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DESIGN OF A MULTIFREQUENCY REED-TYPE SHOCK GAGE WITH AN INERTIA-OPERATED TRIGGER

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BY MILTON MARTIN AND PAUL GOLOVATO

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DESIGN OF A MULTIFREQUENCY REED-TYPE SHOCK GAGE WITH AN INERTIA-OPERATED TRIGGER

by

Milton Martin and Paul Golovato

ABSTRACT

A reed-type shock gage is an instrument designed to measure the motion of a structure subjected to applied shock. It consists of a series of uniform cantilever beams each of which has a concentrated mass at one end and is clamped to the instrument frame at the other end. The natural frequencies of these beams or reeds are chosen so as to span the range of major frequency components of shock motions encountered on ship structures.

The reed-type shock gage described consists of 12 reeds, each having a different natural frequency, supported on a common column, with the free ends provided with scribers which make records of the reed motions on a moving sheet of waxed paper. The gage requires no external source of power, is self-contained, entirely mechanical, and records automatically. The plate holding the waxed-paper record is released when the applied shock actuates an inertia-operated trigger. Means are provided for varying the time duration of the record.

A mathematical analysis is given of the motion of the inertia-operated trigger in which it is assumed that the trigger is a beam constrained to rotate about a fixed pivot when the pivot is subjected to a constant acceleration. The angle $\phi$ through which the trigger will rotate in time $t$ is given by the expression $\phi = \frac{k_1}{k_2} (1 - \cos \sqrt{k_2} t)$ for small angles of rotation, where $k_1$ and $k_2$ represent the lumped physical constants of the system. Detailed consideration of the frictional, inertial, and mechanical forces on the trigger is given. Several approaches to the trigger-design problem are discussed, using the above derived expression.

INTRODUCTION

In the modification of the design of an existing multifrequency reed gage, undertaken at the request of the Vibration Section of the David Taylor Model Basin, several features of interest to designers were incorporated. Most interesting among these, perhaps, was the design and analysis of an inertia-operated trigger. The analysis presented is general enough so that it should serve as a useful guide to designers of other inertia-operated devices. The report first presents the requirements to be met by the gage,
then a general statement of how these requirements are met, and last a more
detailed description and analysis of the inertia-operated trigger.

PRELIMINARY CONSIDERATIONS

The original multifrequency reed gage consists of several flat steel reeds of different natural frequencies arranged one above the other in a single plane and rigidly supported at one end by a common column designed to accommodate from 1 to 12 reeds. The column is rigidly mounted to a steel housing of rugged construction. At the free end of each reed is a scriber which bears lightly against a sheet of waxed paper held against a rigid steel plate. When the instrument is clamped to a structure which is later subjected to shock as a result of which it vibrates, the reeds execute various motions and their maximum displacements are recorded on the waxed paper.

In measuring shock motions, the Vibration Section found that if in addition to obtaining the maximum displacement of these reeds a displacement-time record could be obtained, the shock motion could be more satisfactorily analyzed. By moving the waxed paper during the shock and subsequent vibrations, such displacement-time record could be obtained.

It was desired that the instrument be completely self-contained and independent of external sources of power so that it could be installed in any part of a ship structure without the necessity of electrical wiring or operating personnel. It was also felt that it would be desirable to avoid using an electric drive for the waxed-paper holder and any electrical elements in the triggering device because of the time lags due to the electrical impedance generally inherent in such devices. It was therefore found desirable to have the shock motion itself initiate the motion of the waxed-paper holder directly. Triggering of the gage by random motions of the ship had to be avoided. It was arbitrarily decided to design the gage so that it would not be triggered by an acceleration of less than 1 g with a corresponding displacement of less than 0.01 in. However, provision was made for varying the sensitivity of the trigger if such variation was found necessary.

SPECIFICATIONS

In view of these preliminary considerations it was decided that the modified multifrequency reed gage meet the following requirements:

a. Provision must be made for fastening the gage to decks, bulkheads, and overheads as in the original gage.

b. The arrangement of the reeds and the means of recording should be retained as on the original design.
c. Means for mounting the reeds should be improved.

d. A means for moving the waxed paper and holder through a distance of as much as 2 in. during the occurrence of the shock should be provided.

e. A means of continuously varying the time of motion of the paper from 0.05 sec to 0.50 sec should be provided.

f. A means should be provided for triggering the waxed paper and holder by accelerations applied to the instrument in the direction of measured motion as small as 1 g and corresponding displacements in the same direction as small as 0.01 in. within 15 milliseconds of the inception of the shock.

g. The gage shall be capable of withstanding accelerations of 200 g and corresponding displacements of several inches without damage to any of the component parts.

h. It shall be possible to operate the gage in any position as provided for in requirement a.

i. The modified gage shall be completely self contained and shall not require any external sources of power, operating personnel after initial setting, or the use of electrical devices.

