Manipulation of the Phase-Amplitude Coupling Factor in Quantum Nanostructure Based Devices for On-Chip Chirp Compensation and Low-Cost Applications

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<td>Conducted preliminary research on nonlinear dynamical properties of injection-locked QC lasers. These findings are of prime importance for the performance enhancement of QC lasers, suggesting that an optical injection-locked experiment is now needed for further investigations. Fields of applications include gas spectroscopy, optical countermeasures or free-space communications, requiring stable single-mode operation with a narrow linewidth, high output power and high modulation bandwidth.</td>
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"Manipulation of the Phase-Amplitude Coupling Factor in Quantum Nanostructure Based Devices for On-Chip Chirp Compensation Low-Cost Applications"

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The research project aims to conduct both theoretical and experimental works in the field of nonlinear dynamics and optical injection diode oscillator findings with advanced semiconductor lasers in particular Q-well, Q-dot and Q-cascade (with Q as for Quantum) established with laser gain media to establish the next generation photonic oscillators. The funding provided by the EOARD is currently helping me in developing unique and state-of-the art experiments at Télécom ParisTech which will be used for an in-depth extraction of both static and dynamic features of near-infrared advanced semiconductor lasers operating under self-injection and/or optical injection.

As for self-injection studies, preliminary experiments have demonstrated the potentiality of using the delayed field for the chirp compensation in conventional Q-well distributed feedback (DFB) lasers. Performance of directly modulated DFB lasers are limited by the frequency chirp induced by current modulation. In this first experiment, it is demonstrated that a proper external control laser operation leads to chirp-to-power ratio (CPR) stabilization over a wide range of modulation frequencies as compared to the free-running case. Under experimentally selected optical feedback conditions, the CPR decreases significantly in the adiabatic regime from about 650 MHz/mW in the solitary case down to 65 MHz/mW. Experimental results are also confirmed by numerical investigations based on the transfer matrix method [1].

Increased differential gain is typically realized through strain, quantum confinement, or p-type doping in the active region. These methods have been applied to Q-dots or Q-dashes to raise the differential gain with limited success because the optical gain of these low dimensional systems saturates at modest values. Instead larger differential gain can be accessed at wavelengths blue shifted from the gain peak and close to optical transparency using the threshold shift induced by optical injection. Using
these approaches, greater than 50X improvement in the differential gain is achieved in an injection-locked Q-dash Fabry-Perot laser compared to its free running value. This differential gain enhancement is also connected with a zero linewidth enhancement factor and a rather flat modulation response [2]. From a system viewpoint, high-speed transmissions experiments remain a key-point to be done so as to properly demonstrate the chirp compensation as well as the purification of the eye diagram over the transmission line. To this end, it has to be stressed that long-haul applications rely on the use of single-frequency emitters working at 1550 nm (and not on Fabry-Perot lasers). To this end, a 1550 nm packaged Q-well DFB laser has been recently shipped to the US Air Force Research Laboratory, Dayton, Ohio for demonstrating the chirp compensation. Similar experiments will be conducted at Telecom Paris on a 1550 nm Q-dot DFB laser. Comparison between the two experiments will be extremely helpful to fully validate the chirp reduction which should be obtained by zeroing the linewidth enhancement factor and by increasing the laser’s absorption cross-section. Beside, such a comparison will also lead to a global understanding of the laser’s dynamics in Q-well and Q-dot gain media.

In parallel, in order to fully predict the nonlinear properties of advanced semiconductor lasers, other part of the work is connected to the development of advanced numerical models. To this end, based on a semi-analytical approach, the modulation response of an optically injected Q-dot laser is modeled taking into account the carrier dynamics in the lasing and non-lasing states [3,4]. In order to obtain the stable injection-locking regime, the local bifurcations of the Q-dot laser, namely saddle-node and Hopf bifurcations are investigated. The bifurcations are obtained by an eigenvalue analysis of the fixed point, that is, if a single, real eigenvalue passes through the imaginary axis in the complex plane, one typically finds a saddle-node bifurcation while a pair of complex conjugate eigenvalues passing through the imaginary axis corresponds to a Hopf bifurcation. The stable locking regime is bounded by the supercritical bifurcations. The regime is enlarged under high injection level. The influences of carrier capture and relaxation rates on the modulation response is also investigated. Large capture and relaxation rates are found to be favorable to enhance both the modulation bandwidth and the resonance peak. In addition, in order to further enhance the modulation bandwidth, it is helpful to suppress or even eliminate the pre-resonance dip in the modulation response mostly occurring under positive detuning. Besides, large capture time reduces both the resonance frequency and the damping factor while relaxation time contributes inversely. Since both the carrier rates can be manipulated via band engineering, these recent results can be leveraged to our benefits to engineer the next generation of Q-dot lasers with enhanced dynamical properties under optical injection for radio-over-fibre and cable-access TV applications.

