.9806	14.400	14.400	0	24.905	0	0	2.6476	2.6476	0			
10	905	16.56	12.4	48.7	.77147	.77147	.1250	.9883	9.8559	9.8559	.25875	47.567	1.5969	1.5969	9.4543	9.4543	1.5319			
11	960	46.44	40	137	.54294	.54294	.2500	.9934	13.690	13.690	2.9025	135.20	6.3035	6.3035	21.574	21.574	9.934			
12	1015	17.16	17.3	89.2	.31938	.31938	.375	.9976	1.7504	1.7504	2.4131	88.772	2.0552	2.0552	5.5120	5.5120	6.4719			
13	1070	277.92	913	28.883	.09086	.09086	.5	1.0	2.2944	2.2944	69.480	28.883	12.626	12.626	82.955	82.955	456.5			
14	1125	17.16	17.3	89.2	-.13767	-.13767	.625	↑	.32523	.32523	6.7031	89.2	-1.4765	-1.4765	-2.3817	-2.3817	10.813			
15	1180	46.44	40	137	-.36620	-.36620	.75	↓	6.2280	6.2280	26.123	137	-12.755	-12.755	-14.648	-14.648	30.0			
16	1235	16.56	12.4	48.7	-.59473	-.59473	.875	↓	5.8573	5.8573	12.679	48.7	-8.6176	-8.6176	-7.3747	-7.3747	10.85			
17	1290	4.32	2.7	9.2	-.82326	-.82326	1.0	1.0	2.9279	2.9279	4.32	9.2	-3.5565	-3.5565	-2.2228	-2.2228	2.7			
								Σ =	153.791	64.0899	124.879	29544.6	-3.8240	-3.8240	69.3586	101.220	528.801			

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TABLE 13

AERODYNAMIC INTEGRALS - ANTI-SYMMETRIC CASE

MODES: h, α, β, R

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA.									B-29											
x	b	b^2	b^3	b^4	h_h	h_R	h_β	α_1	h_h^2	$b^2 h_h^2$	$b h_h^2$	$b^3 h_h \alpha_1$	$b^2 h_h \alpha_1$	$b h_h \alpha_1$	$b^2 h_R h_h$	$b h_R h_h$	$b^4 \alpha_1^2$	$b^3 \alpha_1^2$	$b^2 \alpha_1^2$	$b^2 \alpha_1 h_R$
IN.									(6)^2	(3)(10)	(2)(10)	(4)(6)(9)	(3)(6)(9)	(2)(6)(9)	(3)(7)(6)	(2)(7)(6)	(5)(9)^2	(4)(9)^2	(3)(9)^2	(4)(9)(7)
90					-.02	.10588	0	.0083	.0004	.021586	.003136	-.079994	-.010203	-.001135	-.13016	-.016602	.26027	.033197	.004234	.42349
170					-.03	.20	↑	.0190	.0009	.049411	.006669	-.23192	-.031298	-.004224	-.38945	-.04446	1.08838	.14688	.019822	1.54611
250					-.035	.29412		.035	.001225	.059171	.008514	-.41124	-.059171	-.008514	-.49724	-.071545	2.85817	.41124	.059171	3.45587
330					-.025	.38823		.045	.000625	.026406	.004063	-.30896	-.047531	.007313	-.41007	-.063087	3.61483	.55613	.085556	4.62318
410	← SAME AS SYM CASE →				0	.48235		.065	0	0	0	0	0	0	0	0	5.81149	.95430	.15670	7.08165
490					.035	.57647		.090	.001225	.038690	.006885	.55913	.099490	.017703	.63725	.11339	8.08015	1.43775	.25583	9.20911
570					.095	.67059		.135	.009025	.24497	.047020	1.81371	.33530	.066818	1.66553	.33171	13.42818	2.57778	.49470	12.80270
650					.22	.76471		.225	.0484	1.09205	.22990	5.30491	1.11687	.235[illegible]	3.79591	.79912	25.27116	5.42548	1.14225	18.43764
730					.43	.85882	↓	.420	.1849	3.46669	.80062	16.45708	3.78917	.87509	6.92387	1.59904	77.65077	17.93332	4.14165	32.26914
810					.80	.95294	0	.820	.64	9.63456	2.4832	38.31676	9.87542	2.54528	11.47645	2.95793	152.38601	39.27488	10.12231	45.64220
								B-29	Σ =	14.63654	3.59001	61.36968	15.06805	3.71883	23.13367	5.60570	390.94941	63.75056	16.48222	135.99909
								B-29	ΣΔx =	97.582	23.935	407.152	100.659	24.793	154.222	37.373	1939.260	458.360	109.887	906.666
877.5									F-84											
932.5																				
987.5																				
1042.5	← SAME AS SYM. CASE →																			
1097.5									TABLE	8										
1152.5																				
1207.5																				
1262.5																				
F-84	Σ =																			
F-84	ΣΔx =	482.067	1809.36	6930.05						101.391	32.032	158.268	42.016	11.402	101.391	32.032	6896.9	1799.6	479.116	158.268

