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# FINAL REPORT ON CONCRETE PENETRATION

by
Richard A. Beth

NDRC Report No. A-388
OSRD Report No. 6459





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Submitted, January 1946

Walker Bleakney
Princeton University

Approved, March 1946

E. Bright Wilson, Jr.
Chief, Division 2, NDRC
Effects of Impact and Explosion

[redacted]

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## Preface

The work described in this report is pertinent to the projects designated by the War Department as OD-75, "Investigation of the Penetration of Homogeneous and Face-Hardened Armor at Striking Velocities of 3000 ft/sec and Above," and by the Navy Department as NO-11, "Structural Defense, Testing Facilities."

The work was performed under Division 2 Contract OEMsr-260 with Princeton University. This report covers the period from November 1940 through November 1945, and is a final report.

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# FINAL REPORT ON CONCRETE PENETRATION


## Abstract

This report summarizes the work on concrete penetration and perforation which has been done by the Princeton group during the war. The first section outlines the state of knowledge on this subject at the beginning of our work by reprinting H. P. Robertson's original summary of his initial report on Terminal ballistics of January 1941. There follows a listing of our principal reports on concrete since that time with brief notes describing the contents of each report.

Succeeding sections describe the general aspects of the problem and outline an approximate theory of concrete penetration which represents the best recommendation that can be made in the present state of our knowledge. It is shown in the Appendix how this theory is an outgrowth of previous work on this subject. Formulae are derived for computing the resisting force during penetration, the remaining velocity as a function of depth during penetration, and times of penetration. Selected data on the effects of projectile nose shape and mass, and of concrete properties are analyzed to illustrate the application of the proposed theory. Suggestions are made concerning further work on the subject of concrete penetration.


## I. INTRODUCTION: REVIEW AND BIBLIOGRAPHY

Since the latter part of 1940 research has been conducted at Princeton on concrete penetration by bombs and projectiles. The research was initiated by the Committee on Passive Protection Against Bombing (PPAB). This Committee was formed by the National Academy of Sciences for the purpose of carrying out research on protection against bombing for the Office of the Chief of Engineers, U.S. Army. On July 1, 1943, the name of the group was changed to Committee on Fortification Design (CFD), and the formal connection with the Corps of Engineers was terminated as of October 31, 1944. From the beginning this work was very closely integrated with the research program of Division 2, NDRC, in the field of terminal ballistics.

The annotated bibliography which follows is intended to serve both as a historical review of the work since 1940 and as a list of the principal references for the purposes of the remainder of this report. Only the more important items have been included. The state of knowledge on the terminal ballistics of concrete and earth as of 1940 is covered by H. P. Robertson's summary of his report on Terminal ballistics reprinted in the first item of the Bibliography. Attention is drawn to the fact that this report contains extensive bibliographical references to earlier work.

## Bibliography

The abbreviations PPAB and CFD refer, respectively, to the Committee on Passive Protection Against Bombing (July 1, 1940 - June 30, 1943) and its successor, the Committee on Fortification Design (July 1, 1943 - October 31, 1944). The reports of these Committees were made to the Chief of Engineers, United States Army, and their distribution lies in the hands of the Office of the Chief of Engineers, United States Army, Washington, D.C.

1. Terminal ballistics, by H. P. Robertson, PPAB Interim Report, January 1941. Unclassified.

The author's original summary of this initial PPAB report is as follows: The general problem confronting the Committee can be broken down into several subordinate problems, each of which may be attacked more or less independently of the others. This interim report deals with the present state of knowledge concerning one such aspect: namely, the ability of a given material, such as earth or concrete, to withstand the impact of a bomb, insofar as this ability is attributable to the localized properties, such as strength or density, of the material.

Part I deals with theories and data on the penetration of an inert bomb into an unlimited deformable solid or semisolid material. The depth of penetration may be expected to depend upon

a. Properties of the bomb: weight $W$, caliber $d$, shape;
b. Striking conditions: velocity $v_o$, angle of incidence, yaw;
c. Properties of the target material: strength (such as compressive strength or hardness), density, porosity.

Since the latter elements (c) are ultimately responsible for the resistance $R$ to penetration, it is most appropriate to base the investigation on the fundamental equation of motion,

(1.1) $$\frac{W}{g}\frac{d^2x}{dt^2} = -R.$$

In all the theories considered R may be taken as the product $\phi \cdot f$ of two factors, the first $\phi(x)$ depending on the depth x of penetration already achieved, and the other f(v) on the velocity v of the projectile at that stage. In the majority of the investigations found in the literature, this assumption is further specialized (Sec. 2) by taking $\phi$ as a constant proportional to the cross-sectional area A of the bomb. The resistance may then by written

$$R = A\, f(v),$$

and the sectional resistance f(v) may be interpreted as the energy necessary to displace unit volume of target material by a projectile moving with velocity v; in the units adopted in the report f(v) is measured in pounds per square inch, and one of the problems encountered is that of identifying f with some significant stress parameter of the medium. It is shown that such a resistance leads to a maximum depth of penetration $x_1$ of the form

(2.4) $$x_1 = P\, F(v_0),$$

where P = W/A is the sectional pressure of the bomb, and $F(v_0)$ is a function of the striking velocity $v_0$, which can be obtained by integration from the sectional resistance f(v).

The principal sectional-pressure formulae, associated with the names of Euler, Poncelet, and de Giorgi, are discussed in the remaining parts of Sec. 2. Euler (Sec. 2a) assumed the resistance to be a constant, whence

(2.7) $$x_1 = \frac{P}{2\mu g}\, v_0^2;$$

much of the recent German work (Peres) on air-raid protection is based upon the use of this formula. Values of the strength parameter $f = \mu$, for various substances and from various sources, are collected in Note 1 at the end of the report; most of them are, however, quite useless, because of inadequate data or specification.

The second formula considered (Sec. 2b) is based upon the theory developed by Poncelet, supplemented by the admirable experimental program of the Metz Committee (1835). Poncelet's assumptions may be interpreted to mean that the projectile must not only supply energy to disrupt the cohesion of the material (as in the Euler hypothesis), but also energy to remove the detritus from its path; the resistance is then

$$R = A(a + bv^2),$$

where the "shatter strength" a is a measure of the cohesive force, corresponding to Euler's coefficient $\mu$, and the constant b appearing in the inertial term is proportional to the density w' of the target material. The resulting penetration formula is

$$x_1 = \frac{P}{2bg}\log_e\left(1 + \frac{b}{a}v_o^2\right), \qquad (2.10)$$

The Piobert-Morin-Didion values of $a$ and $b$, for various media, are given in the original and in English units in Note 2, and the relation of $b$ to the density of the medium is discussed in Note 3. Much of the modern work on penetration employs a special case of Poncelet's formula, due to Pétry (1910), in which $b/a$ is given a definite numerical value, the same for all media; Pétry's formula has been transcribed here in the form

$$x_1 = KP\log_{10}(1 + v_o^2/215{,}000), \qquad (2.13')$$

where the striking velocity $v_o$ is in feet per second. Values of the parameter $K$ for various materials are listed in Note 4.