Recently, the research work also started for the first time the investigation of the intensity modulation features of an optical injection-locked Q-cascade laser via a three-level rate equation model [6,7]. The locking regime is obtained based on the local bifurcation theory. The stable locking regime is bounded by the supercritical bifurcations. As for injection-locked interband lasers, the injection-locked regime enlarges with the injection ratio. In comparison with interband lasers, let us also note
that the positions of the codimension-two points move to a higher injection level. It is shown that the injection-locked Q-cascade laser exhibits a rather flat modulation response at zero detuning, whose bandwidth increases with the injection level. In contrast to interband lasers, both positive and negative detunings enhance the modulation bandwidth. Besides, a large linewidth enhancement factor can increase the peak amplitude in the response. Moreover, it is found that no frequency dip occurs in the modulation response of injection-locked Q-cascade lasers. Further study will take into account the period of stages to improve the rate equation model. The influence of the carrier lifetimes on the injection-locked properties will be investigated as well.

Main Publications:
The research project aims to conduct both theoretical and experimental works in the field of nonlinear dynamics with near and mid infrared advanced semiconductor lasers operating under self-injection and/or optical injection. The funding provided by the AFSOR/EOARD is currently helping me in developing unique and state-of-the-art basic and applied research at Télécom ParisTech that will be used to establish the next generation of photonic oscillators. Throughout the second year, several experiments and simulations have been conducted as depicted below.

1 Modulation Dynamics of Quantum Dot Lasers

Directly modulated quantum dot laser (QD) is a promising candidate for the next generation high-speed optical networks. However, it is well-known that the intrinsic modulation bandwidth is drastically limited by the low differential gain, the large gain compression factor and the slow carrier capture into the QDs. In this work, the sub-threshold relaxation dynamics is investigated for the first time. Like in figure 1, when reducing the pump current slightly below threshold, the laser’s resonance frequency unexpectedly re-increases, while that of a commercial quantum well (QW) one keeps decreasing. This typical resonance behavior is attributed to the Pauli blocking of the excited state (ES) as predicted by our rate equation model that takes into account the intradot carrier dynamics into the ground state (GS) and ES. As an extension, simulations also show quantitative agreements with the measured resonance behavior. In addition, the damping factor offset is found to be dominated by the product of the Pauli blocking and the carrier escape rate from the GS to the ES, in contrast to the sole carrier effective lifetime in the QW laser case. The offset value is found to be inversely proportional to the carrier relaxation lifetime $\tau_{\text{GS}}$ decreasing from 40 GHz for $\tau_{\text{GS}}=5.0$ ps down to 1.5 GHz for $\tau_{\text{GS}}=100$ ps. Our theory is also supported by various experiments reported in the literature in which offset values are typically in the range from 1.7 GHz up to 17.0 GHz. Finally, it is found that both the resonance frequency and the 3-dB modulation bandwidth decrease almost linearly with...
the increased $\tau_{ES}^{GS}$, while large $\tau_{ES}^{GS}$ values induce a parasitic-like roll-off in the modulation response.

Figure 1 – Squared resonance frequency and damping factor as a function of the normalized bias current. Blue spheres are the experimental results and red stars denote the simulation data. Inset is the case of the commercial QW laser for comparison.

