Electron Kinetics as Applied to Pulsed Power Switching.
Alan Garscadden
Air Force Wright Aeronautical Laboratories
Wright-Patterson AFB, Ohio 45433

SUMMARY
This paper reports on calculations of the electron energy distribution occurring in gases and gas mixtures used in switching. For externally ionized high pressure discharges the optimization of the conductivity as a function of a gas mixture (usually a Ramsauer gas plus a molecular gas) and of E/N is examined. The power loading and pulse length change the vibrational populations of the molecular gas and may reduce the conductivity unless the vibrational relaxation is sufficiently rapid. For high repetition rates we also examine the influence of dissociation on the plasma conductivity and how the decay processes might be influenced by the degree of vibrational excitation.

At low pressures it is more appropriate to treat the switch plasma as a negative glow plasma. At low electron densities, this requires consideration of the ionization and excitation by the fast electrons from the cathode sheath. At high electron densities the electron distribution function tends to a Maxwellian however some electrons with energies greater than the injection energy appear due to velocity diffusion. These features must be included in the traditional treatment of thyatrons and other switches.

INTRODUCTION
In order to calculate the electron conductivity of a plasma switch it is necessary to measure or to be able to calculate the electron spatial and velocity distribution f(v,r,t). The electron density and current are then

\[ n_e(\vec{r}, t) = \int f(\vec{r}, v, t) \, dv \, d^3r \]

\[ \Gamma_e \left( \vec{r}, t \right) = \int v \, f(\vec{r}, v, t) \, dv \, d^3r \]

The electron drift velocity \( \vec{V}_e \) = \( \Gamma_e / n_e \). This velocity is generally small compared to the electron random velocity \( v \). The electron energy distribution function (EEDF) \( f(v,r,t) \) is described by the Boltzmann transport equation. This continuity equation in phase space has the form

\[ \frac{\partial f}{\partial t} + \nabla_v \cdot \vec{V} \cdot f + \nabla_r \cdot \vec{A} \cdot f = \frac{\partial f}{\partial t} \]

where the subscripts on the divergence operators denote the independent variable; \( v \) is the particle velocity and \( a \) is the particle acceleration. The collision term includes the effects of elastic collisions, inelastic collisions with atoms or molecules and electron-electron collisions. The collisions are usually treated as source terms, instantaneously transferring electrons from one volume of velocity space to another. The electron-electron collisions will produce a Maxwellian velocity distribution when the fractional ionization is sufficiently large (\( \pi n_0 \)).

Since the electron-electron Coulomb collision frequency varies roughly as \( n_0 \, v^2 \), these collisions are especially effective in the elastic regime where there are no severe energy loss processes, and they may have an influence at fractional ionization as low as \( 10^{-5} \) for noble gases.

The switch plasmas fall into two main categories: high pressure and low pressure. The approaches for calculating the electron kinetics of these discharges are significantly different. At high pressures the plasma can still be diffuse. This is made possible by external electron beam ionization. Otherwise an arc will form or indeed it may still develop if the pulse length is too long. Criteria for this to occur have been developed using streamer theory or ionization-thermal instability analyses. In the fully developed arc, equilibrium plasma theory can be applied. The spark gap is described by this approach and its principal limitation of low repetition rate is determined by the collisional coupling term between the atoms and the electrons. This coupling maintains conductivity even when the electric field is removed. This means that the electron density decays slowly unless the gas travels across the discharge gap very rapidly.

ELECTRON KINETICS IN THE HIGH PRESSURE DIFFUSE DISCHARGE
In high pressure diffuse discharges that may be suitable for on/off switching the EEDF is usually dominated by electron-neutral collisions. The collision term is then given by the Boltzmann collision integral and a knowledge of the larger elastic and inelastic collision processes is required. If the applied field is not too large (i.e. the energy gained between collisions is small compared to the mean electron energy), perturbation methods for solving the Boltzmann transport equation may be applied. The distribution function is normally expanded in spherical harmonics in velocity space. The first term represents the isotropic distribution and the higher order terms represent the anisotropy induced by the perturbing fields. The electron distribution function is then

\[ f(\vec{r}, t, \theta) = \sum_i f_i(\vec{r}, t) P_i(\theta) \]

