SATURABLE INDUCTORS AS HIGH-POWER SWITCHES*

by

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Abstract

Use of the nonlinear properties of iron in pulse power systems is especially attractive in rep-rated systems. They have been extensively used in the past in radar modulators, as "assist" switches to hydrogen thyratrons, and in many other applications. The failure modes are few; however, the power handling capability of inductors as discharge switches has been at rather low power levels. This paper evaluates the capabilities and limitations of saturable inductors as high-power switches for pulse power systems.

A delay in switching action is inherent in saturable inductor switches. The delay is a design choice, but must be traded against di/dt capabilities. Also, a reset action is usually required for the inductor to recover its switching ability. Reset requirements and tradeoffs affecting switch performance such as jitter, rep-rate, etc. are discussed. In addition, general scaling constraints on various switching parameters are described.

Introduction

Saturable inductors use the nonlinear properties of magnetic materials to achieve a switching action. The ferromagnetic core saturates after a certain flux density, characteristic of the core material, is achieved. The time required to saturate the core is a delay inherent in saturable inductor performance. Core saturation causes a transition from a relatively high inductance to a much lower inductance (typically two to three orders of magnitude difference) providing the impedance transition.

There are many advantages to the use of saturable inductors in repetitive pulse generation. The saturable inductor provides a compact and inexpensive alternative to some switch components. Because failure mechanisms are few, inductors offer the promise of long lifetime. Saturable inductor performance is very predictable with relatively low jitter. In addition, saturable inductors may be used in very-high-power systems. A saturable inductor design is easily scaled from a small prototype to application in a high-power system.

Pulse repetition rates in systems using saturable inductors are limited by the need to reset the core and by the limiting characteristics of magnetic materials and the particular inductor design. Variations in the voltage applied to the inductor will result in variations in switch delay, implying that for low-jitter the saturable inductor is best suited to applications in which the applied voltage is constant or tightly controlled.

Inductor Design Tradeoffs

The design of a saturable inductor consists of choosing an inductor geometry and then determining the number of turns required to achieve the desired switch delay for a specific standoff voltage and core material. Usually, the switch delay and standoff voltage are specified by the application while the core material may be a design choice. However, the core geometry is basically an arbitrary choice. Some geometries provide better saturable inductor performance than others. By examining a simple geometry (toroidal), the switch characteristics and inductor dimensions can be specified and the scaling characteristics determined.

In general, the volts-seconds rating of an inductor may be determined from

\[ \Delta B = \frac{1}{NA} \int_0^{t_d} Vdt \]

where \( N \) is the number of turns, \( A \) is the cross-sectional area of the core, \( \Delta B \) is the change in induction experienced by the core, and \( t_d \) is the switch delay after application of the voltage, \( V \). For most switching applications, the voltage applied to the inductor is approximately a unit-step function, so that

\[ \Delta B = \frac{Vt_d}{NA} \]  

The switch delay and the standoff voltage are design constraints while the change in induction is a magnetic materials constraint.

The number of turns that may be wound on a specific core is limited by the required size of conductor and insulation and by the core window area. If the size of wire wound on a saturable inductor is specified by current requirements (ampacity), then the number of turns may be expressed in terms of the winding thickness and rms current for the toroidal core. For the toroidal core, this expression is

\[ N = \frac{2P J_{\text{max}} a_s (2r_{\text{id}} - a_s)}{\text{Irms}} \]

where \( r_{\text{id}} \) is the window radius and \( P \) accounts for the packing factor and insulation. The winding thickness, \( a_s \), includes conductor.

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insulation, and insulation between layers. For scaling purposes, maximum rms current density in the inductor winding has been limited to $J_{\text{max}} = 2.35 \times 10^6 \text{ A/m}^2$, assuming a copper conductor.\(^1\)

The core area, inner radius of the core, and the winding thickness are interrelated and must be considered simultaneously with switch requirements for a point design. Such requirements might include high $\text{di/dt}$, low loss, low weight, and high efficiency. In general, inductance is given by

$$L_0 = \frac{N^2 \mu_0 N A}{2}$$ \hspace{1cm} (4)

where $N$ is the number of turns, $A$ is the cross-sectional area of the core, $\mu$ is the magnetic length of the core, and $\mu_0$ is the relative permeability of the core. During saturation, permeability of the core approaches that of air, implying that the saturated inductance is dependent upon winding configuration, similar to an air-core inductor. The normalized saturated inductance for a toroidal inductor, $L_{\text{sat}}/L_0$, is shown as a function of winding thickness and core radius in Figure 1\(^2\).