The last of the sectional-pressure formulae discussed in detail is that of Nobile de Giorgi (Sec. 3c). De Giorgi reduced scattered penetration data in accordance with the general formula (2.4) and summarized the resulting $F(v_o)$ in tabular form; his $F$-values, reduced to standard English units, are given in Note 5.

The sectional-pressure formulae of Euler, Poncelet, Pétry, and de Giorgi, for concrete or similar media, are compared graphically in Plates I and II—in Plate I on adjusting the strength parameters to bring the curves into agreement at around $v_o$ = 900 ft/sec, and in Plate II with the original parameter values. The application of the sectional-pressure formulae of Euler, Pétry, and de Giorgi to Aberdeen Proving Ground data on the penetration of projectiles and bombs into earth is given graphically in Plate IV.

In Sec. 3 situations are considered in which the resistance encountered by the projectile may be expected to depend on the depth $x$ of penetration. The first of these, discussed in Sec. 3a, is the case of shallow penetration, in which the actual shape of the nose of the bomb may be expected to play a not inconsiderable role. The most natural modification of the expression for the resistance, traceable back to Morin (1836), leads to the penetration formula

$$V(x_1) = WF(v_o), \qquad (3.4)$$

where $V(x_1)$ is the actual volume swept out by the projectile on penetrating to a depth $x_1$, and $F(v_o)$ is the velocity function previously introduced; this result is illustrated graphically in Plate III for the hypothetical case of a conical-nosed projectile piercing a thin plate.

The second nonsectional pressure theory considered is that of Vieser (1935), who professes to derive the penetration formula

$$x_1 = \left[\frac{E_o}{2\sigma}\right]^{1/3} \qquad (3.10)$$

from known elastostatic results, where $E_o$ is the striking energy of the bomb and $\sigma$ is the compressive strength of the target material. Although there is little or no theoretical or empirical justification for this formula, Vieser's many papers on the subject have had considerable influence on recent German literature on air-raid protection, and it has therefore been thought desirable to include in Sec. 3b an account and an attempted interpretation of his formula.

Part I concludes with a brief discussion of the deviations from the theoretical formula to be expected because of oblique incidence, yaw, and the distortion of the bomb case.

In Part II rupture of the target, by spalling from the front face or scabbing from the back face, is discussed, with emphasis on the latter aspect. The formulae

(5.4) $$s_1^3 = K_g W v^2,$$

(5.6) $$s_1^3 = K_h W v,$$

proposed by de Giorgi and by Heidinger, respectively, for the scabbing depth $s_1$ caused by a projectile of weight $\underline{W}$ and velocity $\underline{v}$ are subjected to a critical examination. The formula

(5.7) $$s_1^2 = K_r W v^2$$

is proposed in the report, as an alternative which seems more plausible on theoretical grounds. But it is to be concluded that there are at present insufficient data available to allow a reliable treatment of scabbing.

Part III is devoted to the question of perforation of a target of given material and thickness $\underline{e}$; in Sec. 6 to the case in which scabbing is inconsiderable, perforation being achieved by pure step-by-step penetration, and in Sec. 7 to the case in which scabbing may play a more important role. In the former, application is made to the problem of determining the "limit velocity" $v_1$ of the plate—that is, the velocity just sufficient to cause perforation. The formula

(6.8) $$v_1 = \sqrt{\frac{\pi a}{2}} \times \frac{de^{\frac{1}{2}}}{m^{\frac{1}{2}}} \times (\text{fn of } \tfrac{e}{d})$$

is derived on the Poncelet hypothesis, and compared with various perforation formulae. The penetration cycle is discussed in Note 6, and the results illustrated graphically in Plate III. Section 7 contains an exposition, following N. de Giorgi, of a method by which the combined effect of penetration plus scabbing may be determined from the separate penetration and scabbing formulae.

The final Part IV reviews briefly the conclusions that may be drawn from the investigation, with particular reference to the dependence of penetration on

a. Weight and size of bomb,
b. Striking velocity,
c. Physical properties of the target.

(a) The data available in the literature seem to warrant the expectation that for similar bombs the depth of penetration (2.4) is proportional to the sectional pressure P of the bomb, with a coefficient $F(v_0)$ which depends on the striking velocity, that is,

(2.4) $$x_1 = PF(v_0).$$

This implies a resistance

$$R = Af(v)$$

proportional to the cross-sectional area A of the bomb.

(b) Again, the available data seem to warrant the expectation that the sectional resistance f(v) decreases as the velocity of the bomb decreases. It is considered significant that the empirical de Giorgi formula agrees closely with a formula of the Poncelet type (Plate I), and it is accordingly suggested that the assumption

(2.8) $$f(v) = a + bv^2, \quad F(v) = \frac{1}{2bg}\log_e\left(1 + \frac{b}{a}v^2\right)$$

be entertained as a working hypothesis.

(c) The need for a more adequate correlation of elements entering into the penetration process with ascertainable physical properties of the target material is stressed. A theoretical argument favoring a sectional resistance F(v) of the form (2.8) is advanced. In brief, it is argued that the actual mechanism of resistance involves both the overcoming of cohesion and the overcoming of the inertial reaction of the resulting detritus; it is contended that the two terms a and $bv^2$ entering into f(v) above do, in fact, represent these two elements, and it is suggested that it may be profitable to determine to what extent they alone may be able to account for the observed phenomena. More specifically, and more tentatively, it is suggested that the former may be related to the hardness of the target material, and that the coefficient b in the latter may be represented (as in exterior ballistics) as the density of the material, modified by a suitable "drag coefficient." Notes 7 and 8 report ballistic and hardness tests which may be relevant to the determination of the "shatter strength" a.

The report concludes with a Glossary of symbols used, and a Bibliography of the principal references consulted in its preparation.

2. PPAB final report for the year ending June 30, 1941, Part I. Confidential.

This report contains the small-caliber penetration data of the first Princeton "Concrete Properties Survey," the object of which was to obtain experimental information on the effect of concrete properties on penetration resistance. The then current theories of concrete penetration are discussed and the data are analyzed in terms of the classical Poncelet theory. The first evidence suggesting the existence of the scale effect for concrete penetration is presented and discussed on page 48 of this report.

