

# Global Assimilative Ionospheric Model

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## LONG-TERM GOAL

The long-term goal of our research is to develop a reliable and accurate global ionospheric weather monitoring and forecast system that can serve as a prototype for an operational system. To achieve this goal, we are developing and validating advanced data assimilation techniques to analyze diverse sources of ionospheric measurements. The assimilation of ionospheric measurements into mature first-principle ionospheric models will produce physically consistent, accurate ionospheric analysis, as well as the determination of important ionospheric drivers. As a result, this allows the generation of more accurate ionospheric weather forecasts. Our research activities maximally leverage the development of data assimilation techniques in the meteorological community. The application of general data assimilation techniques to ionospheric data analysis requires us to develop new mathematical techniques for both ionospheric modeling and optimization. This research will also help improve our understanding of the physics of the ionosphere and its response to magnetic storms, leading to a better characterization of severe space weather effects on power grids, communication, navigation, and other applications.

## OBJECTIVES

Our research effort is focused upon the development of a global assimilative ionospheric model (GAIM). The model under development is based on first-principle theoretical ionospheric models, and can make use of a variety of data sources for the determination of ion and electron densities as well as the ionospheric driving forces. Our objectives in this phase of development are:

1. Augmenting the mid-low latitude model used in the initial phase of the project by adding models for the high latitude region.
2. Developing the algorithms and data management systems to assimilate diverse data sources including both remote sensing and in-situ measurements such as TEC, airglow intensity, electron and ion densities, plasma drifts, winds, etc.

# Report Documentation Page

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14. ABSTRACT <b>The long-term goal of our research is to develop a reliable and accurate global ionospheric weather monitoring and forecast system that can serve as a prototype for an operational system. To achieve this goal, we are developing and validating advanced data assimilation techniques to analyze diverse sources of ionospheric measurements. The assimilation of ionospheric measurements into mature first-principle ionospheric models will produce physically consistent, accurate ionospheric analysis, as well as the determination of important ionospheric drivers. As a result, this allows the generation of more accurate ionospheric weather forecasts. Our research activities maximally leverage the development of data assimilation techniques in the meteorological community. The application of general data assimilation techniques to ionospheric data analysis requires us to develop new mathematical techniques for both ionospheric modeling and optimization. This research will also help improve our understanding of the physics of the ionosphere and its response to magnetic storms, leading to a better characterization of severe space weather effects on power grids, communication, navigation, and other applications.</b>					
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3. Formulating efficient parameterization of important ionospheric driving forces such as ion ExB drift velocity, neutral wind velocity, and ion production rate; developing efficient computation techniques for evaluation of the sensitivity of ionospheric state with respect to the driving forces.
4. Evaluating the quality and the applicability of different data assimilation techniques through Observation System Simulation Experiments (OSSE).
5. Validating GAIM against other measurements and analysis techniques such as ISRs, TEC from TOPEX/Poseidon, Global Ionospheric Maps (GIMs), and ionosonde measurement networks.

## **APPROACH**

Our approach is to employ several data assimilation techniques to address different aspects of the data analysis problem, namely recursive statistical estimation and nonlinear least square optimization. The recursive estimation techniques used are the Kalman filter and approximate Kalman filter methods, including band-limited Kalman and optimal interpolation. The nonlinear least square optimization method used is the 4-dimensional data assimilation (4DVAR) method with the use of the adjoint equation. The recursive estimation technique is primarily devoted to the determination of electron density with a relatively short data assimilation cycle of around 15 minutes. The 4DVAR technique is currently developed to estimate the ionospheric driving forces with a longer data assimilation cycle of around 2 hours. The optimized ionospheric state variables and driving forces are then used in the forward model to produce new forecasts for ionospheric variables.

The evaluation of the approach is first made through Observation System Simulation Experiments (OSSE) in which simulated measurements derived from forward model output are used. After an appropriate scheme for assimilation is established using synthetic data, real measurements are introduced in the evaluation process.