2 Enhanced Dynamics of a Quantum Dot Laser Emitting on the Excited State Transition

Directly-modulated QD semiconductor lasers are promising sources for high-speed telecommunication networks. Owing to the three-dimensional quantum confinement, the QDs consist of discrete GS and ES. Stimulated lasing in the GS has been widely studied, nevertheless it was found that the modulation dynamics is still limited by the slow carrier scattering process, the low differential gain and the non-zero linewidth enhancement factor (LEF or $\alpha$-factor). Although the ES lasing requires a larger threshold current, its faster carrier capture rate from the surrounding carrier reservoir (RS) as well as higher saturated gain could provide enhanced dynamic features. Indeed, the $K$-factor limiting the maximum bandwidth for the ES lasing was proved to be roughly two-fold higher than that of GS lasing. In this grant, we have theoretically demonstrated that the lasing in the ES has additional merits lower chirp-to-power ratio (CPR) and smaller LEF in contrast to the GS. QD lasers can be accurately described utilizing a semiclassical approach within the semiconductor-Bloch framework. In the previous work, we have derived the susceptibility of QD lasers microscopically. It was shown that the carrier population in the off-resonant states induced significant variation of the refractive index while carriers in the resonant state contributed mainly to the gain. Adiabatically eliminating the microscopic polarization, the slowly varying electric field $E_{ES}(t)$ of the ES lasing is given by:

$$\frac{dE_{ES}(t)}{dt} = \frac{1}{2} (\Gamma_p v_g g_{ES} - 1/\tau_p) E_{ES}(t) + jE_{ES}(t) (\Delta\omega_{ES}^N + \Delta\omega_{GS}^N + \Delta\omega_{RS}^N)$$  \hspace{1cm} (1)$$

where the carrier population in the ES, GS and RS induced frequency shifts of the laser field respectively are

$$\Delta\omega_{ES}^N = \frac{1}{2} (\Gamma_p v_g g_{ES} - 1/\tau_p) \alpha_H^{ES}$$  \hspace{1cm} (2)$$

$$\Delta\omega_{GS}^N = \frac{1}{2} \Gamma_p v_g g_{GS} F_{GS}$$  \hspace{1cm} (3)$$

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\[ \Delta \omega^N_{RS} = \frac{1}{2} \Gamma_p v_g g_{RS} F_{RS} \quad (4) \]

with
\[ G_{GS,RS} = \frac{\omega_{ES}}{\omega_{GS,RS}} \left[ \frac{(\omega_{GS,RS} - \omega_{ES}) T_D}{1 + (\omega_{GS,RS} - \omega_{ES})^2 T_D^2} \right] \quad (5) \]

\( g_X \) (\( X \) denotes GS, ES, RS) gives the gain in each state, and \( \omega_X \) is the corresponding frequency. \( \tau_p \) is the photon lifetime in the cavity, \( \Gamma_p \) is the optical confinement factor, \( v_g \) is the group velocity of the light, and \( T_D \) is the polarization dephasing time. \( \alpha^{ES}_H \) originates from the asymmetric inhomogeneous broadening and state filling of the resonant ES, which is treated as a fixed value in this work. Coupling the electric field equation (1) with the QD carrier dynamics, we obtain the described rate equation model for the QD laser. From the standard small-signal analysis, the differential rate equation is derived for the following semi-analytical study. The frequency chirp property is characterized by the Chirp-to-Power ratio (CPR), which is defined as \[ \left| \frac{\delta \phi}{\delta P} \right| \] with \( \delta \phi \) being the phase variation, \( \delta P \) the power variation and \( \omega_m \) the modulation frequency. The \( \alpha \)-factor describing the coupling between the carrier-induced variations of the laser field frequency and the gain is defined as,
\[ \alpha_H = \frac{2}{\Gamma_p v_g} \frac{\delta \left( \Delta \omega^N_{ES} + \Delta \omega^N_{GS} + \Delta \omega^N_{RS} \right)}{\delta g_{ES}} \quad (6) \]

**Figure 2** – (a) 3-dB modulation bandwidth versus the normalized current \( I_{GS,ES}/I_{th-GS,ES} \). Inset is the IM response at \( I_{GS,ES}=1.2 \times I_{th-GS,ES} \) for each lasing state; (b) The CPR comparison at the modulation frequency \( f_m=5 \) GHz. Inset is the CPR versus the modulation frequency at \( I_{GS,ES}=1.2 \times I_{th-GS,ES} \).