\( P_i(\theta) \) is \( i \)th Legendre polynomial then

\[ n_e = \int f_0 \, d\nu = \int f_0 \, 4\pi \, \nu^2 \, d\nu \]

\[ \Gamma_e = \int \nu \, f_0 (\vec{r}, t, \vec{v}) \, d^3\nu = \int \nu \, f_0 (\vec{r}, t, \vec{v}) \, 4\pi \, \nu^2 \, d\nu \]

\[ \frac{n_e}{\Gamma_e} \]
Report Documentation Page

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE  
JUN 1983

2. REPORT TYPE  
N/A

3. DATES COVERED  
-

4. TITLE AND SUBTITLE  
Electron Kinetics as Applied to Pulsed Power Switching

5a. CONTRACT NUMBER  
-

5b. GRANT NUMBER  
-

5c. PROGRAM ELEMENT NUMBER  
-

5d. PROJECT NUMBER  
-

5e. TASK NUMBER  
-

5f. WORK UNIT NUMBER  
-

6. AUTHOR(S)  
-

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  
Air Force Wright Aeronautical Laboratories Wright-Patterson AFB, Ohio 45433

8. PERFORMING ORGANIZATION REPORT NUMBER  
-

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  
-

10. SPONSOR/MONITOR’S ACRONYM(S)  
-

11. SPONSOR/MONITOR’S REPORT NUMBER(S)  
-

12. DISTRIBUTION/AVAILABILITY STATEMENT  
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES  

14. ABSTRACT  
-

15. SUBJECT TERMS  
-

16. SECURITY CLASSIFICATION OF:  
a. REPORT  
unclassified

b. ABSTRACT  
unclassified

c. THIS PAGE  
unclassified

17. LIMITATION OF ABSTRACT

18. NUMBER OF PAGES  
7

19a. NAME OF RESPONSIBLE PERSON  
-

Standard Form 298 (Rev. 8-98)  
Prescribed by ANSI Std Z39-18
The Boltzmann equation may be rewritten in the form of a system of coupled equations
\[
\frac{d\mathcal{f}_0}{dt} + \frac{v}{3} \nabla \cdot \mathbf{f}_0 - \frac{e}{3m} v^2 \nabla (v^2 F \cdot \mathbf{f}_0) = \frac{2}{21r^2} \frac{d}{dv} \left[ v^3 \gamma_m (f_0 + \frac{kT_e}{m} \frac{d\mathcal{f}_0}{dv}) \right]
\]
where \( \gamma_m \) is the momentum loss collision frequency
\[
\gamma_m = \frac{N_0}{\sqrt{2\pi}} \int Q(v,\theta) (1 - \cos \theta) dv
\]
integrated over all solid angles and \( Q(v,\theta) \) is the differential cross-section for electron scattering.

If the applied electric field is not changing rapidly
\[
\frac{\mathcal{f}_0}{\mathcal{f}_1} = -\frac{1}{\gamma_m} \left( v \nabla n_e \mathcal{f}_0 - \frac{eE}{mv} n_e \frac{d\mathcal{f}_0}{dv} \right)
\]
Note that in the absence of electron density gradients,
\[
\frac{\mathcal{f}_0}{\mathcal{f}_1} \propto \frac{1}{\gamma_m} \frac{d\mathcal{f}_0}{dv}
\]
This remark is made here so as to interpret calculations discussed below. If the principal cross-sections are available then it is possible to derive the breakdown condition, the drift velocity and other transport properties. There is some improvement for high fields by going to a Monte Carlo simulation, or using higher order terms, however the spatially averaged Townsend coefficient is a reasonable approximation except in the cathode fall.

All of the above approaches have been cultivated very carefully and used very successfully by those interested in deriving data from swarm experiments. However for discharges of switching interest there are other effects that must be considered and that cause significant correction to the plasma conductivity. Some of these effects lead to some new effects, some of which might be exploitable in pulse power switching. The third difference is that in the discharge any diatomic gas will be vibrationally excited and, especially if the gas is homonuclear, the excited vibrational levels' populations can become a significant percentage of the total gas density. The vibrational populations have two influences: first they can change many of the electron kinetic rates. Some of the excitation or ionization thresholds are altered (usually lowered) and in some cases the rate constant is dramatically modified (e.g. dissociative attachment in H₂, D₂, and HCl). The second influence is that a vibrational population reacts back on the electron gas through collisions of the second kind. This means the electron energy distribution at a given E/N for a power loaded gas is quite different from that for present drift tube experiments. In principle some drift tube experiments in vibrationally excited gases are possible, but no one has attempted to exploit this in this way.