B-29 ~ Δx = 6.667

NOTE: $h_R = h_h$ ALONG THE F-84 STATIONS

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TABLE 13 (CONT.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
STA	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)
x	$b^2\alpha_1 h_R$	$b h_R \alpha_1$	$b h_R^2$	$b^2 h_R^2$					B-29											
IN.	(3)(9)(7)	(2)(7)(9)	$(2)(7)^2$	$(3)(7)^2$																
90	.054017	.006890	.087891	.68907																
170	.20865	.028158	.29640	2.19632																
250	.49724	.071545	.60122	4.17853																
330	.73812	.11354	.97970	6.36803																
410	1.16281	.19094	1.41691	8.28951																
490	1.63865	.29158	1.86763	10.4959																
570	2.45734	.47166	2.34289	12.2064																
650	3.88218	.81728	2.77771	13.1944																
730	7.56795	1.74778	3.19369	13.8287																
810	11.76336	3.03187	3.52341	13.6705																
B-29 Σ:	29.97032	6.77126	17.08745	85.11736																
$\Sigma\Delta x$ =	199.812	45.146	113.922	567.477					F-84											
					$b^3 h_R$	$b^2 h_R$	$b h_R$				$b^2 h_R h_\beta$	$b h_R h_\beta$								
					$b^3 h_h$	$b^2 h_h$	$b h_h$	$\alpha_1 b^4$	$\alpha_1 b^3$	$\alpha_1 b^2$	$b^2 h_h h_\beta$	$b h_h h_\beta$	$b^3 h_\beta$	$b^2 h_\beta$	$\alpha_1 h_\beta b^3$	$\alpha_1 h_\beta b^2$	$\alpha_1 h_\beta b$	$b h_\beta$	$b h_\beta^2$	$b^2 h_\beta^2$
877.5																				
932.5																				
987.5																				
1042.5	←						SAME	AS	SYM	CASE										→
1097.5								TABLE	8											
1152.5																				
1207.5																				
1262.5																				
F-84 $\Sigma\Delta x$ =	42.016	11.402	32.032	101.391	161.399	43.001	11.717	6913.4	1804.4	480.583	-32.054	-11.152	904.67	241.032	903.674	240.759	65.597	65.674	42.175	149.040

B-29 ~ Δx = 6.667

TABLE 14

AERODYNAMIC FORCES - ANTI-SYMMETRIC

$B_{h\dot{\alpha}}$ = SAME AS SYMMETRIC - SEE TABLE 9

$B_{h\alpha}$ = "

$B_{h\beta}$ = "

$$
\begin{aligned}
B_{hR} = {} & \sum_{29} b^2 h_h h_R + b_r K_2(L_h) \sum_{29} b h_h h_R + \sum_{84} b^2 h_h h_R + b_r K_2(L_h) \sum_{84} b h_h h_R \\
& - a \alpha_R \sum_{84} b^3 h_h + b_r K_2(L_\alpha) \alpha_R \sum_{84} b^2 h_h - \left(\tfrac{1}{2} + a\right) b_r K_2(L_h) \alpha_R \sum_{84} b^2 h_h \\
& + b_r^2 K_3(L_\alpha) \alpha_R \sum_{84} b h_h - a \alpha_h \sum_{84} b^3 h_R - b_r \left(\tfrac{1}{2} + a\right) K_2(L_h) \alpha_h \sum_{84} b^2 h_R \\
& + \left(\tfrac{1}{8} + a^2\right) \alpha_h \alpha_R \sum_{84} b^4 + b_r K_2(M_\alpha) \alpha_h \alpha_R \sum_{84} b^3 - \left(\tfrac{1}{2} + a\right) b_r K_2(L_\alpha) \alpha_h \alpha_R \sum_{84} b^3 \\
& + \left(\tfrac{1}{2} + a\right)^2 b_r K_2(L_h) \alpha_h \alpha_R \sum_{84} b^3 - \left(\tfrac{1}{2} + a\right) b_r^2 K_3(L_\alpha) \alpha_h \alpha_R \sum_{84} b^2 \\
& + \left[L_\beta - (c-e) L_z\right] \delta_R \sum_{a} b^2 h_h + \left[T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z\right] \delta_h \delta_R \sum_{a} b^4 \\
& + \left[T_h - (c-e) P_h\right] \delta_h \sum_{a} b^3 h_R
\end{aligned}
$$

$B_{\dot{\alpha}h}$ = SAME AS SYMMETRIC - SEE TABLE 9

$B_{\dot{\alpha}\dot{\alpha}}$ = "

$B_{\dot{\alpha}\beta}$ = "

$$
\begin{aligned}
B_{\dot{\alpha}R} = {} & -a \sum_{29} b^3 \alpha_1 h_R - a \sum_{84} b^3 \alpha_1 h_R - b_r \left(\tfrac{1}{2} + a\right) K_2(L_h) \sum_{84} b^2 \alpha_1 h_R + \left(\tfrac{1}{8} + a^2\right) \alpha_R \sum_{84} b^4 \alpha_1 \\
& + b_r K_2(M_\alpha) \alpha_R \sum_{84} b^3 \alpha_1 - \left(\tfrac{1}{2} + a\right) b_r K_2(L_\alpha) \alpha_R \sum_{84} b^3 \alpha_1 \\
& + \left(\tfrac{1}{2} + a\right)^2 b_r K_2(L_h) \alpha_R \sum_{84} b^3 \alpha_1 - \left(\tfrac{1}{2} + a\right) b_r^2 K_3(L_\alpha) \alpha_R \sum_{84} b^2 \alpha_1 \\
& + \left[M_\beta - \left(\tfrac{1}{2} + a\right) L_\beta - (c-e) M_z + (c-e)\left(\tfrac{1}{2} + a\right) L_z\right] \delta_R \sum_{a} b^4 \alpha_1
\end{aligned}
$$

TABLE 14 (CONT)

$B_{\beta h}$ = SAME AS SYMMETRIC - SEE TABLE 9

$B_{\beta\alpha}$ = "

