The goal of this project is to develop a formal theory of wireless networks providing a scientific basis to understand their fundamental properties and guide their design. Our technical approach is to rely on two aspects that are somewhat inherent to these networks: randomness and optimality. Randomness, in the form of fading, is a defining characteristic of wireless networks. Optimality is a suitable design specification. Wireless network optimization problems are notoriously difficult to analyze and solve. The incorporation of fading – randomness – leads to more complex formulations. However, it is frequently the case that these more complex formulations are in
ABSTRACT

The goal of this project is to develop a formal theory of wireless networks providing a scientific basis to understand their fundamental properties and guide their design. Our technical approach is to rely on two aspects that are somewhat inherent to these networks: randomness and optimality. Randomness, in the form of fading, is a defining characteristic of wireless networks. Optimality is a suitable design specification. Wireless network optimization problems are notoriously difficult to analyze and solve. The incorporation of fading – randomness – leads to more complex formulations. However, it is frequently the case that these more complex formulations are in fact simpler to analyze. Randomness introduces structure making it often possible to infer properties of large-scale stochastic systems even if analogous deterministic counterparts are intractable. In light of the former comments, it should not come as a surprise if random wireless networks exhibit more regular structure than deterministic networks. The research undertaken in the context of this project aims at exploiting randomness to devise solution methodologies for optimal wireless networking problems.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received | Paper
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TOTAL: 5
Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received   Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

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Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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<td>Michael Zargham, Alejandro Ribeiro, Ali Jadbabaie, Asuman Ozdaglar. Accelerated dual descent for network optimization, American Control Conference. 29-JUN-11,</td>
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Sub Contractors (DD882)

Inventions (DD882)
Scientific Progress
Thrust 1: Stochastic Optimization Algorithms for wireless settings. In the context of this project we have worked in the development of adaptive algorithms that operate without knowledge of the channel’s probability distribution. Major findings along this line are the discovery of lightweight algorithms that can be used to solve resource allocation problems in wireless communication and networking. Salient features of these algorithms are that they do not require access to the state’s probability distribution, that they can handle non-convex constraints in the resource allocation variables, and that convergence to optimal operating points holds almost surely.

Thrust 2: Optimal network design with imperfect CSI. We have also worked in the development of optimal resource allocation protocols for random access networks in the presence of fading. These algorithms were first developed for multiple access channels where in each time slot terminals measure the channel to the common access point and, based on the observed channel value, they determine whether to transmit or not and, if they decide to do so, adjust their transmitted power. We showed that the proposed algorithm almost surely maximizes a proportional fair utility while adhering to instantaneous and average power constraints. Important properties of the algorithm are low computational complexity and the ability to handle non-convex rate functions.

These algorithms were then generalized to general ad hoc networks. In this case, the associated optimization problem is neither convex nor amenable to distributed implementation, motivating the introduction of a problem approximation. This approximation is still not convex but it has zero duality gap and can be solved and decomposed into local subproblems in the dual domain. The solution method is through a stochastic subgradient descent algorithm that operates without knowledge of the fading’s probability distribution and leads to an architecture composed of layers and layer interfaces. With limited amount of message passing among terminals and small computational cost, the proposed algorithm converges almost surely in an ergodic sense to the optimal operating point.

We consider optimal random access networks an intermediate step to the development of algorithms to handle imperfect channel state information (CSI) in wireless networks. The interesting aspect of imperfect CSI in a networked setting is that different terminals are likely to have different perceptions on the values of different channels. In this context, collisions become unavoidable as terminals respond to conflicting information on the network’s propagation environment. We have therefore started work on addressing optimal wireless communications in the presence of imperfect CSI.

Our vision for optimal design in the presence of imperfect CSI has coalesced in the development of Cognitive Access Algorithms. We have begun our investigations by considering a multiple access fading channel where each terminal has a different belief about the channel states and adapts its transmission policy to the belief. In this setting, frequency division multiple access (FDMA) and channel aware random access (RA) are two special cases. To find solutions for general cases, we formulate the problem as a Bayesian game in which each terminal maximizes the expected utility based on its belief. We show that optimal solutions for both FDMA and RA are equilibrium points of the game. Therefore, the proposed game theoretic formulation can be regarded as general framework for multiple access channels. Furthermore, we develop a cognitive access algorithm that solves the problem approximately. Numerical results show that the proposed algorithm achieves good performance and is adaptable to different levels of channel beliefs.

