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BY *Arthur E. Creech* USO

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Design of Ground System of Radial Conductors for a VLF Transmitter - and
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A ground system for the Wheeler Mountain FRT-3 vlf antenna was designed, which will produce maximum radiated power per dollar of over-all cost. The fundamental equations are also applicable to any installation operating in the frequency range below 1000 kc. A radial-conductor ground system based on the design parameters utilized will permit vlf antenna efficiency of 50% or better, as compared to efficiencies of 25% or less usually encountered in similar antennas without such a ground system. The technique provides a precise and definite procedure for the design of an optimum ground system for any antenna installation, provided the soil conductivity is known.

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USNEL REPORT NO. 105

PROBLEM NEL 1D2

10 FEBRUARY 1949

FINAL REPORT

**design of ground system of radial
conductors for a vlf transmitter**

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Statement of the Problem:

To design a ground system for the Wheeler Mountain FRT-3 vlf antenna which will result in maximum power radiated per dollar of over-all cost.

Conclusions:

From the design equations developed, a radial-conductor ground system may be constructed which will provide maximum power radiated per dollar of over-all cost. The fundamental equations are also applicable to any installation operating in the frequency range below 1000 kc. A radial-conductor ground system based on the design parameters here presented will permit vlf antenna efficiency of 50 per cent or better as compared to efficiencies of 25 per cent or less usually encountered in similar antennas without such a ground system.

Work Summary:

A technique for achieving optimum design of radial-conductor ground systems was developed and applied to the special case of the Wheeler Mountain FRT-3 antenna in the Puget Sound area.

CONFIDENTIAL 1

CONFIDENTIAL

contents

	<i>page</i>
INTRODUCTION	3
THEORY OF DESIGN	4
APPLICATION OF THEORY TO A SPECIFIC INSTALLATION	6
CONCLUSIONS	8
APPENDIX I — Calculation of the inductance of a wire grid.	9
APPENDIX II — Calculation of ground radial current for 300-meter mono- pole with uniform current $I_0 = 2000$ amperes	11
APPENDIX III — Calculations of power lost due to transverse currents from radial conductors to intervening ground surface	12
APPENDIX IV — Measurement of absorption of radiation at grazing inci- dence by a lossy medium under a conducting grid	14
APPENDIX V — Glossary of symbols	15
REFERENCES	20

list of illustrations

<i>figure</i>		
1.	Radial ground current density versus radial distance from antenna base	6
2.	Optimum grid spacing versus ground current in amperes	6
3a.	Optimum grid spacing versus radial distance from antenna base	7
3b.	Total number of conductors versus radial distance from antenna	7
4a.	Power loss per square meter versus radial distance from antenna	7
4b.	Power loss in an annular ring 1 meter wide versus radial distance from antenna	7
5.	Resistance of a wire in terms of wire radius	9
6.	Ground-charging current density versus radial distance from base of an- tenna	12
7.	Diagrammatic presentation of factors affecting loss in a thin, truncated, triangular conductor	12
8.	Current density at earth surface as supplied by a buried conductor	13
9.	Tapered-tube explanation of current density	13
10.	Power loss due to leakage currents from a radial conductor to earth surface	13

list of tables

1.	Grid spacing versus attenuation	14
2.	Real part of the characteristic impedance of lossy soil under a conducting grid	16
3.	Power loss in watts per square meter versus distance from antenna base	17
4.	Total annual cost versus conductor spacings	18
5.	Values of $J = f(r)$	19

CONFIDENTIAL

introduction

It is an accepted fact that transmitting antennas operating in the very low frequency range of 15 to 30 kc are at best very inefficient radiators. The greatest single factor contributing to this inefficiency is usually the ground system. The rules for ground system design are generally empirical and based on experiments on existing installations.

The problem of obtaining optimum design in a radial-conductor ground system lies in attaining maximum power radiated per dollar of over-all cost. Since the average transmitting antenna operating in the vlf range below 100 kc exhibits an efficiency of 25 per cent or less, and since a ground system designed as set forth in this report permits efficiencies of 50 per cent and more, an adequate solution to this problem has an obvious economic importance. The specific problem undertaken by the U. S. Navy Electronics Laboratory was the design of a radial-conductor ground system for the Wheeler Mountain FRT-3 antenna in the Puget Sound area, but the method is applicable to any vlf antenna problem.

The basic equations presented are valid for any installation, although at the higher frequencies — 10 to 20 Mc and above — the economic factor is usually not of sufficient importance to warrant as exhaustive an analysis as this method provides. The fundamental equations may also be applied to problems involving rhombic antennas, though calculation of ground currents will be modified because of the antenna shape. In applications of the design theory to installations over dielectric earth (frozen earth behaves as a nearly perfect dielectric), the analysis is greatly simplified because of the simplification of the expression for the admittance of the soil.

The design of a radial-conductor ground system is analogous to determination of optimum conductor size by Kelvin's Law.¹ The method offered here is more involved, but provides a precise and definitive procedure for the design of an optimum ground system for any antenna installation, provided the soil conductivity is accurately known. Both calculations and measurements indicate that the use of such properly designed radial conductor ground systems should lead to antenna radiation efficiencies of 50 per cent or more compared to the 25 per cent or less now tolerated.

In the following text the step-by-step analytical procedures are explained, and a typical problem is solved in detail. Appendix I is a derivation of the inductance of a single conductor in the system, a factor required in the analysis. Appendix II is a sample calculation of ground current density in the vicinity of the antenna. Appendix III is an analysis of losses due to currents in the soil transverse to the radial system. Appendix IV describes measurements of absorption of radiation at grazing incidence by a lossy medium under a conducting grid.

CONFIDENTIAL 3

CONFIDENTIAL

theory of design

Optimum design of the ground system, when the design of the antenna and the transmitter is fixed, is accomplished when the total annual expense of such a ground system is at a minimum. The annual expense includes amortization cost of copper and its installation, plus the cost of the power dissipated in the ground. Optimum design, therefore, requires investigation of proper spacing of conductors in any area over which the total ground system current density remains reasonably independent of the ground conductivity.