$B_{\beta\beta}$ = "

$$
\begin{aligned}
B_{\beta R} = {} & \sum_{84} b^2 h_\beta h_R + b_r K_2(l_h) \sum_{84} b h_\beta h_R - a\alpha_R \sum_{84} b^3 h_\beta + b_r K_2(l_\alpha)\alpha_R \sum_{84} b^2 h_\beta \\
& - \left(\tfrac{1}{2}+a\right) b_r K_2(l_h)\alpha_R \sum_{84} b^2 h_\beta + b_r^2 K_3(l_\alpha)\alpha_R \sum_{84} b h_\beta + \left[l_\beta - (c-e) l_z\right]\delta_R \sum_a b^3 h_\beta \\
& - a\alpha_\beta \sum_{84} b^3 h_R - b_r \left(\tfrac{1}{2}+a\right) K_2(l_h)\alpha_\beta \sum_{84} b^2 h_R + \left(\tfrac{1}{8}+a^2\right)\alpha_\beta \alpha_R \sum_{84} b^4 \\
& + b_r K_2(M_\alpha)\alpha_\beta \alpha_R \sum_{84} b^3 - \left(\tfrac{1}{2}+a\right) b_r K_2(l_\alpha)\alpha_\beta \alpha_R \sum_{84} b^3 \\
& + \left(\tfrac{1}{2}+a\right)^2 b_r K_2(l_h)\alpha_\beta \alpha_R \sum_{84} b^3 - \left(\tfrac{1}{2}+a\right) b_r^2 K_3(l_\alpha)\alpha_\beta \alpha_R \sum_{84} b^2 \\
& + \left[T_h - (c-e) P_h\right]\delta_\beta \sum_a b^3 h_R + \left[T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z\right]\delta_\beta \delta_R \sum_a b^4
\end{aligned}
$$

$$
\begin{aligned}
B_{Rh} = {} & \sum_{29} b^2 h_R h_h + b_r K_2(l_h) \sum_{29} b h_R h_h + \sum_{84} b^2 h_R h_h + b_r K_2(l_h) \sum_{84} b h_R h_h \\
& + \left[l_\beta - (c-e) l_z\right]\delta_h \sum_a b^3 h_R - a\alpha_R \sum_{84} b^3 h_h - b_r \left(\tfrac{1}{2}+a\right) K_2(l_h)\alpha_R \sum_{84} b^2 h_h \\
& + \left(\tfrac{1}{8}+a^2\right)\alpha_R \alpha_h \sum_{84} b^4 + b_r K_2(M_\alpha)\alpha_R \alpha_h \sum_{84} b^3 - \left(\tfrac{1}{2}+a\right) b_r K_2(l_\alpha)\alpha_R \alpha_h \sum_{84} b^3 \\
& + \left(\tfrac{1}{2}+a\right)^2 b_r K_2(l_h)\alpha_R \alpha_h \sum_{84} b^3 - \left(\tfrac{1}{2}+a\right) b_r^2 K_3(l_\alpha) - a\alpha_h \sum_{84} b^3 h_R \\
& + b_r K_2(l_\alpha)\alpha_h \sum_{84} b^2 h_R - \left(\tfrac{1}{2}+a\right) b_r K_2(l_h)\alpha_h \sum_{84} b^2 h_R + b_r^2 K_3(l_\alpha)\alpha_h \sum_{84} b h_R \\
& + \left[T_h - (c-e) P_h\right]\delta_R \sum_a b^3 h_h + \left[T_\beta - (c-e) P_\beta - (c-e) T_z + (c-e)^2 P_z\right]\delta_R \delta_h \sum_a b^4
\end{aligned}
$$

TABLE 14 (CONT.)

$$
\begin{aligned}
B_{R\alpha} = {} & -a\sum_{29} b^3 h_R \alpha_1 + b_r K_2(L_\alpha)\sum_{29} b^2 h_R \alpha_1 + b_r^2 K_3(L_\alpha)\sum_{29} b h_R \alpha_1 - a\sum_{84} b^3 h_R \alpha_1 \\
& + b_r K_2(L_\alpha)\sum_{84} b^2 h_R \alpha_1 - (\tfrac{1}{2}+a) b_r K_2(L_h)\sum_{84} b^2 h_R \alpha_1 + b_r^2 K_3(L_\alpha)\sum_{84} b h_R \alpha_1 \\
& + (\tfrac{1}{8}+a^2)\alpha_R \sum_{84} \alpha_1 b^4 + b_r K_2(M_\alpha)\alpha_R \sum_{84} \alpha_1 b^3 - (\tfrac{1}{2}+a) b_r K_2(L_\alpha)\alpha_R \sum_{84} \alpha_1 b^3 \\
& + (\tfrac{1}{2}+a)^2 b_r K_2(L_h)\alpha_R \sum_{84} \alpha_1 b^3 - (\tfrac{1}{2}+a) b_r^2 K_3(L_\alpha)\alpha_R \sum_{84} b^2 \alpha_1 \\
& + [T_\alpha - (c-e)P_\alpha - (\tfrac{1}{2}+a)T_h + (\tfrac{1}{2}+a)(c-e)P_h]\delta_R \sum_{a} b^2 \alpha_1
\end{aligned}
$$

