Distributed Tracking Fidelity-Metric Performance Analysis Using Confusion Matrices

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Abstract – *Distributed target tracking and identification is an important element of operational environments. In this paper, we develop a fidelity metric of track purity assessment using confusion matrix (CM) fusion. We assess individual distributed tracker track purity associations for a multitarget scenario from two platforms. The fidelity metric for each tracker is combined using the CM fusion for track decision-level analysis to aid in the joint assessment of the track quality. CM fusion enables the estimation of the combined quality of the distributed tracking scenario and can be used for any fidelity metrics based on cardinality. In a distributed multisensor multitarget scenario, we demonstrate the fidelity metric CM fusion for enhanced tracking performance evaluation.*

Keywords: *Track Metrics, RMS, Confusion Matrix Fusion*

1 Introduction

In a dynamic targeting scenario, there are hosts of algorithms that affect performance: sensor registration, measurement-to-track (M2T) assignment, track-to-track (T2T) association, sensor management, and ultimately, the user. In many operational contexts, the platform, sensor, and algorithms for target tracking and identification (ID) are designed together which requires novel metrics for distributed tracking [1]. Based on M2T algorithms [2, 3], tracking evaluation [3, 4], T2T developments [5, 6], and simultaneous tracking and ID (STID) approaches [7, 8, 9, 10], we seek a method for distributed tracking evaluation.

The goal of target tracking is to associate measurements of moving objects. There are many tracking approaches that we overviewed in previous publications [11] that included linear and nonlinear as well as Gaussian and non-Gaussian approaches [12]. The focus has been on *comparative analysis* of tracking approaches with interest in metrics and performance. Examples of approaches have been developed for applications [13], radar GMTI and HRRR tracking [14, 15, 16, 17] and the nonlinear-estimation toolbox [18, 19, 20].

In Fusion11, we overviewed many contributors to both tracking approaches and metrics for *tracking performance evaluation (TPE)* [21]. Highlighted were the contributions from K. C. Chang, and S. Mori, and C. Y. Chong [22, 23, 24] along with X. R. Li [25], of which a series of TPE contributions have been reported. In 2011, tracking metrics were overviewed [26] with *fidelity metrics* [27, 28]. Fidelity track metrics include the cardinality rankings as many of the fidelity metrics are normalized without units. The fidelity metrics include such issues as track association that we use here. For the analysis, we use the *track purity* [29] as method for track-to-track association, with the interest of distributed fusion analysis. However, we need to preface the distributed track fusion evaluation concept based on the operational need.

From a collection of tracking information from different platforms (e.g., aerial), there is an operational constraint forcing distributed tracking. From Figure 1, there are three types of fusion capabilities of signal, feature, and decision. [30] While there is an interest to process all the data in *signal-level fusion*, such as image fusion [31], the transmission of the data is limited by communications bandwidth. For feature analysis, there are concerns of feature definitions, classifier coordination, and robust methods of distributed *feature-level fusion analysis*. [32, 33] Recently, Mori and Chong [34] developed a usefully assessment of feature-level fusion for tracking and ID. Since many tracking platforms are designed with the classification and ID analysis processed on-board, information is preprocessed and sent to the fusion center for *decision-level fusion* [35] without sending signal or feature data. The reports would indicate the measurements (e.g., detections), with the notions of allegiance ID.

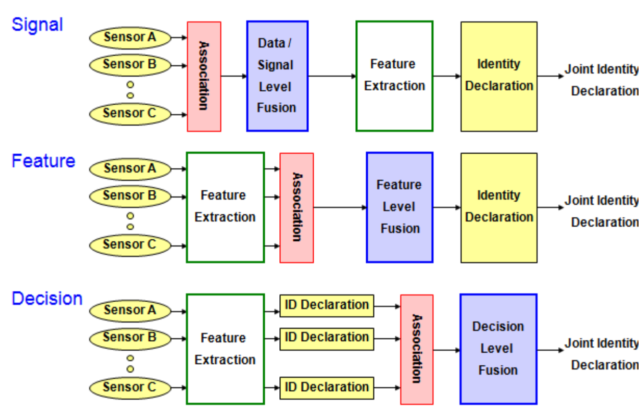

Figure 1. Signal, Feature, and Decision Fusion.

For situation assessment [36], there is a need for distributed TPE of the operating conditions of sensors, targets, environments, and algorithms [37]. In addition, TPE includes target detection, recognition (type), classification (category), and identification (allegiance).

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14. ABSTRACT
Distributed target tracking and identification is an important element of operational environments. In this paper, we develop a fidelity metric of track purity assessment using confusion matrix (CM) fusion. We assess individual distributed tracker track purity associations for a multitarget scenario from two platforms. The fidelity metric for each tracker is combined using the CM fusion for track decision-level analysis to aid in the joint assessment of the track quality. CM fusion enables the estimation of the combined quality of the distributed tracking scenario and can be used for any fidelity metrics based on cardinality. In a distributed multisensor multitarget scenario, we demonstrate the fidelity metric CM fusion for enhanced tracking performance evaluation.

