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CANONICAL DUALITY THEORY AND ALGORITHMS FOR SOLVING SOME CHALLENGING PROBLEMS IN

David Gao  
UNIVERSITY OF BALLARAT

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Final Report

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**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**  
Supported by this grant, the PI and his group have successfully solved a series of challenging problems in computer science, global optimization and applied mathematics, including the well-known NP-hard max-cut problem and sensor location problem in network optimization. An open problem on triality theory left in 2003 has been solved. This theory can be used to identify both global and local extremal solutions and to design powerful algorithms for solving real-world problems. Within the past five years, 2 books, 5 journal special issues, and about 60 papers have been published. Four international conferences have been organized, including the 3rd World Congress of Global Optimization. A unified methodology and algorithm have been developed with real-world applications. This grant has been used to support and co-support three post-doctors, three PhD students, one part-time senior researcher, and more than 15 short-time visitors. The PI has been invited to delivery 18 plenary/keynote lectures at international conferences. The projects proposed in the proposal have been fully completed. The canonical duality theory is now considering as a breakthrough new methodological theory in multidisciplinary fields of applied mathematics, global optimization and nonlinear mechanics.

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**AFOSR Grant FA9550-10-1-0487 Final Report**  
**October 1, 2010 – July 30, 2015**

PI: David Y Gao ; *University of Ballarat and Federation University Australia*  
Title: Canonical Duality Theory and Algorithms for Solving Some Challenging Problems in Global Optimization and Decision Science

**Personnel Supported:**

Post-doctors:

Daniel Morales-Silva, Jiapu Zhang, Ning Ruan

Senior Research Fellow: Dr. Andrei Kelarev

Visiting Scientists:

1. Professor Ray Ogden (FRS). Univ. Glasgow, UK, Nov. –Dec. 2011
2. Prof. Alex Strekalovsky, Russian Academy of Sciences, Jan. 12-Feb. 12, 2012.
3. Associate Professor Ali Mert, Ege University, Turkey, March-April, 2012
4. Dr. S. Bhullar, Univ. of Victoria, Canada,
5. Mr. Vittorio Latorre, Univ. Sapienza of Rome, April 3-October 3, 2012
6. Professor Jinhao Zhu, Tongji University, Shanghai, July 9-July 16, 2012
7. Dr. Xiaoju Lv, Basque Center for Applied Math, Spain, August-November, 2012
8. Professor R. Chen, City Univ of Hong Kong, November –December 2012
9. Dr. Kun Cai, Northwest A&F University, China, August 25, 2012-July 20, 2013
10. Prof. J.H. Fan, Alford University, NY, USA, December –January, 2013.
11. Dr. Vittorio Latorre, Univ. Sapienza of Rome, August – November, 2013
12. Professor H. Netuka, Palacky University, Czech Republic, May –June, 2014
13. Professor J. Machalova, Palacký University, Czech Republic, May –June, 2014
14. Professor S. Migorski, Jagiellonian University, Poland, June-July, 2014
15. Professor M. Shillor, Oakland Univ. Dec, 2014 – Jan. 2015

**Other Senior Collaborators:**

1. Dr. Eldar Hajilarov, Federation University
2. Dr. S.C. Fang, Graduate Alumni Professor of Industrial Engineering, North Carolina State University.
3. Dr. Layne Watson, Professor of Computer Science, Virginia Tech.
4. Dr. Wenxun Xing, Professor of Mathematics, Tsinghua University, China.
5. Dr. Reuy-Lin Sheu, Professor and Chair, National Cheng Kung University, Taiwan.

**Accomplishments/New Findings:**

**Research and Education Activities**

Supported by this AFOSR grant, the PI and his students, post-doctor and co-workers have successfully applied the canonical duality theory and its associated algorithms for solving a large class of nonconvex/nonsmooth/discrete problems in global optimization and decision science. Within five years, he has published **2** books, **5** journal special issues, and about **60** papers (45 are journal papers). The most significant achievement of this project is the solution to an open problem on triality theory left in 2003. This theory can be used to identify both global and local extremal solutions and to design powerful algorithms for solving real-world problems. Additionally, a series of challenging problems have been solved, such as the well-known NP-hard Max-Cut problem in computer science, sensor location problem in network optimization, a complete set of analytical solutions to nonconvex mechanics, and

nonconvex/mixed integer optimization problems in chaotic dynamics, computational biology, decision science, machine learning, neural networks, post-buckling of large deformed structures, industrial and systems engineering, etc. A powerful deterministic method and algorithm have been developed, which can be used for solving efficiently large scale nonconvex/nonsmooth/discrete optimization problems. The projects proposed in the proposal are fully completed.

