Joint Optimal Placement and Energy Allocation of Underwater Sensors in a Tree Topology

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ABSTRACT

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I. INTRODUCTION

Underwater communication is a challenging topic due to its channel characteristics. Recently, there has been a great deal of research effort on the problem of optimal sensor location and energy efficient allocation in under water sensor networks (UWSN) [1], [3].

Recent work on optimal placement of UWSN [1] focuses on tree network topologies, which construct spatial graphs with large characteristic path lengths and small clustering coefficients. So far, the only research dealing with tree-shaped topology has been to find the optimal placement of wireless sensor nodes in order to minimize the destructive effects of shadow zones [2]. In contrast with the work mentioned in [2], in [3] the underwater wireless sensor nodes’ optimal frequency and placement from the information theoretic point of view have been derived.

In UWSN the nodes have small batteries and therefore cannot afford to use much energy to complete the tasks they are supposed to do; such as monitoring and tracking objects. Additionally, their power supplies cannot typically be replenished; that is, once exhausted, the node is discarded. Thus, efficient use of energy is of great importance to long-sustained and well-operated UWSN [4]. Various data transmission mechanisms are proposed in [5] in order to achieve the objective of energy efficiency. The energy efficient protocols and energy costs in underwater acoustic networks (which are different from those in terrestrial radio-based networks) have been discussed in [6].

In this paper, we focus on energy efficient transmission, in conjunction with the nodes’ optimal placements. We achieve the optimal placement of the underwater acoustic sensor nodes with respect to the capacity of the wireless links between the nodes. We assume that the energy consumption of each sensor node is constant and we calculate the vertical and horizontal distances between each sensor node and also between any levels of interest in the tree structured sensor network representing different depths of underwater environment. In contrast with [3], on the same tree structure, we focused on the energy efficient transmission in underwater sensor networks by providing the optimal transmitting energies for the nodes with fixed locations by using KKT conditions.

We assume that our network contains a number of sensor nodes, distributed at different water depth levels, and that the nodes communicate with each other for the purposes of monitoring and tracking. Specifically, we focus on a symmetric tree like multi-hop hierarchical routing topology, which can potentially cover larger areas as we go deeper in the water than flat placement of UWSN [3].

The paper is organized as follows: In Section II we will introduce our topology. In Sections III-VI we investigate different scenarios: finding the optimal placements of wireless sensors with constant and equal power consumption, finding the optimal power consumption for each sensor while the sensors are fixed at their locations, and finally we calculate the optimal placement and energy consumption for the wireless sensors. Numerical results are in Sections VII, VIII and IX. Conclusions /future Section X.

II. UNDERWATER CHANNEL TREE TOPOLOGY

An underwater acoustic channel is characterized by an attenuation that depends on both the distance \( r \)
between the transmitter and the receiver and the signal frequency $f$:

$$\begin{align*}
A[r, f] &= r^k a(f)^r
\end{align*}$$

(1)

where $k$ is the spreading factor ($1 \leq k \leq 2$) and $a(f) > 1$ is the absorption coefficient as defined in [9] and [10]. We consider the empirical model for $a(f)$ as defined as in [9] and [10] with in $f$ kHz, and $a(f)$ can be obtained via Thorp's formula.

The network topology under discussion is a tree structured hierarchical sensor network. We assume a symmetric tree where the total number of nodes is $N$, each parent node has $M$ children, the network has $K$ levels (each sensor node at the bottom level $K$, who does not have any children is called leaf sensor node), and the information rate of each node is equal to $\mu$, as in Fig. 1, where $N = 7, M = 2, K = 3$. It is worth mentioning that we assumed only children can send the information to their parent nodes, therefore, information are sending upward from bottom of the ocean to the surface.