The Boltzmann transport equation calculations are in reasonable order for the pure rare gases, hydrogen, oxygen and several laser gases. Data for mercury is also available. An example for oxygen plasmas is shown in fig 1. Note that these distributions cannot be approximated by either a Maxwellian or a Drude type distribution. In hydrogen however, a Maxwellian distribution is a reasonable approximation at low electron densities and low power loading.

However these analyses assume that the discharge gases are not altered by the power loading. Under high power switch conditions this assumption often breaks down. If the original gas is molecular then it will dissociate with a time constant, Z₀, where
\[
Z_0 = \int n \cdot Q_d(n) f(n) dv
\]
At high powers this effect will tend to compensate the gas depletion caused by ionization. However it is necessary to allow for the fact that the species has changed. This becomes very important for the slower electrons as the effective vibrational cross-sections in a molecular gas are being reduced as dissociation proceeds. There are no equivalent loss processes in the same energy range until the excited state densities become very high. Our calculations for hydrogen indicate that the dissociation rate exceeds the ionization rate by approximately an order of magnitude over most of the E/N range. Therefore the dissociation will occur faster than the ionization growth and must be considered in most pulsed discharges.

Some examples of the effects of dissociation on the EEDF are shown in fig 2 (oxygen) and in fig 3 (hydrogen). Note that the rates for processes controlled by the fast electrons will be significantly altered. The excitation rates derived from the EEDF's for molecular hydrogen are shown in fig 4. At E/N values greater than 20 Td a large fraction of the discharge energy in hydrogen or in deuterium goes into vibrational excitation. This then can change the excitation, ionization and dissociation rates and the EEDF is altered by the effects mentioned above. Under some conditions it is believed the process can be runaway and lead to instability even earlier than thermal heating. Most dramatic, the dissociative attachment rate in hydrogen and deuterium is predicted to be changed orders of magnitude on vibrational excitation. The calculated results for hydrogen are shown in figure 5.

Generally, electron drift velocity experiments have concentrated on simple gases. However Cottrell and Walker in 1965 did measure the electron drift velocity in a large number of polyatomic gases. Some of these gases showed remarkably high drift velocities at low E/N...
values - just the ideal conditions for a switch in the on-condition. For low energy electrons, if we assume isotropic scattering, then on average, the electron motion in the field direction is reduced to zero in a collision. Then $W_D = Ee/2mN_qT$. 

\[ W_D = Ee/2mN_qT \]

where $\lambda$ = electron mean free path = $1/N_qT$ or

\[ W_D = Ee/2mN_qT \]

In steady state $v^2K^* = a/D$, where $K^*$ is the average fractional energy loss in a collision and $a$ is a constant of the order unity then

\[ W_D \propto \left[ \frac{Ee/2mN_qT}{a} \right]^{1/2} \]

While $K^*$ and $a$ (total cross-section) are functions of $E/N$, it is apparent that a high drift velocity is favored by a large fractional energy loss and a small total cross-section (Compare eqn. (9) in that a large drift velocity is favored by a large $\delta f/\delta t$ (caused by a large energy loss) and a small $g_m$, electron momentum transfer cross-section). Thus if a gas has a Ramsauer minimum it satisfies one of these conditions; normally the heavier rare gases do not have inelastic losses near the energy of the Ramsauer minimum and so do not have exceptionally high electron drift velocities. Gases like CH$_4$, C$_2$H$_6$, and SiH$_4$ appear to be Ramsauer gases and also have low energy vibrational modes that are excited by electronic collisions, and so would qualify. It is possible to synthesize a gas with desirable high mobility features by using a rare gas-molecular mixture. Experimental measurements have recently confirmed these models for nitrogen - argon and silane - argon mixtures.