As shown, the saturated inductance increases with an increase in winding thickness. The weight of the inductor also decreases with an increase in winding thickness, but achieves a minimum at $r_c = .5 \text{ r}_{1d}$ and $a' = .5 \text{ r}_{1d}$, as shown in Figure 2.

![Figure 1](image1)

Figure 1  Saturated inductance as a function of winding thickness, $a'/r_{1d}$, and core cross-section radius, $r_c/r_{1d}$, where $r_{1d}$ is the radius of the core window and $r' = r_c/r_{1d}$.

A compromise between the need for low saturated inductance and low weight may be determined for the desired application. This compromise will specify the core radius and winding thickness in terms of the inner radius of the core. An expression for the inner radius of the core can be determined by recognizing that the core cross-sectional area may be written as

$$A = \pi r_{1d}^2 (r')^2$$ \hspace{1cm} (5)

where $r' = r_c/r_{1d}$. By substituting Equation (5) into Equation (2), an expression for $r_{1d}$ may be obtained:

$$r_{1d} = \sqrt{\frac{\Delta B \pi N (r')^2}{\Delta J_{\text{max}} \chi}}$$ \hspace{1cm} (6)

Substitution of Equation (5) into Equation (2) yields an expression for the number of turns:

$$N = \frac{r_{1d}^2 \chi (2 - a') \chi}{\Delta J_{\text{max}} (r')^2 \chi_{\text{rms}}}$$ \hspace{1cm} (7)

where $a' = a/r_{1d}$.

The number of turns has now been expressed in terms of design and materials constraints. The ratios $a/r_{1d}$ and $r_c/r_{1d}$ may be determined from Figures 1 and 2, where a design tradeoff may be made between the need for low weight versus the need for low saturated inductance.
From the number of turns, some of the switch characteristics may be determined. The primary characteristics of interest are saturated inductance and switching di/dt, and physical parameters such as weight and volume. The saturated inductance may be expressed as

\[ L_{\text{sat}} = \frac{N^2 \mu_0 \mu_s A}{l} G \]  

where \( \mu_s \) is the saturated permeability (\( \mu_s = 1 \)), \( l \) is the magnetic length of the core, and \( G \) is the inductive geometry factor. The factor \( G \) accounts for the difference between the self-inductance of an air core winding and the value of inductance obtained by using Equation (3). Note that \( L_{\text{sat}}/L_0 = G \), so that \( G \) is the factor plotted in Figure 1. The saturated inductance may be expressed in terms of design constraints by substituting Equations (6) and (4) into Equation (7) and by expressing the magnetic length in terms of the core and window radii:

\[ L_{\text{sat}} = \frac{\mu_0 G}{3.47} \left( \frac{r'}{a'} \right)^{3/2} \left( 1 + r' \right) \left( \frac{a'}{d} \right)^{5/4} \left( \frac{I_{\text{rms}}}{v} \right)^{3/4} \left( \frac{\mu_0}{\mu_s} \right)^{5/4} \]  

The maximum switching di/dt capability of a saturable inductor is inversely proportional to the saturated inductance; therefore,

\[ \frac{\Delta I}{\Delta t} \propto \frac{3.47}{\mu_0 G} \left( \frac{r'}{a'} \right)^{3/2} \left( 1 + r' \right) \left( \frac{a'}{d} \right)^{5/4} \left( \frac{I_{\text{rms}}}{v} \right)^{3/4} \left( \frac{\mu_0}{\mu_s} \right)^{5/4} \]  

The volume of the toroid core may be determined as

\[ \text{Vol} = \frac{2}{7} \left( \frac{r'}{a'} \right)^{1/2} \left( 1 + r' \right) \left( \frac{a'}{d} \right)^{3/4} \left( \frac{I_{\text{rms}}}{\mu_0} \right)^{3/4} \left( \frac{\mu_0}{\mu_s} \right)^{3/4} \]  

Determination of the core volume provides an idea of the size and weight of the saturable inductor.

