Surface Waves and Turbulence

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LONG-TERM GOALS

To improve our understanding of radar backscatter and microwave emission from the sea surface, and of relevant physical processes, including wave breaking and near-surface turbulence.

OBJECTIVES

This year I have concentrated on the dynamics of jets thrown up by standing waves in mid-ocean and by waves impacting a vertical wall or a ship’s hull. Such waves produce both spray and surface turbulence.

BACKGROUND

Despite conventional belief, standing-wave spectra are quite common in wind-fields over the North Atlantic (1) and other parts of the ocean; also in circular wind-fields and moving depressions. In either situation standing waves can cause severe damage to shipping, and have long been known to be the primary cause of oceanic microseisms (2). They also produce vertical jets and spray which contribute to anomalous radar backscatter (3). Compared to progressive waves, however, standing waves have been little studied. Most experiments and computations have been concerned with purely periodic standing waves, but their energy has an upper limit; see (4). Experiments to determine the sometimes very high pressures due to waves meeting a vertical wall were carried out by Chan and Melville (5). The occurrence of high pressures and accelerations has been confirmed by numerical computation (e.g., Cooker and Peregrine (6)). Laboratory experiments on vertically-forced waves have been conducted by Jiang et al. (7), which revealed the phenomenon of periodic-tripling in which, under conditions of an energy balance between forcing and dissipation, a repeated sequence of three distinct types of wave crest was observed. Longuet-Higgins and Dommermuth (8, 9, 10) have studied numerically the motion resulting from different classes of initial conditions and have found that standing waves initiated from a flat surface by a sinusoidal velocity field tend to produce sharp-pointed single jets (8), whereas waves initially at rest but with a circular-shaped trough tend to produce feat-topped crests, with plunging or spilling breakers occurring to each side (9); see also last year’s Progress Report.

APPROACH

The author’s approach is synergistic, that is it combines (1) a search for simple analytic solutions of free-surface flows using the Euler equations; (2) numerical calculations using boundary-integral
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## Abstract
To improve our understanding of radar backscatter and microwave emission from the sea surface, and of relevant physical processes, including wave breaking and near-surface turbulence.
methods for potential flows; and (3) laboratory experiments making use of the wave tank facilities at the S.I.O. Hydraulics Laboratory or in the author’s own laboratory at the Institute for Nonlinear Science.

WORK COMPLETED

Five papers have been published (see list of Publications, (1) thru (5)), which represent theoretical work summarized in last year’s Progress Report. A sixth paper (6) on the asymptotic form of vertical jets from standing waves has been accepted for publication and will appear in November 2001. A seventh paper, with David Drazen, which describes the experiments carried out in the S.I.O. Hydraulics Laboratory, will be presented at the November meeting of the APS Fluid Dynamics Division in San Diego. Other experimental and theoretical studies are in progress.

RESULTS

1. We have shown theoretically that progressive gravity waves incident on a vertical wall or cliff will produce periodic standing waves only if the incident wave steepness $ak$ is quite small, certainly less than 0.284, whereas the maximum value of $ak$ for a progressive Stokes wave is 0.443. Thus there is a significant range of wave steepness, namely $0.284 < ak < 0.443$, for which the reflected motion cannot be periodic. The same is true of two opposing trains of gravity waves having the same steepness $ak$. The lower limit to $ak$ could be less than 0.284 and must be determined experimentally.

2. With the assistance of David Drazen, laboratory experiments have been carried out in which an incident wave train of almost uniform steepness $ak$ in the range $0.200 < ak < 0.285$ encountered a vertical plane barrier. At wave steepnesses greater than 0.236 an interesting instability was observed in which every third wave crest was steeper than the crest preceding or following it; see Figure 1. The steepest waves in each triplet ultimately developed sharp crests or vertical jets (see Figure 2). The two neighboring crests were rounded, flat-topped or of an intermediate type. The sequence in which these occurred was different from that found by Jiang et al. (7) in their experiments on forced standing waves. The rate of growth of the instability was by a factor of 2.2 for every wave triplet, that is a factor of 1.3 per basic wave period. This growth-rate was found to be almost independent of the wave steepness $ak$, above the lower critical limit.

3. A theoretical solution has been obtained (11) for the vertical jets thrown up by a standing wave (or a wave incident on a barrier). Analytically, the surface profile displays a sharp cusp in the limit as the time $t$ becomes large. In the lowest approximation, the expression for the free surface is cubic in the coordinates $x$ and $y$. In the next approximation, in which quartic terms are added, the solution describes analytically the transition from a rounded crest to a sharp-pointed (cusp-like) jet; see Figure 3. It is expected that an extension to higher approximations will also yield canonical forms for flat-topped wave crests.

IMPACT

These basic results are likely to have implications for our understanding of jets and spray thrown up by standing waves in the open ocean (we reemphasize that such waves are not uncommon) and hence for the consequent anomalous radar backscatter. They also have a direct application to waves impacting vertical cliffs or harbor walls. The dynamics of ships’ bow-waves is probably related.
RELATED PROJECTS

The author has advised and collaborated with Dr. S. Sajjadi on an analytic and numerical calculation of the vorticity generated by pure capillary waves. In Dr. Sajjadi’s absence he presented his paper at the Wind-over-Waves Conference held at Churchill College, Cambridge, England from 3 to 5 September 2001.

REFERENCES


PUBLICATIONS


PRESENTATIONS


Figure 1. Record of the surface elevation $\zeta$ at a vertical wall on which a progressive wave of limiting height 11.0 cm (steepness $ak = 0.252$) is impinging. This shows the growth of a three-period instability. The highest wave in each triplet is marked $S_i$.

Figure 2. Video image of the sharp-crested wave $S_6$ in Figure 1.
Figure 3. Successive profiles of a wave crest as given by the asymptotic expression derived in reference (11). The profile develops smoothly from a rounded crest to a jet-like form with a sharp crest which tends towards a cusp. The profiles are seen in a free-fall frame of reference.