Co-sponsored by Federation University, this AFOSR grant has been used to support three post-doctors, three PhD students, one part-time senior researcher, and about 15 international visitors.

The following international conferences have been organized successfully.

1. Co-Chair, [International Symposium on Interdisciplinary Computation and Optimization](#). December 20-25, 2014, The Yellow Mountains, China.
2. Chair, 3<sup>rd</sup> [World Congress of Global Optimization](#) (WCGO III), July 7-12, 2013. The Yellow Mountains, China.
3. Chair, [International Workshop on Complexity and Data Mining \(IWCDM 2011\), September 24-28, 2011, Nanjing, China](#)
4. Chair, Collaborative Research Network Workshop on Computational Mechanics and Materials, March 4-5, 2012, Univ. of Ballarat

Additionally, the PI is organizing the following two conferences for 2016.

1. Co-Chair, [Emerging Trends in Applied Mathematics and Mechanics](#), May 29 – June 3, 2016, Perpignan, France
2. Co-Chair, Advances in Applied Mathematics, July 10-15, 2016, Wollongong, Australia.

#### **Keynote, Plenary, and Invited Lectures at International Conferences.**

1. Keynote Lecturer, 2<sup>nd</sup> [Intl. symp. on Multiscale Material Mechanics in the 21st Century \(AIFANTIS INTERNATIONAL SYMPOSIUM\)](#), 4 - 9 October 2015, Cornelia Diamond Golf Resort & Spa, Antalya, Turkey
2. Invited Lecturer, [Int'l Conf on Computational Mathematics and Sciences](#), June 6-8, 2015, Xi'an Jiaotong University, Xi'an, China.
3. Plenary Lecturer, [International Symposium on Modern Mathematics and Mechanics](#), February 2-6, 2015 Olomouc, Czech Republic
4. Plenary Lecturer, [The 16<sup>th</sup> Baikal International Triannual School-Seminar Methods of Optimization and Their Application](#), 30<sup>th</sup> June – 6<sup>th</sup> July, 2014, Russia
5. Invited Speaker, [Mixed-Integer Nonlinear Programming](#), June 2-5, 2014, Carnegie Mellon University, Pittsburgh, PA, USA
6. Plenary Lecturer, [International Symposium on Modern Mechanics and Mathematics](#), June 23-28, 2014, Palacky University, Olomouc, Czech Republic.
7. Plenary Speaker, [The 8th International Conference on Bio-Inspired Computing: Theories and Applications \(BIC-TA\)](#), July 12-14, 2013, Huangshan, China.
8. Plenary Speaker, [International Conference and Summer School on Numerical Computations: Theory and Algorithms](#), June 17-23, 2013 Euroldido Hotel, Falerna (CZ) Tyrrhenian Sea, Italy
9. Plenary Lecturer, [59th Workshop of the International School of Mathematics, Nonlinear Optimization](#), June 10-17, 2013, Erice, Italy.
10. Invited Lecturer, Int. Conf. on New Trends in Solid Mechanics: Coupled Fields and Multiscale Modelling, June 24- 28, 2013, Castro, Spain.