For the QD laser under consideration, the threshold current of ES lasing (\( I_{th,ES}=90 \) mA) is about 1.8-fold larger than the lasing in GS (\( I_{th,GS}=49 \) mA). Inset of Fig. 1(a) shows the intensity modulation (IM) response at bias current \( I_{GS,ES}=1.2 \times I_{th-GS,ES} \) for each lasing state. As expected, the ES lasing bandwidth is indeed higher than the GS lasing one because of the larger differential gain and faster carrier capture rate. It is known that the modulation response is controlled by the resonance frequency and the damping factor, both of which increases with the bias current, so is the 3-dB modulation bandwidth as shown in Fig. 2(a). However, the laser reaches the maximum bandwidth when it becomes overdamped. The maximum bandwidth for the ES is about 1.3-fold larger than for the
Figure 3 – LEF variation with pump current for GS lasing ($\alpha_{H}^{GS}=0.5$) and for ES lasing ($\alpha_{H}^{ES}=0.5$), respectively.

GS. Inset of Fig. 2(b) presents the CPR as a function of the modulation frequency. For modulation frequencies less than 1.0 GHz where the adiabatic chirp dominates, the CPR value remains almost constant; while for larger modulation frequencies, it increases almost linearly. Fig. 2(b) shows that the CPR decreases with the increasing bias current. For the modulation frequency at 5 GHz, the ES CPR is only half of the GS one. The LEF below threshold is calculated by the amplified spontaneous emission (ASE) method, while the one above threshold is obtained by the FM/AM method. Fig. 3 shows that both GS and ES LEFs are enlarged by the pump current. Below threshold, the off-resonant states for the GS lasing case increase the LEF (Fig. 2(a)) while it is reduced for the latter case (Fig. 2(b)). Above threshold, the ES LEF is found to be less than 65% of that for the GS. In conclusion, the research conducted in this grant has proved that the ES lasing in a QD laser is more favorable than the GS case in terms of modulation bandwidth, frequency chirp and LEF.

3 Phase-Amplitude Coupling of Optically-Injected Nano-structured Semiconductor Lasers

In semiconductor lasers, the $\alpha$-factor that is defined by the complex optical susceptibility, plays a crucial role in driving various fundamental features of semiconductor lasers such as the spectral linewidth, frequency chirp, mode stability, as well as nonlinear dynamics subject to optical injection and optical feedback. QD lasers have been predicted to exhibit superior dynamical properties compared to their QW counterparts. On the other hand, the QD lasers have more complex carrier dynamics than the latter, which eventually makes the behavior of the $\alpha$-factor different to that of QW lasers. Indeed, the reported $\alpha$-factor of QD lasers varies from 0 up to more than 10, while that for QW lasers is 3 5. My research group has investigated the phase-amplitude coupling of the injection-locked QD lasers taking into account the impacts of carrier dynamics in the carrier reservoir and in the ES.

The rate equation model is based on an improved modeling of the complex electric field, taking into account the contribution of the off-resonant states (ES and RS) on the refractive index change. Direct current modulation of semiconductor lasers induces changes
both in the gain and in the refractive index, the coupling of which can be described by the phase-amplitude coupling of the electric field:

\[ A = \frac{2j\omega \delta\phi}{\Gamma_p v_g \delta g} \]  

(7)

where \( \delta\phi \) and \( \delta g \) are variations of phase and gain under small signal modulation, respectively. \( \omega \) denotes the modulation frequency, \( \Gamma_p \) is the optical confinement factor and \( v_g \) is the group velocity of the light. In solitary semiconductor lasers, the phase-amplitude coupling \( A \) is regularly characterized by the FM/AM ratio \( 2\beta/m \) (\( \beta \) is the frequency modulation (FM) index, \( m \) is the amplitude modulation (AM) index) in measurements, that is,

\[ \frac{2\beta}{m} = 2S \frac{\delta\phi}{\delta S} \approx A \]  

(8)

with \( S \) being the photon intensity. It is noted that both values of \( A \) and \( 2\beta/m \) strongly depend on the modulation frequency. The conventional \( \alpha \)-factor is usually extracted from the minimum of \( 2\beta/m \) at high enough modulation frequencies.