Copper costs are determined by wire diameter and conductor spacing. Wire diameter is dictated by practical considerations. Assume a wire of radius a , costing, when placed a safe distance under the surface, D dollars per meter. Then the cost per square meter is D/d where d is the space between wires measured in meters, and the cost per square meter per year becomes D/nd where n is the number of years over which the cost is to be amortized.

The second factor in the annual expense is the cost of the power dissipated in the ground. This cost is determined by adding the amortization cost of the station, exclusive of the ground system, to the annual operating costs of the station, dividing the sum by the mean antenna input power in watts, and multiplying by the watts dissipated per square meter, w , with a conductor spacing d . This cost term obviously increases with increased spacing of conductors. If this cost is designated by $P(d)$, the total annual cost per meter, T , to be minimized by varying d thus becomes

$$T = \frac{D}{nd} + P(d), \quad (1)$$

in which

$$P(d) = \frac{C_a + C_b}{W} w, \quad (2)$$

where C_a is the annual amortization cost of the station exclusive of the ground system,

C_b is the annual operating cost of the station, and

W is the average input power to the antenna in watts.

The power dissipated per square meter of ground surface is given by the real part of $|J|^2 Z$ in which J is the total current flowing across a square meter of area and Z is the impedance of a square meter. Since the current divides between the embedded conductors and the surrounding soil, it is easier to work with admittances. Thus the loss, w , is expressed as the real part of $|J|^2/Y$ in which Y is the sum of Y_s , the characteristic admittance of the soil, and Y_g , the admittance of the grid of wires on the surface of the meter square, which will be almost purely susceptive.

The characteristic admittance of the soil, Y_s , is given by²

$$Y_s = \sqrt{\frac{\epsilon - j\sigma}{\mu \omega}}$$

In the notation employed,

μ is the permeability of the soil,

σ is the conductivity of the soil,

ϵ is the dielectric constant of the soil,

ω is 2π times the frequency, f , and

j is equal to $\sqrt{-1}$.

CONFIDENTIAL

In the case of a VLF installation, any reasonable soil conductivity makes $\frac{\sigma}{\omega} \gg \epsilon$ and the admittance becomes

$$Y_e \approx \sqrt{\frac{-j\sigma}{\omega\mu}} = \frac{1-j}{\sqrt{2}} \sqrt{\frac{\sigma}{\omega\mu}} = (1-j) \sqrt{\frac{\sigma}{4\pi f\mu}} \text{ mhos.}$$

The grid susceptance to uniformly distributed excitation will be designated Y_g , and is a function of the spacing, d , and radius, a , of the conductors.

It is shown analytically in Appendix I and confirmed experimentally as described in Appendix IV that for $d \ll$ one wavelength, λ , the admittance of soil covered by a grid assumes a simple form. Using the MKS (Meter-Kilogram-Second) units herein employed, it becomes

$$Y = Y_e + Y_g,$$

and further, as is shown in Appendix I,

$$Y_g = -j \frac{1}{f\mu d \log_e \frac{d}{2\pi a}} \text{ mhos.}$$

Thus the total admittance of the soil and grid becomes

$$Y = Y_e + Y_g \approx (1-j) \sqrt{\frac{\sigma}{4\pi f\mu}} - \frac{j}{f\mu d \log_e \frac{d}{2\pi a}} \text{ mhos.}$$

Recalling that the power dissipated per square meter, w watts, is equal to the real part of $|J|^2/Y$ we obtain

$$w = \frac{|J|^2 \sqrt{\frac{\sigma}{4\pi f\mu}}}{\left[\frac{\sigma}{4\pi f\mu} + \left(\sqrt{\frac{\sigma}{4\pi f\mu}} + \frac{1}{f\mu d \log_e \frac{d}{2\pi a}} \right)^2 \right]} \text{ watts/meter}^2. \quad (3)$$

This analytical expression relating watts lost per square meter and the grid spacing, d , permits expression of annual cost of power lost, $P(d)$, and finally the total annual cost associated with one square meter of ground surface, in terms of the grid spacing, d .

The easiest way to determine d for a minimum value of T is by substituting successive values of d in equation (1) and plotting the curve. Unfortunately, in any otherwise completely specified case, the total ground current, J , is a function of radial distance so it becomes necessary to determine the optimum value of d as a continuous function of J . In some cases the total radial current per meter, J , may be measured, in other cases calculated. In any event, if the soil behaves essentially as a conductor and the tilt of the wave front is small, J is practically equal to H_ϕ , the magnetic field at the ground surface.

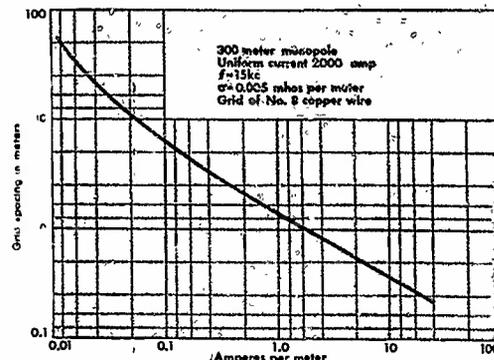
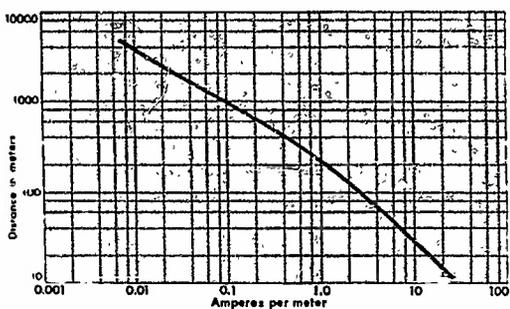
Thus, for preliminary design it becomes necessary to determine H_ϕ at a representative number of points on the ground in the near zone of the antenna. (Near zone may be roughly defined as an area around the antenna base of radius approximating antenna height or 1/6-wavelength, whichever is smaller.) This may amount to a sizeable project in the case of a flat-top antenna suspended over uneven ground. In order to clarify the steps involved in the foregoing analysis, a sample problem is solved below.