**Design Scaling**

As previously mentioned, a basic design may be scaled to larger standoff voltages and conduction currents quite easily. For example, assume that a saturable inductor is required to provide a 1 µs delay. The change in induction available from a representative core material may typically be 1 Tesla. In this analogy, high di/dt and low inductor weight are equally important; so from Figures 1 and 2, the following relations are determined: \( r_c = 0.5 r_{id} \), \( a_c = 0.5 r_{id} \), with \( L_{\text{sat}}/L_0 \) determined from Figure 1 as

\[ \frac{L_{\text{sat}}}{L_0} = G = 2.95 \]

Based on these values, the following equations are determined:

\[ N = \sqrt{\frac{3.53 V}{I_{\text{rms}}}} \]  

\[ L_{\text{sat}} = 1.79 \left( 10^{-9} \right) \frac{V^{5/4}}{I_{\text{rms}}^{3/4}} \]  

\[ \frac{\Delta I}{\Delta t} = 5.59 \left( 10^8 \right) \frac{I_{\text{rms}}^{3/4}}{V^{5/4}} \]  

\[ \text{Vol} = 1.32 \left( 10^{-10} \right) \left( \frac{I_{\text{rms}}}{V} \right)^{3/4} \]

The number of turns as expressed in Equation (13) is shown as a function of \( I_{\text{rms}} \) and of \( V \) in the graph of Figure 3.

As expected, the number of turns increases as the voltage and current increase. For the case where the voltage and current are scaled identically, the number of turns remains constant. This occurs because increasing voltage for a particular design requires an increase in core area. This increase in core area is offset by the increase in core window area necessary for higher currents.
The saturated inductance increases as the design voltage is increased, as indicated in Figure 4.

![Figure 4: Saturated inductance scaled with standoff voltage, $V$, and rms current, $I_{\text{rms}}$, for the case $a_s = r_c = 0.5 r_{\text{id}}$.](image)

This implies that the $\frac{\text{d}I}{\text{d}t}$ capability of the switch decreases with an increase in design voltage, as shown in Figure 5.

![Figure 5: Maximum $\frac{\text{d}I}{\text{d}t}$ scaled with standoff voltage, $V$, and rms current, $I_{\text{rms}}$, for the case $a_s = r_c = 0.5 r_{\text{id}}$.](image)

For a constant or increasing $\frac{\text{d}I}{\text{d}t}$ as the inductor is scaled, the relationship between voltage and current must be such that

$$a I > e^{5/3}.$$  

The constant $a$ is added for the purpose of balancing units.

Figure 6 indicates the change in core volume with respect to current and voltage. The large increase in volume required to maintain a constant or increasing $\frac{\text{d}I}{\text{d}t}$ capability with a scale to large power handling requirements indicates that $\frac{\text{d}I}{\text{d}t}$ versus volume or cost is a major consideration in saturable inductor design.

![Figure 6: Core volume scaled with standoff voltage, $V$, and rms current, $I_{\text{rms}}$, for the case $a_s = r_c = 0.5 r_{\text{id}}$.](image)

Conclusions

The capabilities of inductor switches far exceed what has been achieved in the past. Scaling of simple designs indicates that the size or power handling capabilities of inductor switches can be greatly increased beyond past applications. The scaling results obtained are general because change in design variables affects only the constant of proportionality in Equations (12) through (15). The exponential powers of $V$ and $I$ are independent of the values of $r_c$, $a_s$, and $t_d$.

The scaling presented is based on voltage and current density constraints, and mechanical forces are not included. For very-high-power pulses, the mechanical forces may be sufficient to cause damage, and an analysis of mechanical stresses would be required to verify a design. Temperature restraints in the core and winding insulation were also neglected. The manner of heat deposition and flow in the core should also be examined by determining the distribution of heat in the core, a core volume for a specific design may be determined which safely limits internal heating while minimizing core size.

References