$$
\begin{aligned}
B_{R\beta} = {} & \sum_{84} b^2 h_\beta h_R + b_r K_2(L_h)\sum_{84} b h_\beta h_R - a\alpha_\beta \sum_{84} b^3 h_R + b_r K_2(L_\alpha)\alpha_\beta \sum_{84} b^2 h_R \\
& - (\tfrac{1}{2}+a) b_r K_2(L_h)\alpha_\beta \sum_{84} b^2 h_R + b_r^2 K_3(L_\alpha)\alpha_\beta \sum_{84} b h_R + [L_\beta - (c-e)L_z]\delta_R \sum_{a} b^2 h_R \\
& - a\alpha_R \sum_{84} b^3 h_\beta - b_r(\tfrac{1}{2}+a) K_2(L_h)\alpha_R \sum_{84} b^2 h_\beta + (\tfrac{1}{8}+a^2)\alpha_R \alpha_\beta \sum_{84} b^4 \\
& + b_r K_2(M_\alpha)\alpha_R \alpha_\beta \sum_{84} b^3 - (\tfrac{1}{2}+a) b_r K_2(L_\alpha)\alpha_R \alpha_\beta \sum_{84} b^3 + (\tfrac{1}{2}+a)^2 b_r K_2(L_h)\alpha_R \alpha_\beta \sum_{84} b^3 \\
& - (\tfrac{1}{2}+a) b_r^2 K_3(L_\alpha)\alpha_R \alpha_\beta \sum_{84} b^2 + [T_h - (c-e)P_h]\delta_R \sum_{a} b^3 h_\beta \\
& + [T_\beta - (c-e)P_\beta - (c-e)T_z + (c-e)^2 P_z]\delta_R \delta_\beta \sum_{a} b^4
\end{aligned}
$$

$$
\begin{aligned}
B_{RR} = {} & \sum_{29} b^2 h_R^2 + b_r K_2(L_h)\sum_{29} b h_R^2 + \sum_{84} b^2 h_R^2 + b_r K_2(L_h)\sum_{84} b h_R^2 - a\alpha_R \sum_{84} b^3 h_R \\
& + b_r K_2(L_\alpha)\alpha_R \sum_{84} b^2 h_R - (\tfrac{1}{2}+a) b_r K_2(L_h)\alpha_R \sum_{84} b^2 h_R + b_r^2 K_3(L_\alpha)\alpha_R \sum_{84} b h_R \\
& + [L_\beta - (c-e)L_z]\delta_R \sum_{a} b^3 h_R - a\alpha_R \sum_{84} b^3 h_R - b_r(\tfrac{1}{2}+a) K_2(L_h)\alpha_R \sum_{84} b^2 h_R \\
& + (\tfrac{1}{8}+a^2)\alpha_R^2 \sum_{84} b^4 + b_r K_2(M_\alpha)\alpha_R^2 \sum_{84} b^3 - (\tfrac{1}{2}+a) b_r K_2(L_\alpha)\alpha_R^2 \sum_{84} b^3 \\
& + (\tfrac{1}{2}+a)^2 b_r K_2(L_h)\alpha_R^2 \sum_{84} b^3 - (\tfrac{1}{2}+a) b_r^2 K_3(L_\alpha)\alpha_R^2 \sum_{84} b^2 \\
& + [T_h - (c-e)P_h]\delta_R \sum_{a} b^3 h_R + [T_\beta - (c-e)P_\beta - (c-e)T_z + (c-e)^2 P_z]\delta_R^2 \sum_{a} b^4
\end{aligned}
$$

TABLE 15

EVALUATION OF TABLE 13

$$B_{hh} = 198.4247 + 292.51368\,K_2(L_h) - 1.60445\,K_2(L_\alpha) - 2.10229\,K_3(L_\alpha) + .41185\,K_2(M_\alpha) + 28.44159\,T_h - 1.42208\,P_h + 12.26673\,T_\beta - .61334\,P_\beta - .61334\,T_z + .03067\,P_z + 28.44159\,L_\beta - 1.42208\,L_z$$

$$B_{h\alpha} = 234.512 - 71.0859\,K_2(L_h) + 759.047\,K_2(L_\alpha) + 995.146\,K_3(L_\alpha) - 60.1659\,K_2(M_\alpha) - .036276\,T_\alpha + .036814\,P_\alpha + .220883\,T_h - .011044\,P_h$$

$$B_{h\beta} = -33.1693 - 56.8150\,K_2(L_h) + .41405\,K_2(L_\alpha) + .58682\,K_3(L_\alpha) - .10317\,K_2(M_\alpha) - 7.3867\,L_\beta + .369335\,L_z - 15.1324\,T_h + .756618\,P_h - 3.08769\,T_\beta + .15438\,P_\beta + .15438\,T_z - .007792\,P_z$$

$$B_{hR} = 255.32225 + 362.27233\,K_2(L_h) - .25973\,K_2(L_\alpha) - 1.07667\,K_3(L_\alpha) + .19501\,K_2(M_\alpha) + 17.29777\,L_\beta - .86489\,L_z + 7.46045\,T_\beta - .37302\,P_\beta - .37302\,T_z + .01865\,P_z + 28.44159\,T_h - 1.42208\,P_h$$

$$B_{\alpha h} = 228.53 - 71.0859\,K_2(L_h) + 18.0498\,K_2(L_\alpha) + 25.0461\,K_3(L_\alpha) - 60.1659\,K_2(M_\alpha) - .03628\,M_\beta + .22688\,L_\beta + .03614\,M_z - .011044\,L_z$$

$$B_{\alpha\alpha} = 1458.05 + 1058.76\,K_2(L_h) - 352.919\,K_2(L_\alpha) - 3901.54\,K_3(L_\alpha) + 11763.98\,K_2(M_\alpha)$$

$$B_{\alpha\beta} = 18269 - 374.859\,K_2(L_h) - 4.88267\,K_2(L_\alpha) - 6.67208\,K_3(L_\alpha) + 16.0756\,K_2(M_\alpha) + .197301\,M_\beta - .059196\,L_\beta - .009865\,M_z + .00296\,L_z$$

$$B_{\alpha R} = 481.417 - 68.319\,K_2(L_h) + 8.8275\,K_2(L_\alpha) + 12.249\,K_3(L_\alpha) - 29.425\,K_2(M_\alpha) - .46203\,M_\beta + .13861\,L_\beta + .023101\,M_z - .006730\,L_z$$

$$B_{\beta h} = -33.2842 - 55.7001\,K_2(L_h) - 8.00602\,K_2(L_\alpha) - 11.3660\,K_3(L_\alpha) - .103166\,K_2(M_\alpha) - 15.1324\,L_\beta + .756618\,L_z - 7.38670\,T_h + .369335\,P_h - 3.08769\,T_\beta + .154385\,P_\beta + .154385\,T_z - .007719\,F_z$$

