Abstract
The onset of high voltage electrical breakdown in a vacuum diode is characterized by the "explosive" like formation of cathode spots. A source of high energy density is required to ionize cathode material within nanoseconds. Enhanced field emission of electrons begins from a growing number of small spots on the cathode surface when an increasing voltage is applied. The pre-breakdown field emission current is self-limiting. Its negative space charge in the cathode-anode gap reduces the effective electric field on the cathode surface. The development of a current larger than the field emission current from a cold cathode requires that ions exist in front of the electron emitting spot. The initial ionization must occur in the cathode-anode gap near the electron emitting spot. Ionization of desorbed neutrals by field emitted electrons provides the mechanism. This ionization process requires considerably less current than the ionization of solid material by Joule heating. Ions produced a few μm from the electron emitting spot are accelerated back. Surface heating by these ions is orders of magnitude more efficient than Joule heating. As more ions are produced, a positive space charge layer forms which now increases the effective electric field and thus strongly enhances the field emission current. Experimental results agree with this self-consistent physical model which describes the onset of vacuum breakdown and the formation of a cathode spot plasma.

Field Emission and Onset of Ionization
Electrons will be emitted from spots or "whiskers" on a cold cathode if the applied electric field becomes sufficiently strong, E > 10^7 V/m. The expression "whisker" is meant to represent any electron emitting spot on the surface of the cathode, such as a microprojection, a spot with reduced work function, a grain boundary, etc. The field-emitted current density $j_{FE}$ depends on the work function of an electron emitting spot and the possibly enhanced electric field E on the surface of the spot, Figure 1.

Electrons emitted from a small micron-size "whisker" will remain in the flux tube of vacuum lines of force which ends on the surface of the whisker. Their space charge, present during their time of flight to the anode, will reduce the effective field on the tip of the whisker and hence the field emission current. The emitted electron density $n_e$ will be largest near the emitting spot. If the applied voltage remains constant, any negative space charge in the cathode-anode gap will reduce E on the surface and thus the emitted current density. Increasing the applied voltage would increase both E and the space charge.

Figure 2 shows the results of a simulation of the field emitted current density which is space charge modified [2]. At time $t = 0$, a voltage of 1 MV is applied between anode and cathode which are separated by a gap of 1cm. The applied voltage is kept constant. The Fowler-Nordheim equation is used to calculate the field emission current

$$j_{FE} = \frac{(BE)^3}{\phi} \exp \left[ -c_2 \frac{\phi^{3/2}}{\beta E} \right]$$

Introduction
The initial plasma formation on the surface of a cathode of a vacuum diode, vacuum arc, and many other discharges is highly non-uniform. Micron-sized cathode spots form within nanoseconds. Despite the fundamental importance of cathode spots for the breakdown process, their structure, and the source of the required high energy density were not well understood. The initial phase of the onset of electrical breakdown in a vacuum discharge is characterized by very rapid ionization of surface material which leads to a kind of "explosive" formation of a cathode spot plasma.

One model, the whisker explosive emission model [1], assumes that Joule heating by field emitted electrons provides the energy. Current densities of $j_{FE} = 10^{12} - 10^{13}$ A/m² would be required. However, the pre-breakdown $j_{FE}$ is self-limiting. Once a large electron current begins to flow through the diode, the electric field distribution is altered by the presence of electron space charge and the maximum current density $j_{CL}$ is limited by the Child-Langmuir law. In general one would expect that $j_{FE} < j_{CL}$. The Child-Langmuir current to the anode represents an upper limit.
The onset of high voltage electrical breakdown in a vacuum diode is characterized by the "explosive" like formation of cathode spots. A source of high energy density is required to ionize cathode material within nanoseconds. Enhanced field emission of electrons begins from a growing number of small spots on the cathode surface when an increasing voltage is applied. The prebreakdown field emission current is selflimiting. Its negative space charge in the cathode-anode gap reduces the effective electric field on the cathode surface. The development of a current larger than the field emission current from a cold cathode requires that ions exist in front of the electron emitting spot. The initial ionization must occur in the cathode-anode gap near the electron emitting spot. Ionization of desorbed neutrals by field emitted electrons provides the mechanism. This ionization process requires considerably less current than the ionization of solid material by Joule heating. Ions produced a few J.Lm from the electron emitting spot are accelerated back. Surface heating by these ions is orders of magnitude more efficient than Joule heating. As more ions are produced, a positive space charge layer forms which now increases the effective electric field and thus strongly enhances the field emission current. Experimental results agree with this self-consistent physical model which describes the onset of vacuum breakdown and the formation of a cathode spot plasma.
where \( c_1 = 1.54 \times 10^{-6} \) and \( c_2 = 6.42 \times 10^7 \) are constants if \( j \) is measured in \( \text{A/cm}^2 \), and \( E \) in \( \text{V/cm} \). \( \beta \) is the field enhancement factor of the electron emitting spot, \( \beta = 40 \) for this calculation. The work function is \( \phi = 3 \text{ eV} \). The field emitted charges are calculated for time steps of \( 10^{-13} \text{s} \). Space charge is not considered, as continuous beam of electrons, but rather as a series of infinitely thin, numerous charged disks with varying surface charge density \( \sigma \) [\( \text{C/m}^2 \)].

