Robust Controller Design for Hemispherical Resonator Gyroscope

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ABSTRACT: In this paper, robust H_∞ controller for hemispherical resonator gyroscope is designed. Hemispherical resonator gyroscope(HRG) is one of the coriolis vibratory gyroscope(CVG) which has very stable quartz hemispherical resonator and shows very precise performance. HRG signals are usually modulated at the several kHz of resonant frequency. So the general control scheme cannot be applied directly because general control schemes mainly focused at low frequency range. In this dissertation, H_∞ controller which is suitable for HRG properties is designed. The H_∞ controller for HRG is robust to model uncertainties and has advantages in ease of implementation and analysis. From the experiments, it is verified that the proposed control loop is successfully operated.

Keywords: Hemispherical Resonator Gyroscope, Rebalance loop, H_∞ control

1 INTRODUCTION

The basic fundamentals utilized in the HRG were discovered over a century ago by G.H. Bryan. He observed that a sound wave, or a stress wave to be more exact, set up in a thin shell body is not fixed to the body when the body is rotated, thus providing a gyroscopic reference.[1] HRG shows very precise performance because it is based on inertial properties of the standing wave and it usually use very stable quartz hemispherical resonator.

HRG shows many advantages such as small size, low noise, high performance and no wear-out property and so on. Its versatility will be shown by its use for spacecraft stabilization, precision pointing, aircraft navigation, strategic accuracy systems, oil borehole exploration and planetary exploration.[2]

In this paper, a feedback control loop design using H_∞ control scheme is proposed, which is essential to HRG. The principle of operation of the HRG is introduced and the model equation of the HRG is explained. Then the multivariable H_∞ controller is designed and designed controller is verified from experiment.

2 OPERATING PRINCIPLE

The HRG is a kind of vibratory gyroscope where the resonator is a thin walled axi-symmetric hemispherical shell. The shell resonator is made to vibrate in 2nd order vibration mode. A sketch indicating the nature of the standing wave of 2nd order vibration mode established on the shell resonator is shown in Figure 1. The standing wave of each vibration mode contains four antinodes and four nodes being separated from one another by 45°. The nodes of the two standing waves make an angle of 45° with respect to each other. The resonant frequency of the two vibration modes are the same when the resonator is perfectly symmetric. The operation of HRG is based on the Coriolis Effect. When the shell resonator vibrates in one vibration mode, if an external angular rate is applied to the gyroscopes, then the resonator is forced to vibrate in the other vibration mode. The standing wave is represented by the superposition of the two vibrations.

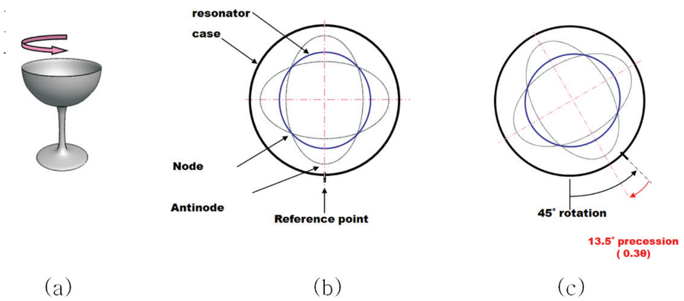

Figure 1. Operating principle of HRG

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3 MODEL EQUATIONS OF HRG

The governing equations of the sensing axis mode of HRG can be expressed as follows. The equation of the HRG can be simply expressed as a second order mass-spring-damper system. The dynamic model equation of sensing axis is given as equation (1),

$$G_y(s) = \frac{Y(s)}{F_{Coriolis}(s)} = \frac{1/m_x}{s^2 + \frac{\omega_y}{Q_y}s + \omega_y^2} \qquad (1)$$

where m_x is the effective mass of the resonator, ω_y and Q_y is the resonance frequency and quality factor of the hemispherical resonator, respectively.

This simplified model is obtained from FEM analysis from the 3 dimensional CAD model like figure 2.