In an actual discharge, if the vibrational manifold becomes saturated, the net energy loss is reduced as there is energy transfer back from the excited states. This will reduce the enhancement of the electron drift velocity. Therefore a triple mixture with a species to relax the vibrational states may be necessary if the energy loading itself does not produce sufficient dissociation to cause the relaxation. (This may partially account for some of the different conductivities reported for Ar N$_2$ mixtures by different experiments).

The EEDF in CO-Krypton mixtures is shown in Fig 7. This gas mixture demonstrates a remarkable drift velocity enhancement over either pure gas. The velocity space diffusion control of $W_D$ is shown by the curves at low E/N where the steep change in the isotropic distribution with energy is caused by the inelastic collisions with CO at energies near the krypton momentum transfer minimum. Thus there is correspondence between the numerical results and the analytical expression (eqn 9).

**ELECTRON KINETICS IN THE LOW PRESSURE DISCHARGE**

For switches such as thyratrons that operate on the low pressure side of the Paschen curve, the derivation of the electron kinetics is more complicated than in the high pressure case. Many of these discharges are characterized as negative glow rather than positive column discharges. Thus $E/N$ is not as unique a scaling parameter. In many of the discharges the EEDF is not in equilibrium with the local value of the electric field. Long and Segur et al. have treated the non-equilibrium regions of a weakly ionized gas. These analyses are very useful in describing the low-pressure cold-cathode discharge. (Actually the non-equilibrium will occur in the high pressure discharge, however the spatial extent is extremely small).

Conscious of the important of non-equilibrium effects, let us examine the hot cathode discharge which appears to be more controllable as a switch and has attracted renewed interest. A Boltzmann type analysis can be used for the case where the mean free path is no longer small compared to the discharge characteristic lengths. A very useful approach to this discharge was introduced by K. R. Emelius and applied by Leckey et al. Additional studies have been made by Aleskovskii.

The cathode sheath injection of fast electrons plays an important role in the thyratron discharge where the pressure is about 0.3 torr and the mean free path is about 0.3cm. Thus the distribution function of ionizing electrons is given as a fixed condition along the cathode boundary of the negative glow (NG). In this case the diffusion of fast electrons in the NG can be described in the $P_1$ approximation by the scalar flux equation

\[ I(\lambda, E) = \int_0^\infty \rho(\lambda, E, \Omega) d\Omega \]  

where $\rho$ is the unit velocity vector, $E$ is the electron energy, $r$ is the position vector and $f(r, E, \Omega)$ is the distribution function. The inelastic collisions are loss terms and the diffusion of electrons from the known boundary source is the gain term. The kinetic equation is then

\[ \frac{D_e}{\nabla^2}n_{e}(\text{fast}) - K n_{e}(\text{fast}) = 0 \]  

\[ D_e \nabla^2 n_{e}(\text{fast}) - K n_{e}(\text{fast}) = 0 \]  

where $D_e$ is the free electron diffusion coefficient

\[ D_e = \frac{N_0 q_r(E)}{E} \]

The condition that the cathode sheath voltage is not much greater than the ionization potential is usually satisfied in a hot cathode discharge. Then

\[ D_e \nabla^2 n_{e}(\text{fast}) - K n_{e}(\text{fast}) = 0 \]  

Where $V > 20$ was needed to describe the.
immediate cathode region. The contours of the fast electron density illustrated in Fig 8 are obtained from:

\[ n(r, \phi) = \sum_{v} \frac{2a_r n_{\infty} J_v (r, \phi)}{A_r [J_v (A_v)]^2} \left( 1 - \frac{3 \lambda}{3R} \right) \]

\[ \frac{\exp (-R^2) - \eta \exp (-2R^2)}{1 - \eta \exp (-2R^2)} \right) J_0 (r, \phi) \]

The spatial distributions of the ionization and excitation rates can then be calculated. The remainder of the discharge potential is distributed across the glow and its spatial distribution is determined by the discharge current continuity requirement. The low energy portion of the EEDF is assumed to be in equilibrium with these E/N values.

