Supplemental Material for:

Examining the roles of the easterly wave critical layer and vorticity accretion during the tropical cyclogenesis of Hurricane Sandy

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1. Introduction
The tropical cyclogenesis model described in Dunkerton et al. (2009), and tested herein for the case of Hurricane Sandy (2012), provides a comprehensive description of the dynamics and thermodynamics that lead to the formation of a tropical cyclone. The model outlines the relevant physical processes and multi-scale interactions and can be summarized by the following three hypotheses (reproduced from Dunkerton et al. 2009):

**H1.** Proto-vortex cyclonic eddies instrumental in TC formation are intimately associated with the parent wave’s critical latitude in the lower troposphere. The critical layer and Kelvin cat’s eye within, formed as a result of the wave’s finite-amplitude interaction with its own critical latitude, contain a region of cyclonic rotation and weak straining/shearing deformation in which synoptic waves and mesoscale vorticity anomalies, moving westward together, amplify and aggregate on a nearly zero relative mean flow. This multi-scale interaction provides a dynamical pathway to “bottom-up” development of the proto-vortex from below.

**H2.** The critical-layer cat’s eye of the parent wave provides a set of quasi-closed material contours inside of which air is repeatedly moistened by convection, protected to some degree from lateral intrusion of dry air and impinging vertical shear, and (thanks to its location near the critical latitude) able to keep pace with the parent wave until the proto-vortex has strengthened into a self-maintaining entity.
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The parent wave is maintained and possibly enhanced by diabatically amplified eddies within the wave (proto-vortices on the mesoscale), a process favored in regions of small intrinsic phase speed.

This cyclogenesis sequence is likened to the development of a marsupial infant in its mother's pouch, where the “juvenile” proto-vortex is carried along and protected by the “mother” wave until it is strengthened into a self-sustaining entity. The cat's eye within the wave critical layer is thus dubbed the “wave's pouch”. Although the dynamical and thermodynamic concepts that are the underpinning of the marsupial paradigm are now familiar to many atmospheric scientists, the terminology used to describe these processes is somewhat unique. For this reason, we have devised this “marsupial primer” as a quick reference material to supplement our analysis of Hurricane Sandy. This primer can be used also as a reference for many of our publications to assist those who are unfamiliar with the concepts of the marsupial paradigm. The material posted herein has been collected and in some cases republished from several sources of peer-reviewed papers, most prominently: Dunkerton et al. (2009), Wang et al. (2010a), and Rutherford and Montgomery (2012).

2. Simple Schematics of the Marsupial Paradigm

Figure 1 is a schematic depicting the operational application of the marsupial paradigm. In this schematic, there are two sets of streamlines, the first, depicted by the green dashed lines, indicate the flow in the resting frame of reference (Fig. 1). Note that in this resting frame, there is no indication of a closed cyclonic circulation and the familiar Inverted-V signature of a tropical easterly wave is present. In the frame of reference moving with the parent wave (i.e., the co-moving frame), the dividing streamline is represented by the thick black line (Fig. 1). This boundary represents the wave pouch. As streamlines (or isopleths of constant streamfunction) are an approximation of particle trajectories in the co-moving frame, particles within the wave pouch are trapped inside and recirculate. Within this recirculating region, deep moist convection is favored and the air is repeatedly moistened by convection (Pink scalloping, Fig. 1). From a dynamical point of view, the wave pouch protects the mesoscale proto-vortex located within from
entrainment of dry air (as indicated in Fig. 1 by the Saharan Air Layer) and impinging vertical wind shear, allowing the proto-vortex to grow. The quasi-closed nature of the wave pouch is also illustrated in Figure 1. While these openings are often times insignificant, recent studies have shown that large openings in the wave pouch can be either detrimental (in the case of ex-Gaston, Rutherford and Montgomery 2012) or beneficial (in the case of pre-Sandy) to tropical cyclogenesis, depending on the properties of the surrounding environment.

Observational studies have shown that, within the wave pouch, tropical cyclogenesis occurs near the sweet spot position within the wave’s pouch. The sweet spot is defined as the intersection of the wave trough (thick black line, Fig. 1) with the critical latitude (thick purple line, Fig. 1). The wave trough is defined as the set of points where the meridional flow is zero \( (v=0) \). The critical latitude is defined as the locus of points where the phase speed of the parent wave is equal to the background zonal flow \( (u= Cp) \). The sweet spot has been shown to be the favored region for low-level organization/concentration of vorticity substance, upscale growth of vorticity, and organization of deep moist convection.

