DEVELOPMENT OF A FRAMEWORK FOR MODEL-BASED ANALYSIS, UNCERTAINTY QUANTIFICATION, AND ROBUST CONTROL DESIGN OF NONLINEAR SMART COMPOSITE SYSTEMS

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Final Report

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**Development of a Framework for Model-Based Analysis, Uncertainty Quantification, and Robust Control Design of Nonlinear Smart Composite Systems**

**Abstract**

This program focused on the development of a synergistic framework for model-based analysis --- model calibration, sensitivity analysis (SA), uncertainty quantification (UQ), validation and verification (V&V), and robust control design --- for a range of nonlinear and hysteretic smart composite systems. A major component of the program focused on the development of this framework in the context of hysteretic macro-fiber composite (MFC) and shape memory alloy (SMA) models. The goal was to develop a theoretical and numerical framework and initial infrastructure to facilitate modeling, design and control for aerospace, aeronautical, industrial, automotive and biomedical systems utilizing smart materials.

**Subject Terms**

Hysteresis, transductive materials, uncertainty quantification, verification, validation, robust control design
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Development of a Framework for Model-Based Analysis, Uncertainty Quantification, and Robust Control Design of Nonlinear Smart Composite Systems

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Objectives
This program focused on the development of a synergistic framework for model-based analysis — model calibration, sensitivity analysis (SA), uncertainty quantification (UQ), validation and verification (V&V), and robust control design — for a range of nonlinear and hysteretic smart composite systems. A major component of the program focused on the development of this framework in the context of hysteretic macro-fiber composite (MFC) and shape memory alloy (SMA) models. The goal was to develop a theoretical and numerical framework and initial infrastructure to facilitate modeling, design and control for aerospace, aeronautic, industrial, automotive and biomedical systems utilizing smart materials.

Accomplishments
We focused on the development and implementation of a modeling, model calibration, sensitivity analysis (SA), and uncertainty quantification (UQ) framework for structures employing macro-fiber composites (MFC) and shape memory alloys (SMA). As illustrated in Figure 1, MFC are comprised of lead zirconate titanate (PZT) fibers embedded in an epoxy matrix and driven by an interdigitated electrode array. The resulting array achieves $d_{33}$ motion which optimizes outputs and is sufficiently flexible to bond to curved surfaces. The composite structures have dual actuator and sensor capabilities and can achieve kHz drive ranges. As detailed in [10, 13, 19] and depicted in Figure 2, MFC-composite structures are presently being investigated for shape modification and flow control, vibration and noise control, health monitoring, and energy harvesting. However, these advantages come at the cost of rate-dependent hysteresis, creep and constitutive nonlinearities that must be addressed in models, designs and control algorithms.

It is noted in [6] that shape memory alloys (SMA) have the highest work density of traditional “smart materials” but typically operate at frequencies ranging from 1-100 Hz due to the thermal convection required relax the actuators. They are being investigated for use as chevrons to improve jet engine performance and decrease aeroacoustic noise, MAV design, and numerous biomedical applications due to the fact that they are biocompatible. However, like PZT-based actuators, they exhibit rate-dependent hysteresis and constitutive nonlinearities at essentially all drive levels.

Figure 1: (a) Macro-fiber composite schematic and (b) $d_{33}$ actuation produced by voltage applied to the patches.
Uncertainty Quantification

Uncertainty quantification is pursued in two steps: (i) determination of densities and uncertainty bounds for inputs consisting of model parameters, boundary or initial conditions, and (ii) development of highly efficient methods to propagate these input uncertainties through models to quantify response uncertainties and errors bounds. The goal is to develop the capability to provide model predictions with quantified uncertainties.

We employed a Bayesian approach to construct parameter uncertainties based on measured data since the consideration of parameters as random variables with associated densities is a fundamental tenet of Bayesian inference. Due to the moderate dimensionality of the parameter space in the MFC and SMA models (e.g., 18 parameters and 36 states), evaluation of posterior distributions using standard cubature formulae was not feasible. Instead, we employed a delayed rejection adaptive Metropolis (DRAM) algorithm whose stationary distribution is the posterior density.