When a QD laser is subjected to external optical injection, the phase-amplitude coupling of the laser field is derived as:

\[ A_{\text{inj}} = \frac{j\omega}{j\omega + k_c \cos \phi} \sqrt{S_{\text{inj}}/S_{\text{GS}}} \left( A_{\text{sol}} + \frac{k_c \sin \phi}{\Gamma_p v_g a_{\text{GS}}} \sqrt{S_{\text{inj}}/S_{\text{GS}}} \delta N_{\text{GS}} \right) \]  

(9)

with

\[ A_{\text{sol}} = \alpha_H^{GS} + \frac{1}{2} F_{ES} a_{\text{ES}} \delta N_{\text{ES}} + \frac{2}{a_{\text{GS}}} F_{RS} a_{\text{RS}} \delta N_{\text{RS}} + 2 F_{RS} a_{\text{RS}} \delta N_{\text{GS}} \]  

(10)

where \( N_X \) and \( a_X \) are the carrier density and differential gain in each state, respectively (\( X \) indicates GS, ES, RS). \( S_{\text{inj}} \) is the injected photon density from the master laser, \( S_{\text{GS}} \) is the density of photons emitted from the slave laser, and \( k_c \) is the coupling efficiency of the master laser into the slave laser. In the bracket of Eq. (9), \( A_{\text{sol}} \) describes the phase-amplitude coupling of the solitary laser. As shown in Eq. (10), the first term \( \alpha_H^{GS} \) denotes the inhomogeneous broadening induced part of the \( \alpha \)-factor. The second and the third terms respectively represent contributions of the ES and the RS, in which the coefficient \( F_{ES} \) (\( F_{RS} \)) is determined by the polarization dephasing time as well as the energy separation.
between ES (RS) and GS. The other term in the bracket is attributed to the optical injection. In contrast, the FM-to-AM ratio of injection-locked laser is given by:

\[
|2\beta/m|_{inj} = A_{sol} \left[ \frac{(j\omega + 1/\tau_p - \Gamma_p v_{gGS}) + k_c \sqrt{S_{inj}/S_{GS}} (\sin \phi/A_{sol} - \cos \phi)}{j\omega + k_c \sqrt{S_{inj}/S_{GS}} (\cos \phi - A_{sol} \sin \phi)} \right]
\]

(11)

with \( \tau_p \) being the photon lifetime. Removing the injection terms in Eqs. (9) and (11) gives the corresponding solutions for the solitary laser, which are almost the same as shown in Figs. 4(a) and (b) (solid lines). However, under optical injection, \( A_{inj} \) becomes quite different to \( |2\beta/m|_{inj} \). At low modulation frequencies less than 1 GHz, \( |2\beta/m|_{inj} \) remains constant instead of reducing with the modulation frequency, while it is increased at high frequencies. In contrast, \( A_{inj} \) increases with the modulation frequency and exhibits a resonance due to the injected field. Beyond the resonance frequency, \( A_{inj} \) begins to decrease. In addition, both behaviors of \( A_{inj} \) and \( |2\beta/m|_{inj} \) strongly depend on the injection ratio and the frequency detuning. Like the solitary laser case, we suggest to define the value of \( A_{inj} \) at large enough frequency as the \( \alpha \)-factor of optically injection-locked lasers. In such way, as shown in Fig. 4(b), the \( \alpha \)-factor is slightly enhanced by optical injection comparing to that of the solitary laser. Fig. 5 shows the experimental measurements of the FM-to-AM ratio for both a solitary QD laser (Fig. 5(a)) and for the laser subjecting to optical injection (Fig. 5(b)). The experiments are qualitatively in good agreement with the simulations depicted in Fig. 4(a).

\[\text{Figure 5 – (a) Measured FM-to-AM ratio for the solitary QD laser and the extracted } \alpha \text{-factor (green line); (b) The FM-to-AM ratio for the laser subjected to optical injection with an injection ratio } R=2.0 \text{ and a frequency detuning of } -4.0 \text{ GHz.}\]

As a conclusion, injection-locked QD lasers exhibit different behavior of the FM-to-AM ratio in comparison with the solitary laser case. For the first time to our knowledge, the research has revealed that the commonly used FM/AM technique is no longer suitable for extracting the \( \alpha \)-factor of injection-locked lasers.