![Fig. 1 A seven nodes (three layered) tree structured hierarchical acoustic sensor network](image)

For a narrowband communication channel, which is our model, with bandwidth $\Delta f$, the channel capacity of the underwater acoustic link between node $i$ and node $j$ is given as [9]:

$$C_i(X_1, \ldots, X_N) = C_i(X) = \Delta f \times \log_2 \left[ 1 + \frac{P_i(f) \left| X_i - X_j(i), f \right|}{A \left| X_i - X_j(i), f \right|} \right]$$

(2)

Where $X_i$ is the position of node $i$ and $X_{j(i)}$ is the position of node $j$ to which node $i$ is transmitting, $|X_i - X_{j(i)}|$ is the distance between nodes $i$ and $j$, $X$ is a vector of the sensors’ placement $(X_1, \ldots, X_n)$ and $r_s$ is the policy providing the set of all nodes transmitting simultaneously in the same transmission time slot.

With narrowband communications assumption, $N(f)$ (the noise), $P_i(f)$ (the power of sensor node $i$), and $A \left| X_i - X_{j(i)}, f \right|$, which we discussed in [3], (the path-loss of link $i$ to $j$) do not change with frequency over the band of $\Delta f$.

Next, we consider the following scenarios:

- **Optimal Sensor Locations**: In this scenario, we assume that the power values of all the sensor nodes are given and we will try to optimize the sensor locations.
- **Optimal Sensor Power Allocation**: In this scenario, we assume that the locations of the sensor nodes are given and we will try to optimize the nodes power values.
- **Optimal Sensor Location and Optimal Sensors Power Allocation**: In this scenario, we derive the optimal placements and also the optimal consuming power for the sensor nodes in the specific UWSN.

### III. OPTIMAL SENSORS LOCATIONS WITH EQUAL SENSOR POWER:

Let assume that for the narrowband underwater acoustic communications, the power values of all the sensor nodes are equal to given value as follows:

$$P_1 = P_2 = \ldots = P_N = P$$

(3)

In a tree structured hierarchical network the destination node is the root node 0 in the tree. Hence, all the information (measurements made by the nodes in the tree structured hierarchical sensor network) is forwarded to the root node 0. In the network in Fig. 1, the total network capacity (total information received by the root node 0) is equal to the summation of capacity of the two links $C_1$ and $C_2$. Therefore, in order to carry the maximum amount of information to the root node, the total capacity of $C_1 + C_2$ has to be maximized. However, the capacities of the other links in the network have to be high enough in order to carry the amount of information in different layers of the tree structured hierarchical network.
IV. THE OPTIMAL PLACEMENT PROBLEM

For the tree structured hierarchical acoustic network in Fig. 1 \((N = 7)\), we would like to find the optimal sensor node positions \((X_1^*, \ldots, X_N^*)\), where

\[X_i^*\] denote the two dimensional coordinate of each sensor node’s position, \((x_i, y_i)\) that maximizes the total network throughput given as below, assuming \(X\) is a vector of sensors’ placement \((X_1, \ldots, X_N)\)

\[
\begin{align*}
\max_{X_1, \ldots, X_N} & \left\{ C_1(X) + C_2(X) \right\} \\
\text{subject to:} & \\
C_1(X) & \leq \delta_1 C_3(X), \quad C_1(X) \leq \delta_1 C_4(X) \\
C_2(X) & \leq \delta_1 C_5(X), \quad C_2(X) \leq \delta_1 C_6(X) \\
C_3(X) & \geq W, \quad C_4(X) \geq W \\
C_5(X) & \geq W, \quad C_6(X) \geq W
\end{align*}
\]  

(4.1)

and:

\[
\begin{align*}
C_3(X) & \geq W, \quad C_4(X) \geq W \\
C_5(X) & \geq W, \quad C_6(X) \geq W
\end{align*}
\]  

