Random fields constitute a corner stone of many probability models to characterize spatially varying uncertainties. This research develops asymptotic theories and numerical methods for computing rare-event probabilities associated with random fields and the associated random elliptic differential equations. The equation has been employed to describe physics systems in areas as diverse as material science, fluid dynamics, neuroscience, fiber optics, astronomy, further civil engineering, engineer design, ocean-earth sciences, and so forth. We perform risk analysis of such systems by investigating the asymptotic behavior of certain interesting rare events. For instance,
ABSTRACT

Random fields constitute a corner stone of many probability models to characterize spatially varying uncertainties. This research develops asymptotic theories and numerical methods for computing rare-event probabilities associated with random fields and the associated random elliptic differential equations. The equation has been employed to describe physics systems in areas as diverse as material science, fluid dynamics, neuroscience, fiber optics, astronomy, further civil engineering, engineer design, ocean-earth sciences, and so forth. We perform risk analysis of such systems by investigating the asymptotic behavior of certain interesting rare events. For instance, we provide assessment and efficient numerical methods for the risk of material's not being able to support a certain amount of external force and of predicting the major causes of material failure. Such an analysis will add substantial value in the development of policies and decision making processes. The output of this research has positive impacts on those areas and also aids other areas of asymptotic analysis and simulation development of random fields and the associated stochastic systems.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

<table>
<thead>
<tr>
<th>Received</th>
<th>Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/29/2014</td>
<td>Xiaoou Li, Jingchen Liu. Rare-event Simulation and Efficient Discretization for The Supremum of Gaussian Random Fields, Advances in Applied Probability, (09 2015): 0. doi:</td>
</tr>
</tbody>
</table>

TOTAL: 3

(b) Papers published in non-peer-reviewed journals (N/A for none)

<table>
<thead>
<tr>
<th>Received</th>
<th>Paper</th>
</tr>
</thead>
</table>

TOTAL:
Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received  Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received  Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received  Paper


TOTAL: 2
### Books

<table>
<thead>
<tr>
<th>Received</th>
<th>Book</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TOTAL:**

<table>
<thead>
<tr>
<th>Received</th>
<th>Book Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TOTAL:**

### Patents Submitted

### Patents Awarded

### Awards

### Graduate Students

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTE Equivalent:</td>
<td></td>
</tr>
<tr>
<td>Total Number:</td>
<td></td>
</tr>
</tbody>
</table>

### Names of Post Doctorates

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTE Equivalent:</td>
<td></td>
</tr>
<tr>
<td>Total Number:</td>
<td></td>
</tr>
</tbody>
</table>
### Names of Faculty Supported

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
<th>National Academy Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jingchen Liu</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

**FTE Equivalent:** 0.25  
**Total Number:** 1

### Names of Under Graduate students supported

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
</table>

**FTE Equivalent:**  
**Total Number:**

### Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period:

- The number of undergraduates funded by this agreement who graduated during this period:      0.00
- The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields: 0.00
- Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00
- Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

### Names of Personnel receiving masters degrees

<table>
<thead>
<tr>
<th>NAME</th>
</tr>
</thead>
</table>

**Total Number:**

### Names of personnel receiving PHDs

<table>
<thead>
<tr>
<th>NAME</th>
</tr>
</thead>
</table>

**Total Number:**

### Names of other research staff

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
</table>

**FTE Equivalent:**  
**Total Number:**

---

**Sub Contractors (DD882)**
Inventions (DD882)

Scientific Progress

In this research, we have developed the following results.

1. Developing closed form asymptotic approximations of the failure probabilities of random material based on the one-dimensional elliptic differential equation. The results are reported in Liu and Zhou (2014).

2. Developing efficient simulation algorithms for computing the tail probabilities of random elliptic differential equation. This is reported in Liu, Lu, and Zhou (2015).

3. Developing efficient simulation algorithms for computing tail probabilities of the supremum of Gaussian random fields. This is reported in Li and Liu (2015).

4. Developing closed form approximations for the exponential integration of Gaussian random fields. This is reported in Li, Liu, and Xu (2014).

5. Developing asymptotic approximation and efficient simulation algorithms for the Korteweg-de Vries Equation. This is reported in Xu, Lin, and Liu (2014).

All the manuscripts/reports have been submitted along with the report.

Technology Transfer