4 Chirp Reduction in an Optically-Injected Quantum Dot Lasers

Direct modulated lasers (DML) are promising transmitters for low-cost optical access networks, because of the absence of external modulators. However, one main drawback of DMLs is the high chirp characteristic due to its destructive interaction with the fibre’s chromatic dispersion. Optical injection is a robust technique to reduce the frequency chirp.
as well as to enhance the laser’s modulation bandwidth. The present work investigates the amplitude (AM) and frequency (FM) modulation characteristics of an injection-locked QD laser operating under direct small-signal modulation. It is found that the phase difference between the FM and AM signals is enlarged by the optical injection, and proper tuning of the injection condition can lead to single sideband (SSB) modulation. Optical injection changes the behaviour of the FM-to-AM ratio depending on the injection strength and the frequency detuning between the master and the slave lasers. In addition, as already pointed out in the previous section, the theoretical analysis also points out that the FM/AM technique cannot properly characterize the linewidth enhancement factor ($\alpha$-factor) of semiconductor lasers subjecting to optical injection.

![Schematic of the experimental setup](image)

**Figure 6 –** Schematic of the experimental setup. PMF: polarization maintaining fibre; HWP: half-wave plate; QWP: quarter-wave plate; (b) An illustration of the measured intensity (black) and chirp (blue) waveforms at a modulation frequency of 6 GHz subjecting to optical injection.

The laser under study is a QD Fabry-Perot laser with light emission peaking around 1550 nm. Fig. 6(a) shows the experimental setup for the time-resolved chirp measurement of the injection-locked QD laser. The injection ratio $R$ and the detuning frequency are controlled by the tunable master laser. The optical discriminator is a fibre-based interferometer, which consists of polarization maintaining fibres (PMF) and a phase control unit (a half and a quarter wave plate (HWP, QWP) and two polarizers). The interferometer enables to convert frequency deviation into amplitude deviation. A high-speed sampling oscilloscope is synchronized with the sine-wave signal from the waveform generator, and is used to record the data. The AM and FM waveforms are extracted based on a Mach-Zehnder interferometer based system. Fig. 6(b) shows an example of the measured intensity (black) and chirp (blue) waveforms. Due to the injected field, these two signals are largely out of phase. The AM ($m$) and FM ($\beta$) indices as well as the phase are obtained by fitting the waveforms. Fig. 7(a) presents the variation of the phase difference as a function of the modulation frequency for both the solitary laser and the laser subject to optical injection.
Generally, high modulation frequency increases the phase difference. For the solitary laser, the intensity and the chirp (FM) are almost in phase at low frequencies (< 0.5 GHz), and increases to be about $0.45 \pm \pi$ at 6 GHz. Thus, the optical spectrum exhibits two symmetric sidebands in the solitary laser. Optical injection enhances the phase difference: at 6 GHz, the difference is $0.9 \times \pi$ for $R=2.0$, and increases to $1.18 \times \pi$ for $R=5.0$. Due to this large phase difference close to $1.0 \times \pi$, the optical spectrum under injection shows very asymmetric sidebands as shown in the inset. Fig. 7(b) illustrates the FM-to-AM ratio of the laser under study. As usual, the ratio of the solitary laser $|2\beta/m|_{inj}$ exhibits giant values at low modulation frequencies, i.e., in the adiabatic chirp regime; and decreases to a constant value beyond 3 GHz, which gives the $\alpha$-factor of the solitary laser. When the laser is subjected to external optical injection, it is found that the behaviour of FM-to-AM ratio becomes quite different. As expected the adiabatic chirp is well suppressed. At an injection ratio of 2.0, $|2\beta/m|_{inj}$ keeps constant for frequencies lower than 0.4 GHz, and then increases to 8.5 at 6 GHz. In contrast for $R=5.0$, $|2\beta/m|_{inj}$ remains constant around 3.0 for the whole modulation range.

![Graph](image)

**Figure 7** – (a) Measured phase difference between the FM and the AM responses. Inset is an optical spectrum with 8.4-dB asymmetric sidebands under optical injection; (b) Measured FM-to-AM ratio versus the modulation frequency. The frequency detuning for (a) and (b) is set at -4.0 GHz.

In summary, optical injection enlarges the phase difference between the chirp and the AM signals of the QD laser by more than $\pi/2$. The variation of the FM-to-AM ratio versus the modulation frequency is also strongly altered by the injected field hence suppressing the adiabatic chirp. However, the former does not provide the $\alpha$-factor of the injection-locked laser with unambiguity.