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The coordination from detection to identification (and fingerprinting) is to assess the target features for target type, category, and allegiance from both the target signature and the target movements to distinguish which target - if there are related signatures. TPE includes many challenging tracking scenarios such as highly maneuvering and dense target environments.

Key developments in methods for STID include the joint-belief probability data association filter (JBPDAF) [10], interacting multiple model (IMM) [38, 39], set-based IMMJPDA [40], multiple hypothesis tracker (MHT) [41], nonlinear methods [42, 43, 44] and evidential reasoning methods [10, 45]. Performance evaluation for current nonlinear methods is needed to address environmental constraints [46, 47], optimal algorithm parameters [48, 49, 50], and methods that aid sensor management [51, 52] such as in a distributed scenario.

The *track-to-track distributed assessment* utilizes a track history (i.e., tracklets or small tracks) [13, 53] which requires association of the small tracks into the general TPE [54]. Distributed tracking can be done from the sensors-to-targets or moving-target to stationary platforms [55]. We thus perform individual track assessments to determine the track purity from each platform, from which we can conduct a distributed track purity assessment using the confusion matrix (CM) fusion.

CMs are used extensively in target ID assessment which occurs in STID methods [56, 57, 58]. For the case of the decision-level fusion (DLF) [35, 37, 59, 60] we have developed a method for confusion-matrix fusion [61] but it can also be used in the track-to-track assessment for distributed applications.

This paper develops the CM distributed fusion TPE using the CM for track purity combination. Section 2 describes the tracking metrics. Section 3 overviews the JBPDAF. Section 4 describes the CM DLF. Section 5 shows a performance analysis for a multisensor multitarget scenario and Section 6 draws conclusions.

2 Tracking and Estimator Metrics

Tracking methods include many opportunities for analysis. Some metrics are listed below: [3]

Metric	**Description**
Absolute Track Quality	Mean square position, velocity, acceleration error
Relative Track Quality	Mean square kinematic error relative to sensor covariance
Track Life-Time	Total time target is in track
Relative Track Life-Time	Total time target in track, relative to length of track-lets
Track Length	Distance over which target is tracked
Relative Track Length	Distance over which target is tracked relative to maneuverability
Track Purity	Percent of associations of dominant track over lifetime
Track Density	Number of targets track per area
Track Continuity	Number of individual targets associated with a given track

We have organized the tracking metrics into two types: *accuracy* and *fidelity* metrics [27]. For information fusion performance evaluation, tracking is one element in object assessment. We plot, in a Fishbone diagram [62, 63] in Figure 2, the five Quality of Service (QOS) information fusion metrics: accuracy, throughput, timeliness, confidence, and cost.

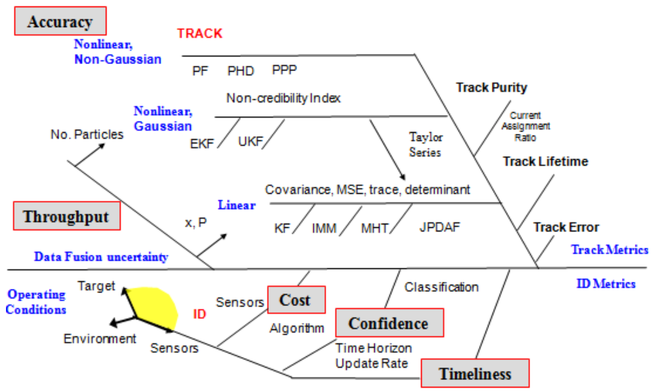

Figure 2. Tracking and Identification Joint Association.

2.1 Track Purity

Track purity **(TP)**, a concept coined by Mori *et. al*. [29], assesses the percentage of correctly associated measurements in a given track, and so evaluates the association/tracking performance. The TP measure of performance (MOP) is not explicitly dependent on detection performance, but it is dependent on the setting of association gates (which depends on the probability of detection P_d) and the ground truth platform density. TP measures the consistency with which a track is updated with measurements from a single ground truth platform or a distributed set of ground truth platforms.