TABLE 15 (CONT.)

$$B_{\beta\alpha} = 182.69 - 324.859\,K_2(L_h) + 124.947\,K_2(L_\alpha) + 1773.87\,K_3(L_\alpha) + 16.0756\,K_2(M_\alpha) + .197301\,T_\alpha - .05919\,T_h - .009865\,P_\alpha + .00296\,P_h$$

$$B_{\beta\beta} = 149.662 + 218.446\,K_2(L_h) + 2.1559\,K_2(L_\alpha) + 2.8583\,K_3(L_\alpha) + .02828\,K_2(M_\alpha) + 4.05504\,T_h - .20275\,P_h + 4.05504\,L_\beta - .20275\,L_z + .82691\,T_\beta - .041345\,P_\beta - .041345\,T_z + .0020673\,P_z$$

$$B_{\beta R} = -32.6 - 57.042\,K_2(L_h) - 3.9165\,K_2(L_\alpha) - 5.5608\,K_3(L_\alpha) - .047134\,K_2(M_\alpha) - 7.3867\,T_h + .36934\,P_h - 9.4959\,L_\beta + .47479\,L_z - 1.9376\,T_\beta + .09688\,P_\beta + .09688\,T_z - .004844\,P_z$$

$$B_{\alpha h} = 255.323 + 362.272\,K_2(L_h) - 1.5403\,K_2(L_\alpha) - 2.1847\,K_3(L_\alpha) + .19503\,K_2(M_\alpha) + 17.298\,T_h - .86489\,P_h + 28.4426\,L_\beta - 1.4221\,L_z + 7.4605\,T_\beta - .37302\,P_\beta - .37302\,T_z + .018651\,P_z$$

$$B_{R\alpha} = 481.417 - 68.319\,K_2(L_h) + 1268.95\,K_2(L_\alpha) + 1547.13\,K_3(L_\alpha) - 29.425\,K_2(M_\alpha) - 46.203\,T_\alpha + .029101\,P_\alpha + .13861\,T_h - .0069304\,P_h$$

$$B_{R\beta} = -32.56512 - 57.03773\,K_2(L_h) + .39724\,K_2(L_\alpha) + .56349\,K_3(L_\alpha) - .04713\,K_2(M_\alpha) - 1.93759\,T_\beta + .09688\,P_\beta + .09688\,T_z - .00484\,P_z - 7.38670\,L_\beta + .36934\,L_z - 9.49590\,T_h + .47479\,P_h$$

$$B_{RR} = 668.677 + 760.850\,K_2(L_h) - .72951\,K_2(L_\alpha) - 1.0347\,K_3(L_\alpha) + .094268\,K_2(M_\alpha) + 4.5373\,T_\beta - .22687\,P_\beta - .22687\,T_z + .011343\,P_z + 17.2978\,L_\beta - .86489\,L_z + 17.2978\,T_h - .86489\,P_h$$

PREPARED G.M.
CHECKED
REVISED

REPUBLIC AVIATION

TABLE 16 — FLUTTER DETERMINANT — ALT. = 20000'