5 Nondegenerate Four-Wave Mixing in a Dual-Mode Injection-Locked Quantum Dot Lasers

Optical wavelength conversion plays an important role in wavelength division multiplexed (WDM) systems. NDFWM in semiconductor gain media is a desirable technique for wavelength conversion due to its ultrafast nature and transparency to the modulation format of the signals. In addition, since the converted signal is the phase-conjugate replica of the input signal, it also provides the possibility for fiber dispersion compensation in long distance transmission systems. In contrast to QW materials, QD ones exhibit various advantages such as a wider gain spectrum, ultrafast carrier dynamics, higher nonlinear gain...
effect and a larger three-order nonlinear susceptibility. In addition, due to the reduced \( \alpha \)-factor, QDs have the possibility of eliminating destructive interference among the nonlinear processes and also offer an enhanced efficiency in the wavelength up-conversion. In order to improve the dynamical performance of semiconductor lasers, the optical injection-locking technique has been widely used to reduce the spectral linewidth, frequency chirp as well as to suppress relative intensity noise and nonlinear distortion. Employing a dual-mode injection-locking technique, our research reports the efficient NDFWM generation in a QD Fabry-Perot laser, in which one tone of the injected continuous-wave (CW) beams is used as the pump wave, while the other one plays the role of the probe wave. Each of these locks longitudinal modes of the FP laser within the stable-locking range.

![Figure 8](image)

**Figure 8** – The light current characteristics of the InAs/InP(100) QD laser. The inset shows the free-running spectra measured at 90 mA (pink) and 110 mA (blue). The resolution of the OSA is set at 70 pm.

Fig. 8 depicts the output power of the solitary QD laser coupled into a lensed optical fiber as a function of the pump current at room temperature. The laser exhibits a threshold current of about 64 mA. Interestingly, when the current increases above threshold, the free-running optical spectrum is broadened as shown in the inset at 90 mA (pink) with a peak centered around 1635 nm, and then splits into two separated peaks. As an illustration, the spectral difference between the split peaks at 110 mA (blue) is 17 nm while it increases up to 23 nm at 160 mA (not shown). The phenomenon that the wavelength detuning varies as a function of the pump current is a specific feature of the QD material and has been attributed to the Rabi oscillation as well as to the state filling effect. Two tunable CW lasers are then injected into the QD laser. The wavelength of the first master laser, which is fixed around the center of the peak located at shorter wavelengths acts as the pump wave, while the other one is tuned to a longer wavelength and acts as the probe wave. The inset of fig. 9 shows an optical spectrum of the dual-mode injection-locked QD laser. Each injected wavelength selects a longitudinal mode within the free-running FP multimodes, while all other modes are well suppressed. M1 and M2 are the stable locked modes. Due to the third-order nonlinear susceptibility, new waves S1 and S2 are generated as the converted conjugate signal of M2 and M1. Assuming the frequency difference \( \Delta f \) between M1 and M2, the FWM process is governed by the carrier density pulsation (CDP) mechanism for \( \Delta f \) within a few GHz while for larger values up to THz range, spectral hole
burning (SHB) and carrier heating (CH) dominates. Fig. 9 shows the normalized conversion efficiency (NCE) for the newly converted signals S1 (red), S2 (blue) as a function of the detuning frequency. The latter is operated from the minimum 57.6 GHz up to 1.72 THz. For even larger detunings up to 5.7 THz, the FWM signal is submerged in the residual FP modes or noise and disappears. It is noted that at a detuning frequency around 1.1 THz, the NCE of the studied QD laser is more than 15 dB larger than that of a QW SOA. These results are of prime importance for the wavelength conversion technique in high-speed WDM systems as well as for the microwave signal generation and radio-over-fiber applications.

![Figure 9](image.png)

**Figure 9** – The normalized conversion efficiency (NCE) for the newly converted signals S1 (red), S2 (blue) as a function of the detuning frequency. The inset shows the FWM optical spectrum where M1,2 are the stably locked modes by the tunable master lasers. The frequency difference between the locked modes is 109.6 GHz.