Thrust 3: Distributed approximate Newton methods for network optimization. A separate research thrust contained in this project is the development of fast method to implement distributed optimization algorithms in networks. In particular, we consider dual descent methods that are commonly used to solve distributed estimation problems in wireless sensor networks. Conventional dual descent implements gradient decent on the dual domain and, as such, exhibits convergence rates that are typically very slow. We have developed a family of accelerated dual descent algorithms (ADD) that use approximate Newton directions to accelerate the convergence rate of conventional dual descent. These approximate directions can be computed using local information exchanges thereby retaining the benefits of distributed implementations. The approximate Newton directions are obtained through matrix splitting techniques and sparse Taylor approximations of the inverse Hessian. We have shown that, similarly to conventional Newton methods, the proposed algorithm exhibits superlinear convergence within a neighborhood of the optimal value. Numerical analysis corroborates that convergence times are between one to two orders of magnitude faster than existing distributed optimization methods. We have also established a connection with recent developments that use consensus iterations to compute approximate Newton directions.

This line of work is currently being extended to deal with stochastic optimization problems that arise in the context of networking. The stochastic ADD algorithm can be thought of as a step towards developing an accelerated method for the backpressure algorithm used to solve networking problems. In our preliminary research we show that a stochastic version of the ADD method can be used to solve network optimization problems with uncertainty in the constraints. Convergence proofs for stochastic ADD, guarantee convergence to an error neighborhood. We also give sufficient conditions on the uncertainty for convergence to the optimal point. Numerical experiments show that stochastic ADD converges in two orders of magnitude fewer iterations than stochastic gradient descent.

Thrust 4: Heuristic rational agents. Significant effort has also been devoted to the development of network optimization algorithms that use agents that perform pseudo-optimal decisions. We consider a network of distributed agents that wants to
minimize a global cost given by a sum of local terms involving nonlinear functions of self and neighboring variables. Agents update their variables at random times by observing the values of neighboring agents and applying a heuristic rule intent on minimizing the local cost with respect to their own variables. The heuristic rules are rational in that their average result is the actual optimal action with respect to the given values of neighboring variables. By identifying the algorithm with stochastic coordinate descent we have shown that all agents visit a neighborhood of the optimal cost infinitely often with probability 1. An exponential bound on the probability of deviation from optimality between visits to near optimal operating points was also derived. The algorithm’s behavior has been exemplified with applications to field estimation in a wireless sensor network, the propagation of opinions in a social network, and herd foraging of animal groups.

Thrust 5: Bayesian Network Games. In this thrust our goal is to study the use of Bayesian games as models of optimal behavior in social and technological networks. These games are characterized by payoff functions that incorporate information externalities – an unknown underlying state – as well as payoff externalities – the actions of all other agents in the network – and are common models of networked economic behavior. In, e.g., trade decisions in a stock market, the payoff that a player receives depends not only on the fundamental (unknown) price of the stock but on the buy decisions of other market participants. Thus, players must respond to both, their belief on the price of the stock and their belief on the actions of other players. Similar games can also be used to model the coordination of members of an autonomous team whereby agents want to select an action that is jointly optimal but only have partial knowledge about what the action of other members of the team will be. Consequently, agents select actions that they deem optimal given what they know about the task they want to accomplish and the actions they expect other agents to take. In the context of this project we are interested in studying the asymptotic behavior of these games and in developing algorithms to let agents compute their equilibrium actions.

Regarding asymptotic behaviors, observe that agents’ stage payoffs capture the kind of trade-off exemplified by the Keynesian beauty contest: each agent’s stage payoff is decreasing in the distance between her action and the unknown state; it is also decreasing in the distance between her action and the average action taken by other agents. The agents thus have the incentive to correctly estimate the state while trying to coordinate with and learn from others. We show that myopic but Bayesian agents who repeatedly play this game and observe the actions of their neighbors over a network that satisfies some weak connectivity condition eventually succeed in coordinating on a single action. The agents also asymptotically receive similar payoffs in spite of differences in the quality of their information. Finally, we show that if the agents’ private observations are not functions of the history of the game, then the private observations are optimally aggregated in the limit. Therefore, agents asymptotically coordinate on choosing the best estimate of the state given the aggregate information available throughout the network.

Regarding algorithmic development, we consider repeated network games where agents have quadratic utilities. Agents play Bayesian Nash Equilibrium strategies with respect to their beliefs on the state of the world and the actions of all other nodes in the network. These beliefs are refined over subsequent stages based on the observed actions of neighboring peers. We have introduced the Quadratic Network Game (QNG) filter that agents can run locally to update their beliefs, select corresponding optimal actions, and eventually learn a sufficient statistic of the network’s state. The QNG filter has been demonstrated on a Cournot market competition game and a coordination game to implement navigation of an autonomous team.