(4.2)

where \(\delta_1 \geq 1\) and are given parameters, \(W\) is the rate of information measured by each sensor node and obviously the link capacities specially of the leaf nodes have to be higher than \(W\). Also, the optimization problem not only optimizes the aggregate information rate in the tree \((C_1, \ldots, C_6)\), it also maximizes the link capacities throughout the tree. In other words, the intermediate links in the tree have to have enough capacities to carry all the information collected in the lower levels of the tree and forward that to the root node. The constraint inequalities \((4.1)\) and \((4.2)\) guarantee that the intermediate links of \(C_1, \ldots, C_6\) have enough capacity to carry the maximum amount of information to the root node. The parameter \(\delta_1\) guarantees that the capacities \(C_1, \ldots, C_6\) are not lower than a specific portion of the maximum aggregate capacity. The policy \(\tau_s\) is provided to us. Furthermore, in the policy \(\tau_s\) we assume that the time axis is chopped into timeslots and the sensor nodes \({1,3,5}\) can transmit on odd timeslots and the sensor nodes \({2,4,6}\) can transmit on the even timeslots. This policy avoids some neighboring sensor nodes to simultaneously transmit over the same timeslot.

Due to the symmetry of the problem, we have the following statements:

\[
\begin{align*}
C_1(X) & = C_2(X) \quad (4.3) \\
C_3(X) & = C_6(X) \quad (4.4) \\
C_4(X) & = C_5(X). \quad (4.5)
\end{align*}
\]

The Simplified Problem:

Earlier observations in \((4.3)\) to \((4.5)\) (and according to the policy \(\tau_s\) which we will specify later) can simplify the optimization problem as follows

\[
\begin{align*}
\max_X & \left\{ C_1(X) \right\} \\
\text{subject to:} & \\
\delta_1 C_3(X) - C_1(X) & \geq 0 \\
\delta_1 C_4(X) - C_1(X) & \geq 0 \\
C_3(X) - W & \geq 0, \quad C_4(X) - W & \geq 0
\end{align*}
\]  

(6.1)

(6.2)

(6.3)

and Complementary Slackness:

\[
\begin{align*}
\mu_1 (\delta_1 C_3(X) - C_1(X)) = 0 \quad (6.5) \\
\mu_2 (\delta_1 C_4(X) - C_1(X)) = 0 \quad (6.6) \\
\mu_3 (C_3(X) - W) = 0, \quad \mu_4 (C_4(X) - W) = 0 \quad (6.7)
\end{align*}
\]
Moreover, looking at the capacity formula (6.8), one observation is that since the log(.) is a monotonic function, we can optimize the signal-to-interference-plus-noise ratio (SINR) instead of the capacity [3,11]:

\[ C = (BW) \log_2(1 + SINR) \quad (6.8) \]

V. THE OPTIMAL POWER ALLOCATION PROBLEM

For the tree structured hierarchical acoustic network in Fig. 1 \((N = 7)\), the locations of the sensors \(X_1, X_2, \ldots, X_N\) are given, \(\overline{P} = (P_1, P_2, \ldots, P_N)\) of the sensor nodes in the network and we would like to find the optimal sensors’ power allocation \((P'_1, \ldots, P'_N)\) that maximizes the total network throughput as shown below:

\[
\max_{P_1, \ldots, P_N} \left\{ C_1(\overline{P}) + C_2(\overline{P}) \right\} \quad (7)
\]

subject to:

\[
C_1(\overline{P}) \leq \delta_1 C_3(\overline{P}), \quad C_1(\overline{P}) \leq \delta_1 C_4(\overline{P})
\]

\[
C_2(\overline{P}) \leq \delta_1 C_5(\overline{P}), \quad C_2(\overline{P}) \leq \delta_1 C_6(\overline{P}) \quad (7.1)
\]

and:

\[
C_3(\overline{P}) \geq W, \quad C_4(\overline{P}) \geq W
\]

\[
C_5(\overline{P}) \geq W, \quad C_6(\overline{P}) \geq W \quad (7.2)
\]

and

\[
0 < P_1, P_2, \ldots, P_N \leq P_{\text{Max}} \quad (7.3)
\]

Using the same approach in the optimal sensors location problem, the optimal sensors power allocation problem can be simplified and solved.