3. Penetration of projectiles in concrete, by R. A. Beth, PPAB Interim Report No. 3, November 1941. Unclassified.

This report suggests the use of an empirical penetration formula for concrete of the form $s_c = KDV^{\alpha}d^{\beta}$, where $s_c$ is the nose corrected penetration in calibers, D is the caliber density of the nondeforming projectile, V is the striking velocity, d is the caliber, and K, $\alpha$, and $\beta$ are constants. The factor $d^{\beta}$ represents the scale effect.

4. AP bomb test--comment, by R. A. Beth, PPAB Interim Report No. 9, April 1942. Unclassified.

Bibliography, data, and discussion of tests with 12-in. AP projectiles, weighing 1000 lb, striking heavily reinforced concrete slabs of three thicknesses (36, 60, and 81 in.) at 1000 ft/sec and 20° obliquity. Caliber .45 penetration data obtained on unreinforced 1-ft cubes of the same concrete are also given and an attempt is made to evaluate the scale effect according to the type of formula suggested in reference 3. This involves a suggested method of making allowance for the density of reinforcing steel on penetration.

5. A brief summary of recent data on penetration in concrete at various scales, by R. A. Beth, PPAB Interim Report No. 18, June 1942. Unclassified.

A summary review of penetration data at caliber .45, 37-mm, 75-mm, 155-mm, 12-in., and 16-in. scales. The data are analyzed and correlated in terms of empirical formulae of the form suggested in reference 3. Scale-effect graphs are given. Some data on sticking, scabbing, and perforation of concrete by inert projectiles are given.

6. Penetration and explosion tests on concrete slabs--Report I: Data, by R. A. Beth and J. G. Stipe, Jr., PPAB Interim Report No. 20, January 1943. Unclassified.

This report contains complete data and some preliminary analyses in the form of graphs of extensive tests on 39 reinforced concrete slabs at calibers .45 and .50, 37-mm, 75-mm, 3-in., and 155-mm scales. Penetrations, perforations, obliquities, and explosions are included.

7. Penetration and explosion tests on concrete slabs—Report II: Crater Profiles, by J. G. Stipe, Jr., PPAB Interim Report No. 21, January 1943. Unclassified.

Eleven large prints of measured crater profile drawings which are reproduced at smaller size in reference 6.

8. Resistance of laminated concrete slabs to perforation, by R. J. Hansen, PPAB Interim Memorandum No. M-9, May 1943. Unclassified.

Report on tests made at 37-mm scale to find the reduction in perforation limit velocity produced by pouring concrete slabs in successive layers rather than monolithically. A lowering of limit velocity by not more than 5 percent per construction joint was found.

9. Terminal ballistics and explosive effects, Appendix to the PPAB final report for the year ending June 30, 1943. Restricted.

This report contains a description of terminal-ballistic phenomena with concrete, steel, armor, and other target materials, together with a compilation of considerable quantitative information on these subjects in the form of tables, graphs, and nomograms. It was originally written to assist the Corps of Engineers in the preparation of a new fortifications manual.

10. Concrete properties survey, by R. A. Beth, J. G. Stipe, Jr., M. E. DeReus, and J. T. Pittenger, CFD Interim Report No. 27, July 1944. Unclassified.

This report consists of three separately bound parts: "Effect of concrete properties on penetration resistance," "Appendix A—Preparation and physical tests of concrete," and "Appendix B—Penetration data."

In order to explore the effect of various concrete properties on penetration resistance 154 1-ft cube targets representing about 75 different concretes were made and tested for penetration resistance using nondeforming hardened-steel caliber .50 model-scale projectiles. Tests were made at normal incidence with striking velocities from 600 to 2000 ft/sec. The earlier Concrete Properties Survey data reported in reference 2 were neither so extensive nor so accurate as these newer data and should therefore be regarded as preliminary or auxiliary to the data of this report. Summary tables of the data and a discussion and analysis of the results are contained in the first part of the report; the two appendices contain complete descriptions and original data on the parts of the work indicated by their titles.

11. Ballistic tests on concrete slabs, by J. G. Stipe, Jr., M. E. DeReus, J. T. Pittenger, and R. J. Hansen, CFD Interim Report No. 28, June 1944. Unclassified.

The separately bound "Appendix A—Tables of data" contains full tabulations of all original ballistic and concrete data.

Perforation, scabbing, and penetration tests were made on 133 concrete slabs in this companion program to the work of reference 10. The same caliber .50 projectiles were used and slabs from 3 to 18 calibers thick were tested. These small-scale tests were planned to supplement the information at larger scales in reference 6, particularly with respect to the effect of slab thickness, concrete strength, aggregate gradation and size, various schemes of reinforcement, scab plates, and obliquity of incidence. The following relations were found: $e/d = 1.23 + 1.07z$ and $s/d = 2.28 + 1.13z$, where $e/d$ and $s/d$ represent the thickness (calibers) that can be perforated and scabbed, respectively, and $\underline{z}$ is the penetration depth (calibers) into massive concrete of the same characteristics at the perforation or scabbing limit velocity. These relations show good agreement with the data except at obliquities above 40°.

12. Repeated fire and edge fire effects on small concrete slabs, by J. G. Stipe, Jr., CFD Interim Memorandum No. M-12, July 1944. Unclassified.

The number of rounds required for perforation of reinforced concrete slabs by repeated fire attack with caliber .50 model-scale projectiles was tested for two thicknesses of concrete, two reinforcing schemes, and for different distances from the slab edge. Tables of ballistic data and many observed crater profile drawings are included.

13. Composite slabs, by J. G. Stipe, Jr., CFD Interim Memorandum No. M-13, June 1944. Unclassified.

A method of estimating the perforation proof thickness of slabs composed of concrete and steel, soil and concrete, and of the three materials is proposed.

14. Penetration theory: Estimates of velocity and time during penetration, by R. A. Beth, Division 2, NDRC Monthly Report OTB-7 (OSRD-4720), February 1945. Confidential.

This paper summarizes the theory of the variation of the resisting force R during projectile penetration for three cases: (1) R is a constant (the Robins-Euler theory), (2) R is a function of the remaining velocity v only (sectional-pressure theories), and (3) R is a function of the penetration depth x only (sectional-energy theories). The functional form of R is not known but there are reasons for believing that the actual curve for R will fall between those predicted by cases (2) and (3). A knowledge of R would be a step toward solving problems of fuze and projectile design and the design of composite targets.

