AN ANALYSIS OF CO-AXIAL PULSE TRANSFORMERS*

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ABSTRACT

The basic idea of using a co-axial cable as an isolation pulse transformer for triggering spark gaps is not new. However, there are several distinct advantages of driving the braid as the primary as opposed to driving the inner conductor. The fundamental advantage is that the ratio of the output voltage to the input voltage is unaffected by the thickness of insulation between the inner conductor(s) and the braid. Thus, the transformer with the braid as the primary works well for isolating high secondary to primary voltages. This and other advantages are demonstrated.

Introduction

Figure 1 shows the co-axial transformer being considered as a one turn loop of co-axial cable. Figure 1 also shows what is meant by the terminology of "driving the braid as the primary (Case I)" and "driving the inner conductor as the primary (Case II)." The relative advantages of Case I over Case II are discussed in this paper.

Figure 2a shows the basic transformer circuit and Figs. 2b and 2c show the equivalent primary and secondary circuits, respectively. Of basic interest, is the voltage transfer ratio $V_{out}/V_{in}$ when the trigger gap is open circuit (i.e. before the gap has broken down, $Z_{TG} = \infty$). It is desirable to have $V_{out}/V_{in}$ as large as possible for reliable triggering. Table I gives the equations for the primary current $I_p$, the ratios of $|V_{out}/V_{in}|$ and $\omega I_s/V_{in}$, the normalized current in the trigger gap when the gap is short circuited (i.e. after the gap has broken down, $Z_{TG} = 0$).

The models in Fig. 2 are valid only for frequencies much less than the speed of light divided by the largest physical dimension of the transformer (i.e. the transformer must be much smaller than one wavelength). In actuality, the performance of the co-axial transmitter is not limited in this respect since transformers as large as a wavelength operate quite well. "Skin depth" effects, "proximity" effects and the electromagnetic radiation from the transformer acting as a magnetic dipole antenna have not been a problem.

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An Analysis Of Co-Axial Pulse Transformers

The basic idea of using a co-axial cable as an isolation pulse transformer for triggering spark gaps is not new. However, there are several distinct advantages of driving the braid as the primary as opposed to driving the inner conductor. The fundamental advantage is that the ratio of the output voltage to the input voltage is unaffected by the thickness of insulation between the inner conductor(s) and the braid. Thus, the transformer with the braid as the primary works well for isolating high secondary to primary voltages. This and other advantages are demonstrated.
Table II gives the parameters of some of the transformers tested and the values of $|\omega I_5/V_{in}|$, which are about the same for either case. The nomenclature used in the table is defined in Figs. 1, 2, and 3. From Fig. 4 it can be seen that $V_{out}/V_{in}$ is independent of $D_{BR}/D_{IC}$ for Case I but not for Case II. Thus, for Case I only, can one add as much insulation between the braid and the inner conductor as desired to isolate the secondary without affecting $V_{out}/V_{in}$. Hence, Case I works very well as a high voltage isolation transformer.

If one looks at a cross-section of one side of the transformer, it can intuitively be seen that all the flux generated by the current in the braid is external to the inner conductor. However, not all the flux generated by the current in the inner conductor is external to the braid (i.e., some of the generated flux is between the inner conductor and the braid). It is this difference that accounts for the results of Fig. 4. Theoretically, one can calculate (or use Ref. 6, p. 52) the inductance of a one turn loop with a major diameter $D$ and a minor diameter $D_{BR}$. This inductance is equal to $L_{BR}$ and equal to $M$ because both inductions come from the same flux linkages. The inductance $L_{IC}$ can be calculated for a one turn loop with a major diameter $D$ and a minor diameter $D_{IC}$. Since $D_{IC}$ is always less than $D_{BR}$, $L_{IC}$ must always be greater than $L_{BR}$. The coefficient of coupling, $k$, can then be calculated from:

$$k = M/L_{BR}L_{IC}.$$ 

These calculations agree quite well with the experimental data in Table II. The fact that $L_{IC} > M$ and that $M = L_{BR}$ accounts for the differences between the two cases in Fig. 4. It should be noted that in Fig. 3b for Case I ($Z_{TG} = \infty$), that the output voltage across $k_{LS}$ is $V_{in}$ but the input voltage is only $kV_{in}$ which is less than $V_{in}$. This "apparent" voltage step-up between $k_{LP}$ and $k_{LS}$ is the result of the perfect coupling between $k_{LP}$ and $k_{LS}$. These arguments, of course, neglect the flux in this cross-section that is generated by the currents flowing in the remainder of the transformer.