CONFIDENTIAL 5

CONFIDENTIAL

Fig. 1. Radial ground current density versus radial distance from antenna base

Fig. 2 (right) Optimum grid spacing versus ground current in amperes.



application of theory to a specific installation

Assume a 1000-kw station feeding a uniform current of 2000 amp throughout a vertical radiator 300 meters high operating at 15 kc. Soil conductivity is 0.005 mhos per meter; soil relative permeability is 1; and absolute permeability is $4\pi \times 10^{-7}$ henrys per meter. Practical considerations dictate that the ground radials be No. 8 copper wire, buried from 1 inch to 1 foot below the surface of the soil. Cost estimates place material and installation expenses at about \$0.10 per linear meter. For reasonable expected life, an annual amortization cost, D/n , of \$0.01 per meter per year seems appropriate.

The station is estimated to cost four million dollars exclusive of the ground system which leads to an amortization cost, C_a , of \$400,000 per year. Operating costs, C_b , are estimated at \$100,000 per year. The average power output, W , is 500 kw. Thus, the annual cost due to w watts per square meter being lost into the ground becomes

$$P(d) = \frac{\$500,000}{500,000} w = w \text{ dollars per year.}$$

Equation (1) thus reduces to $T = \frac{0.01}{d} + w(d)$ dollars per year. Values of d are to be determined such that T is at a minimum at any radial distance, r , from the base of the vertical section. This means that J or H_ϕ must be calculated versus radial distance, r . Figure 1, showing radial ground current density versus radial distance from the base of the antenna, is the result of such calculations for the above case. Details are included in Appendix II.

The real part of the characteristic admittance of the soil will hereafter be designated G , given by

$$G = \sqrt{\frac{\sigma}{4\pi f\mu}},$$

and similarly, for the grid, let

$$jB = Y_g \approx \frac{-j}{f\mu d \log_e \frac{d}{2\pi a}}.$$

Employing this notation, we have for equation (3)

$$w = \frac{|J|^2 G}{G^2 + (G + B)^2} = |J|^2 U(d),$$

where $U(d) = \frac{G}{G^2 + (G + B)^2}$ is the characteristic resistance per square meter of the composite ground. Soil constants given above make

$$G = \sqrt{\frac{0.005}{4\pi \times 15 \times 10^3 \times 1.257 \times 10^{-6}}} = 0.145 \text{ mhos,}$$

$$f\mu = 15 \times 10^3 \times 1.257 \times 10^{-6} = 0.0189,$$

$$a = 0.064 \text{ inches} = 0.001625 \text{ meters, and}$$

$$2\pi a = 0.0102 \text{ meters.}$$

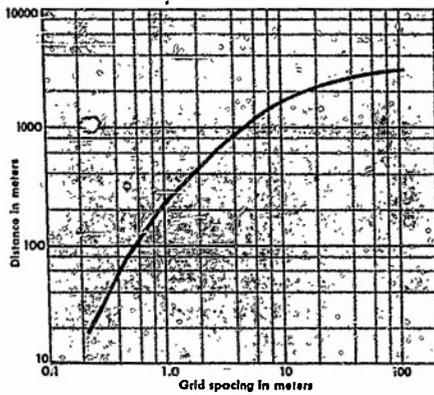


Fig. 3a. Optimum grid spacing versus radial distance from antenna base.

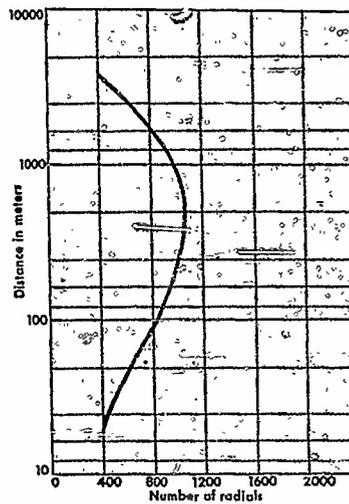


Fig. 3b. Total number of conductors versus radial distance from antenna.

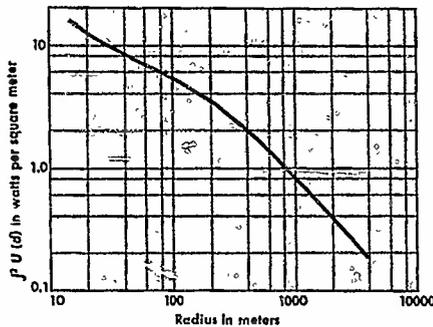
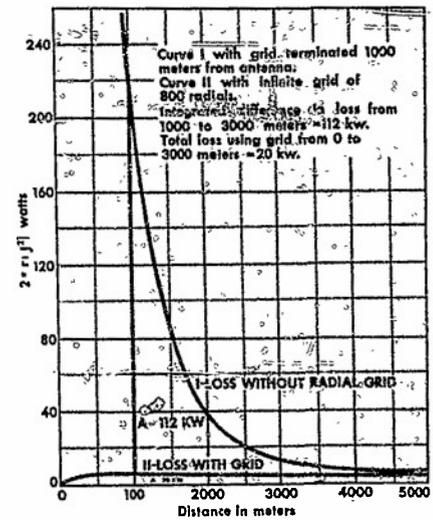


Fig. 4a. Power loss per square meter versus radial distance from antenna.

Fig. 4b. Power loss in an annular ring 1 meter wide versus radial distance from antenna.



Values of $U(d)$ are tabulated in table 2.

Substituting the calculated values of $J(r)$ from figure 1 into equation (3) and letting d take excursions within a reasonable range leads to the values of $|J^2|U(d)$ of table 3.

It has already been shown that

$$P(d) = \$w \text{ per year per square meter, and}$$

$$D/n = \$0.01 \text{ per year per meter.}$$

Equation (1) now becomes,

$$T = \frac{0.01}{d} + |J^2|U(d).$$

Table 4 gives values of T versus distance, r , and spacing, d . Minimum values of T are underlined, and corresponding values of d versus J provide points for the curve of figure 2. Values of $U(d)$ corresponding to minimum values of T have also been underlined. Figure 3a shows radial distance versus optimum grid spacing, and figure 3b shows radial distance versus total number of radials.

