Logistic Regression Model on Antenna Control Unit Autotracking Mode

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Abstract

Over the past several years DoD has imposed constraints on test deliverables, requiring objective measures of test objective results, i.e., statistically defensible test and evaluation (SDT&E) methods and results. These constraints force tester to employ statistical hypotheses, analyses and modeling to assess test results objectively, i.e., on statistical metrics, analytical methods, probability of confidence complemented by, rather than relying solely on expertise, which is too subjective an approach. In this paper we discuss methods of objectifying testing via statistical modeling and decide for a null- or alternative-hypothesis. This paper will present an Antenna Autotracking model using Logistic Regression modeling. This paper presents an example of tools useful for employing a SDT&E methodology.
ABSTRACT

Over the past several years DoD imposed constraints on test deliverables, requiring objective measures of test results, i.e., statistically defensible test and evaluation (SDT&E) methods and results. These constraints force testers to employ statistical hypotheses, analyses and modeling to assess test results objectively, i.e., based on statistical metrics, analytical methods, probability of confidence complemented by, rather than solely on expertise, which is too subjective. In this and companion papers we discuss methods of objectifying testing. We employ an earth coordinate model and statistical modeling of telemetry (TM) tracking antenna employing time-space position information (TSPI) and derived statistical measures for tracking-error and auto-tracking mode. Test data were statistically analyzed via analysis of covariance (ANCOVA) which revealed that the antenna control unit (ACU) under test (AUT) does not track statistically identically, nor as practically or efficiently in C-band while receiving data carriers in both S- and C-bands. The conclusions of this paper add support to that hypothesis. In this third of three papers we use data from a range test, but make no reference to the systems under test as the purpose of this paper is to present an example of tools useful for employing a SDT&E methodology.

KEY TERMS

H₀, tracking mode, tracking error, TM, AGC, ACU, AT, Scan, Slew, GPS, TSPI samples, observables and predictors, ANOVA, ANCOVA, F-test, t-test, PDF, statistical model, inner-product, linear predictor, link function, logit, logistic regression, stochastic filter.

INTRODUCTION

Using statistical analyses we decided to reject our test’s null-hypothesis, H₀ on objective criteria derived from ANOVA and t-test. The test null hypothesis is:

\[ H_0: \text{AUT tracking modes on C-band are statistically independent of data carrier}. \]

The AUT tracks on signal strength sampled via automatic gain control (AGC) circuits, and on controllable conic-scan (Scan) and antenna slew (Slew) rates. The test includes a C-band AUT tracking of C- and S-band carriers. Our measure of tracking is the probability of autotrack mode (AT) sampled from the AUT and regressed against AUT observed AGC and rate controls. Ideally the tracking algorithm should have no dependence on observed angles, and the tracking

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a Rf. 3
b Rf. 1 & 2.
c Rf. 1, 2, 4, 7 & 8.

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algorithm is assumed to control the antenna center pointing angles independent of angle to target, i.e., the conic-scan centering is controlled via scanned signal strength. Our work bears this out: the better models exclude angle data. This paper is an extension of previous analyses of tracking as a synthesis of a tracking model that predicts autotracking based on tracking modes. A full predictor stochastic filter can be built on the predictor updates and angle errors of a linear model, where error is based on difference of observed and calculated angles of a tracking-error model. This paper presents an auto-tracking, or mode model.

LOGISTIC REGRESSION MODEL

TM signal power is modeled as bounded continuous parameters received and sampled from automatic gain control (AGC) circuits of the AUT. As well, azimuth and elevation angles are continuous parameters. The AUT employs two categorical tangential or rate control parameters we denote Scan and Slew. Available data includes these parameters and others from the tracker ACU, as well as target TSPI, i.e., GPS and INS. Since a tracking mode should not depend on its local angle we do not include angles in the model. This was not merely a choice, but the result of model reduction from the full model. We tried various models before settling on this ‘best model’ using a linear predictor based on ACU parameters independent of target. Auto-track holds if both azimuth and elevation modes auto-track, all other modes are considered not-auto-track. Thus we have two response states, which we map into the response set, \( y: \{0,1\} \). This two-state response is the reason for a binary response, which is transformed to a continuous response \( p: [0,1] \) and regressed against a linear predictor for our model. The tracking-error model has continuous response, expected error that is linearly regressed against angle and signal strength parameters.

