ELECTRODE-EXPANSION MHD IN A PLASMA-FILLED ROD PINCH

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Abstract

The Gamble II plasma-filled rod pinch (PFRP) produces an order-of-magnitude higher x-ray dose and smaller FWHM source size than achieved with a vacuum rod pinch. Intense electron deposition at the tip produces a high-energy-density expanding-tungsten plasma, the bremsstrahlung from which creates a low-intensity halo around the source core that increases the effective source size for some applications. The plasma motion has been measured with a holographic interferometer. Using the distribution of electron deposition derived from linespread measurements, the observations are compared with zero-and one-dimensional self-similar MHD modeling of the plasma motion. Results demonstrate that magnetic-field and ohmic-heating effects during electron deposition are important to understanding PFRP dynamics.

II. DISCUSSION AND CONCLUSIONS

The experimental configuration has been described elsewhere.[6,7] Here, electrical, Schlieren, interferometry, on-axis and side-viewing rolled-edge measurements for PFRPs with 1-mm-diam tapered-tungsten rods are employed in the analysis. Figure 1 shows the diode current, diode voltage, and bremsstrahlung x-ray traces for a typical shot. Time is measured from the start of the x-ray pulse, when voltage first appears at the rod tip. Negative times correspond to the run-down phase during which the injected plasma is displaced by \( J \times B \) forces, sweeping open a vacuum gap along the rod.[6,7] Voltage at negative times is then associated with \( d(LI)/dt \), where \( L \) is the inductance associated with the evolving vacuum gap. Using the variations of \( V \) and \( I \) shown in Fig. 1, \( L \) is calculated to be close to linearly rising from the start of the current pulse, reaching 22 nH at \( t = 0 \). This variation

![Figure 1. Diode voltage, current, and x-ray traces.](image-url)
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corresponds to the opening of a cylindrical volume around
the rod with a 1.2-cm radius and length increasing from 0
to 3 cm during the run-down. For \( t > 0 \), the voltage at the
tip \( V_{\text{tip}} \) is then given by \( V = 2.2 \times 10^4 \frac{dI}{dt} \), so that the
electron-beam power heating the full 3-cm length of the rod is
taken to be \( IV_{\text{tip}} \) in the analysis.

Figure 2 compares Schlieren images of vacuum (50-kA)
and plasma-filled (500-kA) rod pinches 60 to 70 ns after
the start of x-radiation. The image scales are identical and
the rod tips are aligned (initial rod image shown for the
PFRP). For the PFRP, high-energy-density tungsten
plasma expands from the tip during bremsstrahlung emis-
sion, creating a low-intensity halo around the x-ray-source
core that is manifested as extended wings in the on-axis
line spread (Fig. 3). Also shown in Fig. 2 is the PFRP 90°
line spread demonstrating that time-integrated e-beam
deposition along the rod correlates with the observed plasma expansion.

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tations for the PFRP that have two distinct spot-size scales.


![Figure 2. Schlieren images for vacuum and plasma-filled RPs 60-70 ns after start of x-ray emission.](image)

Figure 3 shows typical 0° edge- and line-spread functions for the PFRP that have two distinct spot-size scales.
The 0.4-mm FWHM is consistent with the 90° line-spread
distribution of Fig. 2 showing concentration of electron
deposition on the tapered tip. The 1.7-mm LLNL definition
source size [1] reflects the extended line-spread wings of Fig. 3, which, in turn, reflect the observed rod-plasma expansion of Fig. 2.

The zero-dimensional MHD analysis seeks to explain
the observed line-spread wings by assuming a cylindrical self-similar expansion of plasma from the tapered tip. A
Gaussian tungsten-density distribution \( n(r,t) \) of width \( R(t) \)
is assumed.

\[
n(r,t) = \frac{N}{\pi R^2(t)} \exp\left(\frac{-r^2}{R^2}\right) \tag{1}
\]

Here, \( N/(\text{cm}) \) is the fixed line density given by

\[
N = \frac{\pi}{3m_W} R_b^3 \rho_0 \tag{2}
\]

\( m_W \) is the mass of a tungsten ion, \( \rho_0 \) is the solid rod mass density, \( R_b = L/20 \) is the radius of a cone of rod-tip length
\( L \) assumed to be heated by the e-beam. Based on Fig. 2, \( L \approx 3-4 \text{ mm} \) is reasonable. The continuity equation is then satisfied when the radial fluid velocity takes the form

\[
v(r,t) = \frac{r}{R(t)} \frac{dR}{dt} \tag{3}
\]

Substituting these forms into the fluid equation of motion,

\[
\frac{d^2 R}{dt^2} = \frac{2(1 + Z_k) kT}{m_W R} \tag{4}
\]

where \( T(t) \) is the temperature and \( Z(T) \) is the ionization level. Temperature is determined from radially-integrated
energy balance per unit length given by

\[
E_{\text{int}}(t) = \int_0^t \left[ P_h - \epsilon \sigma T^4 - 2 \pi R^2 \right] dt - \frac{m_W N}{2} \left( \frac{dR}{dt} \right)^2 \tag{5}
\]

In Eq. (5), \( E_{\text{int}} \) is the internal energy, the last term represents kinetic energy, \( P_h \) is the beam-heating power and
black-body-like radiation \( P_{bb} \) from the characteristic
radius \( R \) is assumed. The quantities \( T(E_{\text{int}}) \) and \( Z(T) \) are
determined from SESAME [10] equation-of-state data for
tungsten. Measured x-ray dose from the rod tip [6] indicates that \( P_h = IV_{\text{tip}}/3L \) is appropriate; a piece-wise linear fit to it is used to solve Eqs. (4) and (5). Note that \( L \) and emissivity \( \epsilon \) are free parameters.