15. Concrete penetration, by R. A. Beth, NDRC Report A-319 (OSRD-4356), March 1945. Confidential.

An attempt is made to revive the Poncelet hypothesis by postulating a force law of the form $R = a(x) + bv^2$ for concrete penetration, and a(x) and b are evaluated from caliber .50 penetration data in reference 10 and some additional data on the effect of projectile mass and nose shape given in an appendix. Calculations of resisting force, time, and remaining velocity during penetration are made. The theoretical consequences of a further generalization of the Poncelet force law, $R = a(x)v^{2\lambda} + b(x)v^2$, in which the first term is able to take account of the concrete scale effect, are worked out in an appendix. Later developments along this line are described in the appendix and text of the present report.

16. An electromagnetic method for measuring projectile velocity during penetration, by R. A. Beth, and E. J. Schaefer, NDRC Report A-329 (OSRD-5175), June 1945. Confidential.

The method consists in magnetizing the projectile and recording the electromotive force induced in suitably disposed coils during deceleration in a nonmagnetic and nonconducting target material like concrete by means of a cathode-ray oscillograph equipped with a linear time sweep. The report outlines the theory and design of the coils, the equipment used, and describes preliminary experimental work including the methods of stabilizing the magnetic moment of the projectiles against the effects of impact.

17. Penetration theory: Separable force laws and the time of penetration, by R. A. Beth, NDRC Report A-333 (OSRD-5258), June 1945. Confidential.

This report considers the consequences of assuming a separable force law of the form $R = c \cdot g(x) \cdot f(v)$ as an alternative to the generalized Poncelet force law of reference 15. General formulae are given for penetration as a function of striking velocity, remaining velocity as a function of depth during penetration, and for time of penetration. A number of special cases are tabulated, including all of the classical theories of penetration. A separable force law for perforation leads to a relation between limit, striking, and residual velocities of the form

$$F(v_\ell) = F(v_o) - F(v_r),$$

which is independent of the projectile mass and the target strength under certain plausible assumptions.

18. Ballistic tests on concrete slabs, II: Effect of nose shape, by J. G. Stipe, Jr., Division 2, NDRC Memo. A-112M (OSRD-6638).

This report contains unanalyzed data on a continuation of the work of reference 11. Perforation, scabbing, penetration, and ricochet tests were made with caliber .50 projectiles of various nose shapes at normal incidence and obliquities up to 60° on 41 reinforced concrete slabs from 3 to 18 calibers thick.

## II. THE PROBLEM OF CONCRETE PENETRATION[1]

In attempting a rational approach to the complicated problems connected with the interaction of a projectile and target it is important to distinguish between penetration and perforation, and to analyze the effect of obliquity by comparison with the corresponding case of normal incidence.

In a strict sense the term <u>penetration</u> is reserved for the entry of a projectile into a target without passing through it. Penetration into a massive target or, simply, massive penetration is often used when we wish to emphasize the fact that there is no yielding or rupture of material at the back face of the target.

The term <u>perforation</u> is used specifically when the projectile passes completely through the target slab or plate with a finite residual velocity upon emergence from the back face. The lowest striking velocity $v_0$ for which the projectile will just perforate the target is called the limit velocity $v_\ell$.

In the transition region between massive penetration and perforation the presence of the back face permits a progressively greater penetration than would be obtained with the given striking velocity in a massive target.

For a rational approach it is also desirable to consider first only inert and nondeforming projectiles. The effects of an explosive projectile depend on the stage during the penetration cycle at which the projectile detonates. For maximum effect the detonation should take place after the maximum inert penetration depth has been reached. If this is determined by a time fuze setting, the instant when the decelerating force becomes sufficient to initiate the fuze, and the time to maximum inert penetration thereafter need to be estimated. Both of these needed items of information depend on an understanding of the phenomena of inert penetration.

With target materials like concrete a steel projectile almost never breaks, while against hard armor there is a competition between the plate

1/ A more complete discussion is given in Ref. 9 including diagrams, photographs, and graphs.

and the projectile which, especially at high striking velocities or obliquities, often results in projectile deformation or rupture. Even against concrete the ballistic cap or windshield with which many service projectiles are provided, is crushed and swept away. Thin-shelled HE projectiles or GP bombs may also be deformed or ruptured against concrete, especially as the striking velocity is increased.

Thus, for the problem of concrete penetration, the aim has been to find first the laws governing the penetration of an inert, uncapped, nondeforming projectile of conventional form penetrating a massive concrete target without yaw in a direction normal to the target face (zero obliquity). This has been considered the primary problem and the phenomena of oblique penetration, of normal and oblique perforation, of projectile deformation and explosion, and so forth, are to be elucidated with reference to it.

Most practical problems on concrete penetration involve one or more of the factors of perforation, obliquity, deformation, explosion, and so forth. In time of war the seeming urgency of finding direct answers to specific practical problems tends to work against the attempt to pursue a logical sequence of experimentation. Unless interrelations among various parts of the problem, such as those described, are kept clearly in mind in planning tests it may easily become impossible to correlate the results from different programs because too many variables were changed from one test to another. Such ad hoc experimentation is, in the end, very wasteful and time-consuming because it prevents the attainment of a quantitative over-all view which would provide immediate answers to many practical problems without the expense and delay involved in making further tests. For this reason the Princeton group has made every attempt to plan concrete penetration and perforation tests so that the interrelations between various experimental programs may be exploited to shed light on the general problem.2/

For a given projectile and concrete target the maximum penetrations are not proportional to the striking kinetic energy, as they would be if the

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2/ See pp. 1-11 in Ref. 6.

force resisting the projectile at each instant were constant and the same for all penetrations. This suggests that the resisting force depends on depth or velocity or both during penetration.[3/]

Experiments furthermore show that a "scale effect" exists for penetrations into concrete. This may be described as follows. Consider similar projectiles (same shape and same caliber density) striking a given concrete target at a given velocity $v_o$; then it is found that the maximum caliber penetrations $z_1$ are not constant but increase in a regular way with the caliber of the projectiles. This phenomenon suggests that the resisting force per unit area of the projectile must depend on the caliber d.

The physical origin or cause of the scale effect is not understood. Among the suggestions that have been made are the following:

(a) The resisting pressure may be a function of a pure number ratio s/d, where s is a characteristic length associated with the structure of the concrete, such as an aggregate size parameter. The length s should probably be defined to take due account of the relative proportions of various aggregate sizes present in the concrete. In a general way this suggestion is in agreement with the observed fact that increasing the fineness modulus of the aggregate used tends to increase the penetration resistance of concrete. The physical mechanism underlying this suggestion should depend on the difference in crushing strength between the aggregate particles and the interstitial cement in some way. If this is the case, then the scale effect should be greater for weak than for strong concretes. This has not been observed, but perhaps sufficiently accurate data are not yet available.