MODES: h, α, β, R

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1/K →		1.	25	2.	50	5.	00	10.	00	16.	67								
			R	I	R	I	R	I	R	I	R	I								
(1)	1 +	.1977x10^-4 B_{hh}	1.00168	-.0082078	.99649	-.018062	.98064	-.040407	.92783	-.087950	.80926	-.15110								
(2)	1.5793 +	" $B_{h\alpha}$	1.54591	-.028748	1.41901	-.036465	.84255	.017486	-1.23666	.31349	-8.23019	.92394								
(3)	-.05967 +	" $B_{h\beta}$	-.059862	.0016102	-.058720	.0035154	-.055027	.0077616	-.042010	.016600	-.012117	.028113								
(4)		.1977x10^-4 B_{hR}	.0025955	-.010191	-.0025888	-.022707	-.016322	-.051847	-.054811	-.11580	-.13284	-.20317								
(5)	.003426 +	.4288x10^-7 $B_{\alpha h}$	.0034346	.0000058	.0034293	.0000139	.0034011	.0000358	.0032652	.0000932	.0029147	.0001834								
(6)	1 +	" $B_{\alpha\alpha}$	1.00038	-.0003555	1.00146	-.0008967	1.00635	-.0025724	1.028266	-.0075334	1.083472	-.015989								
(7)	.01790 +	" $B_{\alpha\beta}$	.017913	.0000219	.017923	.0000491	.017949	.0001133	.018012	.0002562	.018127	.0001533								
(8)	.002348 +	" $B_{\alpha R}$	.0023685	.0000048	.0023667	.0000113	.0023544	.0000251	.0022902	.0000696	.0021208	.0001326								
(9)	-.03062 +	.1014x10^-4 $B_{\beta h}$	-.030393	.0010212	-.028706	.0019952	-.021770	.0035410	.0075558	.0051251	.017895	.0051822								
(10)	4.2345 +	" $B_{\beta\alpha}$	4.20262	-.022792	4.08851	-.022698	3.56526	.040926	.19874	.21743	-4.21941	.94740								
(11)	1 +	" $B_{\beta\beta}$	1.00077	-.0031338	.99923	-.0069892	.99523	-.015986	.98440	-.035786	.96272	-.063914								
(12)	-.03062 +	" $B_{\beta R}$	-.030560	.0009258	-.029490	.0019134	-.025368	.0038664	-.0086933	.0072055	.031280	.010656								
(13)		.8237x10^-5 $B_{R h}$	.0009881	-.0042675	-.0014927	-.0094402	-.0082015	-.021302	-.031459	-.046871	-.080950	-.081265								
(14)	.45099 +	" $B_{R\alpha}$	.43001	-.021818	.34724	-.030137	-.022309	-.0033759	-1.69966	.17253	-5.90724	.54545								
(15)	-.02486 +	" $B_{R\beta}$	-.024932	.00066926	-.024456	.0014654	-.022918	.0052363	-.017494	.0069298	-.0050442	.011729								
(16)	1 +	" B_{RR}	1.00354	-.0087327	.99965	-.019583	.99050	-.045114	.96943	-.10215	.93274	-.18097								
(17)	(16) x	(16)	1.007169	0	.999683	0	.983125	0	.950229	0	.903254	0								
(18)	(16) ÷	(17)	.99640	.00867	.99997	.01959	1.00750	.04588	1.02021	.10750	1.03382	.20047								
(19)	(13) x	(18)	.0010215	-.0042435	-.0013080	-.0094691	-.007790	-.0218610	-.02705	-.0512001	-.06735	-.1001926								
(20)	(14) x	(18)	.42865	-.0180113	.34782	-.0233337	-.02736	-.0046541	-1.75256	-.0066966	-6.21283	-.6206546								
(21)	(15) x	(18)	-.024848	.00045069	-.024484	.00098626	-.023238	.0022091	-.018593	.0051892	-.0075631	.011107								
(22)	(4) x	(19)	-.0000406	-.0000213	-.0002115	.0000542	-.0010063	.0007607	-.0044464	.0059587	-.0114094	.0269931								
(23)	(4) x	(20)	.0009290	-.0044151	-.0014303	-.0078376	.0002053	.0014945	.0952841	.2033155	.6992139	1.3447084								
(24)	(4) x	(21)	-.0000599	.0002544	.00006562	.0005534	.00049383	.0011688	.0016200	.0018686	.0032613	.00006114								

$\frac{\pi\rho}{A_{hh}} = .1977 \times 10^{-4}$ $\quad \frac{\pi\rho}{A_{\beta\beta}} = .1014 \times 10^{-4}$ $\quad \frac{A_{h\alpha}}{A_{hh}} = 1.5793$ $\quad \frac{A_{\alpha h}}{A_{\alpha\alpha}} = .003426$ $\quad \frac{A_{\alpha R}}{A_{\alpha\alpha}} = .002348$ $\quad \frac{A_{\beta\alpha}}{A_{\beta\beta}} = 4.2345$ $\quad \frac{A_{R\alpha}}{A_{RR}} = 45099$

$\frac{\pi\rho}{A_{\alpha\alpha}} = .4288 \times 10^{-7}$ $\quad \frac{\pi\rho}{A_{RR}} = .8237 \times 10^{-5}$ $\quad \frac{A_{h\beta}}{A_{hh}} = -.05967$ $\quad \frac{A_{\alpha\beta}}{A_{\alpha\alpha}} = .01790$ $\quad \frac{A_{\beta h}}{A_{\beta\beta}} = -.03062$ $\quad \frac{A_{\beta R}}{A_{\beta\beta}} = -.03062$ $\quad \frac{A_{R\beta}}{A_{RR}} = -.02486$