In each time interval, the forces on the disks within the gap are computed, the disks are moved, and a new disk is created at the cathode. When a disk reaches the anode, it is no longer used in any further calculations. In the final steady state, all of the disks will have the same charge density; for every time step just as much charge is lost at the anode as is created at the cathode.

The initial current density at time \( t = 0 \) is \( j_0 = 2 \times 10^5 \text{ A/cm}^2 \). The transient oscillations at the start are caused by the first injected charged disks which contain large \( \sigma \). Their arrival at the anode reduces the space charge remaining in the gap and the effective \( E \) on the cathode surface increases correspondingly. The dashed line shows the approach to the final steady state current density of \( j_{FE} = 1.2 \times 10^3 \text{ A/cm}^2 \). This is more than two orders of magnitude less than the initial \( j_0 \). Indeed, the field emission current density is self-limiting. The Child-Langmuir current density for this example is \( j_{CL} = 2.3 \times 10^3 \text{ A/cm}^2 \).

The field emitting spot cannot deliver \( j_{CL} \) without turning itself off, since the negative space charge caused by \( j_{CL} \) reduces \( E \) to 0 at the cathode surface. Hence, it must be that the vacuum \( j_{FE} < j_{CL} \). The development of a current with \( j > j_{FE} \) (vacuum) requires that ions exist in front of the electron emitting spot. Of course positive ions cannot be emitted from the surface of the whisker, the enhanced electric field would hold them back. The initial ionization must occur in the cathode-anode gap near the electron emitting spot. Ionization of desorbed neutrals provides the mechanism which allows an increasing electron current to be emitted from the whisker. Actually this ionization process requires considerably less current than the ionization of the solid material of the entire whisker by Joule heating. Besides Joule heating of the electron emitting spot, the emission of electrons itself, and the enhanced electric field, all stimulate desorption of weakly bound adsorbates [3]. A suddenly released monolayer of \( 2 \times 10^{19} \text{ molecules/m}^2 \) forms a dense expanding neutral gas cloud.

The relatively large concentration of neutrals above the electron emitting spot will to some extent become ionized by the field emitted electrons. The electron mean free path length \( \lambda = 1/(n_0 \sigma_0) \) depends on the ionization cross section \( \sigma_0 \) which in turn is a function of the electron energy. For many gases the ionization cross section has maximum value for electrons with an energy around 100 \text{ eV}. Within the "ionization region" the ionization cross section and neutral density are sufficient to provide a reasonable probability for ionization of neutrals. The diode used for this experiment has a gap of \( 2.5 \text{ cm} \), and 1.0 MV potential was applied. At \( z = 2.5 \mu \text{m} \) the potential is 100 \text{ V} and the field emitted electrons begin to ionize the desorbed neutrals. The ionization region is approximated as having a thickness of \( d = 1 \mu \text{m} \) and is centered on the 100 \text{ V} potential. On average the ions are produced at \( z = 2.5 \mu \text{m} \). These ions are accelerated toward the whisker and deliver 100 \text{ eV} of energy each to the surface layer. The ion-heated surface layer may be \( k \) monolayers thick. The surface layer of depth \( L \) is heated by ion bombardment as well as by Joule heating. The power dissipated per unit volume in the surface layer of the whisker due to Joule heating is given by [4]:

\[
\text{Joule Power} = \frac{\rho P}{V} = \frac{\rho}{A L} = \frac{j_e}{p} \frac{\text{watt}}{\text{m}^3}
\]