11. Tutorial Speaker, 3<sup>rd</sup> World Congress of Global Optimization, July 7-12, 2013, the Yellow Mountains, China
12. Plenary Speaker, the 4<sup>th</sup> [International Conference on Optimization, Simulation and Control](#), Ulaanbaatar, Mongolia, July 1-4, 2013, Ulaanbaatar, Mongolia.
13. Plenary Speaker, 19th International Conference on Neural Information Processing (ICONIP2012), Nov. 12-15, 2012, Doha, Qatar.
14. Invited speaker, 2012 International Workshop on Optimization and Engineering Systems, National Cheng Kung University, Taiwan, June 27-30, 2012
15. [Main Speaker, International Conference on Numerical Analysis & Optimization-Theory and Application](#), December 17-21, 2011, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia.
16. Keynote Speaker, [1<sup>st</sup> International Symposium on Optimization and Complex Systems](#), Dec. 13-16, 2011, Shanghai University, Shanghai, China
17. Invited Lecturer, [Int. Workshop on Global Optimization](#), July 11-12, 2011, Izmir University of Economics, Turkey.
18. Invited Lecturer, [International Workshop on Optimization and Scientific Computation](#), May 28-29, 2011, Nanjing Normal University, China.
19. Plenary Lecturer, [52th Workshop on Nonlinear Optimization, Variational Inequalities, and Equilibrium Problems, International School of Mathematics “G. Stampacchia”](#), Erice, Italy, July 2-10, 2010.
20. Plenary Lecturer, International Conference on Optimization and Control (ICOCO-10), Gueiyang, China, July 18-23, 2010.
21. Plenary Speaker, [International Conference on Optimization, Simulation and Control, Ulaanbaatar, Mongolia](#), July 25 - 28, 2010

### **Findings:**

#### **1) A breakthrough theory in global optimization and decision science**

Global optimization plays fundamental roles in real-world applications. Due to nonconvexity with discrete decision variables, most of global optimization problems are considered as NP-Hard. However, by the canonical duality theory, many real-world global optimization problems can be converted to a unified concave maximization problem over a convex feasible set in continuous space, which can be solved efficiently to obtain not only global minimal solution, but also local extremal solutions. Extensive applications have been illustrated which show that many real-world global optimization problems can be solved deterministically in polynomial time if the canonical dual feasible set is not empty. Otherwise, such problems could be really NP-hard, which a conjecture is proposed by the PI in 2007. This conjecture shows that the canonical duality theory can be used to identify NP-hard problems. This theory has been challenged by a set of 11 papers since 2011. These challenges show a huge gap between mathematical optimization and mathematical physics and lead to a complete solution to an open problem left on the triality theory. Recent review article [1] shows that the canonical duality theory can be used not only for modeling complex systems within a unified framework, but also for solving a large class of challenging problems in multidisciplinary fields of computer science, applied math, industrial and systems engineering.

#### **2) Deterministic Algorithms for solving “NP-Hard and NP-complete Problems”.**

Based on the canonical duality-triality theory, a powerful canonical primal-dual algorithm has been developed for solving general global optimization problems with real-world applications. In my recent papers [10,17,33,34] joined with three other co-workers, a series special perturbation methods have been proposed. Our results show that a large class of so-called NP-complete problem can be actually solved in polynomial times. Even in the worst

case that the canonical dual feasible set is empty, the integer programming problem can still be reformulated as a continuous unconstrained Lipschitzian global optimization problem, which can be solved by a deterministic method proposed in my recent paper (see [31]) joined with four computer scientists. This paper shows that in either case, the NP-hard problems can be solved by deterministic methods (but may not be in polynomial time in the worst cases).

3) **Canonical duality theory for solving chaotic dynamical systems.**

It was realized by the PI in his review article published in 2003 that the so-called chaotic phenomena in nonlinear dynamical systems are mainly due to the nonconvexity of the total potential of the system. By using the methods of finite difference and least squares, it is shown in the PI's recent paper [25] joined with Dr. N. Ruan that the well-known logistic equation in population dynamical systems can be reformulated as a global optimization problem, which could have at most  $2^n$  local solutions. The so-called chaotic solutions obtained by traditional direct approaches are actually certain combination of these local solutions. By the canonical duality theory, global optimal solution is obtained for the first time. This paper should have important impacts to the nonlinear dynamical systems.

4) **Global optimal solutions to Radial Basis Neural Networks**

Radial Basis Function Neural Networks (RBF NN) are a tool largely used for regression problems. The principal drawback of this kind of predictive tool is that the optimization problem solved to train the network is highly non-convex and cannot be solved by traditional direct approaches. By using the sequential canonical duality theory, the PI and his visiting PhD student V. Latorre from University "Sapienza" of Rome solved this challenging problem to obtain global optimal solution for the first time. The methodology developed and the optimal solution obtained can be used to design new neural networks. Our paper [24] has been published in Neuralcomputing.