Correctional local MOPs, such as TP, measure how well the tracks are being associated with measurements of ground truth platforms. The TP MOP is based on the calculation of a confusion matrix C for which the elements C_{ji} are constructed by counting reports. Given the tracks t_1 , ..., t_b and a set of ground truth platforms g_1 , ..., g_a , C is:

	Targets			
	g_1	g_2	...	g_a
	C_{01}	C_{02}	...	C_{0a}
t_1	C_{11}	C_{12}	...	C_{1a}
t_2	C_{21}	C_{22}	...	C_{2a}
Tracks	:	:		:
.	:	:		:
.	:	:		:
t_b	C_{b1}	C_{b2}	...	C_{ba}

Here, C_{ji} is the number of reports originating from ground truth platforms g_i which were assigned to track t_j *(i* = *1, ..., a*; *j* = *1, ..., b*) by the tracker. Also, C_{0i} (the "ambiguity vector") consists of the number of reports that could not be assigned to any ground truth platform *(i* = 1,

..., a). When C_{ji} is large, a strong association between t_j and g_i is implied.

The TP measure can be estimated for each single track, but is more meaningful when statistics of the TP quantity are calculated. A recommended statistic is the *Weighted Average of Track Purity* (WATP) [21] taken over all tracks and ground truth platforms. The WATP statistic should be calculated separately for each platform. It has a particularly convenient form if the weight given to each track is the number of measurements for that track, and if the weight given to each ground truth platform is the number of measurements originating from that ground truth platform. The resulting definition of the WATP, for track t_j, is as follows:

$$\text{WATP}\,[t_j] = \frac{\max_{1 \le j \le b} {}_i \; C_{ji}}{\sum_{i=1}^{b} \sum_{i=1}^{a} C_{ji}} \qquad (1)$$

The following elements are needed to compute Track Purity or WATP: the list of correct (CO) track numbers for which TP will be computed (provided by the operator), the valid time and the ground truth platform number to which the CO track is attached, and the time stamp and the ground truth platform number.

The CM is the starting point of many MOPs and its construction requires a lot of computation. Basically, we have to associate each correct track report to a target in the ground truth. The choice of association can be determined from positional and/or ID data. The association will take as argument a track T at the time t, and the complete lists of tracks and ground truth's targets resulting in a CM. Here is a procedure to construct the confusion matrix:

a. Collect data to have all CO track reports for each track and each history point of all targets in the ground truth,
b. Initialize the CM by filling each entry with zeros,
c. For each track, process all CO track reports by:
   1) Using an association function, find the corresponding target in the ground truth, and
   2) Adding 1 to the related entry of the confusion matrix.

3 Track and ID Data Filtering

Kalman filters are the baseline for tracking and are optimal if the process and measurement equations are linear and the noise is Gaussian. To enhance the tracker performance in clutter, detection can be improved with classification information; however there is a need to associate measurements to multiple tracks. We thus use the JBPDAF using classification information from evidential reasoning for a belief filter to determine ID.

3.1 Belief Filter for Simultaneous Tracking and ID

Consider an environment in which a multiple platforms are monitoring multiple moving targets with stationary clutter. By assumption, the tracking sensor is able to detect target signatures. Assume that the 2-D region is composed of T targets with f features. Dynamic target measurements z are taken at time steps k, which include target kinematic and identification features $\mathbf{z}(\text{k}) = [x_t\ (\text{k}), f_1, \ldots f_\text{n}]$. Any sensor can measure independently of the others, and the outcome of each measurement may contain kinematic or feature variables indicating any target. A final decision is rendered as to which $[x, y]$ measurement is associated with the target-type. The multilevel feature fusion problem is formulated and solved by using the JBPDAF [10]. For the symmetric-target case, the "association rule" uses the measurement with the highest target probability.

The target *state* and *true measurement* are assumed to evolve in time according to:

$$\mathbf{x}(k+1) = \mathbf{F}(k)\,\mathbf{x}(k) + \mathbf{v}(k) \qquad (2)$$

$$\mathbf{z}(k) = \mathbf{H}(k)\,\mathbf{x}(k) + \mathbf{w}(k) \qquad (3)$$

where $\mathbf{v}(k)$ and $\mathbf{w}(k)$ are zero-mean mutually independent white Gaussian noise sequences with known covariance matrices $\mathbf{Q}(k)$ and $\mathbf{R}(k)$, respectively. We assume each target has a separate track (multiple state equations), initialized at an initial state estimate $\mathbf{x}(0)$, contain a known number of targets determined from the scenario, and have associated covariances.

The JBPDAF devotes equal attention to every validated kinematic or ID measurement and cycles through measurements until a believable set of object IDs is refined to associate one object per track. The belief measurement $\mathbf{Bel}_\text{k}^t = \mathbf{M} \bullet \mathbf{Bel}_\text{k-1}^t$, derived from the classification data, represents the belief update states of the ID measurements. The $\mathbf{M}$ matrix is the Markov transition matrix, which represents the similarity of objects. The similarity of objects represents how the belief in an object type may be related to other objects of the same or different type.