TABLE 16 (CONT.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1/K	→	1.25		2.50		5.00		10.00		16.67									
			R	I	R	I	R	I	R	I	R	I								
(25)	(1) -	(22)	1.00172	-.0081865	.99670	-.018116	.98165	-.041168	.93228	-.093869	.82117	-.17809								
(26)	(2) -	(23)	1.54498	-.024533	1.42044	-.028627	.84234	.015991	-1.83194	.11018	-8.92940	-.42077								
(27)	(3) -	(24)	-.059802	.0013558	-.058786	.0029620	-.055521	.0065928	-.043630	.014731	-.015378	.028052								
(28)	(8) x	(19)	.0000024	-.0000100	-.000030	-.000024	-.0000177	-.0000517	-.0000584	-.0001191	-.0001296	-.0002314								
(29)	(8) x	(20)	.0010153	-.0000406	.0008234	-.0000513	-.0000642	-.0000116	-.0040132	-.0001373	-.0130939	-.0021401								
(30)	(8) x	(21)	-.00005885	.00000095	-.00005796	.00000206	-.00005477	.00000462	-.00004294	.00001059	-.00001751	.00002255								
(31)	(5) -	(28)	.0034322	.0000158	.0034323	.0000363	.0034188	.0000875	.0033236	.0002123	.0030445	.0004048								
(32)	(6) -	(29)	.99936	-.0003149	1.00064	-.0008454	1.00641	-.0023608	1.03228	-.0073961	1.09656	-.0138490								
(33)	(7) -	(30)	.017972	.00002095	.017981	.00004704	.018004	.00010868	.018055	.00024561	.018145	.00043075								
(34)	(12) x	(19)	-.0000273	.0001306	.0000567	.0002767	.0002821	.0005245	.0006041	.0002502	-.0010727	-.0037018								
(35)	(12) x	(20)	-.0130829	.0009473	-.0102126	.0013536	.0007121	.0000123	.0152838	-.0125699	-.1908300	-.0859283								
(36)	(12) x	(21)	.00075894	-.00003678	.00072015	-.00007593	.00058096	-.00014589	.00012424	-.00017908	-.00035871	.00027239								
(37)	(9) -	(34)	-.030366	.0008906	-.028763	.0017185	-.0220521	.0030165	.0069517	.0048747	.0809677	.0090840								
(38)	(10) -	(35)	4.21570	-.023739	4.09872	-.024052	3.56455	.040914	.18346	.23000	-4.52858	1.03333								
(39)	(11) -	(36)	1.00001	-.0030970	.99851	-.0069133	.99465	-.015840	.98428	-.035607	.96328	-.063186								
(40)	(39) x	($\overline{39}$)	1.00003	0	.99707	0	.98958	0	.97007	0	.93190	0								
(41)	($\overline{39}$) -	(40)	.99998	-.0030969	1.00144	.0069336	1.00512	.016007	1.01465	.036706	1.03367	.067803								
(42)	(41) x	(37)	-.030368	.00079654	-.028816	.0015215	-.022213	.0026790	.0068746	.0052015	.083078	.014880								
(43)	(41) x	(38)	4.21569	-.010683	4.10479	.0043323	3.58215	.098181	.17771	.24010	-4.75112	.76107								
(44)	(27) x	(42)	.0018150	-.00008881	.0016895	-.00017480	.0012156	-.00029519	-.00037656	-.00012567	-.0016950	.0021017								
(45)	(27) x	(43)	-.25209	.0063545	-.24132	.011904	-.19953	.018165	-.011290	-.0078577	.051713	-.14498								
(46)	(25)-(44)	= a_{11}	.99991	-.0080977	.99501	-.017941	.98043	-.040873	.93266	-.093743	.82287	-.18019								
(47)	(26)-(45)	= a_{12}	1.79707	-.030887	1.66176	-.040531	1.04187	-.0021740	-1.82065	.11804	-8.98111	-.27579								
(48)	(33) x	(42)	-.00054579	.00001368	-.00051821	.00002600	-.00040021	.00004582	.00012284	.00009560	.0015010	.00030578								
(49)	(33) x	(43)	.075765	-.00010368	.073208	.00027099	.064482	.0021570	.0031496	.0043787	-.086537	.011763								

TABLE 16 (CONT.)

P = 2.439

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1/k	→	1.25		2.50		5.00		10.00		16.67									
			R	I	R	I	R	I	R	I	R	I								
(50)	(31)-(48) = a_{21}		.0039280	.00000212	.0039505	.00001030	.0038190	.00004168	.0032008	.00011670	.0015435	.00009902								
(51)	(32)-(49) = a_{22}		.92359	-.00021132	.92683	-.0011164	.94193	-.0047178	1.02913	-.011775	1.18310	-.025612								
(52)	$a_{11} \times a_{22}$		.92351	-.0026502	.92219	-.017739	.92330	-.043125	.95872	-.10746	.96892	-.23426								
(53)	$a_{21} \times a_{12}$		.0071488	-.00011706	.0065652	-.00014300	.0039790	.00003512	-.0058413	.00016535	-.013835	-.0013150								
(54)	[(52)-(53)] ÷ P		.12318	-.0018178	.12308	-.0023654	.12358	-.0058019	.12966	-.014468	.13211	-.031314								
(55)	$a_{11} + 2P + a_{22} + 2 = \lambda$		.52900	-.00064988	.53029	-.0017641	.53687	-.0051061	.57726	-.012188	.64686	-.024917								
(56)	λ^2		.27984	-.00068757	.28120	-.0018710	.28820	-.0054826	.33308	-.014071	.41781	-.032236								
(57)	λ^2 - (54)		.15666	.00033023	.15812	.00049440	.16462	.00031930	.20342	.00039700	.28570	-.00092200								
(58)	$\sqrt{(57)}$		.39580	.00041716	.39764	.00062166	.40573	.00039348	.45102	.00044011	.53451	-.00086247								
(59)	λ + (58) = $x_1 + iy_1$		.9248	-.00023272	.92793	-.0011424	.94260	-.0047126	1.02828	-.011048	1.18137	-.025779								
(60)	.373 $(1/k/\sqrt{x_1})$; iy_1/x_1		.4848	-.0002516	.9680	-.001231	1.921	-.004999	3.678	-.01142	5.721	-.02182								
(61)	λ - (58) = $x_2 + iy_2$		.1332	-.0010670	.13265	-.0023858	.13114	-.0054996	.12624	-.012628	.11835	-.024055								
(62)	.373 $(1/k/\sqrt{x_2})$; iy_2/x_2		1.278	-.008011	2.560	-.01799	5.150	-.04194	10.50	-.1000	18.55	-.2141								
P = 2.0																				
(54)	[(52)-(53)] ÷ P		.45818	-.0037856	.45781	-.0082980	.45966	-.021580	.48223	-.053813	.49138	-.11647								
(55)	$a_{11} + 2P + a_{22} + 2 = \lambda$		.91177	-.0021300	.91217	-.0050435	.91607	-.012577	.94773	-.029323	.99727	-.057853								
(56)	λ^2		.50661	-.0030321	.50716	-.0071837	.51260	-.018012	.55824	-.043851	.63229	-.092249								
(57)	λ^2 - (54)		.04843	.0007535	.04935	.0016143	.05294	.003568	.07596	.009962	.14091	.024221								
(58)	$\sqrt{(57)}$		.22007	.0017120	.22245	.0036334	.23022	.0077536	.27620	.018073	.37677	.032262								
(59)	λ + (58) = $x_1 + iy_1$		.93184	-.000418	.93462	-.0014101	.94629	-.0048234	1.02393	-.01125	1.17404	-.025591								
(60)	.373 $(1/k/\sqrt{x_1})$; iy_1/x_1		.4830	-.0004486	.9646	-.001509	1.917	-.005097	3.686	-.01099	5.739	-.02180								
(61)	λ - (58) = $x_2 + iy_2$		.49120	-.003842	.48972	-.0086769	.48585	-.020331	.47153	-.047396	.4205	-.090115								
(62)	.373 $(1/k/\sqrt{x_2})$; iy_2/x_2		.6649	-.007814	1.333	-.01772	2.676	-.04185	5.432	-.1005	9.589	-.2143								