It is of considerable interest to plot the underlined values of $|J^2|U(d)$ and r of table 3 and, from the resulting curve shown in figure 4a, to calculate the total power dissipated in the soil. The loss in an annular ring of radius r becomes $2\pi r |J^2|U(d)$ and is shown in figure 4b.

CONFIDENTIAL

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Without a ground radial system, the loss per unit area becomes the real part of $w = |J|^2/Y_s$. Recalling that $Y_s \approx (1 - j)\sqrt{\frac{\sigma}{4\pi f\mu}}$, the loss per unit area becomes the real part of $|J|^2/(0.14 - j0.14)$ or $0.14|J|^2/(0.02 + 0.02) = 3.5|J|^2$. The loss in an annular ring 1 meter wide is $2\pi \times 3.5|J|^2r$ or $22|J|^2r$.

Values of $22|J|^2r$ for r greater than 1000 meters are plotted in figure 4b, and the loss in power due to termination of the grid at 1000 meters is determined from the area under the curve. In this specific installation, the extension of the grid to 3000 meters would accomplish an annual saving of 112 kw. The cost of this power, at \$1 per watt per year, is \$112,000 annually. Capitalized at 10 per cent, the saving in power would justify an expenditure of \$1,120,000. The actual cost of the extension, however, would approximate only \$160,000.

There is an additional loss due to currents to the ground surface from the radial grid wires. This current is the charging current which supplies a uniformly distributed charge to the ground surface. In Appendix III it is shown that this loss factor is inconsequential if an optimum radial grid system is employed.

conclusions

It has been shown that a properly designed ground system for the VLF antenna considered above can lead to ground loss of only 2 per cent of the transmission output. The copper loss in the antenna and feeder is of about this same order of magnitude. The radiation resistance of an installation such as the one analyzed above usually lies between 0.1 and 0.2 ohms. Insulator losses can be neglected. Loading coil loss approximates 0.1 ohm.

The following is a summary of these losses for the installation:

Ground loss	0.01 ohm
Radiation resistance	0.10 ohm
Copper loss	0.01 ohm
Efficiency without load coil	83 per cent
Load coil loss	0.1 ohm
Efficiency with load coil	45 per cent

These values of efficiency are far above measured values for installations of similar VLF transmitting antennas which do not have ground systems of optimum design. A lower value of radiation resistance results from calculations based on remote field strength measurements than from calculations based on geometric shape of the antenna. A shielding factor is usually introduced to compensate for this difference. However, for the installation described above, extending the ground system the full 3000-meter distance prescribed in the foregoing analysis should largely eliminate such discrepancy. In directions over which effective coverage is unimportant, the system of conductors should probably be reduced somewhat in length.

Finally, some idea of the initial cost of such a ground system is probably of interest. Using the unit figures given earlier of \$0.10 per meter for buried No. 8 copper wire, the total cost for 800 radials 3000 meters long would be about \$240,000. The cost of the copper would account for half the total. Such a system, as has been demonstrated, permits operation of the station at an efficiency of 45 per cent. If a station not having this type of ground system but similar in all other respects were operated at a typical efficiency of only 25 per cent, the loss in power would approximate \$300,000 per year under the same operating conditions. Thus it appears that first-year savings could pay for the cost of installing such a ground system under the conditions established for the station described in this report.

appendix I. calculation of the inductance of a wire grid³

Assume an infinite grid of perfectly conducting wires. Let E equal the rms longitudinal impressed electric field at the surface of a conductor.

Let d equal the spacing between wires (assumed $\ll \lambda$),

a equal the radius of the wires (assumed $\ll d$), and

λ equal the wavelength of radiation.

On the surface of each wire a field, E , is produced by the current in the wires.

Let J_z be the surface current density on a conductor.

Then I equals the total current in any wire (equal to $2\pi a J_z$). Further, let

μ equal the permeability of the medium,

ϵ equal the dielectric constant of the medium,

ω equal 2π times the frequency, f ,

γ equal the propagation constant of the medium $= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$,

A equal the magnetic vector potential due to a total current, I , per wire, and

j equal $\sqrt{-1}$.

The inductance per unit length of a wire is defined as

$$L = \frac{E}{\omega I} = \frac{\text{Longitudinal field}}{2\pi (\text{frequency}) (\text{current per wire})}$$

In MKS units, $E = j\omega\mu A$, and

$$A = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{2\pi} J_z \frac{e^{-\gamma R}}{4\pi R} dz a d\phi,$$

the summation being carried over all the wires.

We are concerned only with the value of E on the surface of a conductor.

The value of r for the wire considered becomes, as shown in figure 5,

$$R_1 \approx \sqrt{z^2 + (2a \sin \frac{1}{2}\phi)^2}, \text{ and}$$

$$I = \int_0^{2\pi} J_z a d\phi.$$

For all other wires,

$$R_n = \sqrt{z^2 + n^2 d^2},$$

and the current,

$$I = \int_0^{2\pi} J_z a d\phi,$$

is considered to flow along the axis of the conductor. For simplicity, let $\psi = \frac{\phi}{2}$

and we have for E ,

$$E = \frac{j\omega\mu}{4\pi} \left[\int_{-\infty}^{\infty} \int_0^{\pi} \frac{I e^{-\gamma(x^2 + 4a^2 \sin^2 \psi)^{\frac{1}{2}}}}{\pi(x^2 + 4a^2 \sin^2 \psi)^{\frac{1}{2}}} d\psi dz + 2 \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{I e^{-\gamma(x^2 + n^2 d^2)^{\frac{1}{2}}}}{(x^2 + n^2 d^2)^{\frac{1}{2}}} dz \right]. \quad (4)$$

Consider the expression

$$F = \int_{-\infty}^{\infty} \frac{e^{-\gamma(x^2 + \eta^2)^{\frac{1}{2}}}}{(x^2 + \eta^2)^{\frac{1}{2}}} dz.$$

Let $\sqrt{(x^2/\eta^2) + 1} = \cosh z$.