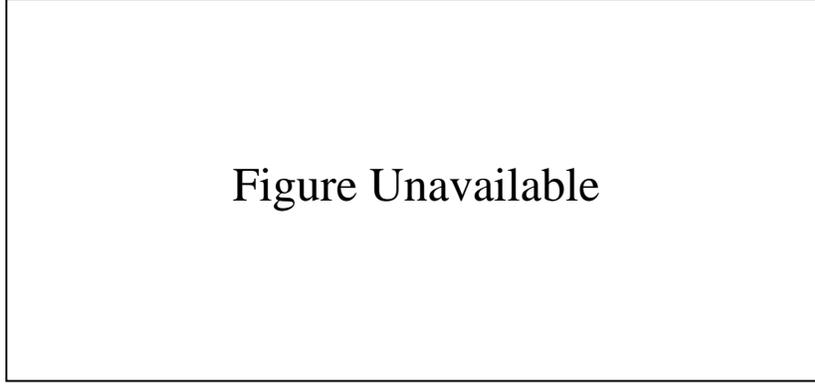
## **WORK COMPLETED**

Over the past 12 months, we have made significant progress in the following areas:

1. Forward Modeling. A primary focus of our effort in this phase of the research is to extend the coverage of our forward model to the high latitude region. We have completed the development of a global 3-dimensional ionospheric model covering the polar, high latitude, mid-latitude, and equatorial regions. This development is critical for the assimilation of several important data types such as the ground GPS TEC measurements and the GPS occultation measurements. Our basic model uses an Earth-fixed Eulerian reference frame. The boundaries of the spatial elements follow either the geomagnetic field lines or geomagnetic potential lines given by a dipole model of the geomagnetic field. The model grid is non-uniform and provides higher resolution in areas of higher variability in electron density.

The newly constructed GAIM is capable of taking inputs from empirical models or direct measurements of auroral precipitation energy and electric potential to compute ion production and plasma convection, respectively (the latter is required to drive the model dynamics at high latitudes). The GAIM has been exercised for global modeling with the high-latitude inputs coming from a few empirical models, such as the Heppner-Maynard convection model

[Heppner and Maynard, 1987] and auroral precipitation energy patterns [Fuller-Rowell and Evans, 1987]. Various levels of high latitude activity, which determine the quantitative inputs and MLT-MLAT patterns of the energy and convection, have been tested with the new code. Figure 1 shows an example of such tests, in which global TEC is obtained using inputs of solar EUV flux, thermosphere, thermospheric winds, low-latitude dynamo electric field and high-latitude magnetospheric electric field, as well as auroral precipitation energy for specified geophysical conditions. Preliminary validation of the forward model is carried out by comparing the model generated vertical TEC map to the data driven Global Ionospheric Map (GIM) developed at JPL. The global extension of the GAIM has been reported at the January MURI review meeting, 2001 (Boulder, Colorado), and it has been exercised for the 4DVAR data assimilation approach [Pi et al., 2001a and 2001b].



**Figure 1. Example of model grid and vertical TEC generated by GAIM forward model**

2. Nonlinear Least Square Minimization Approach and Adjoint Equation. The nonlinear least-square approach formulates the data assimilation problem as a minimization of the nonlinear functional:

$$J(n_0, \theta) = \sum_{k=1}^N \left\| R_k^{-1/2} (T_{Obs,k} - H_k n_k(n_0, \theta)) \right\|^2 + \iiint_{\Omega} \left\| P^{-1/2} (n_0 - \bar{n}_0) \right\|^2 d\Omega + \left\| W^{-1/2} (\theta - \theta_0) \right\|^2$$

where the first term on the right hand side represents the overall difference between the data  $T_{obs,k}$  and the model predicted observation  $H_k n_k$ . The second and the third terms represent the regularization of the optimization variables  $n_0$  and  $\theta$  which are the initial estimates of ionospheric state and driving force vector, respectively. The regularization terms allow us to moderate the amount of adjustment to the optimization variables to reflect our level of confidence in the empirical values. At this phase of our research program, we have focused on the development of a capability to estimate a large number of ionospheric driving forces. We have developed efficient parameterization techniques that allow us to estimate equatorial ExB drift, neutral wind, and ion production rates. The parameterization of the production and neutral winds was accomplished in a manner that allowed for a significant reduction in the number of parameters that had to be estimated. In essence, we defined the driving forces on a coarser grid than was used for the ion densities. The grid for the driving force is also in a Sun-fixed spherical coordinate system. We also took advantage of the basic structure of gradient computation via the adjoint method to significantly reduce the amount of memory required. The adjoint method employs ideas from control and optimization theory to significantly reduce the computational effort required to compute the gradient of the least squares cost functional. More

precisely, with the adjoint method, the computational effort necessary to compute the gradient in each iteration of the optimization process does not significantly increase with the number of parameters being estimated. This is in contrast to a finite difference based approach to computing partial derivatives which requires a forward integration of the model equations for each parameter being estimated. By combining the coarse grid parameterization of the driving forces with the adjoint method for computing the gradients, we have been able to keep the memory and CPU requirements of the 4DVAR data assimilation technique within tractable bounds.