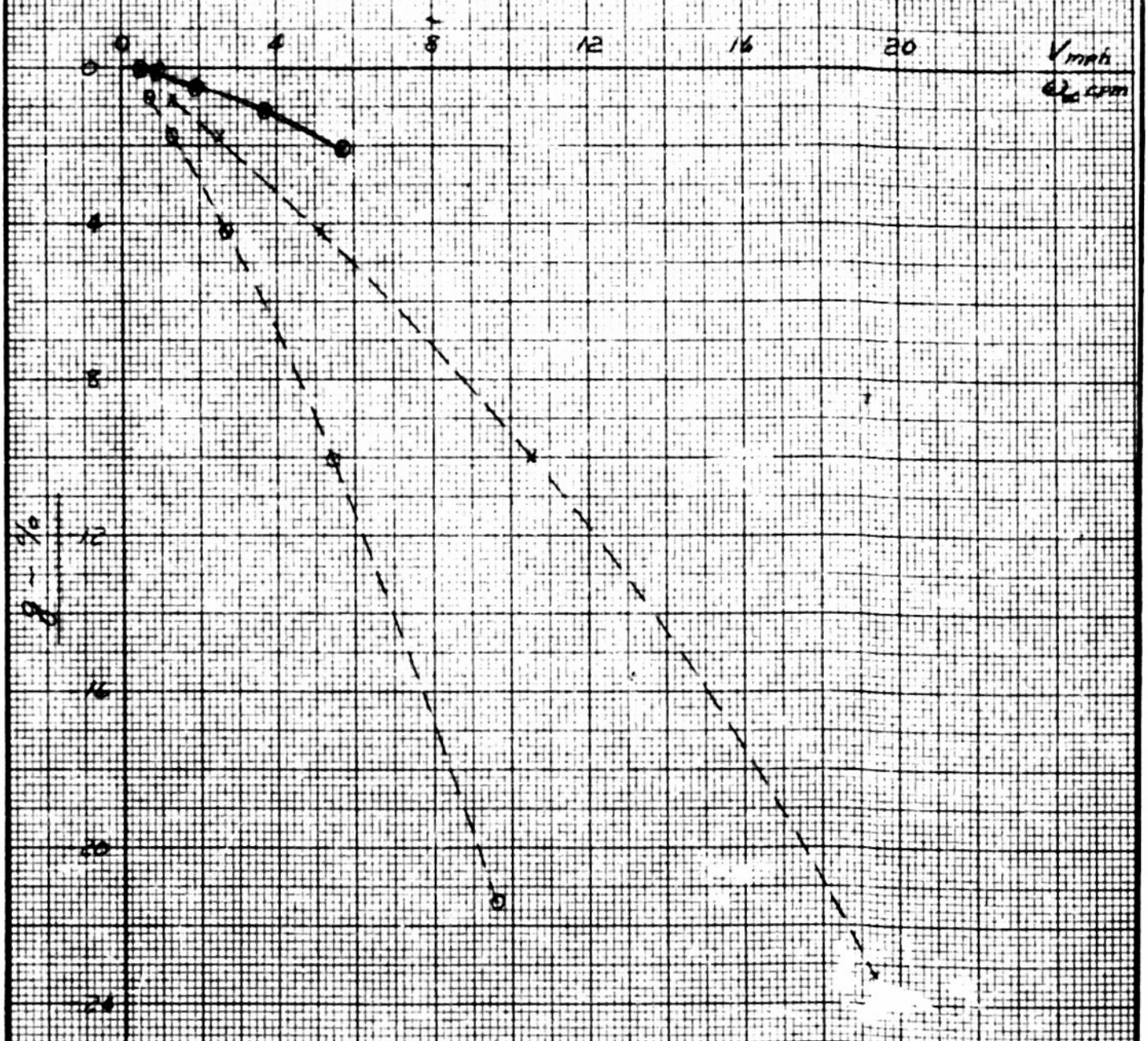
$$\frac{V}{\omega_\alpha} = \frac{b_r}{14}\,\frac{1/k}{\sqrt{\mu}}$$

$$P = \left(\frac{\omega_h}{\omega_\alpha}\right)^2$$

$b_r = 5.21$

FIGURE 11

ANTI-SYMMETRIC BENDING - TORSION - ROLL - FLAPPING FLUTTER



AD-A800 195

3C 2 3CC U See Reverse

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REPUBLIC AVIATION CORP., FARMINGDALE, L.I., N.Y.
(REPORT NO. E-SAF-MX-1016-1)

VIBRATION AND FLUTTER ANALYSIS OF THE B-29 - F-84
TIP-TO-TIP CONFIGURATION

S. PINES; ROBERT S. LEVY 57 PP. TABLES, GRAPHS

STRUCTURES (7) | AIRPLANES - FLUTTER
AEROELASTICITY (8) | AIRPLANES - WING-TIP COUPLING
PROJECT MX-1016

P 1/2

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