LOGIT of PROBABILITY:

The auto-tracking model is built on a linear predictor formed as an inner-product of an observable-control space and its regressed vector. The predictor form is

\[ \beta \cdot x = \beta_0 + \Sigma \beta_k x_k \equiv bx. \]  

This resolves to scalar. \( \beta \) designates regressed coefficients and \( x \) are sampled vectors from the observable-control space; \( \beta_0 \) designates the predictor intercept. The term, \( bx \) is a standard statistical representation of this inner-product. In the literature \( bx \) is also designated as \( \eta \). The link function is defined to yield a linear response against the predictor (which we verify when checking the model). Our link is the natural logarithm of odds:

\[ \text{link}(y) \equiv \ln(O(p)) = \ln(p/1-p); p \in [0,1]. \]  

This transforms the binary set \( y: \{0,1\} \) to a smoothed step function, or ‘S-curve’ bounding response to predictor to the interval \([0,1]\). Thus probability \( p \) (or of expectation \( \mu \)) of the link of \( y \) is the logarithm of odds, or log-odds, also known as the logit of probability. Our model derives the logit of probabilities as the linear predictor of regression. We represent this:

\[ \text{link}(y) \]
\[ y \leftrightarrow \text{logit}(p) \sim \text{lm}(bx). \]  

The odds is easily recovered via exponentiation:

\[ O(p) = \frac{p}{1 - p} = e^{bx}. \]  

From this we derive a probability of odds, which is our probability of response to observation:

\[ p(O) = \left(1 + e^{-bx}\right)^{-1} = p(f(y)). \]  

For tracking observation, \( y \) estimates the autotrack mode parameter \( AT \in \{1\}^+ \), which is regressed against other ACU parameters to transform a binary state to continuous state space \( y : \{0,1\} \rightarrow p : [0,1] \). Regression parameters are collated in an R \textit{dataframe}, shown in figure 1. We separate data by band. The probabilities (column ‘ProbAT’) are predicted by the model and appended to the frames post analysis. Each frame contains time from ACU and GPS-INS samples to align tracker and target data. We correlate at 1Hz. The observed angles and AGC values come from the ACU, the calculated angles and LoS are derived via geometric model\(^8\) that translates TSPI global position, the attitude data is from the INS, the control data and AT, which is derived from the AUT Az/El track modes.

For each band we have the two categorical tangential control parameters\(^h\): \textit{conical scan rate} encoded in two levels and planar \textit{slew rate} encoded in four levels: \( \text{Scan} \in \{2\}^+; \text{Slew} \in \{4\}^+ \). In the models categorical parameter levels are typed as \textit{factors}. The sampled data also contains continuous angle and AGC parameters. All angles are bounded modulo 360º: \( \{\text{Az.Obs, El.Obs, Yaw, Pitch, Roll}\} \subset [0, 360] \), and AGC, which is a derived parameter from truncated minimum and maximum power, ranging from \( 0 \leq \text{dB} \leq 52 \), of two tracking receivers for both C- and S-band signals. The \textit{full general linear model (glm)}, with link from the binomial family, is represented in R as:

\[ \text{glm}(AT \sim AGC + \text{Slew} + \text{Scan}, \text{family} = \text{binomial}). \]  

Time alignment correlates ACU and TSPI samples at 1Hz. Observed angles and AGC levels come from the ACU, the expected angles and line-of-sight (LoS) are derived via a

\(^8\) Rf. 1.  
\(^h\) Rf. 7 & 8.  
\(^i\) \( \{n\}^+ \equiv \{1, \ldots, n\}; \{n\} \equiv \{0, \ldots, n-1\} \).
that translates TSPI global position to local azimuth and elevation angles of LoS between tracker and target. For reasons stated earlier, we do create our full model from all parameters, and employ different parameters creating the linear error model discussed in the companion paper on tracking-error. Figure 2a shows a statistical summary of the data, sans time and angles. We include angle errors for perspective. You can see that the mean AT count is consistent with probability (ProbAT) of the model, for both bands. It is also apparent why we do not use Yaw in modeling the target signal pattern – it is constant.

A simple count of the auto-tracks (sum ‘AT’) over the categorical parameters yields the table shown in figure 2b. Of the 5021 samples culled from the source data we have 968 auto-tracks (AT = 1) and 4053 non-auto-tracks (AT = 0). A quick estimate of probability of auto-track is $968/5021 \approx 0.20$.  

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*\textsuperscript{1}Rf. 1.*
This quick look indicates about a 20% probability of being in autotrack, thus odds of $\frac{1}{4}$, over all scan and slew rate combinations. Figure 2a shows the average autotrack over both bands is $(0.2730 + 0.1550)/2 \approx 0.2140$. Thus predicted probability is consistent with the AT counts over both and across band samples. From this table we can calculate the probability over bands, scan and slew. We see in figure 2b that at Scan level 1, Slew level 4 there are no tracks. So we don’t need this level in our model. Also at Scan level 2 only Slew levels 2 and 3 auto-track and only over C-band carrier. Rather than eliminate these, we allow the regression and ANOVA\textsuperscript{k} table to reveal their insignificance. To decide on ‘the best’ model we build our model and reduce it. We reduce by eliminating or recategorizing parameters with a high $p$-value, or a low probability of information. Note that the $p$-value is not the probability of track, but a probability of significance derived from a $t$-test (displayed as $t$ or $z$ value), that the modeled parameter coefficient contributes informative rather than random model results. Once we have our reduced model, we test it against statistical metrics to determine our model’s ‘goodness of fit’.