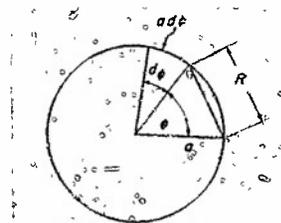


Fig. 5. Resistance of a wire in terms of wire radius.

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Then $z/\eta = \sinh t$, and $dz = \eta \cosh t dt$. For $z = \infty$, $t = \infty$, and the integral becomes

$$F = \int_{-\infty}^{\infty} e^{-\gamma\eta \cosh t} dt.$$

As shown by Watson,⁴ this is equal to the Hankel function, $j\pi H_0^{(1)}(j\gamma\eta)$. Thus we have

$$E = \frac{j\omega\mu}{4\pi} I \left[\int_0^\pi jH_0^{(1)}(2j\gamma a \sin\psi) d\psi + j2\pi \sum_{n=1}^{\infty} H_0^{(1)}(jn\gamma d) \right]. \quad (5)$$

To evaluate the first term of equation (5), we recall that $H_0^{(1)}(z) = J_0(z) + jN_0(z)$, where J_0 is a Bessel function of zero order and N_0 is a Weber function of zero order. Since we are concerned with cases where $|j\gamma a| \ll 1$, we may write $J_0(z) \approx 1$, and $N_0(z) \approx (2/\pi)(\log_e z + C - \log_e 2)$, where C is Euler's constant, 0.5772. Let $C = \log_e k$. Then

$$N_0(z) \approx \frac{2}{\pi} \log_e \frac{kz}{2}.$$

We have for the first term in (5),

$$\begin{aligned} j \int_0^\pi H_0^{(1)}(2j\gamma a \sin\psi) d\psi &= j \int_0^\pi \left[1 + \frac{2j}{\pi} \log_e(j\gamma k a \sin\psi) \right] d\psi \\ &= j \left[\pi + 2j \log_e j\gamma k a - 2j \log_e 2 \right] = j\pi - 2 \log_e \frac{j\gamma k a}{2}. \end{aligned} \quad (6)$$

To evaluate the second term in equation (5) we again express $H_0^{(1)}$ in parts as $H_0^{(1)}(z) = J_0(z) + jN_0(z)$. It has been shown by von Ignatowsky^{5,6} that for $d \ll \lambda$

$$\begin{aligned} \sum_{n=1}^{\infty} J_0(jn\gamma d) &= \frac{-j}{\gamma d} - \frac{1}{2} \text{ and,} \\ \sum_{n=1}^{\infty} N_0(jn\gamma d) &= \frac{-1}{\pi} \log_e \frac{j\gamma k d}{4\pi} \text{ so that,} \\ 2j\pi \sum_{n=1}^{\infty} H_0^{(1)}(jn\gamma d) &= 2j\pi \left[\left(\frac{-j}{\gamma d} - \frac{1}{2} \right) - \frac{j}{\pi} \log_e \frac{j\gamma k d}{4\pi} \right]. \end{aligned} \quad (7)$$

Substituting (6) and (7) in (5), we obtain

$$\begin{aligned} E &= \frac{\omega\mu I}{2} \left(\frac{j}{\gamma d} + j \frac{2}{2\pi} \log_e \frac{d}{2\pi a} \right), \text{ or} \\ E &= J \left(-\frac{1}{2} \eta_0 + jf\mu d \log_e \frac{d}{2\pi a} \right), \end{aligned} \quad (8)$$

in which $J = I/d$ is the current in a strip of unit width,

$\eta_0 = -j\omega\mu/\gamma$ is the characteristic impedance of the media, and

$f = \omega/2\pi$ is the excitation frequency.

The first term, $-\frac{j\eta_0}{2d}$, leads to the energy absorbed by the space on either side of the grid when the excitation field, E , is applied to the grid alone. The admittance, Y , of a unit area of grid and surrounding media in parallel becomes the sum of their individual admittances. Thus

$$Y = \frac{1}{\eta} - j \frac{1}{f\mu d \log_e \frac{d}{2\pi a}} = Y_s - j \frac{1}{f\mu d \log_e \frac{d}{2\pi a}}$$

in which Y_s is the soil admittance.

appendix II. calculation of ground radial current for 300-meter monopole with uniform current $I_0 = 2000$ amperes

Let A_z represent the vertical and sole component of magnetic vector potential due to a uniform current, I_0 , in a vertical monopole of height $z = b$. Then if r is the horizontal distance from the base of the monopole to a point under consideration, and all the other symbols are as defined in Appendix I, we have

$$A_z = 2 \int_0^b \frac{I_0 e^{-\gamma R}}{4\pi R} dz,$$

where

$$R = \sqrt{r^2 + z^2},$$

$$A_z = 2 \int_0^b \frac{I_0 e^{-\gamma(r^2 + z^2)^{\frac{1}{2}}}}{4\pi(r^2 + z^2)^{\frac{1}{2}}} dz, \text{ and,}$$

$$H_\phi = \frac{\partial A_z}{\partial r} = \frac{I_0}{2\pi} \int_0^b e^{-\gamma(r^2 + z^2)^{\frac{1}{2}}} \left[\frac{-\gamma r}{r^2 + z^2} - \frac{r}{(r^2 + z^2)^{\frac{3}{2}}} \right] dz.$$

Since $z < b \ll \lambda$, the phase factor, $e^{-\gamma(r^2 + z^2)^{\frac{1}{2}}}$, is given to sufficient accuracy by $e^{-\gamma r}$ so that

$$H_\phi = \frac{I_0 e^{-\gamma r}}{2\pi} \left[-\gamma \tan^{-1}(z/r) - \frac{rz}{r^2(r^2 + z^2)^{\frac{3}{2}}} \right]_0^b,$$

and since $J = H_\phi$ at the ground,

$$J = \frac{I_0}{2\pi} \left(-\gamma \tan^{-1}(b/r) - \frac{b}{r(r^2 + b^2)^{\frac{3}{2}}} \right) = \frac{I_0}{2\pi} (jU - V),$$

where $2\pi rJ$ is the magnitude of the total radial ground current at a distance, r , from the antenna. Therefore

$$\frac{I_0}{2\pi} = 319; \quad j\gamma = 3.14 \times 10^{-4}.$$

Values of $J = f(r)$ are given in table 5.