The chain and density for a representative SMA parameter are plotted in Figure 3 along with joint sample points for two parameters. We first note that the density is non-Gaussian so asymptotic methods limiting to Gaussian distributions will be ineffective. Moreover, the parameters are highly correlated which significantly limits available options for uncertainty propagation.

Stochastic Galerkin, collocation and discrete projection methods all rely on the assumption that parameters are mutually independent or that one has a representation for the joint density for all parameters. These assumptions are violated for this model as well as many physical models arising in general applications. To propagate uncertainties and construct prediction intervals, we thus sample by index from the DRAM chains used to construct input densities. This yielded the prediction intervals shown in Figure 4. The application of Bayesian model calibration and uncertainty quantification techniques to MFC are reported in [9,10] whereas SMA results are reported in [1,3,4].

Figure 2: (a) MFC for shape modification and flow control, and (b) experimental configuration used for energy harvesting.

Figure 3: (a) Chain and (b) density for a representative SMA parameter. (c) Joint sample points demonstrating correlation between parameters.
Robust Control Design

The goal in this component of the program was to use model uncertainties, quantified using the previously discussed Bayesian and sampling techniques, to construct robust control designs having improved performance. We initially investigated this in the context of the sliding mode control (SMC) design depicted in Figure 5 and detailed in [1, 5, 15, 16]. The actuators were SMA tendons with strains $\varepsilon(t)$ generated by input temperatures $T(t)$. This was modeled using the homogenized energy model (HEM) framework in [6] with the model inverse discussed in [14]. The objective was to track a reference angle trajectory $\theta_r(t)$ by heating or cooling the actuator.

Since temperatures are not directly measured, we employed the observer

\[
\dot{T}_0(t) = -h[T_0(t) - T_\infty] + \gamma u(t) + H x(t) + L[\theta(t) - \theta_0(t)] \\
= \hat{f}(t, \bar{h}, \bar{H}) + \Delta f(t, \Delta h, \Delta H) + (\bar{\gamma} + \Delta \gamma) u(t) + L[\theta(t) - \theta_0(t)]
\]

where $h = \bar{h} + \Delta h, H = \bar{H} + \Delta H$ and $\gamma = \bar{\gamma} + \Delta \gamma$ are model parameters with means $\bar{h}, \bar{H}, \bar{\gamma}$ and $2\sigma$ uncertainties $\Delta h, \Delta H, \Delta \gamma$ as illustrated in Figure 3(b). Furthermore, $x(t)$ are summed phase fractions from the model, $L$ is the observer gain, and $u$ is the control input which ultimately consists of the voltage applied to the actuator. The terms

\[
\hat{f}(t, \bar{h}, \bar{H}) = -\bar{h}[T_0(t) - T_\infty] + \bar{H} x(t) \\
\Delta f(t, \Delta h, \Delta H) = -\Delta h[T_0(t) - T_\infty] + \Delta H x(t)
\]
isolate the mean and uncertain components of the observer. For the sliding surface

$$s(t) = \lambda \int [T_0(\tau) - T_r(\tau)] d\tau + [T_0(t) - T_r(t)],$$

use of the Lyapunov candidate

$$V(t) = \frac{1}{2} s^2(t)$$

yielded the control $u(t) = u_{eq}(t) + v(t)$ where

$$u_{eq}(t) = \frac{1}{\gamma} \left[ -\lambda [T_0(t) - T_r(t)] + \dot{T}_r(t) - \dot{f}(t, \bar{h}, \bar{H}) - L[\theta(t) - \theta_0(t)] \right]$$

$$v(t) = -\frac{1}{\gamma} \left[ \Delta f(t, \Delta h, \Delta H) + \Delta \gamma u_{eq}(t) + \rho_0 \tanh(s) \right].$$

The experimental performance of this sliding mode control design is compared with proportional-integral (PI) control in Figure 6. The errors in the square wave tracking performance illustrate an inherent physical limitation of SMA actuators for high frequency applications since convective cooling is a relatively slow process. As detailed in [1], the sliding mode design generally provides superior performance without the necessity of experimentally tuning gains. This provides an illustration of how uncertainty quantification can improve robust control design. We are presently extending these techniques to alternative robust control designs and other SMA and PZT-based actuators.

