NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORTORY
NEW LONDON, CT 06320-5594

Technical Memorandum

A RELATIONSHIP BETWEEN OFF-SPECULAR SCATTERING AND TIME SPREAD PHENOMENA

Date: 17 September 1986

Prepared: E. Y. T. Kuo
Code 3322

E. P. Jensen
Code 3332

Approved for public release; distribution unlimited
A Relationship Between Off-Specular Scattering and Time Spread Phenomena

Acoustic signals received from near sea surface explosive detonations were observed to spread in time of arrival due to bottom interaction. The time spread extends considerably beyond the pulse width. In this study, off specular scattering, due to bottom roughness which is thought to be a possible mechanism causing the spreading, is investigated. The scattering theory of Marsh-Kuo is utilized in an attempt to predict the observed time spreading.
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A Relationship Between off-Specular Scattering and Time Spread Phenomena

1. Introduction

Acoustic signals received from near sea surface explosive detonations were observed to spread in time of arrival due to bottom interaction. The time spread extends considerably beyond the pulse width. In this study, off specular scattering due to bottom roughness which is thought to be a possible mechanism causing the spreading, is investigated. The scattering theory of Marsh-Kuo is utilized in an attempt to predict the observed time spreading.

2. Selected Geometry

For many experiments the source and receiver are approximately at the same distance \( H \) from the bottom. For this geometry, off-specular arrivals at the same time, \( t \), are reflected from an elliptical ring:

\[
\left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = 1 ,
\]

(1)

where

\[
a^2 = \frac{(ct/2)^2(t^2 - t_s^2)}{(t^2 - t_d^2)} ,
\]

(2)

\[
b^2 = \frac{(ct/2)^2(t^2 - t_s^2)}{t^2} ,
\]

(3)

\[t_s = \text{specular arrival time},\]

\[t_d = \text{direct arrival time},\]

\[C = \text{acoustic speed} .\]

3. Model Description

Utilizing a method of perturbation, Kuo \(^1\) generalized the scattering theory of Marsh \(^2\) and predicted reflected field characteristics from a rough transmitting bottom. To the second order of perturbation, the reflected field consists of specular and scattered components.

The specular component is a concentrated beam in a forward specular direction. Its energy level depends on Rayleigh's reflectivity and the amount of scattered energy lost in all other
directions. The scattered energy lost in all directions depends on the entire range of the bottom roughness spectrum.

The scattered component is diffused and its energy level in any direction is dependent on a particular part of the bottom roughness spectrum. Therefore, the scattering process in an off-specular direction is roughness wave number selective.

Given locations of the point source and receiver as indicated in Figure 1, each ray emerging from the source is assumed to be approximately plane at the scattering boundary. The first arrival after the direct arrival is the specular component which is reflected at the boundary point with coordinates \((0,0,0)\). The succeeding arrivals, at a particular time, are all off-specular components arriving simultaneously from a particular elliptical ring depicted in the same figure. Figure 1 indicates one such off-specular component arrival. To find the received level at a particular time, one must sum up all contributions from such arrivals along a particular elliptical ring.

Kuo's \(^1\) scattering strength, \(m''\), predicts off-specular components to be proportional to the product of the scattering function and the roughness spectral level at a selected wave number. The scattering function is independent of the roughness but is dependent on the incident/reflected wave and media properties. The selected wave number is the difference wave number between incident wave number \(k_1 = \frac{2\pi f}{c}(\alpha, \beta)\) and the reflected wave number \(k_r = \frac{2\pi f}{c}(\lambda, \mu)\) where \(f = \text{acoustic frequency}, \ (\alpha, \beta) = \text{incident wave direction cosines with respect to } (x, y), (\lambda, \mu) = \text{reflected wave direction cosines with respect to } (x, y)\). The magnitude of the selected wave number can be expressed in terms of geometrical quantities as given by the following expression.

\[
|k_1 - k_r| = k_1 \left[ \frac{x}{n} \left( \frac{1}{n^2} + \frac{1}{n^2} \right) + x_0 \left( \frac{1}{n^2} - \frac{1}{n^2} \right) \right]^2 + y^2 \left( \frac{1}{n^2} + \frac{1}{n^2} \right)^2
\]  

(4)

where 

\[
\eta^2 = (x + x_0)^2 + y^2 + H^2,
\]

\[
\eta_r^2 = (x_0 - x)^2 + y^2 + H^2,
\]

\[
k_1 = \frac{2\pi f}{c}
\]

\(x, y\): satisfies equation (1).

The selected wave number given by (4) will vary from zero at specular point \((0,0,0)\) to \(2k_1\) at \(\infty\). The latter selected wave number is also the selected wave number in the back-scattering direction. The extent of variation in the selected wave number along each ellipse was investigated. It was found that the magnitudes of the selected wave numbers along any ellipse vary as much as a factor of 2 within the area between the source and receiver. However, outside of that area variations were small.
Received level (RL) can be expressed in terms of the scattering strength level, $10 \log m''$, by the following expression:

$$RL = 10 \log m'' + SL - TL + 10 \log A$$  \hspace{1cm} (7)

where

$SL =$ source level,

$TL =$ total transmission loss level,

$A =$ insonified area.

For negligible water medium absorption loss, the total transmission loss becomes the double spherical spreading loss given by the following expression.

$$TL = 20 \log r_1 + 20 \log r_2$$  \hspace{1cm} (8)

The received level at a given time, $t$, is considered to be contributed to by a sonified elliptical ring area of one pulse width. The corresponding selected wave number contained in the scattering strength varies over this insonified ring area and the effect of each elementary area on $RL$ should be integrated over the entire elliptical ring. However, for this study, a much simpler approximate method described below is utilized.