(b) The resisting pressure may be a function of the actual depth x in the concrete target rather than of the caliber depth z. This would imply an inhomogeneity with depth in the target in the sense that the deeper layers should be less resistant to penetration.[4/]

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3/ See Ref. 14.

4/ See pp. 33, 34 in Ref. 10.

(c) The resisting pressure may depend on the time rate of deformation of the target material. This would imply that the resistance should increase with a quantity like v/d which has the dimensions of a rate of strain.

## III. RECOMMENDED APPROXIMATE PENETRATION THEORY

Time and space do not now permit a full discussion of all aspects of the theory of concrete penetration; for this reference is made to previous reports. Instead only the best solution that can be recommended at the present stage of the work will be described. A critical discussion of the most recent previous theoretical report and the path that has led to the present recommendation is given in the Appendix.

We base the recommended penetration theory on the following assumptions for a nondeforming projectile of conventional form penetrating a massive concrete target without yaw in a direction normal to the target face (zero obliquity):

Assumption I. The force per unit area resisting the forward motion of the projectile in the target can be represented to a very good approximation by a separable force law[5]

$$R = c \cdot g(z) \cdot f(v), \tag{1}$$

where $z$ is the depth of nose penetration measured in calibers and $v$ is the remaining velocity at each instant.

Assumption II. The depth dependence of $R$ can be approximated by

$$g(z) \begin{cases} = z/2 \text{ for } 0 \leq z \leq 2.00 \text{ calibers.} \\ = 1.00 \text{ for } z \geq 2.00 \text{ calibers.} \end{cases} \tag{2}$$

This assumption is an attempt to take account of the entry of the pointed nose of the projectile into the target and the effect that the escape of target material during crater formation may have on $R$.

Assumption III. The velocity dependence of $R$ can be approximated by

$$f(v) = v^{2-\alpha}, \tag{3}$$

where $\alpha$ is a constant. Both the fractional exponent needed for fitting the data and the fact that $f(v)$ then goes to zero with $v$ are un-

[5] See Ref. 17 for a general treatment of this case.

satisfactory from a physical point of view. These defects are associated with our basic lack of knowledge concerning the physical causes of the velocity dependence of R.

Assumption IV. The constant c is inversely proportional to

$$K \eta d^{\beta} , \qquad (4)$$

where K is the "penetrability" of the concrete, $\eta$ is a nose shape factor for the projectile, d is the caliber, and $\beta$ a numerical exponent. The form of the assumed scale-effect dependence on d is again unsatisfactory, but until we attain a better understanding of the underlying physical phenomena (see end of Part II) it seems difficult to improve this formulation.

Assumption V. An excellent representation of concrete penetration data at all scales is obtained by assigning the values

$$\alpha = 1.80 \text{ and } \beta = 0.20,$$

or

$$2 - \alpha = \beta = 0.20. \qquad (5)$$

According to these five assumptions the law of force for concrete penetration becomes[6]

$$R = \frac{g(z)}{K\eta} \left(\frac{v}{d}\right)^{0.20} \times \text{const.} \qquad (6)$$

The equation of motion can be integrated by separation of the variables.[7] Using a hybrid system of units[8] that is convenient for practical numerical computations, the resulting penetration formula can be put in the form

$$\boxed{G(z_1) = K\eta d^{0.20} DV_0^{1.80}} \qquad (7)$$

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6/ The numerical values of $\alpha$ and $\beta$ given in Assumption V were found from concrete penetration data without assuming $2 - \alpha = \beta$. The fact that they lead to a force law in which both the velocity dependence and the scale dependence can be combined in the factor $(v/d)^{\beta}$ may be significant in connection with the problem of the cause of the scale effect as discussed at the end of Part II.

7/ The details of the integration are not repeated here. They are amply covered in previous reports, especially Refs. 1 and 17.

8/ Compare p. 11 in Ref. 15.

$$G(z_1) = \int_0^{z_1} g(z)\,dz \begin{cases} = z_1^2/4 & \text{for } 0 \le z_1 \le 2.00 \text{ calibers} \\ = z_1 - 1.00 & \text{for } z_1 \ge 2.00 \text{ calibers} \end{cases}$$

$z_1$ = final maximum nose penetration (calibers) of the projectile, a pure number

$K$ = "penetrability" of the concrete. Units are such as to make $G(z_1)$ a pure number

$N$ = nose factor for the projectile, a pure number

$d$ = caliber or maximum diameter of the projectile (in.)

$D$ = ratio of weight of projectile (lb) to the cube of the caliber $d$ (in.), "caliber density" of the projectile (lb/in.$^3$)

$V_o = v_o/1000$ = striking velocity of the projectile ($10^3$ ft/sec).

In these units the resisting force per unit maximum cross-sectional area of the projectile, Eq. (6), is

$$R = \frac{263{,}820}{KN}\left(\frac{V}{d}\right)^{0.20} g(z) \text{ lb/in.}^2, \tag{8}$$

where $V$ is the instantaneous remaining velocity of the projectile in thousands of feet per second, and $d$ is in inches. The relation between $V$ and $z$ during penetration is given by[9/]

$$\frac{G(z)}{G(z_1)} + \left(\frac{V}{V_o}\right)^{1.80} = 1. \tag{9}$$

The time $\sigma$ (msec) from the instant of impact to any depth $z$ during penetration may be computed from

$$\frac{V_o\sigma}{x} = \frac{1}{z}\int_0^z \frac{dz}{\left[1 - \frac{G(z)}{G(z_1)}\right]^{1/1.80}} = \text{a pure number, a function of } z \text{ and } z_1 \tag{10}$$

where $x$ (ft) is the nose penetration [$x = zd/12$]. The total time of penetration, $\sigma_1$ msec, can be determined from

9/ Compare Eq. 15 and Fig. 2 in Ref. 17.

$$\frac{V_o \sigma_1}{x_1} = \frac{1}{z_1} \int_0^{z_1} \frac{dz}{\left[1 - \frac{G(z)}{G(z_1)}\right]^{1/1.80}} = \text{a pure number, a function of } z_1 \text{ only} \qquad (11)$$

where $x_1$ (ft) is the maximum nose penetration [$x_1 = z_1 d/12$]. The right-hand sides of Eqs. (10) and (11) are universal functions for concrete, independent of the target and projectile parameters K, n, d, and D. Thus, by numerical evaluation of the integrals, a graph[10] can be made for finding values of $V_o\sigma/x_1$ and $V_o\sigma_1/x_1$. Then the determination of $\sigma$ or $\sigma_1$ is reduced to simple slide-rule operations for any particular case.

10/ Analogous to Fig. 11 in Ref. 15.