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appendix III. calculations of power lost due to transverse currents from radial conductors to intervening ground surface

The radial ground charging current decreases rapidly with distance from the antenna so that leakage currents flow transverse to the radial conductors. The charging current per unit area is given by

$$K_z = \frac{1}{2\pi r} \cdot \frac{d}{dr} (2\pi r J_r) = \frac{J_r}{r} + \frac{dJ_r}{dr}$$

Values of J_r/r are calculated from the curve of figure 1, and corresponding values of dJ_r/dr are obtained by differentiating the same curve. Figure 6 presents the resulting curve of J_r versus r . The following analysis was undertaken to determine the power loss due to this current.

Consider first the loss in a thin, truncated, triangular conductor one meter wide carrying a current, I , from a surface at O to one at B separated a distance, s , as in figure 7. Let the depth be unity and the area at B be A . Let a_0 be the area at O . Complete the triangle with a cap of height, b . Then $a_0/b = A/(s+b)$ and $a_0(s+b) = bA$ so that $b = a_0s/(A - a_0)$. The area, a_x , at X is related to the area at B by $a_x/A = (b+x)/(b+s)$, or

$$\frac{a_x}{A} = \frac{\frac{a_0s}{A - a_0} + x}{\frac{a_0s}{A - a_0} + s} = \frac{a_0s + x(A - a_0)}{As}$$

The power loss, W , is given by:

$$W = \int_0^s \frac{I^2}{a_x \sigma} dx = \int_0^s \frac{I^2 s}{\sigma [a_0s + (A - a_0)x]} dx.$$

Further simplifying,

$$W = \frac{I^2 s}{\sigma(A - a_0)} \log_e [a_0s + (A - a_0)x]_0^s = \frac{I^2 s}{\sigma(A - a_0)} \log_e \frac{As}{a_0s} = \frac{I^2 s}{\sigma(A - a_0)} \log_e \frac{A}{a_0}$$

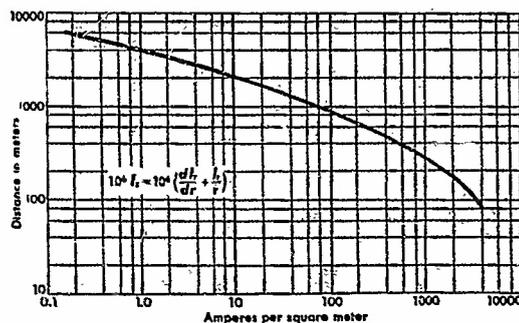


Fig. 6. Ground-charging current density versus radial distance from base of antenna.

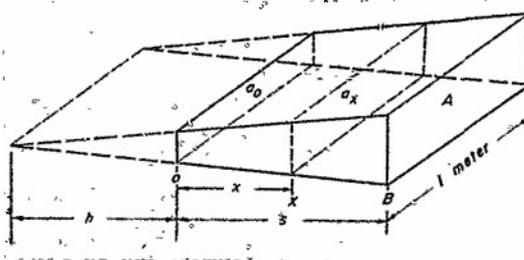


Fig. 7. Diagrammatic presentation of factors affecting loss in a thin, truncated, triangular conductor.

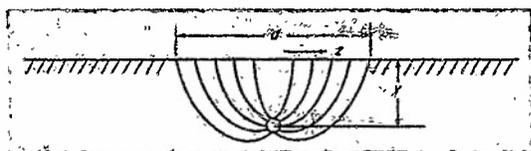


Fig. 8. Current density at earth surface as supplied by a buried conductor.

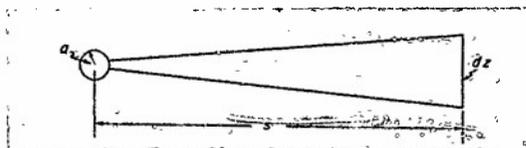


Fig. 9. Tapered-tube explanation of current density.

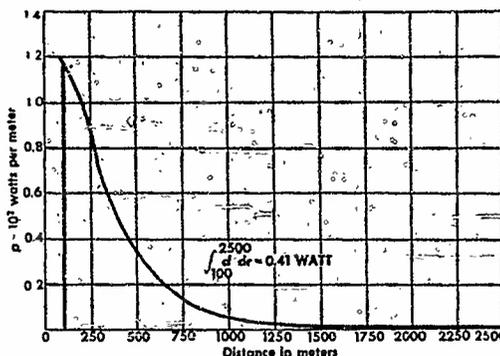


Fig. 10. Power loss due to leakage currents from a radial conductor to earth surface.

Now consider a conductor y meters below the surface and one meter long supplying a uniform current to the surface between $z = -d/2$ and $z = d/2$ as in figure 8. The length, s , of each imaginary tapered conducting tube is approximately $y + z$. The wire radius is a . The ratio A/a_0 for all tubes is $d/2\pi a$. The entire loss from $-d/2$ to $d/2$ now becomes

$$p = 2 \int_0^{d/2} W dz.$$

i , the current in any tube, is related to K_z , the current density, by $i = K_z dz$, or $i^2 = K_z^2 dz^2$ as in figure 9. Since A has now become dz ,

$$p = 2 \int_0^{d/2} \frac{K_z^2 \left(\log_e \frac{d}{2\pi a} \right) (y + z)}{\sigma \left(1 - \frac{2a}{d} \right)} dz = 2 \frac{K_z^2 \left(\log_e \frac{d}{2\pi a} \right)}{\sigma \left(1 - \frac{2a}{d} \right)} \left(y \frac{d}{2} + \frac{d^2}{8} \right).$$

This represents the loss per linear meter for each radial. Values of p versus distance are shown in figure 10. The radius, a , of No. 8 wire is 0.001625 meters. Let y be 0.1 meters. The factor, $2\pi a/d$, in the denominator in the expression given above for p can be neglected compared with unity. Using values of K_z versus r from figure 6, a curve of p versus r is derived and is shown in figure 10. The area under the curve between $r = 100$ meters and $r = 3000$ meters was planimeted. The power loss due to supplying the charging current from the radials using 800 radials becomes

$$W_r = 800 \int_{100}^{3000} p dr \text{ watts.}$$

This was found to be 0.4 watts per radial and 320 watts total for the 800 radials. This is an insignificant loss.