Figure 1. Schematic of the marsupial paradigm. Adapted from Wang et al. (2010a).
Lagrangian flow fields can provide a more accurate representation of the wave pouch and recirculation region because they account for the time dependence of the flow field. For the case of a neutral easterly wave in the hypothetical form of a single normal mode, the simple co-moving analysis would yield an exactly time invariant flow. However, easterly wave disturbances in the real world are generally not simple neutral normal mode disturbances devoid of cumulus convective activity within, and the environmental flow is generally time varying. Figure 2 provides a schematic of the intersection of the stable (red) and unstable (blue) manifolds during a typical tropical cyclogenesis sequence. In general terms, the stable manifolds represent a repeller to particles (as indicted by the thin black arrows) while the unstable manifolds act as an attractor to particles. In this co-moving frame, a clear view of the recirculating flow geometry is seen by a saddle point often appearing outside of the pouch, and stable and unstable manifolds forming the pouch boundary (Figure 2, Rutherford and Montgomery 2012). The unstable manifold encloses the wave pouch, while the region between the stable and unstable manifolds is a pathway for environmental air to reach the circulation center. In the case that the flow is steady and the stable and unstable manifolds of a single fixed point coincide, a teardrop-shaped boundary is formed and blocks any transport into or out of the pouch (Rutherford and Montgomery 2012). If the manifolds of a pair of fixed points coincide, a cat’s eye boundary is formed (Rutherford and Montgomery 2012).
3. Glossary of Scientific Terms

This glossary is republished from Dunkerton et al. (2009).

**bottom-up development:** A pathway to tropical cyclogenesis that assigns logical and chronological precedence to phenomena in the lower troposphere prior to the upward growth of a tropical depression-strength vortex. These phenomena may occur at meso-α (e.g., critical layer formation in an easterly wave), meso-β (e.g., low-level convergence due to a convective heating profile, Ekman convergence induced by boundary-layer drag) or meso-γ (convective heating profile associated with one or more vortical hot towers).

**coherent structure:** In geophysical (quasi-2-D) turbulence, an unusually large or long-lived flow feature (e.g., solitary wave, isolated horizontal eddy, or train of Kelvin cat’s eyes in a nonlinear critical layer) that disrupts locally or episodically the turbulent aspects
of the flow: viz., disorder, particle dispersion, and energy or enstrophy cascades. The Great Red Spot on Jupiter is a well-known example.

**co-moving frame:** A frame of reference translating horizontally at the velocity of the parent wave or proto-vortex. Streamlines or stream function appear different in resting versus co-moving frames, unless the translation speed is zero trivially. The proper choice of translation speed may vary with height owing to vertical shear and wave-vortex interaction.

**critical layer:** The region surrounding a nonlinear wave’s critical latitude or level in shear flow. In the enclosed Kelvin cat’s eye, particles are trapped and recirculate, rather than being swept one way of the other by the surrounding shear. Reversible undulations of particles immediately adjacent to the cat’s eye on either side are included in the definition.

**critical points:** In 2-D flow, the points where both components of horizontal velocity vector vanish. The nature of a critical point is determined by the local velocity gradient tensor. Two examples are (i) a gyre center, to or from which particles may converge or diverge; (ii) a saddle, or hyperbolic point, where a closed gyre is connected to the outer flow, and particles simultaneously converge (along one axis) and diverge (along the orthogonal axis).

**dividing streamline:** In steady 2-D flow, the streamline intersecting a nearby saddle point. Particles on opposite sides of the dividing streamline belong to different manifolds of the flow.

**easterly wave:** A prominent synoptic-scale feature of the summer tropics, centered ten or more degrees off the equator just poleward of the ITCZ, originating and propagating westward across tropical Africa, the tropical Atlantic, and eastern or central tropical Pacific. They are usually accompanied by cloudiness near the trough, and have been
documented in radiosonde data and satellite imagery since the early 1950s and 1970s, respectively.