The *M2T association probabilities* are computed across the objects and these probabilities are computed only for the latest set of measurements. The conditional probabilities of the joint track-ID association events pertaining to the current time k are defined as θ_{jot_k}, where θ_{jot_k} is the event that object center-of-gravity measurement j originated from object o and track t, j =1, ..., m_k; o = 0, 1, …, O_n, where m_k is the total number of measurements for each time step and O_n is the unknown number of objects. Note, for purposes of tracking and ID, we define i = 1, … , m_k for the entire measurement set while j = 1,…, m_k is for tracking and o = 1…, m_k is for object ID.

A validation gate for each object bounds the believable joint measurement events, but not in the evaluation of their probabilities. The *plausible validation matrix*: $\mathbf{\Omega} = |\,\boldsymbol{\omega}_{jt}\,|$ is generated for *each* object of a given track which comprises binary elements that indicate if measurement j lies in the validation gate of track t. The index t = 0 represents "the empty set of tracks" and the corresponding column of $\boldsymbol{\Omega}$ includes all measurements, since each

measurement could have originated from clutter, false alarm, or true object [10].

For a track event, we have:

$$| \hat{\omega}_{jt}(\theta)| \triangleq \begin{cases} 1 & \text{if } \theta^{i}_{jt} \in \theta;\ [\mathbf{z}]^{i}_{k} \subset t \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where measurement $[\mathbf{z}]^{i}_{k}$ originated from track t

For an ID-belief event, which is above a predetermined ID threshold,

$$| \hat{\omega}_{oO}(\theta)| \triangleq \begin{cases} 1 & \text{if } \theta^{i}_{oO} \in \theta;\ [\mathbf{Bel}]^{i}_{O_k} \Leftrightarrow o \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where measurement $[\mathbf{Bel}]^{i}_{O_k}$ is associated with object *o*.

Since the JBPDAF is tracking multiple objects, *o*, assuming one for each track, *t*, it has to determine the ID-belief in each object from a known database comparison. While these IDs are processed over time to discern the object, for each measurement, JBPDAF must determine if the track-ID measurements are plausible. JBPDAF uses the current ID-beliefs to update the association matrix. If the belief in the object is above a threshold, JBPDAF declares the measurement *i*, to be plausible for the target.

3.2 Data Association

Since we have assessed the continuous-kinematic information and the discrete-classification event, we can now assess the intersection of kinematic and ID information for STID. Note, ID goes beyond object detection, recognition, and classification, to associate two objects of the same class with a specific track. A kinematic-ID *joint association event* consists of the values in Ω corresponding to the associations in θ_{jot},

$$| \hat{\omega}_{jot}(\theta)| \triangleq \begin{cases} 1 & \text{if } \theta^{i}_{jot} \in \theta\ * \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where (*) measurement $[\mathbf{z}]^{i}_{k}$ originated from track *t with* a $[\mathbf{Bel}]^{i}_{o_k}$ for a given O_{ot} and

$$\hat{\omega}_{jot}(\theta) = \hat{\omega}_{jt}(\theta) \oplus \hat{\omega}_{oO}(\theta). \tag{7}$$

Note, we define the indices as *jot* since *O* is the number of objects which is equal to the number of tracks.

These joint events will be assessed with "β" weights [2] to determine the extent of belief in the associations. To process the believability of track associations, augmented with the ID information, we set up a matrix formulation. For example, we have a set of kinematic measurements $\mathbf{z}_i$ with a $\mathbf{Bel}_o$ and put them into the event association matrix as illustrated in Figure 3. The upper left of a box represents the track information where a "1" indicates the kinematic measurement lies within a gated position measurement. The lower right represents the belief in an object type of any class except the unknown class where a believable object receives a "1". Columns are for tracks and rows for measurements. These generalized equations propagate ID-filtered, predicted ID measurements in time.

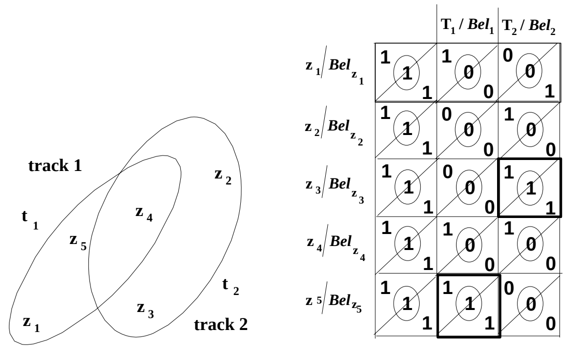

Figure 3. Tracking and Classification Joint Association.

JBPDAF processes event matrices with an "AND" function in the case of joint association allowing for plausible events from either the track or classification. To determine the event plausibility, JBPDAF uses the validation region for track measurements and uses a threshold, or *classification gate,* to determine a target-type ID match associated with a given track. Figure 4 illustrates the "AND" function. Note, JBPDAF rejects non-believable measurements and measurements that lie outside the kinematic validation gate.