## IV. DATA EVALUATIONS

This outline is sufficient to indicate the scope of the recommended solution based on the Assumptions I through V. It remains to show how K and n may be evaluated.

If $G(z_1)$ is plotted against $V_o^{1.80}$ for a given set of observed data (same projectile on the same concrete target), the points will be found to fall on or near a straight line through the origin. The slope is equal to $K n d^{0.20} D$ according to Eq. (7). Both Eqs. (7) and (9) can be conveniently represented as straight lines if special graph paper is prepared on which the abscissa distances are proportional to $V^{1.80}$ and are labeled with values of V and on which the ordinates are proportional to $G(z)$ and labeled with values of z.[11]

Usually the weights of the otherwise similar projectiles used in obtaining a given set of data vary somewhat. For numerical calculation it is therefore better to compute $DV_o^{1.80}$ separately for each shot. In combining the data from a number of shots in evaluating K and n it is suggested that an appropriate weighting of the data is obtained by summing the values of $G(z_1)$ and the values of $DV_o^{1.80}$. Thus the deeper penetrations will have a proportionately greater influence on the computed results.

This principal was used in recomputing the caliber .50 nose-shape data given in a previous report.[12] The results are listed in Table I where $Kn$ was computed from

$$K n = \frac{\sum G(z_1)}{d^{0.20} \sum D V_o^{1.80}}. \tag{12}$$

The nose factor n is defined to be unity for a 1.500-caliber radius tangent ogive nose. With reference to this the value of n for the other nose shapes used was computed by simple division. These values of n are plotted against the nose height h (calibers) in Fig. 1. For a tangent ogive of n calibers radius, n is easily shown to be

11/ This is a particular application of the principles described on and in Fig. 2 of Ref. 17.

12/ See Appendix C in Ref. 15.

$$h = \sqrt{n - 0.25} \text{ calibers.} \qquad (13)$$

The straight line drawn on the figure is

$$\mathcal{N} = 0.72 + 0.250\ h. \qquad (14)$$

It is dashed from h = 0 to h = 0.50 because, strictly speaking, a tangent ogive is impossible with $h$ less than 0.50. This straight line is closely related to the similar straight line obtained from the same data in a previous report,[13/] but the values have been recomputed in accordance with

Table I. Evaluation of $\mathcal{N}$ from caliber .50 nose shape data.

| Target Cube No. | Ogival Nose Radius $n$ (calibers) | Nose Height $h$ (calibers) | Number of Shots | $\Sigma G(z_1)$ | $d^{0.20}\Sigma D V_o^{1.80}$ | $K\mathcal{N}$ | $\mathcal{N}$ |
|---|---|---|---|---|---|---|---|
| B3B 3- 7 | 1.50 | 1.118 | 10 | 40.15 | 9.042 | <u>4.440</u>[a/] | 1.000 |
| B3B 3- 7 | Flat | 0 | 9 | 25.32 | 7.852 | 3.225 | 0.726 |
| B3B13-17 | 1.50 | 1.118 | 6 | 25.47 | 5.103 | <u>4.991</u>[a/] | 1.000 |
| B3B13-17 | 0.50 | 0.500 | 6 | 18.79 | 4.411 | 4.260 | 0.853 |
| B3B13-17 | Flat | 0 | 6 | 19.68 | 5.548 | 3.547 | 0.71i |
| B3B20-24 | 1.50 | 1.118 | 6 | 18.19 | 4.419 | <u>4.116</u>[a/] | 1.000 |
| B3B20-24 | 3.10 | 1.688 | 7 | 29.57 | 6.290 | 4.701 | 1.142 |

a/ The underlined value is the penetrability $K$ for the concrete since $\mathcal{N}$ = 1.000 by definition for the projectile with n = 1.50.

Eqs. (7) and (12). More data on the effect of nose shape are needed: it is important that $K$, $d$, and $D$ be held constant in obtaining comparative data on the effect of two nose shapes on penetration. Until better information is secured it is felt that the relation given by Eq. (14) should be used as a basis for estimating $\mathcal{N}$ even at larger calibers.

Table II lists such estimates of $\mathcal{N}$ for some of the projectiles on which extensive data are available. In estimating $\mathcal{N}$ for nontangent and composite ogives, the actual nose height from tip to bourrelet was not used directly, but $h$ was adjusted to correspond to a best-fit tangent ogive approximating the actual nose shape as shown on the projectile drawings. The values of

13/ See pp. 18-20, 25 in Ref.15. The script $\mathcal{N}$ is used in this report [illegible] it from the N of Ref. 15.