where \( A \) is the surface area of the whisker, \( \rho \) is the resistivity of the whisker material and \( j_0 \) is the current density flowing through the whisker. The power dissipated per unit volume in the surface layer due to ion bombardment with ion flux density \( j^+ / \epsilon \) is given by

\[
\text{ION POWER} = \frac{(j^+ / \epsilon) A}{V} = \frac{\rho P}{V} = \frac{(j^+ / \epsilon) A}{L}
\]

where \( j^+ \) is the ion return current density. \( j_0 \) depends on the ionization of neutrals by field emitted electrons with current density \( j^+ \). For a neutral cloud thickness \( d \), \( \lambda / d \) electrons will produce one ion. The ratio of current densities is given by
where the current density of electrons emitted from the surface of the whisker and is the time of flight for an emitted electron to travel to the ionization region, see Figure 3. Thus \[ j^+ = \frac{e/(\text{area})\Delta t}{e(\lambda/d)/\text{(area)}\Delta t} = \frac{d}{\lambda} \] where \( j^+ \) is the current density of electrons emitted from the surface of the whisker and \( \Delta t \) is the time of flight for an emitted electron to travel to the ionization region, thus \( j^+ \) = \( j^+ \) = \( j_0 \sigma_0 \). For every ion that falls back to the surface of the whisker there is an image electron that travels within the whisker to meet and recombine with the ion. Therefore the total current density inside the whisker is \( j = j^- + j^+ \). In terms of the emitted electron current density \( j = j^- + j^+ = j^- (1 + \rho_0 \sigma_0) \).

The ratio of Joule plus ion heating to Joule heating in the surface layer is then given by

\[
R = \frac{j^2 P + j^+ (V)/L}{j^2 P} = 1 + \frac{V\rho_0 \sigma_0}{j L (1 + \rho_0 \sigma_0)^2}
\]

It is important to note that the return ions deposit their energy into a highly localized, thin surface layer. Ion bombardment is especially important at low current densities since it leads efficiently to further desorption of neutrals from the surface and hence increases the neutral density which in turn increases the ionization rate.

The energy required to liberate a metal atom from the surface of the cathode is approximately 5 eV, the binding energy. The energy (eV) per atom deposited in the whisker due to Joule heating is

\[
\text{energy (Joule heating)} = \frac{j^2 \rho \Delta t / 16 \times 10^{-19} [\text{eV} / J]}{n_w}
\]

where \( n_w \) is the number density of the whisker material and \( \Delta t \) is the duration of the current flow in seconds. The total energy per atom deposited in the surface layer of the whisker due to both joule and ion heating is

\[
\frac{\text{total energy}}{\text{atom}} = R \frac{\text{energy (Joule heating)}}{\text{atom}}
\]

Inserting the values \( d = 1 \times 10^{-6} \text{ m}, \sigma_0 = 1 \times 10^{-20} \text{ m}^2, \rho = 7 \times 10^{-8} \Omega-\text{m}, L = a_0 k, \) where \( a_0 \) is the lattice parameter for stainless steel \( (2.9 \times 10^{-10} \text{ m}), \rho \) is the resistivity for stainless steel, \( k \) is the number of monolayers in the surface layer of the whisker through which the ions penetrate, and \( V = 100 \text{ volt}, n_w = 8.2 \times 10^{28} \text{ atoms/m}^3 \) for steel, and expressing the total current density in terms of the field emitted electron current density gives

\[
\frac{\text{total eV (surface atom)}}{\text{atom}} = R \left( 5 \times 10^{-18} \right) \frac{[j^-]^2 \times (1 + 10^{-26} n_0)^2}{\Delta t}
\]

Values of eV/atom are plotted in Figure 4 as a function of \( j^-, n_0, k, \) and for \( \Delta t = 10^{-9} \text{ s}. \) The curve in the lower right corner represents solely Joule heating. These calculations demonstrate that the heating of the surface due to return ion current is far more efficient than the heating of the whisker bulk due to Joule heating. The ions provide the high energy density required to explode a "whisker" within nanoseconds.
The return ion bombardment leads to further desorption of neutrals thus increasing the neutral density. The increased neutral density will reduce the mean free path for ionization of neutrals by electrons thus increasing the ionization rate. As more ions are produced, the positive space charge presence, (see Figure 3), enhances the electric field and this strongly enhances the field emitted electron current. Also due to the enhanced electric field, the 100 volt equipotential surface moves closer to the cathode surface into a region of higher neutral density. This effect also increases the ionization rate. The whole process rapidly becomes unstable.