Adding extra insulation between the braid and the inner conductor also decreases the capacitance "C" (see Fig. 3) between the braid and the inner conductor. It is desirable to have this "C" as small as possible as it is a source for unwanted common mode noise. Thus, if Case I is used (see Fig. 4), one can make "C" as small as necessary to reduce the common mode noise by adding extra insulation without affecting $V_{out}/V_{in}$, the performance of the transformer.

Figure 5 shows one transformer ($D_{BR}/D_{IC} = 21.3$) response to a square wave for Case I and Case II. One can see that for Case I, $V_{out}$ is approximately $V_{in}$, but for Case II $V_{out}$ is as indicated in Fig. 4. The "ringing" response of $V_{out}$ has not caused problems in triggering spark gaps.

By "threading" one end of the inner conductor $N$ times through the braid, one can obtain a $1:N$ step-up transformer. In order to prevent the turn to turn capacitance of the secondary from degrading the response, extra insulation must be added between the turns to decrease this capacitance. Neither the $N$ secondary turns nor the extra insulation between them has a noticeable affect on the transformer coupling (i.e., $V_{out} = NV_{in}$ for $Z_{TG} = \infty$). The experimental evidence of this is not shown here, but the reasoning is the same as the explanation given for Fig. 4.
As an example, a 1:2 step-up transformer (D ≈ 0.40 m, Db ≈ 0.03 m, l ≈ 0.10 m, inner insulation and conductor of RG-8/U coaxial cable with extra insulation) is presently being used to provide a 300 kV, 30 ns rise-time trigger to a rail gap for the TeePee 18 Theta Pinch from a 150 kV, 13 stage Marx bank. This step-up transformer was needed (1) to provide the high voltage trigger pulse required for reliable switching (2) to prevent damaging the Marx bank modules by isolating them from a reverse high energy common mode noise pulse of 75 kV when the rail gap was fired and (3) to break a ground loop.

Acknowledgement

We thank Dr. J.P. Craig for his helpful suggestions.

References

Fig. 2. Transformer circuit theory (Ref. 6, p149).

(a) Inductively coupled circuit.

(b) Equivalent primary circuit.

(c) Equivalent secondary circuit.

Table I. Variables of interest
Fig. 3. Transformer circuits used in Table II. (Ref. 6, p151).

<table>
<thead>
<tr>
<th>Case I:</th>
<th>Case II:</th>
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<tbody>
<tr>
<td>$P \rightarrow Br$; $S \rightarrow IC$</td>
<td>$P \rightarrow IC$; $S \rightarrow Br$</td>
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<tr>
<td>$M = L_{Br}$</td>
<td>$L_{IC} &gt; M = L_{Br}$</td>
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<td>$V_{out} = V_{in}, Z_{TG} = \infty$</td>
<td>$V_{out} \leq V_{in}, Z_{TG} = \infty$</td>
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<table>
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<tr>
<th>$D_{Br}/D_{IC}$</th>
<th>$D_{Br}$ (cm)</th>
<th>$L_{Br} (\mu H)$</th>
<th>$k$</th>
<th>$M (\mu H)$</th>
<th>$L_{IC} (\mu H)$</th>
<th>$\omega L_{in}/V_{in}$ for $Z_{TG} = 0$</th>
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<tr>
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<td>Single Turn Inner Conductor</td>
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Table II: Measured transformer parameters.
a) Test setup.

Case I

Case II

b) Typical results ($D_{BR}/D_{IC} = 21.3$).

Fig. 5. Co-axial pulse transformer response to square wave input.