**Hydrodynamic instability of ITCZ:** In theoretical literature the Intertropical Convergence Zone (ITCZ) is idealized as an east-west vortex strip which, once perturbed, may undergo subsequently a (dry or moist) form of barotropic instability, resulting in excitation of zonally propagating waves and coherent vortices of finite horizontal size embedded therein. According to the marsupial paradigm, a quasi-closed gyre generated by large-scale instability may become the “mother pouch” of an incipient tropical cyclone if environment and mesoscale processes allow.

**Marsupial paradigm:** A conceptual framework introduced in this paper to describe how a hybrid diabatic Rossby wave/vortex (existing in a wave critical layer or in isolation) may become a tropical depression and thereafter, a tropical cyclone. The marsupial paradigm embraces three new hypotheses regarding (H1) the vortical organization of the critical layer at meso-α, (H2) moisture supply to the proto-vortex and protection from dry or dusty air outside, and (H3) a mutually beneficial interaction of wave and vortex. The simpler term pouch theory has been introduced to encompass H2.

**Multi-scale interaction:** A mathematical and theoretical concept describing how processes on different scales interact simultaneously, and in some cases, synergistically to support one another. An important result of multi-scale interaction is that turbulent cascades are interrupted or “shortcircuited” by instantaneous communication across widely separated scales. Another is long-range interaction, whereby waves communicate information to large distances, faster than is possible in advective turbulence. The co-existence and possible synergy of waves and turbulence (or of waves and vortices) is arguably the most important form of multiscale interaction in geophysical fluid dynamics.

**Okubo-Weiss parameter:** A measure of the shape-preserving component of a vortical flow, in comparison to (i) the shape-destroying component associated with shearing deformation, and (ii) straining deformation. The sign convention is arbitrary: “positive”
may be chosen to emphasize the former, in shape-preserving phenomena (e.g., TC genesis) or the latter, in filamentary phenomena (e.g., stirring and mixing).

**organized deep moist convection**: Tropical convection, particularly over the oceans, is often organized on subseasonal time scales by atmospheric dynamics rather than by fixed features such as coasts and topography. The form of organization varies widely, and may be associated with mesoscale convective systems, density currents, gravity waves, equatorial waves, and extratropical intrusions. When convection is phase-locked to the underlying dynamic structure to such an extent that this structure cannot be understood without the effect(s) of the convection, we have a convectively coupled disturbance.

**proto-vortex**: The initial vortical structure within a hybrid diabatic Rossby wave/vortex, which may subsequently grow to a tropical depression-strength vortex. The fine structure of a proto-vortex is seldom observed directly; even the best meteorological analyses visualize such a disturbance as if “under a pane of knobbly glass” in the famous words of Michael McIntyre.

**saturation fraction**: A column-integrated measure of moisture relative to what the column can hold, given the observed temperature versus pressure in the column.

**translating closed gyre**: A structure that may be identified in meteorological analyses of stream function, streamlines or trajectories, by viewing a tropical wave critical layer in its co-moving frame, i.e., a frame of reference moving at the phase speed of the wave. For stream function, the translation can be achieved directly by adding the appropriate gauge function.

**tropical depression**: Although no formal definition can be found, a tropical depression-strength vortex is thought to exist when circular horizontal winds approach 30 knots and deep, persistent convection is well organized by a putative closed circulation. It can be said that forecasters regard a proto-structure with features loosely resembling a mature hurricane as a good candidate for a “depression” – the more such features, the better.
Their classification evidently guarantees (in all but a few instances) subsequent growth to a named tropical storm. It is not only the statistical narrowness of the tropical depression classification, but a physical threshold for instability leading to intensification, that is responsible for their success. In other words, “tropical depression” is a physically meaningful, albeit imprecisely defined, concept.

**vortical hot towers**: Deep moist convective clouds that rotate as an entity and/or contain updrafts that rotate in helical fashion (as in rotating Rayleigh-Benard convection). Although early observations suggested that VHTs are neither necessary nor sufficient for tropical cyclogenesis, it is becoming increasingly clear from cloud representing numerical simulations that moist vortical updrafts are the essential building blocks of the tropical storm within the rotating proto-vortex. These hot vortical plume structures amplify pre-existing cyclonic vorticity and equivalently induce concentrations of vorticity substance much larger than that of the aggregate vortex.