![Figure 6: Tracking performance of the sliding mode control (SMC) and PI control for a (a) 0.1 Hz sine wave, (b) 0.2 Hz sine wave and (c) 0.1 Hz square wave.](image)

Quantification of Model Discrepancy

The previously discussed model calibration and prediction interval results were obtained using statistical models of the form

$$Y_i = f(t_i, q) + \varepsilon_i$$

where $f(t_i, q)$ denotes the parameter-dependent model response and measurement errors $\varepsilon_i$ are assumed to be independent and identically distributed (iid) and normal distributed with $\varepsilon_i \sim N(0, \sigma^2)$. However, this assumption is often violated in dynamic applications. To illustrate, we model the thin beam displacement data shown in Figure 7 using the Euler–Bernoulli beam model

$$\int_0^L \left[ \rho(x) \frac{\partial^2 w}{\partial t^2} + \gamma \frac{\partial w}{\partial t} \right] \phi dx + \int_0^L \left[ YI(x) \frac{\partial^2 w}{\partial x^2} + cI(x) \frac{\partial^3 w}{\partial x^2 \partial t} \right] \phi'' dx = k_p V(t) \int_{x_1}^{x_2} \phi'' dx$$
which holds for all test functions $\phi \in V = \{ \phi \in H^2(0, L) \mid \phi(0) = \phi'(0) = 0 \}$. The density, stiffness and damping relations
\[
\rho(x) = \rho h + \rho_p h_p h_p \chi_p(x) \quad YI(x) = YI + Y_p I_p \chi_p(x)
\]
\[
cI(x) = cI + c_p I_p \chi_p(x)
\]
reflect the differing geometry and material properties in the region covered by the patch. The reported density $2700$ kg/m$^3$ for aluminum yielded $\tilde{\rho}_b = \rho_b = 0.08775$ which is fixed to ensure that the remaining parameters are identifiable. The parameter set is
\[
q = [\tilde{\rho}_b, \gamma, YI_b, YI_p, cI, cI_p, k_p]
\]
where $\tilde{\rho}_p = \rho_p h_p h_p, YI_p = Y_p I_p, YI_b = YI, CI_p = C_p I_p$ and $CI_b = C_b I_b$. The model response is the displacement $y(t_i, q) = w(t_i, \bar{x}; q)$ evaluated at the point $\bar{x} = 128$ mm. Details regarding the experimental setup are provided in [18].

Calibration over the time interval $[0, 1.0]$ yielded the fit, residuals, and 95% prediction intervals shown in Figure 7. It is observed that the residuals, and hence measurement errors, are clearly correlated, and that the 95% prediction interval does not contain the correct percentage of experimental measurements. This is due to unaccommodated model errors, or discrepancies, $\delta(t_i)$ in the statistical model
\[
Y_i = f(t_i, q) + \delta(t_i) + \varepsilon_i.
\]
Damping and frequency analysis established that calibration on the interval $[0.25, 1.25]$ can reduce the model discrepancy as indicated by the residuals plotted in Figure 8(a). In this case, the 95%
Figure 8: (a) Residuals for calibration on the temporal interval [0.25, 1.25]. 95% prediction intervals and experimental data in (c) the calibration regime and (d) extrapolatory prediction interval [2.0, 2.5] seconds.

The prediction interval shown in Figure 8(c) contains the correct percentage of future data points, thus extending the validation regime for the model. The development of statistical techniques, such as periodic Gaussian process representations, to quantify model discrepancies for extrapolatory prediction constitutes a future research direction. Details regarding issues pertaining to the quantification of model discrepancy can be found in [18].