Figure 4 shows solutions of the above equations for
\( \epsilon = 0.3 \) and two values of \( L = 3 \) and \( 4 \text{ mm} \). For each trace, the smaller value of \( L \) yields the upper curve. The single \( P_h \) curve is for \( L = 4 \text{ mm} \); that for 3 mm is 4/3 larger. At peak
temperature, \( Z \) in the 14 to 16 range is calculated. Of greatest interest here is \( R(t) \) as it determines the wings
doing the 0° line spread. This radius is found to depend
weakly on \( \epsilon \) in the range 0.03 to 0.3. At each instant, the bremsstrahlung radiation \( P_{bb}(r,t) \) is assumed to scale like \( IV_{\text{tip}}^3/2 \) in time with a radial dependence proportional to
\( n(r,t) \) of Eq. (1). Integrating \( P_{bb} \) in time over the power pulse
then provides the 0° point spread function \( PS(r) \) used to calculate the line spread function \( LS(y) \) in Eq. (6).
The ability of the zero-dimensional model to predict tip expansion is demonstrated by the fits of Fig. 5, and consistency between the values of $R(t)$ at 70 ns in Fig. 4 with the Fig. 2 Schlieren boundary at the tip. It is desired to extend the analysis to a one-dimensional axial self-similar model in order to better understand rod-plasma expansion away from the tip. Equation (1) is generalized to include the axial variation of line density associated with the local solid-density rod radius.

$$N(z) = \pi R_{rod}^2(z) \rho_0 / m_w$$  

(7)

Local rod heating $P_{rod}(z,t)$ follows the $90^\circ$ line spread of Fig. 2 in space and $IV_{ip}$ in time. The local beam current density $J_{br}(z,t)$ entering the rod plasma follows this line spread in space and $I$ in time. Conservation of charge requires that the axial return current in the rod $I_z(z,t)$ is

$$I_z = 2\pi \int_0^z J_{br}(z',t) dz'$$  

which is therefore proportional to the $90^\circ$ edge spread.

With the addition of magnetic pressure, Eq. (4) is modified to

$$d^2 R(z) / dt^2 = 2 / m_w R \left[ (1 + Z - k T) I_z^2 / 200N - I_z^2 / 100R \right]$$  

(9)

in cgs units except for $I_z$ in A. Energy-balance now takes the form

$$E_{rad}(t) = \int_0^L \left( P_{hc} - P_{bb} + 7.4 \times 10^8 \eta(T) I_z^2 / 2\pi R^2 - I_z^2 / 100R \right) dt$$

$$- m_w N \left( dR / dt \right)^2$$  

(10)

where $\eta$(Ohm-cm) is the Spitzer resistivity. The ohmic heating and work on the plasma in Eq. (10) derive from the radial dependence of return-current density that preserves self-similarity in Eq. (9), i.e., $j_z \times B_0 \sim n(r,t)$.

$$j_z(r,z,t) = \frac{I_z(z,t)}{2\pi R^2} \frac{d}{du} \left[ \left( 1 + u^2 \right) \exp \left( -u^2 \right) \right]^{1/2}$$  

(11)

In Eq. (11), $u = r/R(z,t)$. The self-similar axial-current density resembles a Gaussian with characteristic radius 1.2R. Note that formally, Eqs. (8)-(10) are only valid for $dR/dz << 1$, so that results near the rod tip are suspect.

Figure 6 shows the applied axial variations of line density, beam heating and return current, and the computed $R(z)$ at 110 ns from the solution of Eqs. (9) and (10). For $z < 0.6$ cm, beam heating of the rod plasma dominates over ohmic heating, and magnetic pressure is much less than kinetic pressure, so that the zero-dimensional model can be employed. For larger values of $z$, axial-return current has accumulated high values and temperature is lowered due to reduced beam deposition and higher mass. Under these conditions, ohmic heating dominates and magnetic pressure retards plasma expansion, so that the return-current effects in the one-dimensional model are...
In conclusion, measured Schlieren images and line-spread distributions compare well with simple zero- and one-dimensional self-similar MHD modeling of tungsten-plasma expansion for the plasma-filled rod pinch. Close to the tip, beam heating of the rod plasma and kinetic pressure dominate the dynamics so that the zero-dimensional model can be employed to estimate how expansion alters the radiographic source. Behind the tip, accumulated axial-return current and reduced beam deposition lead to high ohmic heating and a magnetic pressure sufficient to retard plasma expansion, so that the more sophisticated analysis is required.

Future plans include 2-dimensional MHD and particle-in-cell (PIC) simulations of the PFRP. Challenges for this objective will be to model: rod return-current-heating effects during the run-down phase, effects of adsorbed gases in the rod, a self-consistent transition between run-down and plasma-opening phases, and beam- and plasma-current distributions in the expanding-rod plasma after opening. It is hoped that additional modeling will help in the development of geometries that reduce the MHD wings in the line spread to enhance the radiographic applications of the plasma-filled rod pinch.

### III. REFERENCES


[8] Installed at NRL by J. Moschella and C. Vidoli, HY-Tech Research Corp.