**Contributions:**

**Contributions within Discipline:**

The canonical duality theory and algorithm can be used for solving a large class of challenging problems in decision science and complex systems.

**Contributions to Other Disciplines:**

Canonical duality theory has been used successfully in neural networks optimization, sensor communication systems, filter design, signal processing, machine learning, chaotic dynamical systems, decision making, supply chain, scheduling problems, and computational mechanics, etc.

**Impacts to the communities:**

The *canonical duality theory* is now considering as a breakthrough new methodological theory in multidisciplinary fields of applied mathematics, global optimization and engineering mechanics. Two special journal issues have been published dedicated to this theory.

The application of the canonical duality theory in continuum mechanics solved a well-known open problem and the associated pure complementary energy principle is recognized as the Gao principle.

The large deformed nonlinear beam model proposed by PI is now known as the nonlinear Gao beam.

**Publications:**

**Books/Special Issue Published:**

1. Gao, D.Y. and Motreanu, D., Handbook on Nonconvex Analysis and Applications, International Press, 2010, 680pp.

2. Fang, S-C, Floudas, C., and Gao, D.Y., Recent Developments in Global Optimization, Special Issue of Journal of Global Optimization. 2011
3. Gao, DY, Ruan, N. and Xing, WX. *Advances in Global Optimization*, Springer, 2015, 537pp.
4. Gao, DY, Special issue of 3<sup>rd</sup> World Congress of Global Optimization, Journal of Global Optimization, Springer, 2015
5. Gao, DY, Latorre, V. and Ruan, N. *Advances in Canonical Duality Theory*: Special Issues of Mathematics and Mechanics of Solids, 2015.
6. Yu-Bo Yuan, David Yang Gao and Shan Zhao, Machine Learning in Intelligent Video and Automated Monitoring, The Scientific World Journal, Volume 2015 (2015). <http://dx.doi.org/10.1155/2015/570145>

**Papers to appear:**

1. Gao, D.Y., Ruan, N., and Latorre, V. (2015). [Canonical duality-triality: Bridge between nonconvex analysis/mechanics and global optimization](#), *Math. Mech. Solids*.
2. Gao, DY (2015) Remarks on analytical solutions in nonlinear elasticity and anti-plane shear problem, to appear in *ZAMP*, <http://arxiv.org/abs/1507.08748>
3. [DY Gao](#), [Patrizio Neff](#), [Ionel Roventa](#), [Christian Thiel](#) (2015) On the convexity of nonlinear elastic energies in the right Cauchy-Green tensor, <http://arxiv.org/abs/1508.05721>
4. Gao, DY (2015). Analytic solutions to general anti-plane shear problems in finite elasticity, *Continuum Mech. Thermodyn*, Published online at <http://link.springer.com/article/10.1007%2Fs00161-015-0412-y>
5. Gao, D.Y. and Hajilarov, E. On analytic solutions to 3-d finite deformation problems governed by St Venant–Kirchhoff material. *Math. Mech. Solids* (to appear) (2015)
6. [Vittorio Latorre](#), [DY. Gao](#), Canonical duality for solving general nonconvex constrained problems, *Optimization Letters*, published online at <http://arxiv.org/abs/1310.2014>
7. Cai, K., Gao, D.Y., and Qin, Q.H.(2015) Effects of interpolation schemes of dual stress on post-buckling solutions of Gao-beam by canonical dual finite element method, to appear in *Mathematics and Mechanics of Solids*.
8. D.Morales-Silva, DY Gao, On the minimal distance between two surfaces, *Math Mech Solids*, <http://arxiv.org/abs/1210.1618>
9. D. M. Morales Silva, D.Y. Gao (2015), Canonical duality theory and Triality for solving general unconstrained global optimization problems, *Math. Mech. Complex Systems*, <http://arxiv.org/abs/1210.0180>
10. Fang, S.-C., Gao, D.Y., Lin, G-X, Sheu, R-L, and Xing, W.X. [Double well potential function and its optimization in the n-dimensional real space: part I](#) *Mathematics and Mechanics of Solids*
11. C. Wu, C. Li, DY Gao Canonical Primal-Dual Method for Solving Non-convex Minimization Problems, *J. Global Optimization*, <http://arxiv.org/abs/1212.6492>
12. Zhou, XJ, Gao, DY, Yang, CH. (2015). Canonical dual approach to continuous-time constrained optimal control, *J. Global Optimization*
13. Zhou, XJ, Gao, DY, Yang, CH. (2015). Improved Canonical Dual Algorithms for the Maxcut Problem,