ENTIAL

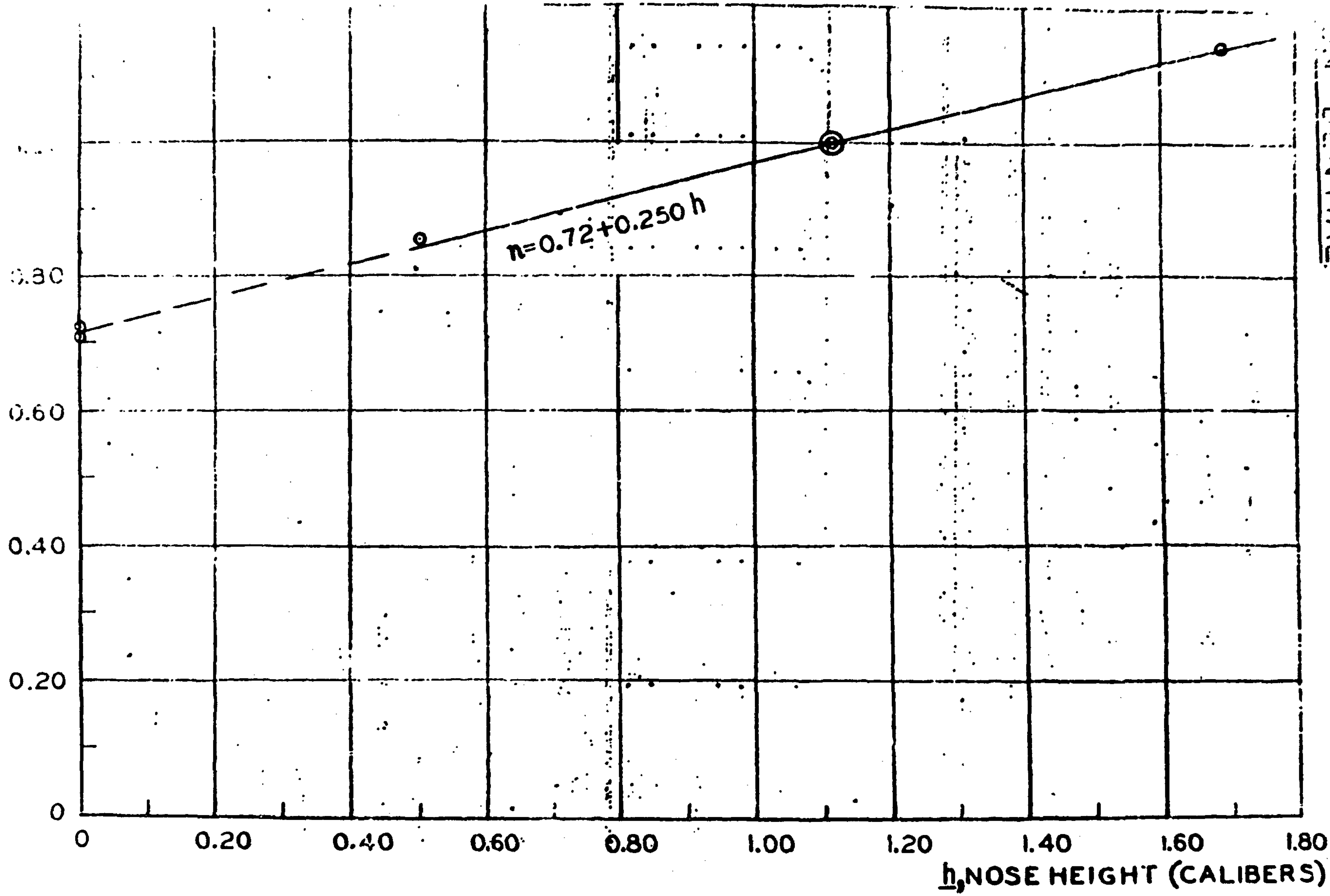


FIG. 1. PLOT OF NOSE FACTOR n VERSUS NOSE HEIGHT.

d, $d^{0.20}$, and $nd^{0.20}$ are also listed for each projectile in Table II. These give a representative picture of the kind of variations involved in the nose and scale effects.

Table II. Nose and scale factors for selected projectiles.

| Projectile | Data Reference | Caliber d (in.) | Estimated h (calibers) | Estimated n | Scale Factor $d^{0.20}$ | $nd^{0.20}$ |
|---|---|---|---|---|---|---|
| Cal .50 E-6 | 10, 11, 15 | 0.4985 | 1.118 | 1.000 | 0.870 | 0.870 |
| 37-mm M80 | 6 | 1.453 | 1.169 | 1.012 | 1.078 | 1.091 |
| 37-mm M74 | 6 | 1.453 | 1.123 | 1.001 | 1.078 | 1.079 |
| 75-mm M61 | 6 | 2.945 | 1.004 | 0.971 | 1.241 | 1.205 |
| 3-in. M79 | 6 | 2.995 | 1.192 | 1.018 | 1.253 | 1.268 |
| 155-mm M112 | 6 | 6.092 | 1.098 | 0.995 | 1.435 | 1.428 |

The effect of projectile mass on penetration is uniquely determined by Assumptions I, II, and III. The principal available data are analyzed in Table III. The results shown in the last column suggest that the values of K as computed from data for similar projectiles of different mass on the same concrete may tend to increase with D. However, the best set of data (37-mm) with the largest number of shots shows practically no effect of this sort. Insofar as this trend is real these data indicate the degree of approximation involved in Assumptions I, II, and III. It may be that the theory would be improved by adding a Poncelet-type inertia term to the velocity-dependence assumption but the present data are felt to be insufficient for the evaluation of such a term.[14] Further data, holding K, n, and d constant, are needed.

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14/ See the Appendix of this report and Appendix B in Ref. 15.

Table III. Penetrability determined from data for similar projectiles of different mass on the same concrete.

| Data Reference | Target | Projectile | Average Caliber Density D (lb/in.$^3$) | Number of Shots | $\sum G(z_1)$ | $\sum Dv_c^{1.80}$ | $nd^{0.20}$ | K |
|---|---|---|---|---|---|---|---|---|
| 15 | B3B8-12 | Cal .50 Hollow | 0.361 | 4 | 14.21 | 3.493 | 0.870 | 4.676 |
| 15 | B3B8-12 | Cal .50 E-6 | 0.517 | 5 | 21.74 | 5.187 | .870 | 4.818 |
| 15 | B3B8-12 | Cal .50 Carboloy | 1.044 | 5 a/ | 22.18 a/ | 5.181 a/ | .870 | 4.921 a/ |
| 15 | B3B8-12 | Cal .50 Carboloy | 1.044 | 8 | 50.92 | 11.092 | .870 | 5.277 |
| 6 | Slab 4B | 37-mm M80 | 0.538 | 14 | 60.156 | 23.061 | 1.091 | 2.392 |
| 6 | Slab 4B | 37-mm M74 | .623 | 14 | 77.361 | 29.892 | 1.077 | 2.399 |
| 6 | Slab 6B | 155-mm M112 | .378 | 4 | 16.605 | 4.225 | 1.428 | 2.752 |
| 6 | Slab 6B | 155-mm M112 lead filled | .430 | 3 | 15.050 | 3.630 | 1.428 | 2.903 |

a/ Omitting three doubtful shots included in the next line.

Some scale-effect data are analyzed in Table IV. The agreement between the values of penetrability K as derived for the same concrete from penetration data obtained with different calibers d indicates the degree to which the scale-effect factor $d^{0.20}$ is able to represent the facts. Unfortunately both D and n factors are involved in all of the available data. Scale-effect data in which K, n, and D are held strictly constant are greatly needed. Nevertheless the agreement secured is felt to justify the use of the present formulation until better information becomes available.

No systematic analysis of the effect of various concrete properties on the penetrability K has yet been made. A considerable amount of data in which n, d, and D were held constant is available for this purpose[15] and the work should be carried out. A number of typical values of K in the range $2.0 < K < 5.0$ are shown in Tables I, III, and IV. It will be noted that the practical system of units suggested in connection with Eq. (7) leads to convenient numerical values of K without the use of awkward powers of ten.

15/ [illegible] Ref. [illegible]. Older data are given in Ref. [illegible].

Table IV. Penetrability determined from data for projectiles of different caliber on the same concrete.

(Data from Ref. 6)

$\pi d^{0.20} = 1.091$ for 37-mm M80 projectile.