Because of the intractability of the finite difference technique, we were forced to develop an alternative method for verifying the correctness and accuracy of our computed gradients. This was accomplished by relying upon the well-known mean value theorem for scalar fields. Indeed, we have

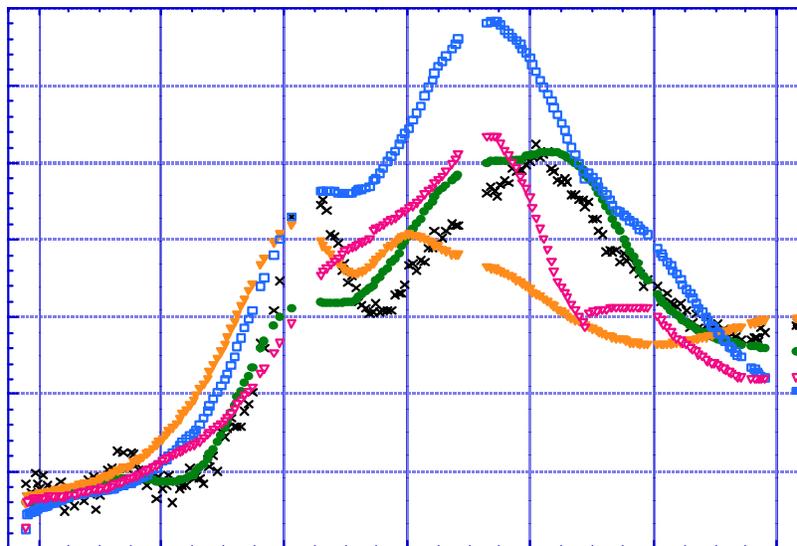
$$J(\bar{P}) - J(\bar{Q}) = \bar{\nabla} J((1 - \alpha)P + \alpha Q) \bullet (\bar{P} - \bar{Q})$$

where  $J$  is the scalar field whose gradient is to be computed,  $P$  and  $Q$  are two points in the domain of  $J$ , and  $\alpha$  is a number between 0 and 1. By comparing the left hand side of this equation with the right hand side using our adjoint based computation of the gradient at chosen values for  $P$  and  $Q$  and values of  $\alpha$  ranging between 0 and 1, we are able to check the validity of our computed gradient.