**Personnel Supported**

- Nathan Burch Postdoc, NCSU, Raleigh, NC
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- John Crews Postdoc, NCSU, Raleigh, NC
- Jerry McMahan Graduate Student and Postdoc, NCSU, Raleigh, NC
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- Lucus Van Blaircum Graduate Student, NCSU, Raleigh, NC

**Book**

Publications


**AFRL Points of Contact**

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**Transitions/Interactions**

*Transitions: Bayesian Analysis for Parameter and Model Uncertainty – DOE CASL:* Aspects of the Bayesian analysis and sampling-based techniques to construct prediction intervals were transitioned to the DOE Consortium for Advanced Simulation of Light Water Reactors (CASL) Energy Innovation Hub. This built upon work initiated in the AFOSR Program and provided techniques employed in the last two years of the CASL program.

**Conference, Colloquia and Workshop Presentations**

- Invited Presentation: 7th Workshop on Control of Distributed Parameter Systems (CDPS 2011), Wuppertal, Germany, July 20, 2011.
- Seminar: Center for Control, Dynamical Systems, and Computation, University of California, Santa Barbara, Santa Barbara, CA, October 21, 2011.
- Seminar: Mechanical Engineering Department, University of California, Santa Barbara, Santa Barbara, CA, October 24, 2011.
• Invited Presentation: Second Monterey Workshop on Computational Issues in Nonlinear Control, Monterey, CA, November 8, 2011.


• Invited Presentation: 50th IEEE Conference on Decision and Control, Orlando, FL, December 13, 2011.


• Invited Presentation: SIAM Conference on Uncertainty Quantification, April 5, 2012.


• Invited Presentation: International Workshop on Acoustic Transduction Materials and Devices, Penn State University, State College, PA, May 18, 2012.


• Invited Presentation: 8th International Conference on Differential Equations and Dynamical Systems, Waterloo, Ontario, August 1, 2012.


• Colloquium, Department of Mathematical Sciences, University of Montana, Missoula, MT, October 1, 2012.

• Invited Presentation, IEEE Conference on Decision and Control, Maui, HI, December 10, 2012.

• Presentation: SPIE’s 20th Symposium on Smart Structures and Materials, San Diego, CA, March 11, 2013.

• Invited Presentation: SIAM Conference on Control and Its Applications, July 9, 2013.


• Colloquium: Department of Mathematics and Statistics, Missouri University of Science and Technology, Rolla, MO, November 15, 2013.


• Invited Presentation: IMA Hot Topics Workshop on Uncertainty Quantification in Materials, Minneapolis, MN, December 16, 2013.

• Colloquium: Mechanical Engineering Seminar, UCSB, February 3, 2014.
• CRSI Seminar, Sandia National Laboratories, February 6, 2014.
• Colloquium: Department of Mathematics, VA Tech, February 21, 2014.
• Colloquium: Department of Mathematics, University of Alabama at Birmingham, October 24, 2014.
• Colloquium: Department of Mechanical & Aeronautical Engineering, Clarkson University, November 7, 2014.

Honors and Awards
None

Patents
None

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References


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**Abstract**  
This program focused on the development of a synergistic framework for model-based analysis --- model calibration, sensitivity analysis (SA), uncertainty quantification (UQ), validation and verification (V&V), and robust control design --- for a range of nonlinear and hysteretic smart composite systems. A major component of the program focused on the development of this framework in the context of hysteretic macro-fiber composite (MFC) and shape memory alloy (SMA) models. The goal was to develop a theoretical and numerical framework and initial infrastructure to facilitate modeling, design and control for aerospace, aeronautic, industrial, automotive and biomedical systems utilizing smart materials.  

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Archival Publications (published) during reporting period:


J.A. McMahan and R.C. Smith, "Data-Driven Design of Sliding Mode Controllers for Ferroelectric Actuators," Proceedings of the IFAC World Congress, Cape Town, South Africa.


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Research Objectives
Technical Summary

Funding Summary by Cost Category (by FY,$K)

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