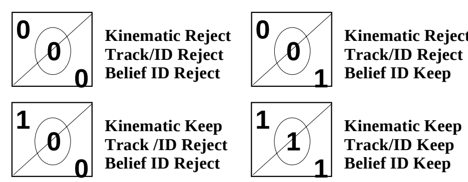

Figure 4. Believable Events for the association matrix.

JBPDAF sets up the state and probability values for the determination of the weights assigned to these associations. A *track-ID association event* has [2]

i) A single object-type measurement from a source:

$$\sum_{o=0}^{O_n} \hat{\omega}_{jot}(\theta^{i}_{jot}) = 1 \quad \forall\, j \tag{8}$$

ii) And at most one object-type measurement ID originating from a object for a given track:

$$\delta_t(\theta) \triangleq \sum_{j=1}^{m_k} \hat{\omega}_{jot}(\theta^{i}_{jot}) \leq 1 \tag{9}$$

The event matrices, $\hat{\Omega}$ for each track, corresponding to ID events can be done by scanning $\boldsymbol{\Omega}$ and picking one unit/row and one unit/column for the estimated set of tracks except for t = 0. In the case that JBPDAF has generated event matrices for an estimated number of tracks with different object types, JBPDAF needs to assess the combination of feature measurements to infer the correct number of tracked objects that comprise the set. The binary variable $\delta_t(\theta_{jot_k})$ is called the *track detection indicator* [2] since it indicates whether a measurement is

associated with the object o and track t in event θ_{jot_k}, i.e. whether it has been detected. The *measurement association indicator*

$$\tau_j(\theta_{jot_k}) \overset{\Delta}{=} \sum_{j=1}^{m_k} \hat{\omega}_{jot}(\theta_{jot_k}) \quad (10)$$

indicates measurement j is associated with the track t in event θ_{jot_k}. The *false measurements* in event θ are:

$$\phi(\theta) = \sum_{j-1}^{m} [\,1 - \tau_j(\theta)\,] \quad (11)$$

The *joint association event probabilities* are, using Bayes' formula:

$$\begin{aligned} P\{\theta(k)|\mathbf{Z}^k\} &= P\{\theta(k)|\mathbf{Z}(k),m(k),\mathbf{Z}^{k-1}\} \\ &= \frac{1}{c}\, p[\mathbf{Z}(k) \mid \theta(k),m(k),\mathbf{Z}^{k-1}]\; P\{\theta(k) \mid m(k)\} \\ &= \frac{1}{c} \prod_{j=1}^{m(k)-\phi(k)} V\, \{f_{t_t}(k)\, [\mathbf{z}_j(k)]\}^{\tau_j} \end{aligned} \quad (12)$$

where c is the normalization constant.

The number of *M2T assignment events* $\theta(k)$ is the number of targets to which a measurement is assigned under the same detection event $[m(k) - \phi]$. The *target indicators* $\delta_t(\theta)$ are used to select the probabilities of detecting and not detecting events under consideration.

3.3 Fused Track and ID State Estimation

Assuming the targets conditioned on the past observations are mutually independent, the decoupled state estimation uses the *marginal association probabilities*, which are found from the joint probabilities by summing all the joint events in which the marginal track and classification events result. The beta weights [2] are:

$$\beta^t_{jo_k} \overset{\Delta}{=} P\{\theta_{jot_k} \mid \mathbf{Z}^k\} = \sum_{\theta} P\{\theta_{jot_k} \mid \mathbf{Z}^k\}\hat{\omega}_{jo}(\theta_{jot_k}) \quad (13)$$

JBPDAF decomposes the object-state estimation with respect to the object *location* of the latest set of validated belief-set and kinematic-set measurements. For each object measurement, we use the total probability theorem to get the *conditional mean* of the state at time k as:

$$\hat{\mathbf{X}}^t_{k|k} = \sum_{i=0}^{m^o_k} \hat{\mathbf{X}}^{ti}_{k|k}\, \beta^{ti}_k\,, \quad (14)$$

where $\hat{\mathbf{X}}^t_{k|k}$ is the updated state conditioned on the event that the i^{th} validated object measurement is correct for track t. The covariance propagation for each track t is:

$$\mathbf{P}^t_{k|k-1} = \mathbf{F}^t_{k-1}\,\mathbf{P}^t_{k-1}\,(\mathbf{F}^t_{k-1})^T + \bar{\mathbf{Q}}^t_{k-1}, \text{ where } \bar{\mathbf{Q}}_k = \begin{bmatrix} \mathbf{Q}_k & 0 \\ 0 & \mathbf{B}_k \end{bmatrix}$$

We can obtain the innovation covariance $\mathbf{S}_k$ with the associated $\mathbf{R}_k$ and measured $\mathbf{D}_k$ by:

$$\mathbf{S}^t_k = \mathbf{H}^{o_t}_k\,\mathbf{P}^t_{k|k-1}\,(\mathbf{H}^{o_t}_k)^T + \bar{\mathbf{R}}^t_k, \text{ where } \bar{\mathbf{R}}_k = \begin{bmatrix} \mathbf{R}_k & 0 \\ 0 & \mathbf{D}_k \end{bmatrix}$$

Since $\mathbf{S}_k$ is the innovation covariance update, we can use $\mathbf{S}_k$ to gate measurements based on the uncertainty with the associated track and IDs.