4. Results for a Rigid Boundary

The mathematical model presented by Kuo\(^1\) in 1964, was intended for scattering from a non-shear supporting sedimentary layer of an infinite thickness. As such, the model can not really predict scattering from basement material. A model development for shear supporting sediment and basement layers is underway. One approximation that can be made at this time is to let the density of the sediment become very large in comparison with that of water. Effectively this causes the sediment to approach a rigid boundary and simulates solid basement material to some extent.

For this approximation, the scattering strength ($m''$) is simplified to the following expression:

$$m'' = \frac{2\kappa''}{\pi} \left(1 - \frac{\gamma_1 \gamma_2}{\kappa''} \right) \frac{E \left(1 \gamma_1 \gamma_2 \gamma_3 \right)}{\left|\gamma_1 - \gamma_3 \right|}$$  \hspace{1cm} (9)
where \( E \) represents a boundary roughness spectrum. In deriving the above expression, bottom roughness is assumed locally isotropic near the selected wave number region - on the order of 0.1 (meter\(^{-1}\)). Spofford/Holmes/Spiess show measurements indicating approximate isotropic roughness down to wave number of about 10\(^{-1}\) (meter\(^{-1}\)). For the following calculation, the power law spectrum, 
\[
E(k) = E_0 k^{-m}, \quad m = 0 \text{ to } 3,
\]
will be assumed. 

Because of geometrical symmetry and symmetry of \( \mathcal{R}_y \) and \( \mathcal{R}_x \) in equation (9), scattering strength varies only in one quadrant. Scattering strengths at three locations in one quadrant are calculated and averaged. The averaged value is then used in equation (7) as applicable to the entire elliptical ring area. Source level is assumed zero dB.

Figure 2 depicts the time spread in received levels (propagation loss) given by off-specular scattering. Signal frequency and pulse length are 500 Hz and 20 ms, respectively. The specular arrival time, is calculated to be 13.849 seconds. The first off-specular arrival level is calculated at 30 ms after the specular arrival. The succeeding off-specular levels are calculated at 20 ms intervals. Each curve represents the result of using a different power law for the roughness spectrum.

In the same figure, the received specular path loss, based only on the spherical spreading, is 86 dB.

Figure 2 predicts a higher received off-specular level and a faster decay rate for a larger spectral power law. Over the range of arrival time, the selected wave numbers are in the range of about 0.09 to 0.3 (meter\(^{-1}\)), i.e. magnitudes of less than one. The selected spectral shape 
\[
E(k) = E_0 k^{-m},
\]
indicates that the case of higher power law assumes a higher spectral level due to the magnitude of \( k \) being less than one. Therefore, the greater received level for a larger \( m \) value is due to the higher spectral level used. For the same spectral level, \( E_0 \) should be adjusted for different \( m \). However, the higher decay rate for larger \( m \) is physically caused by the faster drop in the roughness spectral level as the arrival time increases which increases the magnitude of the selected wave number.

According to measurements of Spofford et al\(^3\), the value of \( m \) is about 3. For this power law, the off-specular received level is found to be independent of frequency. For \( m = 2 \), frequency dependence is depicted in Figure 3.

5. **Results for Sedimentary Boundary**

Time spread caused by off-specular scattering from a transmitting rough boundary is investigated here. For this case, the full scattering strength equation given by Kuo\(^1\) is utilized. The general equation is such that an average over two quadrants is needed. Otherwise computations are similar to the rigid
boundary case.

Figure 4 presents received levels from various kinds of bottom boundaries represented by various combinations of bottom to water density ratio $f$ and wave number ratio $k$. A sediment represented by $(f, k) = (1.7, 0.8)$ was previously used by Kuo because it presented the best fit to earlier experimental back-scattering data. This sediment type together with a basalt type bottom, gives received off-specular levels which are very close to those of the rigid boundary investigated previously. A sediment Soup type bottom (expected in the area far away from continental shelf) gives very low off-specular levels.

Figure 5 presents frequency dependence. Though time decay rate is higher in general, frequency dependence is similar to that of the rigid boundary case.

Figure 6 depicts dependence on the grazing angle. In all cases, the total transmission loss is geometrically adjusted to have the same value.

6. Concluding Remarks

The results presented here represent a first attempt to model the time spreading due to scattering using perturbation theory. Detailed comparison with experimental data is not performed. However, private communication with experimentalists do indicate that the predicted decay rate is representative for some cases. There are some experimental data which show a much slower decay rate. For these cases further studies are needed.

The possible causes of a much slower decay rate are many. A few possibilities are suggested below.

1) Multi-rough boundaries associated with multi-layers, can cause multi-off-specular returns and cause a slower decay rate compared to a single rough boundary.

2) The combination of boundary and volumetric scattering can also cause a slower decay.

3) A bottom material that supports shear can alter both the decay rate and the off-specular scattering levels considerably.
References


Figure 1. Scattering Geometry
FIG. 2 RECEIVED OFF-SPECULAR LEVEL (RIGID)
FIG. 3 RECEIVED OFF-SPECULAR LEVEL (RIGID)
FIG. 4 RECEIVED OFF-SPECULAR LEVEL (NON-RIGID)

\[ \eta = 3 \]
\[ f = 200 \text{ Hz} \]

\[ (f, \lambda) = (2.7, 2.7) \]
\[ (f, \lambda) = (4.7, 4.8) \]

(\( f, \lambda \)) = (1.3, 1.2)
FIG. 5
RECEIVED OFF-SPECULAR LEVEL (NON-RIGID)
FIG. 6. RECEIVED OFF-SPECULAR LEVEL (NON-RIGID)
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E. Y. T. Kuo
Mechanical Engineering and Acoustic Applications Division
E. P. Jensen
Environmental Acoustics Research and Applications Division
Surface Ship Sonar Department
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