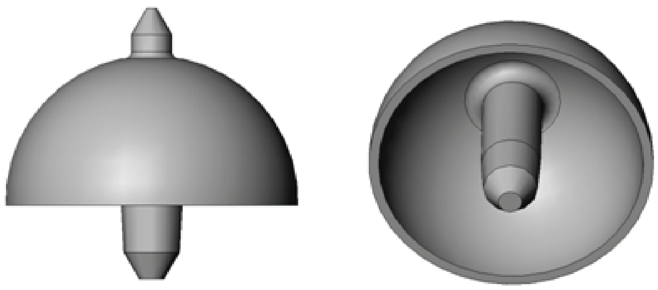


Figure 2. 3D modeling of HRG

4 CONTROL LOOP DESIGN

A feedback controller, or a rebalance loop, is very important to achieve the performances such as stability, bandwidth, linearity, dynamic range, and robustness. Especially, to obtain wide bandwidth, the feedback controller is indispensable, because the HRG is very high Q factor system and has small open loop bandwidth. Moreover, the feedback controller reduces the effect of the driving noise, measurement noise, variation of driving signal, and instability of the driving and sensing mode, because it tracks Coriolis force due to the external angular rate instead of sensing signal. To satisfy these performances, a controller more systematically, H_∞ controller design technique is used in this paper.

In this section, the H_∞ based controller for an HRG is proposed. When a controller is applied, the angular rate can be estimated by modulating the controller output because the controller output is proportional to the input Coriolis force.

To design a H_∞ controller, a system model should be transformed into two-port system model, which is illustrated in Figure 3. The state space model equation of two-port system is given by The two-port realization including scale factor models is given by equation (2), where variables are given by equation (3). In equation (2),(3), the x_p=[x_1 x_2] is a

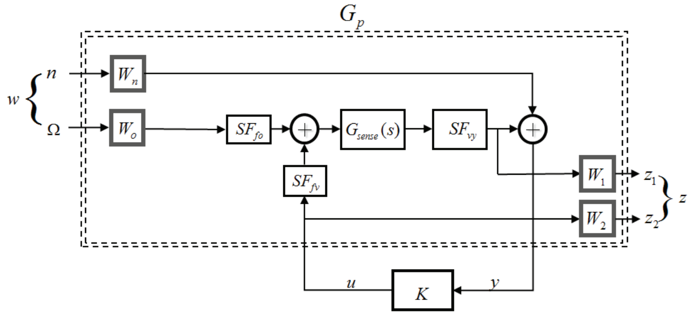


Figure 3. Two port system of HRG.

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_{p1} w + B_{p2} u \\ z &= C_{p1} x_p + D_{p11} w + D_{p12} u \\ y &= C_{p2} x_p + D_{p21} w + D_{p22} u \end{aligned} \qquad (2)$$

$$\begin{aligned} x_p &= [x_1 \quad x_2]^T \triangleq [x \quad \dot{x}]^T \\ z &= [z_1 \quad z_2]^T = [W_1 SF_{vy} x_1 \quad W_2 u]^T \\ w &= [w_1 \quad w_2]^T = [\Omega \quad n]^T \\ y &= SF_{vy} x_1 + W_n n \end{aligned} \qquad (3)$$

state vector which includes the position state and the velocity state. The w=[w_1 w_2]=[Ω n] is an exogenous input vector composed of input rate and measurement noise, and the u is control input voltage. The output, y, is a sensing voltage which is proportional to the displacement of the resonator. The controlled output z=[z_1 z_2] is selected to include the output and control input vector. Then the plant is given by

$$G_p(s) = \left[\begin{array}{c|c:c} A_p & B_{p1} & B_{p2} \\ \hline C_{p1} & D_{p11} & D_{p12} \\ \hdashline C_{p2} & D_{p21} & D_{p22} \end{array}\right] = \left[\begin{array}{cc|cc:c} 0 & 1 & 0 & 0 & 0 \\ -\frac{K}{M} & -\frac{C}{M} & \frac{SF_{fo}}{M} W_0 & 0 & \frac{SF_{fv}}{M} \\ \hline W_1 SF_{vy} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W_2 \\ \hdashline SF_{vy} & 0 & 0 & W_n & 0 \end{array}\right] \qquad (4)$$

Where K is the effective spring constant, C is the effective damping factor, SF_{fo} is scale factor between rate input to induced coriolis force, SF_{fv} is scale factor between control input voltage to electrostatic force and SF_{vy} is scale factor between resonator displacement to pickoff output voltage.