Validation: At k, two measurements are available for object o for a given track t: $\mathbf{z}^T_{k-1}$, and $\mathbf{z}^T_k$, from which position, velocity, pose, and ID features can be extracted from the belief track vectors. Validation, based on track and ID information, is performed to determine which track-belief measurements fall into the kinematic region of interest:

$$(\mathbf{z}^t_k - \hat{\mathbf{z}}^{lt}_{k|k-1})^T\,[\mathbf{S}^t_k]^{-1}\,(\mathbf{z}^t_k - \hat{\mathbf{z}}^{lt}_{k|k-1}) \le \gamma \quad \text{for } l = 1 \ldots m^o_k \quad (15)$$

where γ is a validation threshold obtained from a χ^2 table and $\mathbf{S}_k$ stands for the largest among the predicted track belief covariance, i.e., $\det(\mathbf{S}_k) \ge \det(\mathbf{S}^t_k)$ for $t = 1, 2,...,n$ where n is the number of states. The combined predicted track belief, $\hat{\mathbf{z}}_{k|k-1}$, is given by $E\{\mathbf{z}_k|\{\beta^s\}^s_{o=1}, \mathbf{Z}^{k-1}\}$ where s is the set of object beliefs for a track.

Data association for β^{ti}_l : Data association performed for each belief object-track is similar to that in PDA and the details can be found in [2] for the association probabilities for l validated object measurements m^o_k, P_G assessing the probability that augmented belief track measurements fall into the *validation region,* and P_D representing a detection probability. For the JBPDAF case, we vary the innovation covariance ($\mathbf{S}_k$), P_D, P_G proportionally to the sensor manager collection resolution (i.e., higher resolution $\rightarrow$ higher P_D, higher P_G, and lower $\mathbf{S}_k$). The lower $\mathbf{S}_k$ for the higher resolution is a result of changing the prediction, which results after a few track instances. The *volume of the validation gate* is

$$V_k = \mathbf{C}_d\, \gamma^{d/2}\, |\mathbf{S}_k|^{1/2}, \quad (16)$$

where $\mathbf{C}_d$ is the unit hypersphere volume of dimension d, the dimension of the augmented belief-track measurement.

Kinematic belief-probabilistic update: The object belief-probabilistic track update is performed as a full rate system to combine the state, innovation, and covariances.

$$\hat{\mathbf{X}}^t_{k|k} = \hat{\mathbf{X}}^t_{k-1|k-1} + \mathbf{W}^t_k \sum_{l=1}^{m^o_k} \beta^t_{lk}\, \nu^t_{lk} \quad (17)$$

and $$\mathbf{P}^t_{k|k} = \beta^t_0\, \mathbf{P}^t_{k|k-1} + (1 - \beta^t_0)\, \mathbf{P}^*_{k|k} + \mathbf{W}^t_k \left[\sum_{l=1}^{m^o_k} \beta^t_{lk}\, \nu^t_{lk}\, [\nu^t_{lk}]^T - \nu^t_k [\nu^t_k]^T \right] (\mathbf{W}^t_k)^T \quad (18)$$

where, $$\mathbf{P}^*_{k|k} = \left[\, I - \mathbf{W}^t_k\, \mathbf{H}^{o_t}_k \right] \mathbf{P}^t_{k|k-1} \quad (19)$$

$$\mathbf{W}^t_k = \mathbf{P}^t_{k|k-1}\, [\mathbf{H}^{o_t}_k]^T\, (\mathbf{S}^t_k)^{-1} \quad \text{and } \nu_k = \sum_{l=1}^{m^o_k} \beta^t_{lk}\, \nu^t_{lk} \quad (20)$$

where $\mathbf{H}_k^{o_t}$ is the measurement matrix that is calculated for each object pose, ϕ, and estimated position of track t.