**DESCRIPTION OF THE GAGE**

In the description of the modified design of the multifrequency reed gage an attempt is made to point out how these requirements were met. Figure 1 is a schematic drawing of the gage.

**Frame**

The frame is an open-ended box of welded steel plate. A diagonal member and flanges along the sides and top provide additional stiffness. Four bolt holes on the rear flanges and four on the bottom flanges allow the gage to be bolted in any desired position.

**Reeds**

The reeds are spring steel strips 1/2 in. wide of different thicknesses and lengths as required. A 33/64-in. hole is drilled through one end to fit on the supporting column. To the other end is silver-soldered a small brass weight. At the bottom of the brass weight is soldered a small phosphor bronze cantilever spring at the end of which is punched a small dimple which forms the scriber point. The cantilever spring provides just sufficient pressure to allow the point to scratch a fine line on the waxed paper without excessive friction.
Figure 1 - Schematic Drawing of Reed-Type Shock Gage in Cocked Position
Reed-Supporting Column

The reed-supporting column consists of a steel shaft 1/2 in. in diameter on which the reeds and spacers are mounted alternately. Each spacer is keyed to a reed and to an adjacent spacer. The bottom spacer is larger than the rest and is fixed to the frame. The top spacer is keyed and slides in a hole in the top of the frame. The shaft passes through a hole in the bottom of the frame and through the stack of reeds and spacers with the threaded end projecting through the top spacer. A head is provided at the lower end of the shaft and a nut at the top for clamping the assembly of reeds and spacers. The reeds may be removed by unscrewing the nut and pulling the shaft out from the bottom.

Recording Waxed-Paper Holder

To provide a means of moving the recording waxed paper it was decided to fasten a precut sheet of waxed paper by means of C-type clamps at opposite edges to a sliding paper holder. The paper holder consists of a 1/4-in. aluminum plate to which is welded an aluminum boss at each corner for housing a plain bronze bushing. These bushings run on two hardened ground-steel shafts mounted in the ends of the frame. Provision has been made for substitution of reciprocating ball bushings for the bronze bushings if it is desired to reduce bearing friction. A support for each shaft is also provided in the center to prevent excessive deflection of the shaft at high shock loads. A compression-coil drive spring mounted on each shaft between the center-shaft support and the forward paper-holder boss provides the means for driving the paper holder. The spacing is such that a stroke of 2 in. may be obtained. A small rubber bumper is provided at the end of each shaft. The paper holder is prevented from bouncing back at the end of its stroke by a plate-return friction ratchet.

Dashpot

To provide means of varying the speed of the paper holder, a small air dashpot is attached by one end to the backside of the paper holder and the other end to the frame. A loose-fitting joint was provided at each point of attachment to prevent binding of the piston in the cylinder because of possible misalignment at assembly. A needle valve provides easy means of adjustment of the damping and hence of the speed of the paper holder.
Inertia-Operated Trigger

A light aluminum beam, pivoted in the middle on two small ball bearings rigidly mounted to the frame, serves as the triggering beam. The beam has a 2.8-lb lead seismic weight fastened to one end and a 0.04-in. by 0.06-in. flat surface ground on a hardened insert fastened to the other end. Two soft adjustable helical springs fastened to each side of the seismic weight are provided to center the beam. The hardened steel flat bears against the outer race of a small trigger ball bearing attached to the backside of the waxed-paper holder with a force of 25 lb due to the action of the drive springs. The plate is thus held in the cocked position ready for release. The trigger is designed so that an acceleration as small as 1 g with a corresponding displacement of 0.01 in. on the frame of the gage, and consequently the beam pivot, produces sufficient rotation of the seismic trigger beam to release the paper-holder plate within 15 milliseconds and allow it to be driven forward by the drive springs. A detailed analysis of the design of the trigger is given later in this report.

Impact Resistance

It was relatively easy to design the elements of the gage rugged enough to withstand accelerations as high as 200 g. However, buffers had to be designed to absorb the rotational energy of the trigger beam and prevent the beam from becoming damaged during the test. Two buffers, each similar in principle to a multiple-disk friction clutch, were therefore provided. One was located 1/4 in. above and the other 1/4 in. below the seismic weight.
The buffers, shown in Figure 2, consist of a movable stack of 1/32-in. rectangular brass disks threaded on two steel pins which are mounted in a channel-shaped block. Each disk is set between a pair of stationary disks. The stationary disks are also threaded on two steel pins which are anchored to the gage frame. When the buffer is to be used the movable stack of disks is pulled out part way from the stationary stack of disks. All the disks are then clamped together by a screw bearing against an elliptical spring which in turn bears against the end stationary disk. A fixed pressure is thus applied between the stack of movable disks and the stack of stationary disks. Varying the load on the elliptical spring varies the disk pressure, and hence the capacity of the buffer to absorb energy. On the occurrence of a shock motion the seismic weight strikes a leather insert mounted in the channel-shaped block holding the movable disks and drives the stack of movable disks farther into the stack of stationary disks. The seismic weight is prevented from bouncing back by a friction-type ratchet which wedges it against a stationary plate located opposite the ratchet.