Technology Transfer
Objective

The goal of this project is to develop a formal theory of tactical wireless networks providing a scientific basis to understand their fundamental properties and guide their design. The proposed technical approach is to rely on two aspects that are somewhat inherent to these networks: randomness and optimality. Fading, i.e., random variations in channel gains, is a unique property of wireless communications. Therefore, randomness is not only inherent but a defining characteristic of wireless networks. As for optimality the goal of a tactical network is to administer given resources to support information flows between generating sources and intended destinations. This general statement of purpose still leaves considerable leeway prompting the question of which out the many different ways to support such information flows is preferable. It is thus natural to design networks that are optimal according to predefined criteria.

Approach

Wireless network optimization problems are notoriously difficult to analyze and solve. The incorporation of fading – randomness – leads to more complex formulations. However, it is frequently the case that these more complex formulations are in fact simpler to analyze. This fact, if surprising, is a well-known and fundamental property of probability theory. Gnedenko and Kolmogorov stated this in pithiest form by saying that the “epistemological value of the theory of probability is based on this: that large-scale random phenomena in their collective action create strict, nonrandom regularity.” Randomness introduces structure making it often possible to infer properties of large-scale stochastic systems even if analogous deterministic counterparts are intractable. In light of the former comments, it should not come as a surprise if random wireless networks exhibit more regular structure than deterministic networks. The research proposed here aims at exploiting randomness in discovering fundamental properties of optimal wireless communication networks.

An example of the desired outcomes is recent results concerning the optimality of separating wireless networking problems in layers and per-fading state subproblems. The goal of a communication network is to administer given resources to support information flows with some required level of service. In conventional wired networks the resource given is a set of physical connections between nodes. Supporting information flows requires finding routes between source and destination, determining link sharing strategies, and controlling the amount of traffic injected into the network. It was an early design specification to separate these problems in layers – routing, link and transport for the problems in the previous sentence – that operate more or less independently, and interact through standardized interfaces. While this was mostly a matter of ensuring inter-operability it is remarkable that this separation can be optimal. Specifically, it is possible to define separate per-layer optimization problems whose outcome coincides with the solution of a joint non-layered optimization. Mathematically, separability comes from the fact that the wired networking problem is convex – a linear program in fact. The Lagrangian dual problem can thus be solved instead. As it often happens, the Lagrangian exhibits a separable structure, which, as it turns out coincides with the conventional layers. In a wireless network, the given resources are not connections but bandwidth and power. Therefore, on top of routes, link shares and rate control, a wireless networking problem entails determining which connections, among those possible for the given bandwidth and power, should be established to support the required level of service. Earlier approaches to
wireless networking migrated the conventional layers and defined the power and frequency assignment as physical layer subproblems. This yields poor results though, and over time lead to the surge of cross-layer design as synonym of joint optimization across layers. Ultimately, the poor performance of layered wireless networks stems from the non-convexity of the joint cross-layer networking problem. As in wired networks, the Lagrangian exhibits a separable structure that can be mapped to layers. But non-convex problems have positive duality gap explaining the poor results of layered wireless networks. In what constitutes an interesting example of structure introduced by randomness, it has been proved that general wireless networking problems in the presence of fading, while non-convex, have zero Lagrangian duality gap. Exploiting the separability of the Lagrangian, this result yields the following principles:

**First separation principle of wireless networking.** This principle pertains to the separability of wireless networking problems into layers. It states that it is possible to define separate optimization problems to obtain optimal routes, optimal link capacity allocations, and optimal power/frequency assignments.

**Second separation principle.** Another difficulty in optimal wireless networking is the need to optimize jointly for all fading states. Given that fading coefficients take on a continuum of values, this is a variational problem that requires finding optimal functions of the fading coefficients. This principle states that network optimization is further separable in per-fading-state subproblems. The practical importance of this result is that it is not necessary to find optimal functions but only the values of the functions for those channels actually observed.

In the context of this project the separation principles are provided as motivation for part of the proposed research and to exemplify the feasibility of the proposed technical approach. The research agenda is to formulate generic optimization problems that model wireless networks in a variety of settings. Questions about such optimization problems are then posed and corresponding answers translated to fundamental properties of wireless communication networks. The analysis relies heavily on statistical signal processing tools to exploit structure generated by randomness. The proposed research comprises the following topics.

**Resource allocation at the physical layer.** A consequence of the separation principles is that optimal wireless network design can be achieved if optimal resource allocation problems at the physical layer can be solved. Algorithms to find exact and approximate solutions for different physical layer models will be pursued.

**Adaptive algorithms.** Optimal networks depend on the probability distributions of fading. But these distributions vary from link to link and are not known beforehand. Adaptive algorithms and protocols to learn fading’s probability distributions while searching for optimal operating points will be developed.