It is worth mentioning that in this section we have an additional constraint on the sensors consuming power, indicating the fact that sensors especially underwater sensors, have very limited power. (In the capacity equation, for large values of sensor power, the ambient noise level can be neglected. Thus, any scalar multiplied by the optimal power allocation would be still optimal)

VI. OPTIMIZATION OF SENSOR PLACEMENT AND SENSOR CONSUMING POWER

In this section we combine both problems, the optimization of sensor nodes’ location in the tree structured multi-hop hierarchical UWSN and sensor nodes consuming power optimization and solve it through the same approach mentioned above. In other words, we try to maximize the total network throughput

\[
\max_{X_1, \ldots, X_N, P_1, \ldots, P_N} \left\{ C_1(\overline{X}, \overline{P}) + C_2(\overline{X}, \overline{P}) \right\} \quad (8)
\]

subject to:

\[
C_1(\overline{X}, \overline{P}) \leq \delta_1 C_3(\overline{X}, \overline{P}) \quad (8.1)
\]

\[
C_1(\overline{X}, \overline{P}) \leq \delta_1 C_4(\overline{X}, \overline{P}) \quad (8.2)
\]

\[
C_2(\overline{X}, \overline{P}) \leq \delta_1 C_5(\overline{X}, \overline{P}) \quad (8.3)
\]

\[
C_2(\overline{X}, \overline{P}) \leq \delta_1 C_6(\overline{X}, \overline{P}) \quad (8.4)
\]

and:

\[
C_3(\overline{X}, \overline{P}) \geq W \quad (8.5)
\]

\[
C_4(\overline{X}, \overline{P}) \geq W \quad (8.6)
\]

\[
C_5(\overline{X}, \overline{P}) \geq W \quad (8.7)
\]

\[
C_6(\overline{X}, \overline{P}) \geq W \quad (8.8)
\]

and

\[
0 < P_1, P_2, \ldots, P_N \leq P_{\text{Max}} \quad (8.9)
\]

Using the same approach as in the previous sections we can simplify and solve the problem.

VII. NUMERICAL RESULTS FOR OPTIMAL SENSOR LOCATION

Let us assume that the system operating frequency \(f\) and the system bandwidth \(\Delta f\) are equal to 10 KHz and 100 Hz, respectively. This is consistent with the narrowband assumption. Also, let us assume that the sensor powers are all identical to each other and equal to 1 Watt. We also assume that the spreading factor for the ambient noise is \(\alpha = 1[9]\). Let us assume that the minimum measurement information rate at each sensor node is \(W = 10 \text{ bps}\). We also assume that \(\delta = 3\).

Figure 2 shows the optimal sensor placement.
Table I shows all the other optimized parameters when all the nodes have the same consuming power.

Note that under the discussed policy, the two links of $C_1$ and $C_2$ are not transmitting simultaneously, thus the aggregate bit rate is equal to $C_1$ or $C_2$. It is interesting to notice that although the tree structured sensor network is symmetric with respect to the root node, the optimal locations of nodes 3 and 4 are not at the same depth. As expected, since sensor node 3 has a fewer number of neighbor nodes and is less affected by the interference of the other sensor nodes, the optimal distance between child node 3 and the parent node 1 (16.8 km) is larger than the optimal distance between child node 4 and the parent node 1 (14.2 km). It is also interesting to note that in this scenario, $C_3^* \approx C_4^*$.

VIII. NUMERICAL RESULTS FOR OPTIMAL SENSORS’ POWER ALLOCATION

First Scenario:

All the parameters are the same as in the previous section except the power of each sensor is not known, but, its location is given. Table II shows the coordinates of each sensor node, optimal transferring energy, optimal SINR for each sensor node and the optimal links’ capacities.

Similar to the optimal sensors’ location scenario, it is also interesting to note that in this scenario, $C_3^* \approx C_4^*$.

Second Scenario:

Now, let us obtain the optimal sensor power allocation in another network which is asymmetric compared to the previous graph, where the nodes are located in the following locations (Fig. 3). Table III also depicts all the related parameters of this scenario.