$\pi d^{0.20} = 1.428$ for 155-mm M112 projectile.

| Target | Projectile | Average Caliber Density D (lb/in.$^3$) | Number of Shots | $\sum O(z_1)$ | $\sum DV_o^{1.80}$ | K |
|---|---|---|---|---|---|---|
| Slab 6A | 37-mm | 0.554 | 12 | 62.409 | 20.115 | 2.345 |
| Slab 6A | 155-mm | .379 | 3 | 13.084 | 3.261 | 2.810 |
| Slab 6B | 37-mm | .554 | 8 | 39.127 | 12.624 | 2.842 |
| Slab 6B | 155-mm | .401 a/ | 7 a/ | 31.655 a/ | 7.855 a/ | 2.822 |
| Slab 6C | 37-mm | .538 | 11 | 52.639 | 16.334 | 2.946 |
| Slab 6C | 155-mm | .394 | 6 | 21.380 | 5.147 | 2.909 |
| Slab 6D | 37-mm | .554 | 11 | 48.460 | 19.674 | 2.248 |
| Slab 6D | 155-mm | .378 | 4 | 16.860 | 4.559 | 2.591 b/ |
| Slab 6E | 37-mm | .538 | 10 | 56.373 | 18.037 | 2.866 |
| Slab 6E | 155-mm | .391 | 4 | 14.201 | 3.776 | 2.633 |
| Slab 10 | 37-mm | .554 | 10 | 43.946 | 15.275 | 2.638 |
| Slab 10 | 155-mm | .395 | 4 | 11.315 | 2.955 | 2.681 |

a/ Combined 155-mm data from Table III.

b/ Graph of 155-mm data for this case suggests possibility of systematic error in penetration measurements with transit (see pp. 17, 18 in Ref. 6).

[redacted]

## V. CONCLUDING SUGGESTIONS.

Further work on the problems of concrete penetration and perforation should be pursued along some or all of the following lines.

(a) The most promising direction for future fundamental work on penetration lies in finding and exploiting experimental methods of observing the phenomena during the penetration cycle. A promising electromagnetic method of recording projectile velocity as a function of time during penetration in a concrete target was suggested and described in a previous report.16/ It should be possible to devise other methods or to adapt those that are being used in the study of armor penetration. Measurements of the total time of penetration or of residual velocities after perforation would also be very useful, particularly in assaying the accuracy of any proposed theory.

The important thing in this type of work is to go beyond the traditional measurement of only striking velocity and penetration. The latter may lead to certain empirical formulations, but does not reach the heart of the problem thus leaving us with only the vaguest qualitative notions when a new form of problem presents itself.

(b) The analysis of existing concrete data should be continued along the lines illustrated in the previous Part. A study of the effect of concrete properties on K has already been suggested. An analysis of the accuracy of the penetration formula, Eq. (7), should be made by statistical methods.

On the basis of the improved knowledge of normal penetration, existing data on perforation and on the effects of obliquity for both penetration and perforation should be evaluated again.17/

Graphs for practical applications should be prepared to aid the designers of weapons and fortifications.

16/ See Ref. 16.

17/ See Refs. 4, 6, 11, and 18.

[redacted]

 [redacted]

(c) The analysis of existing data will also serve to emphasize the need for additional observations on certain points: the need for better nose-effect, mass-effect, and scale-effect data has already been mentioned. On the whole a thorough knowledge of existing data including its defects is desirable in order to plan further tests for maximum effectiveness and value.

[redacted]

[redacted]

## APPENDIX

### Poncelet-Type Force Laws for Concrete Penetration

In a previous report on concrete penetration[18] a force law of the generalized Poncelet form

$$R = a(x) + bv^2 \qquad \text{(A-1)}$$

was postulated. Caliber .50 data were used to evaluate a(x) and b and a very good fit was achieved. Larger-caliber data were not considered.

However, these results and their extension to larger calibers do not seem to be completely satisfactory. It is felt that the following difficulties may be directly attributed to the strong increase of the $bv^2$ term with v in the force law, Eq. (A-1):

(a) Figure 10, page 31 in Ref. 15, shows that the crushing resistance a(x) has a maximum at about 3 calibers penetration and then decreases to about 50 percent of its maximum value at 15 calibers. There are reasons connected with the curing of the concrete[19] for a possible decrease of crushing resistance with depth in concrete, but it is hard to believe that the decrease is really as great as is shown by this curve.

(b) The theory of Ref. 15 gives a basis for computing the change in the penetration curve (striking caliber energy, $DV^2$, as a function of maximum penetration) with the caliber density D of the projectile. For large D the computations lead to a downward or negative curvature of the penetration curve as is illustrated by the computed curve for the tungsten carbide projectiles in Fig. 8, page 27 of Ref. 15. If this were true, the penetration would increase more rapidly than the striking energy under certain circumstances. The tungsten carbide data which seem to support this are rather weak as is shown by the "Remarks" in the data sheet.[20] The data for larger calibers tend to deviate from this computed caliber-density dependence in a way which suggests that the penetration curve actually does not tend to curve downward for larger D values.

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[18] Ref. 18.

[19] See pp. 33, 34 in Ref. 10.

[20] [illegible] Ref. 15.



[redacted]

[redacted]

[redacted]

The computations for the effect of projectile mass ... penetration on the basis of Eq. (1) in ... awkward and ... numerical differentiations and integra... While this ... ment against the possible correctness ... assumption, if an equally good representation of e... ng data could ... of a simpler or more manageable appro... ...en to the forc...

It is not felt that the theoretica... inertia term, $bv^2$, in Eq. (1) is sufficien... strong, part... projectile is being rapidly decelerated, t... vercome the ... culties enumerated in (a) and (b) above.

In Appendix B of Ref. 15 the theoretical consequences ... force law of the form

$$R = a(x)\, v^{2\lambda} + b(x) ...$$

are worked out. It is suggested that this may be made to ... effect by writing the first term in the form $a(z) \cdot (v/d)^{2\lambda}$ ... caliber of the projectile and $z = x/d$ calibers. Values of $\lambda$ ... of 0.10 to 0.12 may be deduced from penetration data of diff... on the same concrete. It is surprising and probably signifi... such a value of $\lambda$, selected on the basis of the observed scal... things happen:

(i) The inertia term, $b(x)v^2$, is no longer needed to ... for the effect of changes in projectile mass, and then ...

(ii) the coefficient, $a(z)$, rises to a maxim... about ... calibers, and then remains sensibly con... is much more plausible than the ... above $z$ = ... computed on the basis of Eq. (A-...).

These observations lead directly to the theory recommen... sent report which corresponds to Eq. (A-2) with the last te... o...

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ABSTRACT


A theory of perforation and penetration of concrete which represents the best recommendation that can be made in the present state of our knowledge of concrete penetrations is presented. Formulas are derived for computing the resisting force during penetration and the remaining velocity as a function of depth during penetration. Selected data on the effects of projectile nose shape and mass and on concrete properties are analyzed to illustrate the application of the proposed theory.


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