![Diagram](image)

**Figure 5.** Sequence of events leading to the formation of a cathode spot plasma. (a) Field emission. (b) Enhanced field emission. (c) The whisker "exploded", forming the cathode spot plasma by unipolar arcing. Ion bombardment forms the cathode spot crater. The dense plasma shields the cathode spot from the externally applied field. \( j_{\text{ext}} \) = electron field emission current density; \( j_{\text{CL}} \) = electron Child-Langmuir space-charge limited current density; \( j_{\text{arc}} \) = electron current density of unipolar arc; \( \beta \) = electric field-enhancement factor; \( E_s \) = sheath electric field.

Formation of Cathode Spots

The localized build-up of plasma above the electron emitting spot naturally leads to pressure and electric field distributions which ignite unipolar arcs [5]. As the ionization rate of neutrals increases in the ionization region in front of an electron emitting whisker, a kind of double layer forms between the electrons moving toward the anode and the ions moving towards the cathode, Figure 3.

The positive space charge produced by these ions increases the effective electric field on the tip of the whisker Figure 5 (b), and thus strongly enhances the field emitted electron current. Finally, the ionization rate becomes so large that the electron current density to the anode becomes the space charge limited \( j_{\text{CL}} \). A sufficiently dense plasma blob has formed which shields the tip of the whisker from the externally applied electric field Figure 5 (c). However, the potential of this cathode spot plasma is positive with respect to the cathode surface. The dynamics of the cathode spot plasma is now determined by plasma pressure gradients, associated internal electric fields, and the sheath electric field. A sheath forms as the radially expanding cathode spot plasma sweeps over the metallic surface. The increasing plasma pressure \( P \) above the electron emitting spot leads to a pressure gradient and an electric field \( E \) in radial direction, tangential to the surface, Figure 6.

Without any current flowing this field would be the ambipolar electric field \( E_{\text{amb}} = - \frac{V_{\text{p}}}{e_0} \). Associated with this field the plasma potential decreases in radial direction. Consequentially, the plasma sheath potential \( V_{\text{s}} \) also decreases in a ring-like area around the cathode spot crater. The distribution of \( V_{\text{s}} \) will be such that the quasineutrality of the plasma is assured. At some radial distance \( r_F \) from the cathode spot crater the sheath potential will be equal to the floating potential, \( V_F = \frac{\sqrt{2m_e k}}{2e} \ln \left( \frac{m_e V_F}{2m_p} \right) \), providing equal ion and electron flow rates to the surface at this location, i.e., the net current through the sheath is zero. At distances \( r > r_F \) the plasma potential, and thus \( V_{\text{s}} < V_F \) and more electrons than ions will return to the cathode surface. This closes the electron current loop of the unipolar arc. The increasing flow of electrons to the surface at \( r > r_F \) due to \( V_{\text{s}} < V_F \) results also in a reduction of the negative radial space charge distribution within the cathode spot plasma.

This leads to a reduction of the radial electric field \( E_r \) (r) = - \( \nabla V_A < E_{\text{amb}} \). Therefore, the radially outward directed electron pressure gradient force becomes larger than the electric field force which holds the electrons back \( -P_{\text{e}} \beta = -e_0 E_r \). A net force \( F_{\text{net}} \) acting on the electron fluid is pointing outward in radial direction

\[
F_{\text{net}} = -P_{\text{e}} \beta - e_0 E_r
\]

This is the driving force of the unipolar arc, [5]. The unipolar arc current density,
which is driven by the radial plasma pressure gradient, can be orders of magnitude larger than the Child-Langmuir space charge limited diode current density. This high current density and the associated surface heating by ions provide the "explosive" like formation of a cathode spot plasma.

Conclusions
This paper presents a self-consistent physical model describing the onset of breakdown. It starts with field emission from single spots, it leads to the build up of positive space charge due to ionization of desorbed neutrals which further enhances field emission. The increased surface heating by ion bombardment provides further desorption. As the ionization zone propagates into the dense vapor, more ionizations occur. The build up of plasma pressure gradients and sheath electric field leads naturally to the formation of an unipolar arc. A cathode spot is formed by unipolar arcing.

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References