In this work, we have experimentally investigated the NDFWM in a QD laser employing the dual-mode injection-locking technique. Taking advantage of the two-peak lasing features of the free-running laser, an efficient NDFWM is demonstrated from a detuning frequency of 58 GHz up to 1.7 THz under a weak optical injection level.

6 Nonlinear Dynamics of Quantum Cascade Lasers

Quantum cascade (QC) laser covers a wide wavelength ranging from mid-infrared up to terahertz, which is tailored by the subband engineering. Thus, the QC lasers have extensive applications in gas sensing, telecommunication, spectroscopy and imaging. A unique feature of QC lasers is the ultrafast carrier relaxation time (in the order of several picoseconds), which is dominated by the LO-phonon assisted scattering process. This property makes QC lasers quite suitable for high-speed operation. Generation of ultrafast pulses has been demonstrated by the gain switching and mode locking techniques. Furthermore, the QC laser shows a broadband modulation response with a 3-dB bandwidth of tens of gigahertz in contrast to several gigahertz for the case of interband semiconductor lasers.

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A particular feature of QC lasers is the absence of relaxation oscillations. Relaxation oscillations are observed in most semiconductor lasers and result from the relatively large carrier lifetime compared to the photon lifetime. A slight external perturbation such as optical feedback or optical injection is usually enough to induce sustained pulsating intensities. As such, no relaxation oscillation arises in the QC laser modulation dynamics. In order to boost the dynamical characteristics of semiconductor lasers, optical injection is a favourable technique to produce plenty of nonlinear dynamics. Our theoretical studies of injection-locked QC lasers have shown that both instability and bistability appears in the local bifurcation diagram. Especially before investigating the modulation properties, it is important to ensure that the operation is located in the stable locking regime. The regime is bounded by the local bifurcations including saddle-node bifurcation and Hopf bifurcation. These bifurcations can be obtained by the eigenvalue analysis and continuation computation. In the complex space, if a single, real eigenvalue passes through the imaginary axis, one finds a saddle-node bifurcation while a pair of complex conjugate eigenvalues passing through the imaginary axis corresponds to a Hopf bifurcation. Figure 10 is calculated by the continuation package Matcont in this work. High injection ratio enlarges the stable locking region, and a total detuning range of about 30 GHz is obtained at an injection ratio of 10. The asymmetry of the locking diagram is due to the nonzero linewidth enhancement factor (0.5) of the laser.

![Figure 10](image-url)

**Figure 10** – Local bifurcation diagram of the injection-locked QC laser with a linewidth enhancement factor of 0.5. The stable locked regime is bounded by both the saddle-node bifurcation (solid line) and Hopf bifurcation (dashed line).

In our previous work, the intensity modulation behaviour of an injection-locked QC laser have also been analyzed. It was shown that a kind of resonance appears in the positive detuning side of the stable-locked regime. The intensity modulation response of the laser depicted in figure 11(a) shows that the optical injection indeed enhances the modulation bandwidth in comparison with the free-running laser (solid line). In addition, both positive and negative frequency detunings lead to a peak in the modulation response. From the analysis of the eigenvalues, it is found that a kind of resonance is induced for the positive frequency detuning case, which originates from the interaction between the locked field and the shifted cavity-resonance field. Figure 11(b) presents the 3-dB modulation bandwidth as a function of the detuning frequency. Each frequency detuning enhances the modulation...
bandwidth. Particularly, the positive detuning increases the bandwidth by more than 30 GHz.

![Figure 11](image)

**Figure 11** – Frequency detuning effect of the injection-locked QC laser with an injection ratio of 5.0. (a) Intensity modulation response; (b) The 3-dB modulation bandwidth versus the detuning frequency.

In conclusion, the grant has allowed to conduct preliminary research on nonlinear dynamical properties of injection-locked QC lasers. These findings are of prime importance for the performance enhancement of QC lasers, suggesting that an optical injection-locked experiment is now needed for further investigations. Fields of applications include gas spectroscopy, optical countermeasures or free-space communications, requiring stable single-mode operation with a narrow linewidth, high output power and high modulation bandwidth.

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- **Journal Papers**
  7. C. Wang, F. Grillot, V. Kovanis and J. Even, *Rate Equation Analysis of Injection-

• Invited Talks

• Conference Papers