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appendix IV. measurement of absorption of radiation at grazing incidence by a lossy medium under a conducting grid

Although the expression derived in Appendix I for the inductive reactance of a grid is generally accepted, question was raised as to its applicability in the case of radiation incident at a grazing angle when the grid was on a surface dividing two media of widely different electromagnetic properties.

In order to remove all doubt, a slotted copper wave guide was filled with sea water except for the top section which was excited with the E field normal to the water surface. The air space above the water had a cross section 4.5 inches by 9 inches with the water surface providing the bottom 9-inch face. The excitation frequency was 1000 Mc. The attenuation was determined by insertion of a probe in the slot. In such an arrangement the absorption of energy by the copper walls becomes negligible. The expression for the attenuation as a function of the real part of the characteristic impedance of an absorbing wall is well known.⁷ From the observed attenuation of 4.8 db per meter, the conductivity of the water was determined to be about 4 mhos per meter. The experiment was repeated with copper wire grids covering the water surface. The attenuation was not measurably altered until the grid spacing was less than a wavelength in the water. The wire diameter was 0.027 inches.

Values tabulated in table 1 give the calculated and measured attenuation for each grid spacing employed. The real part of the characteristic impedance of the grid and water surface is the co-factor of I^2 in equation (3) of the text.

TABLE I.
GRID SPACING VERSUS ATTENUATION.

<i>Grid Spacing in inches</i>	<i>Attenuation in db per meter</i>	
	<i>Calculated</i>	<i>Observed</i>
infinite	4.8	5.0
1 1/2	4.8	4.9
1	3.6	3.4
3/4	2.8	2.4
3/8	1.0	1.1

These measurements preclude the possibility of serious error in the analysis. The measurements mentioned above are completely described in an article in preparation and will be available on request.

appendix V. glossary of symbols

<i>Symbol</i>	<i>Page of First Ref.</i>	<i>Definition</i>
<i>a</i>	4	Radius of wire used in grid.
<i>A</i>	9	Magnetic vector potential. Area at <i>B</i> in figure 7.
<i>B</i>	6	Susceptance of the grid in mhos.
<i>C_a</i>	4	Annual amortization cost of the station, exclusive of the ground system.
<i>C_b</i>	4	Annual operating cost of the station.
<i>d</i>	4	Grid spacing in meters.
<i>D</i>	4	Cost of installed grid in dollars per meter.
<i>E</i>	9	RMS longitudinal impressed electric field at the surface of a conductor in volts per meter.
<i>f</i>	5	Frequency of incident r-f energy in cycles per second.
<i>G</i>	6	Real part of characteristic admittance of the soil in mhos.
<i>H_φ</i>	5	Magnetic field at the ground surface in amperes per meter.
<i>I</i>	9	Total current flowing in a conductor in amperes.
<i>j</i>	4	$\sqrt{-1}$.
<i>J</i>	4	Surface current density in amperes per meter.
<i>J_z</i>	9	Surface current density on a conductor in amperes per meter.
<i>K</i>	12	Volume current density in amperes per square meter.
<i>log_e</i>	5	Natural log.
<i>L</i>	9	Inductance per meter of a wire = $\frac{E}{\omega I}$.
<i>n</i>	4	Proportionality factor.
<i>P(d)</i>	4	Cost of power dissipated in ground.
<i>r</i>	6	Radial distance from base of vertical section of radiating element in meters.
<i>T</i>	4	Total annual cost per square meter.
<i>U(d)</i>	6	Characteristic resistance of the composite ground.
<i>w</i>	4	Watts dissipated per square meter with conductor spacing, <i>d</i> .
<i>W</i>	4	Average input power to the antenna in watts.
<i>Y</i>	4	$Y_s + Y_g$.
<i>Y_g</i>	4	Admittance of the grid of wires on the surface in mhos.
<i>Y_s</i>	4	Characteristic admittance of the soil in mhos.
<i>Z</i>	4	Characteristic impedance of the ground surface in ohms.
<i>γ</i>	9	Propagation constant of the medium.
<i>ε</i>	4	Dielectric constant of soil in farads per meter.
<i>λ</i>	5	Wavelength of exciting r-f energy in meters.
<i>μ</i>	4	Permeability of the soil in henrys per meter.
<i>σ</i>	4	Conductivity of the soil in mhos.
<i>ω</i>	4	$2f$.

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TABLE 2.
REAL PART OF THE CHARACTERISTIC IMPEDANCE
OF LOSSY SOIL UNDER A CONDUCTING GRID

$$U(d) = \frac{0.145}{0.0211 + \left(0.145 + \frac{53/2.301}{d \log_{10} 98d}\right)^2} \text{ ohms}$$