**Channel state information.** Adaptation to varying fading coefficients requires channel estimation and percolation of channel state information (CSI) through the network. Two issues arise in this context, incorporation of the cost of sensing into optimality criteria and operation with imperfect and outdated CSI. Note that imperfect CSI arises due to estimation errors and outdated CSI appears because of communication delays.

**Protocols.** Protocols are defined as mechanisms to exchange variables between neighboring terminals to determine optimal operating points. They are a particular type of algorithm in which only certain operations, those between nearby nodes, are allowed. They will be developed drawing from tools used in parallel and distributed optimization.
Scientific Barriers

Progress in optimal design of wireless networks is hampered by the complexity of the associated optimization problems. They are highly dimensional and non convex, therefore falling on the unlucky side of the watershed division between easy and difficult. The novel technical approach is to use statistical signal processing tools to exploit structure introduced by fading. It is well known that when correctly taken into account fading can be beneficial for wireless communications. The related fact that resource allocation problems are simpler to solve in the presence of fading, however, is sometimes overlooked. Even though it might seem counterintuitive that seemingly more complicated resource allocation problems in the presence of fading are easier to solve that their deterministic – i.e., without fading – counterparts, science is full of examples where randomness yields a simpler problem. This observation is consistently leveraged through the technical development of this research project.

Significance

The last decades of the 20th century witnessed the emergence of networks that have revolutionized our combat systems. At the start of the 21st century developing a scientific understanding of networks remains an unresolved intellectual endeavor. In characterizing the current understanding of networks the National Research Council report on Network Science compares the situation with metallurgy in the Renaissance period. Steel forming technology of the day was sufficiently well developed so as to support the industrial revolution, but the scientific understanding of metallurgy was limited. When a science of metallurgy was finally developed in the late nineteenth century, better alloys and novel processes ignited the industrialization surge of the early twentieth century. In the last fifty years we have been able to engineer networks that have certainly transformed our world. Yet, while we obviously master the pertinent technology, our scientific understanding of fundamental issues arising in networked systems remains limited. Examples of technological surges following the development of formal theories abound. The closest to our field is the development of digital communications in the 1960’s and 1970’s. Success in developing a formal theory of wireless networks has the potential to ignite an important technological surge in the filed.

Accomplishments

Thrust 1: Stochastic Optimization Algorithms for wireless settings. In the context of this project we have worked in the development of adaptive algorithms that operate without knowledge of the channel’s probability distribution. Major findings along this line are the discovery of lightweight algorithms that can be used to solve resource allocation problems in wireless communication and networking. Salient features of these algorithms are that they do not require access to the state’s probability distribution, that they can handle non-convex constraints in the resource allocation variables, and that convergence to optimal operating points holds almost surely.

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team.

**Dissemination of results**

**Thrust 1: Stochastic Optimization Algorithms for wireless settings.** Findings are reported in the following journal paper that was published in December 2010:


Our work on optimal design of wireless systems has resulted in an invitation to publish a tutorial article on a special issue on "Recent Advances in Optimization Techniques in Wireless Communication" that appeared in the EURASIP Journal on Wireless Communications and Networking:


**Thrust 2: Optimal network design with imperfect CSI.** Findings regarding the optimal design of random multiple access channels and networks are reported in the following conference and journal papers:


Findings regarding optimal adaptation to imperfect CSI are reported in two conference publications and a journal paper that appeared in the IEEE Transactions on Signal Processing:


A preliminary description of Cognitive Access Algorithms is available in the following conference submission:


**Thrust 3: Distributed approximate Newton methods for network optimization.** Findings are reported in a journal paper and three conference papers:


Extensions to stochastic optimization problems are reported in the following conference publications:


**Thrust 4: Heuristic rational agents.** Findings in this thrust are reported in a journal paper and two conference papers:

**Thrust 5: Bayesian Network Games.** Results on asymptotic behaviors have been submitted to Operations Research and have also appeared in three separate conference venues:


Results on algorithmic development are reported in a manuscript which is under consideration for publication in the Transactions on Signal Processing. Preliminary results have also appeared in three separate conference proceedings:


A tutorial article detailing our progress in this project has also appeared in the Signal Processing magazine:


**Journal Publications**


Conference Publications


**Collaborations and Leveraged Funding**

- Collaboration with A. Jadabaie (University of Pennsylvania) and A. Ozdaglar (Massachusetts Institute of Technology) on fast methods to implement distributed optimization algorithms in networks.

- Collaboration with R. Guerin (University of Pennsylvania) on optimal wireless networking implementations.

- Leverage National Science Foundation CAREER award (CCF-0952867) to get a second student working on wireless networking problems.