At higher current densities the fast electron energy loss will be dominated by Coulomb collisions. The Boltzmann equation then reduces to the Fokker - Planck equation. Hiskes and Karo\(^{14}\) obtain

\[ \frac{\partial f}{\partial E^2} (Dv) + \frac{\partial}{\partial E} (q \cdot f) = - \frac{J(E)}{\text{volume}} \]

where \( f \) is the EEDF for the slowing electrons, \( q \) is the electron drag rate and \( J \) is the injected current/unit energy. The electron drag is evaluated assuming that it is dominated by the near Maxwellian slow group. If the injected current is constant over a range of energy \( E_0 < E < E_F \)

\[ f = f_M + P \left( \frac{E}{E_F} \right)^{1/2} \beta_1 \]

\[ \beta_1 = 1, \quad E < E_0 \]

\[ \beta_2 = \left( \frac{E_F - E}{E_F - E_0} + (1 + \frac{kT}{E_F - E_0}) \right) \left( 1 - \frac{E - E_0}{E_F - E} \right) \]

\[ E_0 < E < E_F, \]

\[ \beta_3 = \left[ 1 + \frac{kT}{E_F - E_0} \right] \left[ 1 - \frac{(E_F - E_0)}{E_F - E} \right] \]

\[ \frac{E_F - E_0}{E_F - E} \]

\[ \beta_3 = \left[ 1 + \frac{kT}{E_F - E_0} \right] \left[ 1 - \frac{(E_F - E_0)}{E_F - E} \right] \]

\[ E > E_F \]

Hiskes and Karo have applied this analysis to the Berkeley 56kW hydrogen ion source. Fig 9 shows the calculated distribution for this low pressure hydrogen discharge.

Arguments have usually been made for a Maxwellian EEDF at an ionization fraction above 10\(^{-4}\). This holds for the body of the distribution but not for the high energy tail of the EEDF. While the deviation shown in Fig 9 is quite small, it is nevertheless very important because the ionization rate is controlled by those electrons with energies above the ionization energy.

Summarizing the points made above: to describe the kinetics of the low pressure discharge, it is necessary to consider the spatial decay of the cathodic beam at low currents and the departure from a Maxwellian even at high electron densities.

When the discharge is switched off, the ionization decays by diffusion and recombination. The singlet excited states in \( \text{H}_2 \) decay down to vibrationally excited levels of the \( \text{H}_2 \left( \chi \Sigma^+ \right) \) ground state. The vibrational levels\(^*\) appear to have a de-excitation coefficient of about 0.1 for wall collisions. The electron temperature decay is therefore much faster than the vibrational temperature decay. The electron energy equation for a non-flowing plasma after the electric field is removed, reduces to

\[ \frac{dE_e}{dt} = -k_{ea} (E_e - T_e) - k_{ev} (E_e - T_v) \]

\[ T_e > T_v + (T_F - T_v) \exp \left( -k_{ev} t \right) \]

Therefore in some discharges after switch-off, the electron temperature will decay to the vibrational temperature and the subsequent decay will then depend on the relaxation time for vibrational energy. This sequence would appear to increase the diffusion losses provided that vibrational excitation does not cause enhanced dissociative attachment in the gas studied.

The above notes have treated in a general manner some non-equilibrium effects—that are important when considering pulsed power switching. Wave particle interaction and scattering will be important in some low pressure switches. The electron kinetics modeling assists in establishing trends and limits of different approaches. However, the most comprehensive model that does not have any assumptions and contains all of the physical processes and boundary conditions, is still a well-defined experiment.

REFERENCES


FIG 3. Electron Energy Distribution Functions in Hydrogen. \((E/N = 40 \text{ Td})\) Dissociation as Noted.

FIG 4. Electron Excitation Rates in Molecular Hydrogen. \(k_v(01)\) First Vibrational Level.
- \(k_D\) Dissociation; \(k_{ex}\) Other Electronic Excitation.
- \(k_i\) Ionization; \(k_{DA}\) Dissociative Attachment.

FIG 5. Influence of Vibrational Temperature on Dissociative Attachment Rate in Hydrogen.

FIG 7. Calculated Electron Energy Distribution Functions in CO - Krypton (2:98)

FIG 8. Calculated Attenuation and Scattering of Fast Electrons in Hydrogen (0.3 torr) Injection Energy 20eV, Cathode Radius 1.5 cm

FIG 9. Calculated Electron Energy Distribution in Hydrogen at High Electron Density. (ref 19) $E_0 = 40eV$, $E_F = 51.5eV$ (Solid Curve) Dashed Curve is 5eV Maxwellian. Modified Dash: $E_0 = 49.5eV$