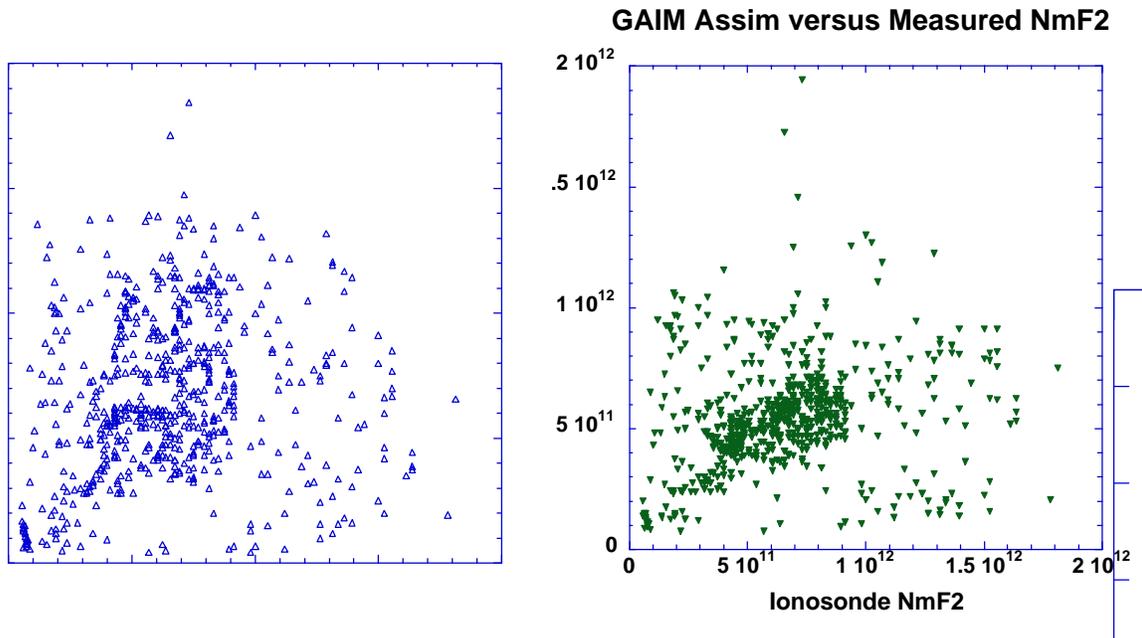
3. Recursive Estimation Approach: Band Limited Kalman Filter. The most significant limitation of the Kalman filter approach for ionospheric data assimilation is its computational complexity. Several approximate versions of the Kalman filter have been tested over the past two years. During the last 12 months, we have developed a new approach that allows us to reduce significantly the computational cost of the algorithm but retain most of the information in propagating the covariance matrix. This approach truncates the covariance matrix by eliminating the correlation between two spatially distant elements. The resulting covariance is, strictly speaking, sparse and it is only a band limited matrix for a uniform grid. This approach allows us to select the length of correlation in a physically meaningful fashion. The number of computational steps required for the implementation is nearly a linear function of the dimension of the model. The software contains a new efficient storage scheme for the positive definite band-limited covariance matrix and a set of new methods for matrix computation to efficiently implement the Kalman filter.
4. Model Validation. The GAIM assimilation mode which uses a band-limited Kalman filter has been validated by assimilating real input data from ground and space-based GPS receivers and then comparing the retrieved electron density field to independent ionospheric measurements. For a typical run on 2001/06/13, GAIM assimilated 183,000 TEC links from 96 ground GPS sites, 4260 TEC occultation links from the CHAMP satellite, and 45,000 upward looking TEC links from CHAMP. The 3D density grid consisted of 13,107 elements with a resolution of 5 degrees in latitude, 15 degrees in longitude, and 80 km in altitude. The GAIM electron density retrieval was validated by four comparisons to independent measurements of ionospheric density, vertical TEC, and slant TEC as follows:
  - 1) Compared vertical integrations of the GAIM densities to measurements of vertical TEC from the dual-frequency TOPEX altimeter;

- 2) Compared GAIM-derived global maps of vertical TEC to JPL's GPS data-driven maps of vertical TEC (GIM 2D shell algorithm);
- 3) Compared line-of-sight (slant) TEC measurements from independent GPS sites to the slant TEC predicted by integrating through the GAIM density field;
- 4) Compared GAIM density profiles to measurements of the peak F2-layer density (NmF2) from 19 globally distributed ionosonde sites.

For this day, the output of the forward model without using any input data, the GAIM "climate", exhibited densities that were larger than reality. However, after assimilating the input GPS data the retrieved densities were much closer to reality as illustrated by the good comparisons to the independent measurements. Figure 2 shows a comparison of GAIM vertical TEC to TOPEX measurements before and after assimilating the input data. The agreement of GAIM with the vertical TEC from TOPEX and GIM is quite good in the northern hemisphere where there were ground GPS sites near the TOPEX ground track. Figure 3 shows a comparison of the GAIM "climate" and "assimilation" to a global set of NmF2 data from ionosondes. After data assimilation, the GAIM specifications of NmF2 agree much better with the measurements as seen by the "tightening up" of the distribution along the 45 degree line. Some of the remaining outliers are due to inaccurate ionosonde points while others represent real differences between the GAIM retrieval and the measured NmF2.



*Figure 2. Comparison between GAIM, GIM, and TOPEX*



*Figure 3. Distribution of NmF2 in GAIM climatology and assimilation results*

## RESULTS

The preliminary validation of GAIM using GIM, TOPEX, and ionosonde demonstrates that the approaches we have taken in data assimilation can significantly increase the accuracy of ionospheric specification. In particular, use of the large volume of ground GPS TEC alone can significantly increase the accuracy of the specification of electron density profiles.

## IMPACT/APPLICATIONS

This work will have a strong impact on the implementation of an operational space weather model for ionospheric specification and forecast. The results of this study have been well received by the ionospheric research communities. Several research groups including AFRL and NRL have shown strong interest in future collaborative research.

## TRANSITIONS

Our project is still in its initial stage. No software has been transitioned to other institutions or agencies yet.

## RELATED PROJECTS

Several key investigators on our team are also key members in the ionospheric and atmospheric remote sensing group at JPL working on improving and maintaining JPL's ionospheric mapping capabilities, which have been used and supported in part by grants from the Air Force.

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