Note that the H_∞ control problem is to find a controller which makes the infinity norm of the transfer function from w to z minimum. Because z is composed of the output and the control input, the H_∞ controller minimizes the output and control input for the external angular rate, which is indispensable to achieve the wide bandwidth and dynamic range.

With the proper input weighting function W_n, W_o and output weighting function W_1, W_2 that reflects the characteristics of signals of HRG, 4th order H_∞ controller is given by equation (5).

$$K_c(s) = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right] \quad (5)$$

where

$$A_c = \begin{bmatrix} -642.3 & -2.059e^{12} & 5.581e^{-3} & 599.2 \\ 1 & -2.029e^{6} & 0 & 0 \\ 0 & -1.728e^{17} & -64.77 & -1.049e^{9} \\ 0 & -5.835e^{11} & 1 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 4.936e^{12} & 4.865e^{6} & 4.145e^{17} & 1.399e^{12} \end{bmatrix}^T$$

$$C_c = \begin{bmatrix} -1.294e^{-4} & -4.156e^{-2} & -1.125e^{-9} & -4.225e^{-1} \end{bmatrix}$$

$$D_c = 0$$

5 EXPERIMENTAL RESULTS

Using 40mm diameter model of HRG sample, rebalance control loop experiments were carried out. Configuration of rebalance loop test is shown in Figure 4. In order to fully oscillating the HRG sample, it is needed that vacuum environment under 10^{-6} Torr. In this study, we used the sputter equipment for obtain high vacuum environments. Figure 5 shows vacuum chamber for HRG sample test. With some interface board, pickoff and driving board, 4th order H_∞ controller which is implemented with DSP has tested.

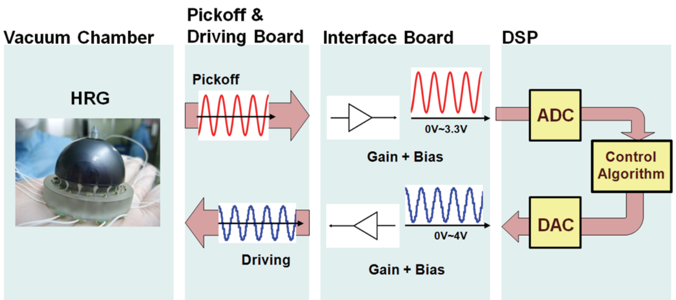
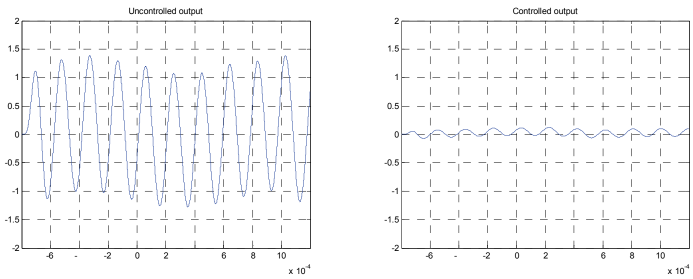


Figure 4. Configuration of rebalance loop test

Figure 5. Vacuum chamber for HRG

Figure 6 shows the test result of H_∞ controller rebalance loop test. Figure 6(a) is output displacement of the sensing axis which is resonating. And Figure 6(b) is output displacement of the H_∞ controller is applied. The controlled output is regulated about 5% of the amplitude of the uncontrolled resonating output. From this result, you can see that the designed and digitally implemented 4th order H_∞ controller is successfully operates with the HRG sample.



(a) (b)

Figure 6. Experimental result of rebalance loop control

6 CONCLUSION

To operate the HRG well, the feedback control loop is indispensable. In this paper, H_∞ control scheme is adopted.

In general, it is not easy to design a conventional PID controller since the HRG are operated at the resonant frequency of several kHz, which needs a high derivative gain and can be very sensitive to the high frequency noise. The H_∞ control enables us to have systematic approach for the controller design, which effectively reject the noise and regulates the sensing axis resonance.

From the experimental results, the designed controller is successfully operates with the HRG sample.

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