\begin{table}[h]
\centering
\begin{tabular}{cccc}
Scan & Slew & Band & \hline
1 & 1 & C & 1484 455 \\
 & & S & 190 127 \\
 & 2 & C & 385 1 \\
 & & S & 170 134 \\
 & 3 & C & 243 44 \\
 & & S & 234 82 \\
 & 4 & C & 0 0 \\
 & & S & 271 41 \\
2 & 1 & C & 0 0 \\
 & & S & 0 0 \\
 & 2 & C & 272 14 \\
 & & S & 0 0 \\
 & 3 & C & 577 14 \\
 & & S & 0 0 \\
 & 4 & C & 0 0 \\
 & & S & 307 56 \\
Total & & & 4053 968 \\
\end{tabular}
\caption{Auto-track Counts Summary}
\end{table}

\textsuperscript{k} Rf. 1.
The regression coefficients for our reduced model for the conic-scan antenna ACU autotracking on C-band for S- and C-band carriers are shown in figure 3a and b respectively. The full model for received C-band had significant interaction terms of the Scan level 2 and Slew level 2. You can see that these are statistically significant. Slew level 4 was excluded due to singularities and the model results show that Slew level 3 is not statistically significant. We could exclude this Slew factor level from the model, but will not as it would eliminate samples of angles and gain. We keep the models as is. The full S-band carrier model shows Slew levels 3 and 4 are statically insignificant. Note that each model has very different predictor coefficients indicating the parameters’ influences in tracking are distinct to receiver band. These means of autotrack counts are the first indication of an inherent difference in tracking mode in distinct bands.
We could build a logistics model in which we eliminate the continuous parameters and instead categorize the AGC, as higher power levels should increase probability of autotrack. We examined such models and example results are shown in figure 4 for C-band carrier. It is clear that probability of autotrack mode is more sensitive to higher signal levels. But this
categorization of AGC does not improve the model’s explanatory power, as is evidenced by the AIC score (a measure of the relative quality of the statistical model) compared to the AIC for the continuous AGC in figure 3b.; but does show sensitivity to autotrack increases with gain, which is useful information. For plotting purposes of estimating the probability density and cumulative density functions we employ continuous parameters. Figure 5a and b show these plots for S- and C-band carriers, respectively.

Note the probability density (PDF) for both bands show negative probabilities; this is due to ‘kernel smoothing’ of the histogram. We can eliminate this by employing a histogram which shows the same information. I prefer to leave it as is, and realize the high density near probability zero is ‘smoothed’ into negative probabilities. Note too that receiving the S-band carrier results in a clear multimode PDF, with concentrations around $p = 0$ and $p = 1$, as expected given the high slope of the ‘S- curve’ (probability of autotrack). The C-band PDF has a less steep S-curve, with a greater dispersion. The box-plot shows a slight overlap in auto-track vs. not-auto-track near 10dB AGC.
Note the appearance in Figures 5a and 5b of what appears to be four probability curves for both bands. For S-band carriers there are actually five; one is hidden by overlap. With four slew and two scan rates we’d expect eight curves, but not all combinations were used. These separate curves are partitioned over combinations Slew and Scan for each AGC. A plot of a probability and logit vs. AGC at slew levels 1 and 2 at scan 1 is shown in figure 5c. The logit plot being linear over the AGC is a verification that the log-odds is linear w.r.t. the AGC predictor partitioned over the control set predictors. This linearity of the logit vs. predictor is an assumption essential to our model. Not only can we infer a difference in the ACU’s ability to track over the two bands, but we can also infer that the ACU tracked C-band carrier less often in autotrack mode than it did S-band carrier. The number of auto-track states, as well as the gentle slope of the probability curve and the overlap of ‘Not-Auto’ ‘Auto’ around 10dB indicate a poor tracking profile in C-band of C-band carrier.
SUMMARY AND CONCLUSION

This paper presents a rather standard synthesis of a tracking filter using observed data and antenna tracking controls to model an autotracking predictor model that explores statistical measures on observations to decide the probability of autotracking, based on received signal strength and the state of control of the ACU. That there is an error in tracking is evidenced by the data, statistics and models developed in this and the preceding paper mentioned above. Our analysis and models confirm our conclusions reached in the companion paper that we reject the null, $H_0$ and accept the alternative hypothesis:

$$H_1: \text{AUT autotacks statistically differently on C-band while receiving S- or C-band carriers.}$$
There is also a practical difference. The probability of autotrack was far more decisive while receiving an S-band carrier. As to the cause, we make no hypothesis. The purpose was to determine if there was a significant statistical difference in the probability of autotracking in C-band when receiving C- and S-band carriers. The antenna modes are not close to identically probable within or across bands. This is not a general conclusion, but rather particular to the test configuration. We could extend our synthesis: this model, along with a tracking-error model, together form essential systems of a stochastic tracking filter. Such work must await another opportunity.

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