Operation of Gage

After the gage is mounted on the structure to be tested, the springs supporting the seismic weight are adjusted by means of a nut provided on one end of a threaded shaft to which one of the springs is anchored in order to center the trigger. After the trigger is centered the reeds are temporarily lifted away from the paper holder by loosening the clamping nut, waxed paper is inserted, and the paper holder is pulled back 2 in. by hand until it is engaged by a paper-holder latch provided at the end of the stroke. The trigger is accurately aligned by a locating pin which engages the seismic weight to the frame. The paper-holder latch is designed so that by turning a screw the paper holder will be allowed to move forward slowly for 1/16 in. until the trigger bearing rests against the flat of the trigger insert. The latch is disengaged by turning the screw until it comes to a stop. The gage is then ready to record, as shown in Figure 3. The gage in the released position is shown in Figure 4.

MATHEMATICAL ANALYSIS OF THE INERTIA-OPERATED TRIGGER

Figure 5 is a schematic diagram of the inertial trigger shown rotated through angle $\phi$ by an acceleration $A$ on the frame of the gage. The springs supporting the trigger are located approximately over the center of gravity of the trigger and are adjusted to support the trigger in the initial position $\phi = 0$. The trigger is pivoted at point 0 on a bearing $H$, and its center of gravity is located at point $G$. The trigger is shown with a bearing
Figure 3a - Rear View

Figure 3b - Front View

Figure 3 - Reed-Type Shock Gage in Cocked Position
Figure 5 - Schematic Diagram of the Inertia-Operated Trigger

Insert held in contact with the trigger bearing E by the paper-holder drive springs. Both the bearing insert and the trigger bearing are shown exaggerated in size. This spring force acts in a line connecting the centers of bearings E and H.

The mathematical symbols used in the analysis of the trigger motion are defined as follows:

- \( M \) is the mass of the trigger.
- \( A \) is the acceleration of point 0 in the direction of \(-x\).
- \( F \) is the \( x \)-component of the pivot reaction force on the inertial trigger.
- \( P \) is the \( y \)-component of the pivot reaction force on the inertial trigger.
- \( t \) is the time.
- \( I \) is the mass moment of inertia of the trigger about its center of gravity.
- \( \phi \) is the angle measured from the \( y \)-axis, positive clockwise. 
- \( y \) and \( x \) are measured positive to the right and downward respectively.
- \( F_1 \) is the force exerted by the paper-holder drive springs.
- \( F_2 \) is the sliding friction force between bearing E and the bearing insert.
- \( F_3 \) is the force at the bearing face due to bearing E frictional torque.
- \( F_4 \) is the force at the bearing face due to bearing E inertial resistance to rotation.
\( F_s \) is the force exerted by the seismic springs.
\( M_1 \) is the frictional torque in bearing H.
\( M_2 \) is the frictional torque in bearing E.
\( I_d \) is the mass moment of inertia of the outer race and balls of bearing E.
\( k \) is the spring constant of the seismic springs.
\( l \) is the distance between the pivot point 0 and the face of the bearing insert.
\( r \) is the distance between the pivot point 0 and the center of gravity, point G.
\( R \) is the radius of bearing E (\( R_1 \) shown exaggerated in Figure 5).
\( R_2 \) is the radius of curvature of the trigger insert face.

The forces \( F_3 \) and \( F_4 \) can be expressed as:

\[
F_3 = \frac{M_2}{R_1} \\
F_4 = \frac{I_d}{R_1^2} \phi
\]

The moment on the trigger due to force \( F_1 \) acts in a direction so as to resist the rotation of the trigger. The value of this moment is given by:

\[
T = \frac{F_1}{\cos \alpha} (R_2 - 1) \sin (\phi - \alpha)
\]

where

\[
\alpha = \sin^{-1} \left( \frac{R_2 - 1}{R_1 + R_2} \sin \phi \right)
\]

It is apparent that by varying \( R_2 \) alone the value for \( T \) may be varied from zero, the case where \( R_2 = 1 \), to rather large positive values. Hence by the proper selection of \( R_2 \) almost any desired stability may be obtained. However, practical considerations limit the designer to some extent. For the trigger described in this report and in the analysis to follow, a bearing insert with a flat face, \( R_2 = \infty \), is used. This choice simplifies the analysis considerably. The resisting moment for this insert is given by:

\[
T = F_1 (R_1 + 1) \frac{\tan \phi}{\cos \phi}
\]

The width of the bearing tip required is then \( 2(R_1 + 1)\tan \phi \)

where \( \phi_r \) is the desired release angle.