1	2	3	4	5	6	7	8	9
Grid Spacing <i>d</i>	98 <i>d</i>	log ₁₀ 98 <i>d</i>	<i>d</i> log ₁₀ 98 <i>d</i>	$\frac{23}{\text{Col. 4}}$	0.145 + Col. 5	(Col. 6) ²	0.0211 + Col. 7	$\frac{0.145}{\text{Col. 8}}$
0.15	14.7	1.167	0.175	131.5	131.645	17300.	17300.0211	84 × 10 ⁻⁷
0.25	24.5	1.3982	0.347	66.3	66.445	4400.	4400.0211	33 × 10 ⁻⁶
0.33	32.4	1.5105	0.498	47.0	47.145	2225.	2225.0211	65.2 × 10 ⁻⁶
0.50	49.0	1.6902	0.846	27.2	27.345	748.	748.0211	19.4 × 10 ⁻⁵
0.67	65.5	1.8162	1.21	19.0	19.145	366.	366.0211	39.6 × 10 ⁻⁵
1.00	98.0	1.9912	1.99	11.54	11.685	136.	136.0211	0.001065
1.33	130.3	2.1149	2.81	8.19	8.335	69.2	69.2211	0.00210
2.00	196.0	2.2922	4.58	5.03	5.175	26.7	26.7211	0.00545
2.67	264.0	2.4216	6.39	3.60	3.745	14.05	14.0711	0.0103
4.0	392.0	2.5932	10.76	2.14	2.285	5.22	5.2411	0.0277
5.5	540.0	2.7324	15.0	1.53	1.685	2.845	2.8661	0.0615
8.0	785.0	2.8949	23.2	0.991	1.136	1.290	1.3111	0.1108
11.0	1080.0	3.0334	33.4	0.690	0.835	0.698	0.7191	0.201
16.0	1570.	3.1959	51.1	0.450	0.595	0.355	0.3761	0.385
22.0	2160.	3.3344	73.3	0.314	0.459	0.211	0.2321	0.624
32.0	3140.	3.4969	112.0	0.205	0.350	0.1225	0.1436	1.01
44.0	4320.	3.6355	159.5	0.1442	0.289	0.0835	0.1046	1.38
64.0	6280.	3.7980	243.0	0.0949	0.2399	0.0574	0.0785	1.85

TABLE 3. POWER LOSS IN WATTS PER SQUARE METER VERSUS DISTANCE FROM ANTENNA BASE

r, meters	1000 x P/U(d)																
	0.25	0.33	0.5	0.667	1.0	1.33	2.0	2.66	4.0	5.5	8.0	11.0	16.0	22.0	32.0	44.0	64.0
3000	0.000196																
2500	0.00034																
2050	0.000783																
1675	0.0015																
1400	0.00314																
1100	0.0064																
900	0.0125																
750	0.0256																
630	0.050																
520	0.107																
400	0.230																
330	0.490																
245	1.00																
190	1.85																
140	4.0																
105	7.85																
75	16.0																
55	31.4																
40	64.0																
26	125																
20	256																
14.5	501																

VALUES OF U(d) ASSOCIATED WITH MINIMUM VALUES OF T ARE UNDERLINED.

TABLE 4. TOTAL ANNUAL COST VERSUS CONDUCTOR SPACINGS $T = \frac{0.01}{d} + |P|U(d)$

Parameters	T	10 ⁴ /A	30.3	0.35	0.5	0.667	1.0	1.33	2.0	2.66	4.0	5.5	8.0	11.0	16.0	22.0	32.0	44.0	64.0	1000 T			
																				10 ⁴ /A	10 ⁴ /U(d)		
3800	0.01	0.0001																					
3100	0.014	0.000196																					
2500	0.02	0.0004																					
2050	0.028	0.000785																					
1675	0.04	0.0016																					
1400	0.056	0.00314																					
1100	0.075	0.00664																					
900	0.112	0.0135																					
750	0.16	0.0256																					
630	0.24	0.05																					
520	0.32	0.187																					
400	0.50	0.490																					
330	0.70	1.00																					
245	1.0	1.96																					
190	1.4	4.00																					
140	2.0	10.0																					
105	2.5	16.25																					
75	4.0	16.4																					
55	5.6	31.4																					
40	8.0	64.0																					
26	11.2	125.0																					
20	16.0	256.0																					
14.5	22.4	501.0																					

MINIMUM VALUES OF T ARE UNDERLINED.

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TABLE 5. VALUES OF $J = f(r)$

$$J = \frac{I_0}{2\pi} \left[-\gamma \tan^{-1}(b/r) - \frac{b}{r(r^2 - b^2)^{1/2}} \right] = \frac{i_0}{2\pi} (jU - V)$$

$$\frac{I_0}{2\pi} = 319 \quad \left(\frac{i_0}{2\pi} \right)^2 = 1.02 \times 10^5 \quad j\gamma = \pi \times 10^{-4}$$

r	b/r	$\tan^{-1}b/r$	$U^2 = -\gamma^2 (\tan^{-1}b/r)^2$	$r^2 + b^2$	$(b/r)^2$	V^2	$V^2 + U^2$	J^2	J
10	30.00	1.54	0.235×10^{-6}	9×10^4	900.0	1.0×10^{-2}	1.0×10^{-2}	1.0×10^3	31.60
20	15.00	1.50	0.224×10^{-6}	9×10^4	225.0	2.5×10^{-3}	2.5×10^{-3}	2.5×10^2	15.80
40	7.50	1.44	0.205×10^{-6}	916×10^2	56.0	6.1×10^{-4}	6.1×10^{-4}	6.2×10	7.90
80	3.75	1.31	0.171×10^{-6}	964×10^2	14.0	1.5×10^{-4}	1.5×10^{-4}	1.5×10	3.86
160	1.88	1.08	0.116×10^{-6}	1156×10^2	3.5	3.0×10^{-5}	3.0×10^{-5}	3.1	1.76
320	0.94	0.73	0.530×10^{-7}	1924×10^2	0.88	4.6×10^{-6}	4.6×10^{-6}	4.7×10^{-1}	0.66
640	0.47	0.44	0.190×10^{-7}	4996×10^2	0.22	4.4×10^{-7}	4.6×10^{-7}	4.7×10^{-2}	0.217
1280	0.24	0.23	0.053×10^{-7}	1730×10^3	0.055	3.1×10^{-8}	3.6×10^{-8}	3.7×10^{-3}	0.061
2560	0.12	0.12	0.183×10^{-8}	6650×10^3	0.014	2.1×10^{-9}	3.5×10^{-9}	3.6×10^{-4}	0.019
5120	0.06	0.06	0.344×10^{-9}	3633×10^4	0.0035	9.9×10^{-11}	4.4×10^{-10}	4.5×10^{-5}	0.0067

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20 CONFIDENTIAL