4 Decision Level Fusion (DLF) Method

The WATP decisions are stored in a CM. For initial track performance, these estimates are treated as priors [61]. Decisions from multiple platforms with different geometric perspectives are fused using the Decision Level Fusion (DLF) technique. Assume that we have two platforms each with a WATP described in a CM designated as C^A and C^B. The elements of a CM are c_{ij} = Pr{WATP decides track object o_j when track object o_i is true}, where i is the true object track, j is the assigned track class, and i = 1, …., N for N true tracks. The CM elements can be represented as probabilities as $c_{ij} = \Pr\{ z = j \mid o_i\} = p\{ z_j \mid o_i\}$. To determine an track declaration, we need to use Bayes' rule to obtain $p\{o_i \mid z_j\}$ which requires the track priors, $p\{o_i\}$. We denote the priors and likelihoods as column vectors:

$$p(\bar{o}) = \begin{bmatrix} p(o_1) \\ p(o_2) \\ \vdots \\ p(o_N) \end{bmatrix} \; ; \; p(z_j \mid \bar{o}) = \begin{bmatrix} p(z_j \mid o_1) \\ p(z_j \mid o_2) \\ \vdots \\ p(z_j \mid o_N) \end{bmatrix}. \quad (21)$$

For M decisions, a confusion matrix would be of the form

$$C = \begin{bmatrix} p(z_1 \mid o_1) & p(z_2 \mid o_1) & .. & p(z_M \mid o_1) \\ p(z_1 \mid o_2) & p(z_2 \mid o_2) & .. & p(z_M \mid o_2) \\ \dots & \dots & \ddots & \dots \\ p(z_1 \mid o_N) & p(z_2 \mid o_N) & .. & p(z_M \mid o_N) \end{bmatrix}. \quad (22)$$

The joint likelihoods are similar column vectors, where we assume independence for two confusion matrices A and B (denoted here as superscripts),

$$p(z_j^A, z_k^B \mid \bar{o}) = \begin{bmatrix} p(z_j^A \mid o_1) \bullet p(z_k^B \mid o_1) \\ p(z_j^A \mid o_2) \bullet p(z_k^B \mid o_2) \\ \dots \\ p(z_j^A \mid o_N) \bullet p(z_k^B \mid o_N) \end{bmatrix}, \quad (23)$$

where k is used to distinguish between the different assigned object tracks between the two confusion matrices when the CMs are not symmetric. The independence assumption is valid if the sensors, decision analysis, or the noise from sensor-to-target perspectives are different.

Using the priors and the likelihoods, we can calculate *a posteriori* from Bayes' Rule

$$p(\bar{o} \mid z_j^A, z_k^B) = \frac{p(z_j^A, z_k^B \mid \bar{o}) \; p(\bar{o})}{\sum_{i=1}^{N} p(z_j^A, z_k^B \mid \bar{o}) \; p(\bar{o})}. \quad (24)$$

Note that there are similar column matrices for the posteriors $p(\bar{o} \mid z_j)$ and $p(\bar{o} \mid z_j^A, z_k^B)$. A decision is made using the maximum likelihood estimate

$$d_i = \underset{j;\,k}{\operatorname{argmax}} \; p(o_i \mid z_j^A, z_k^B) \;, \quad (25)$$

where the final decision of the true object track i is determined from the largest value from the vector.

Note that the subscripts indicate the value of a variable and the superscripts indicate the track source. For example, $z^A = z_3$ indicates that tracker A made a decision z_3; where tracker A might be the first track and decision z_3 might be track type. The absence of a superscript implies an unspecified single source. We represent the particular states from each tracker with the subscripts a and b such as $z^A = z_a^A$ indicating that tracker A's decision was z_a.

For the developments of the pseudo code, shown below in Figure 5, we shorten the notation to $z^A = z_a$, while keeping an update of the CM source A or B. Inputs to the fuser are the decisions of trackers A and B, i.e., z_a and z_b respectively. The output decision d is based on a maximum a posteriori probability (MAP) decision rule, where $p(\bar{o} \mid z_a, z_b)$ is posterior, $p(\bar{o})$ is the prior probabilities, and CA and CB are the WATP CM (one for each source).

```
function [d, pObarZaZb] = fuseCMdecisions(za, zb, Obar)
        CA = getConfusionMatrix(1);
        CB = getConfusionMatrix(2);
        pZaObar = CA(:,za);
        pZbObar = CB(:,zb);
        pZaZbObar = pZaObar .* pZbObar;
        posteriorNum = pZaZbObar .* pObar;
        posteriorDen = sum(posteriorNum);
        pObarZaZb = posteriorNum / posteriorDen;
        [junk, d] = max(pObarZaZb);
        return
```

Figure 5. Pseudo code for DLF with Confusion matrix.