Summing up the forces in the x and y directions, we have respectively
\[ P + kr \sin \phi - F_1 \tan \phi - (F_2 + F_3 + F_4) \cos \phi = -M \ddot{x} \] [5]

\[ P - F_1 + (F_2 + F_3 + F_4) \sin \phi = -M \ddot{y} \] [6]

Summing up the moments about the center of gravity of the trigger we have

\[ F_1 \cos \phi - Pr \sin \phi - F_1 (R_1 + l) \frac{\sin \phi}{\cos \phi} - M_1 - (F_2 + F_3 + F_4)(l + r) = -I \ddot{\theta} \] [7]

The \( x \) and \( y \) components of the acceleration of point \( G \) may be written as:

\[ \ddot{x} = -A + \dot{\phi} r \cos \phi \] [8]

\[ \ddot{y} = -\dot{\phi} r \sin \phi \] [9]

Substitute Equations 5 and 6 in 7 to eliminate \( F \) and \( P \). Using Equations 8 and 9 to eliminate \( \ddot{x} \) and \( \ddot{y} \) and substituting for \( F_3 \) and \( F_4 \) their values from Equations 1 and 2, we obtain the following equation:

\[ \left( I + Mr^2 + \frac{I_1^2}{R_1^2} \right) \ddot{\phi} + kr^2 \sin \phi \cos \phi - M_1 r \cos \phi + F_1 (R_1 + l) \frac{\sin \phi}{\cos^2 \phi} 
\]

\[ + (F_2 + F_3 + F_4)(l + r) = 0 \] [10]

In general the angle \( \phi \) will be small so that we may put \( \cos \phi \approx 1 \) and \( \sin \phi \approx \phi \). Equation 10 can then be simplified to

\[ \ddot{\phi} + k_2 \phi - k_1 = 0 \] [11]

where

\[ k_1 = \frac{M_1 r - M - F_1 l - \frac{M_1 l}{R_1}}{I + Mr^2 + \frac{I_1^2}{R_1^2}} \]

\[ k_2 = \frac{kr^2 + F_1 (R_1 + l)}{I + Mr^2 + \frac{I_1^2}{R_1^2}} \]

Using the initial conditions \( \phi = \dot{\phi} = 0 \) when \( t = 0 \), the solution of the differential Equation 11 is

\[ \phi = \frac{k_1}{k_2} \left( 1 - \cos \sqrt{k_2} t \right) \] [12]

The choice of physical constants for the inertial trigger involves considerable manipulation in order to obtain a practical solution to the problem. In order to give an idea of the relative magnitudes required, the physical constants of the trigger used in the reed gage are herein stated.
M = 0.00768 lb-sec²/in., A = 386 in./sec², P₁ = 25 lb, k = 9 lb/in., I = 0.0235 lb-in.-sec², \( I_b = 4.066 \times 10^{-7} \) lb-in.-sec², r = 2.078 in., l = 5.515 in., and \( R_1 = 0.25 \) in. \( M_1, M_2, \) and \( F_2 \) were negligible. The solution obtained in Equation 12 is shown graphically in Figure 6, using the constants given above.

![Figure 6 - Time versus Angle of Rotation](image)

The design specifications for an inertia-operated trigger generally will give an allowable time \( t \) for the trigger to release. The value of \( t \) will depend on the amount of the initial portion of the shock record which can be sacrificed. The choice of its value will also be determined by the minimum shock excitation with which it is desired to trigger the recording motion. This minimum excitation might be specified as an acceleration \( A \) and a displacement \( S \) necessary to trigger. The value of \( t \) may then be determined from the kinematic relation
If we decide that the trigger shall release on rotating an angle $\phi$, we can use the values of $t$ and $\phi$ to solve for the $k_1$ and $k_2$ in Equation 12. The dimensions and inertia of the trigger may then be based on the values $k_1$ and $k_2$.

On the other hand, if we guess at a design of the trigger and calculate $k_1$ and $k_2$ first, then Equation 12 gives us $\phi$, the angle through which the trigger must rotate to release in the specified time.

In an attempt to increase the sensitivity of the trigger one might try to get a very short time $t$ for release and a consequent small angle of rotation $\phi$ to the release point. However, if the angle is made too small the trigger becomes difficult to set, since it must be set inside the angle $\phi$. An angle of about $0.25^\circ$ seemed to be a suitable angle to use in the reed gage design.

**BIBLIOGRAPHY**


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