14. Fang, SC, Gao, DY, Lin, GX, Sheu, RL, and Xing, WX (2015). [Double Well Potential Function and Its Optimization in The n-dimensional Real Space](#), to appear in *Mathematics and Mechanics of Solids*
15. Liu, G.S., Gao, DY and Wang, SY (2015) [Canonical Duality Theory for Solving Non-Monotone Variational Inequality Problems](#), to appear in *Mathematics and Mechanics of Solids*
16. Ruan, N. and Gao, D.Y. Global Optimal Solution Computation of a Quadratic Integer Programming Problem with Linear Inequality Constraints, *J. Global Optimization* <http://arxiv.org/abs/1205.0856>
17. Latorre, V. and Gao, D.Y.(2015). [Canonical duality for solving general nonconvex constrained optimization problems](#). *Optimization Letters*
18. Latorre, V., Ruan, N. and Gao, D.Y.(2015). Canonical duality methodology for solving general global optimization problems, *Mathematics and Mechanics of Solids*
19. Zhou, XJ, Gao, DY, and Yang, CH (2015), [Global solutions to a class of CEC benchmark constrained optimization problems](#), *Optimization Letters*, DOI 10.1007/s11590-014-0784-0
20. Yi Chen, DY. Gao, [Global Optimal Solutions to General Nonconvex Optimization Problems with Sum of Double-Well and Log-Sum-Exp Functions](#), *J. Global Optimization* DOI 10.1007/s10898-014-0244-5 DOI 10.1007/s10898-014-0244-5

**Papers published in international journals:**

21. D.Y. Gao, J. Machalová, H. Netuk, 2015, Mixed finite element solutions to contact problems of nonlinear Gao beam on elastic foundation, *Nonlinear Analysis: Real World Applications*, 22, 537-550.
22. Ruan, N. and Gao, D.Y. Global Optimal Solutions to Nonconvex Fractional Programming Problems, *Applied Math and Computation*, Volume 255, Pages 66-72 , March 2015 <http://dx.doi.org/10.1016/j.amc.2014.08.060>
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26. N. Ruan and D.Y. Gao (2014) [Canonical duality approach for non-linear dynamical systems](#), *IMA J. Appl. Math*, 79 (2): 313-325. First published online: October 11, 2012 doi:10.1093/imamat/hxs067
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### **Interactions/Transitions:**

Supported by this grant, the PI has been invited to present invited talks at the following international conferences and institutions

1. Colloquium Lecture at Basque Center for Applied Mathematics, Bilbao, Spain, September 7, 2015. Title: Complete Sets of Analytical Solutions to a Class of Challenging Problems in Nonconvex Analysis and Complex Systems.
2. Colloquium Lecture at College of Airspace Engineering, Xi'an Jiaotong University, June 5, 2015. Title: Advances in Nonconvex Mechanics and Canonical Duality-Triality Theory
3. Colloquium Lecture at State Key CAD Laboratory, Zhejiang University, June 10, 2015. Title: Canonical Duality-Triality: Unified Modeling, Theory and Solution to Challenging Problems in Complex Systems
4. Colloquium Lecture at College of Airspace Engineering, Zhejiang University, 12, 2015. Title: A Breakthrough Theory Unifying Large Deformation Theory and Computational Mechanics