Pseudo code for DLF is represented as:

- za = z_a and zb = z_b are the integer decisions between 1 … M of trackers A and B, respectively
- pObar = $p(\bar{o})$ is a vector of priors, represented as either constants or input variable
- CA = C^A and CB = C^B are the confusion matrices derived from trackers A and B, respectively
- pZaObar = $p(z_a \mid \bar{o})$ and pZbObar = $p(z_b \mid \bar{o})$ are the likelihoods as extracted columns from the confusion matrices [pZaObar = CA(:,za); and pZbObar = CB(:,zb)]
- pZaZbMbar = $p(z_a, z_b \mid \bar{o})$ is the joint likelihood derived from the point-wise product of the tracker likelihoods (pZaZbObar = pZaObar .* pZbObar);
- pObarZaZb = $p(\bar{o} \mid z_a, z_b) = \dfrac{p(z_a, z_b \mid \bar{o}) \; p(\bar{o})}{\sum_{i=1}^{N} p(z_a, z_b \mid \bar{o}) \; p(\bar{o})}$
  - the numerator is:
    posteriorNum = pZaZbObar .* pObar;
  - the denominator is:
    posteriorDen = sum(posteriorNum);
  - pMbarZaZb = posteriorNum / posteriorDen;

- d = max(pObarZaZb), which is the fused decision, $d_i \ni p(o_i \mid z_a, z_b) \geq p(o_i \mid z_a, z_b)\ \forall\ i, j$ where $i, j \in 1, \ldots, N$.

5 Distributed Performance Analysis

For this analysis, we first developed a toolbox of performance evaluation methods [18-20]. We compared multiple scenarios for analysis of closely spaced targets with linear and nonlinear movements. Here we present the case of the nonlinear motions to demonstrate a distributed track fusion assessment using the belief filter. Figure 6 shows the scenario with clutter and Figure 7 shows the resulting track outputs from one sensor.

The scenario was generated using the trajectories shown and variations in the clutter. Two elements of clutter can be induced from the spurious measurements for a sensor. Since distributed sensors have different perspectives, the measurement clutter was altered relative the perspective. For example, Sensor 2 has a better perspective of Target 3, which has a higher WATP.

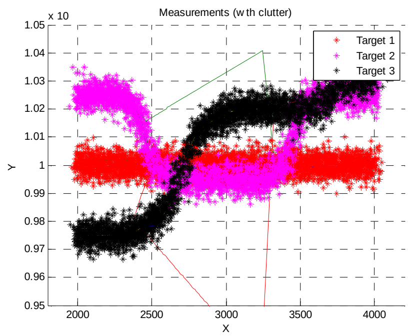

Figure 6. Scenario with Clutter.

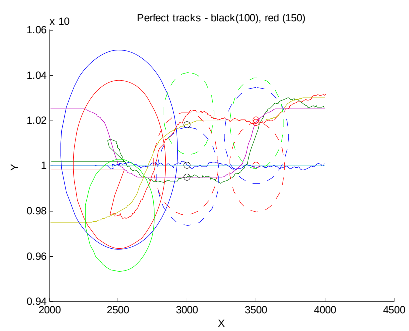

Figure 7. Sensor 1 track result with covariances.

Figure 8 and 9 show the CM for the individual sensors and the DLF combined result (using the method shown in Section 4 [61]) which improves the distributed track purity assessment. It is noted that the use of the CM fusion does improve the overall assessment (sum of diagonals), but may result in poorer performance for a case in which a closer sensor has a better STID analysis (as noted from Sensor 1, Track 1 going from 0.98 to 0.97). Future work requires a more intelligent method of score fusion over different scenarios to improve distributed analysis based on the credibility of the sensor/track outputs. For example, more significant degradations happen when fusing a result with 0.98 and 0.50, of which relying on 0.98 should be the choice for a more credible sensor. Other concerns relate to different CM sizes, sensor update rate, and track algorithm choice, which all affect a distributed track fusion analysis.

Sensor 2

	ARGE 1	ARGE 2	ARGE 3
RACK 1	0.98	0.015	0.005
RACK 2	0.03	0.9	0.07
RACK 3	0.005	0.065	0.93

Sensor 1

	ARGE 1	ARGE 2	ARGE 3
RACK 1	0.97	0.025	0.005
RACK 2	0.015	0.92	0.065
RACK 3	0.01	0.04	0.95

Figure 8. WATP CM from Sensor 1 and Sensor 2.

Fused CM

	ARGE 1	ARGE 2	ARGE 3
RACK 1	0.97	0.023	0.0052
RACK 2	0.017	0.92	0.061
RACK 3	0.0042	0.039	0.96

Figure 9. Fused WATP.

6 Discussion and Conclusions

Building on many developments in track performance evaluation, we developed a metric for distributed track fusion assessment by integrating track purity from track segments from distributed platforms. We used the novel confusion-matrix fusion approach for the analysis. Future work will explore metrics for sensor management, net-centric solutions, nonlinear trackers, and exploration of non-physics-based tracking scenarios such as social networks [64], of which newer methods are needed.

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