As we expected, the nodes located further to their parents need more power than the ones closer to the receiver nodes.

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Table III. Optimized energy allocation and other related parameters when the sensor nodes are fixed in their locations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 0</td>
<td>(8.50, 4.00)</td>
<td>1.00</td>
<td>12.96</td>
<td>437.56</td>
</tr>
<tr>
<td>2 to 0</td>
<td>(-8.50, 4.00)</td>
<td>-0.79</td>
<td>87.5</td>
<td></td>
</tr>
<tr>
<td>3 to 1</td>
<td>(20.00, 12.00)</td>
<td>0.35</td>
<td>2.35</td>
<td>144.34</td>
</tr>
<tr>
<td>6 to 2</td>
<td>(-20.00, 12.00)</td>
<td>0.84</td>
<td>2.32</td>
<td>143.55</td>
</tr>
<tr>
<td>4 to 1</td>
<td>(6.5, 18.00)</td>
<td>0.35</td>
<td>2.35</td>
<td>144.34</td>
</tr>
<tr>
<td>5 to 2</td>
<td>(-6.5, 18.00)</td>
<td>0.84</td>
<td>2.32</td>
<td>143.55</td>
</tr>
</tbody>
</table>

IX. NUMERICAL RESULTS FOR OPTIMAL SENSORS PLACEMENT AND SENSORS’ POWER ALLOCATION:

We take the same network parameters we had for previous sections to solve (8). Figure 4 shows the optimal sensor locations and also optimal power allocation for each node at the tree structured network explained earlier.
According to Fig. 4 and also the calculated information inserted in Table IV, we see that the optimal tree structured UWSN is the one that is symmetric. This could play an important role in the network configuration design. For the UWSNs with more levels, we can consider a symmetric structure and then find the optimal locations and transmitting power for each sensor node.

Table IV also shows the other related parameters in the network. It is clear that the power consumption in the levels closer to the water surface is dramatically larger. One possible solution for this huge difference between the power consumption, channel capacity and SINR of acoustic links at different levels would be to deploy more sensors in the levels closer to the water surface. This can be led to change the tree structured configuration of the network so that we start with a specific shape and we can end up with another structure in order to improve the network parameters.

Table IV. Optimized sensors’ locations and optimized energy allocation and other related parameters when the both are unknown

<table>
<thead>
<tr>
<th>Wireless Links (node to node)</th>
<th>Optimal Locations Km</th>
<th>Optimal Power Watt</th>
<th>Optimal SINR dB</th>
<th>Optimal links’ Capacity bps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 0 2 to 0</td>
<td>(7,2) (-7,2)</td>
<td>1.11</td>
<td>48.4230</td>
<td>562.71</td>
</tr>
<tr>
<td>3 to 1 6 to 2</td>
<td>(12,3) (-12,3)</td>
<td>0.01</td>
<td>2.6500</td>
<td>186.79</td>
</tr>
<tr>
<td>4 to 1 5 to 2</td>
<td>(6,3) (-6,3)</td>
<td>0.01</td>
<td>2.7013</td>
<td>188.80</td>
</tr>
</tbody>
</table>

X. CONCLUSIONS

The problem of sensor node placements and their power allocations for a tree shaped topology multi-hop underwater wireless system has been investigated using KKT conditions in various scenarios. We have been able to calculate vertical and horizontal distances between each sensor nodes/interest levels.

For future work, we may consider the coverage area for each sensor node in the tree structure UWSN. Each sensor node can detect a specific area around it and this issue will diminish the area of coverage by the UWSN. Furthermore, we can investigate the effect of adopting different kinds of policies on the optimal sensor location and also optimal sensor power allocation. For instance, we may want to see the optimal locations of sensor nodes in the tree structured sensor network when all the nodes are transmitting their information at the same time. Comparing the results with the ones which we calculate from our policy would be interesting. Consequently, we would find the optimum policy according to the energy consumption by each sensor node and also comparing channels capacities for the acoustic links in the tree structured UWSN.

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