5. Colloquium Lectures at Institute for Computational Engineering and Sciences, University of Texas, Austin. Title: Canonical Duality and Triality: Unified Understanding Bifurcation, Chaos and NP-Hard Problems in Nonconvex Analysis/Mechanics and Global Optimization. Lecture I: Analytical Solutions to Certain Nonlinear PDEs in Nonconvex Analysis and Mechanics, Nov. 6, 2014; Lecture II: Canonical Dual FEM for Solving NP-Hard Problems in Global Optimization and Computer Mechanics, Nov. 11, 2014.
6. Distinguished Colloquium Lecture at Department of Mechanical Engineering, University of Texas, San Antonio, Nov. 14, 2014. Title: Canonical Duality and Triality: Unified Understanding and Complete Solutions for Bifurcation, Chaos, and NP-Hard Problems in Large Deformation Mechanics and Global Optimization
7. Colloquium Lecture at College of Arts and Science, Shanghai Maritime University, December 21, 2014. Title: Canonical duality-triality principle: Unity in art, science and philosophy.
8. Colloquium Lecture at No 2. Military University of Medicine, Dec. 25, 2014. Title: Mathematical Triality Principle in Art, Philosophy, and Traditional Chinese Medicine.
9. Colloquium Lecture at Department of Engineering Mechanics, Dalian University of Technology, July 8, 2014. Title: A Breakthrough Theory Bridging Computational Mechanics and Global Optimization.
10. Colloquium Lecture at Center for Computer Aided Design, Huazhong University of Science and Technology, July 11, 2014. Title: Unified Modelling in Art, Science and Complex Systems.
11. Colloquium Lecture at Faculty of Mathematics and Computer Science, Jagiellonski University, Krakow, Poland, June 18, 2014. Title: Canonical Duality Theory and Finite Element Solution to NP-hard Unilateral Post-Buckling Problem of Large Deformed Beam on Rigid Foundation.
12. Invited Lecture at Department of Electrical and Electronic Engineering, University of Melbourne, May 2, 2014. Title: Canonical duality-triality: Unified solutions to nonconvex/nonsmooth/discrete problems in complex systems with applications in signal processing and sensor networks optimization.
13. Invited Lecture at Department of Mechanical Engineering, University of Melbourne, April 3, 2014. Title: Canonical duality-triality: Unified understanding for bifurcation, chaos, and NP-hard problems in computational mechanics and global optimization.
14. Colloquium lecture at University of California, Berkeley, January 24<sup>th</sup>, 2013. Title: Analytical Solutions and Canonical Dual Finite Element Method for a Class of Challenging/NP Hard Problems in Nonconvex Mechanics and Complex Systems.
15. Colloquium lecture at Central South University, Changsha, China, July 6, 2013. Title: Fundamental issues and challenges in global optimization.
16. Colloquium lecture at Shanghai University of Finance, July 17, 2013.
17. Colloquium lecture at China Renming University, July 22, 2013

18. Colloquium lecture at S.-T. Yau Center, National Chiao Tung University, Taiwan, June 21, 2012. Title: Analytical Solutions and Primal-Dual Algorithms for Certain Challenging Problems in Complex Systems
19. Colloquium lecture at College of Management, National Chiao Tung University, June 22, 2012. Title: Unified Framework in Global Supply Chain and Decision Science.
20. Colloquium lecture at Centre for Intelligent Systems Research (CISR), Deakin University, April 16, 2012. Title: Unified Framework in Modeling and Simulation of Complex Systems.
21. Colloquium Lecture at the Department of Math, University of Melbourne, Sept. 12, 2011. Title: Canonical Duality and Triality: Unified Understanding and Analytical Solutions for Nonconvex, Nonsmooth and Discrete Problems in Complex Systems
22. Colloquium Lecture at the Department of Mechanical Engineering, Zhejiang University, May 31, 2011. Title: Advances in Nonconvex/Nonsmooth Mechanics Canonical Duality Theory
23. Colloquium Lecture in Institute of Automatic Control, Harbin Institute of Technology, June 11, 2011. Title: Modeling, Optimization, and Control of Complex Systems: Canonical Duality Approach
24. University lecture in Anhui University of Science and Technology, June 16, 2011. Title: Duality-Triality Unify Human-Understanding in Arts, Sciences and Religions