# Data Fusion Methods for Small Arms Localization Solutions

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*Abstract*— **In this paper several methods and models for improving small arms localization are investigated. Each acoustic sensor is placed at a disparate location and it is assumed that each system may or may not return an estimated range and/or azimuth shooter. Various simple geometric based data fusion methods are proposed and their performance evaluated. Models of localization errors are also proposed and these models are used herein to develop a maximum likelihood approach to data fusion. The parameters of these statistical distributions are estimated from real world data. Comparing / contrasting the results of both methods side by side, it can be shown that while the maximum likelihood based approach performs the best, decent results can be achieved with the simpler geometric based approach.**

*Keywords – Gunfire / Small Arms Locization, Data Fusion, Maximum Likelihood, Acoustic Sensors, Impulsive Events*

## I.    INTRODUCTION

Acoustic localization of small arms fire is accomplished by measuring the time and direction of arrival of two sounds produced by a supersonic bullet. The two sounds generated by small arms fire are the supersonic shockwave (crack) and the muzzle blast (bang). The shockwave is the sound produced by supersonic flight of the round and the muzzle blast is the sound produced during expulsion of the bullet from the gun. Microphones are placed at several locations and by noting how a given wave-front traverses the microphones, a direction can be inferred. By measuring the time differences of arrival between the shockwave and muzzle blast and the angle between their directions of arrival it is possible to calculate range to the shooter (assuming round type is known) [1].

Typically, gunfire detection systems attempt to estimate caliber from the received shockwave [2]. Once caliber is determined, one can begin to make an estimate of the round type (which is required to determine the range to shooter). Since this may or may not always be successful, estimates of range are subject to errors. Some systems may not return an estimate of range when it is suspected that the underlying data doesn't meet various sanity checks [3].

The goal of the present work is to devise methods, whereby measurements of azimuth and range for different sensors placed at disparate location can be combined to produce an estimate that is, on average, better than each individual sensor alone.

Using real world data, various models for errors in azimuth and range are proposed and evaluated. Additionally, simple geometric fusion techniques are proposed and their performances evaluated. These models provide a basis for the development of a maximum likelihood based approach for a data fusion solution for small arms fire. The performance of both the geometric based fusion approach and the maximum likelihood fusion approach are demonstrated [4].

## II.    DATASET

In order to develop a fusion approach for acoustic based small arms fire localization data was needed. Identical gunfire detection systems were placed at disparate locations and shots were fired from different ranges. The purpose of the test was to furnish realistic data. The data collection was conducted with positions simulating sensor locations that might be found in a squadron or platoon during a patrol mission. Additionally, this data collection also provided information to start modeling system level errors that could potentially affect the fused solution [3], [4], [5].
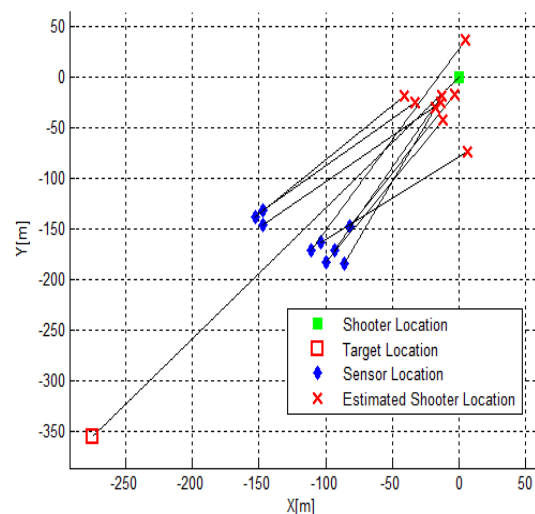


Figure 1. Small-arms fire event

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14. ABSTRACT
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. The data collection consisted of several different shooter/target/sensor geometries. In order to facilitate comparisons between different fusion algorithms, all coordinates translated so as to place the shooter at the origin. Typically, the distance between each sensor and the shooter was between 100 and 400 meters. Figure 1 illustrates a typical scenario for small arms fire event. The eight sensors denoted with diamonds are emplaced in a wedge formation commonly seen during patrolling missions [5]. The shooter represented with a solid square fires through the middle of the sensor formation towards a target signified by an open square. In the figure 1 the estimated shooter location by each sensor is represented with an 'x'.

### III. GEOMETRIC METHODS FOR DATA FUSION

#### A. Intersection of basic solutions

Initially, estimates of possible shooter locations were obtained by various simple geometric methods, namely possible shooter locations were generated by intersecting circles and rays. If a system reports a given range then this is understood as the shooter residing on a circle, centered at the sensor location, with radius equal to the reported range. A system returning a bearing or azimuth is interpreted as the shooter residing on a ray emanating in the reported direction with origin at the sensor location.

It is feasible to generate additional possible shooter locations by intersecting different combinations of circles and rays. Figures 2a-c demonstrates how these points can be generated. Figure 2a exploits intersecting circles, figure 2b demonstrates intersecting rays, and figure 2c uses circle/ray intersections to identify possible shooter location solutions depicted by dots [7].
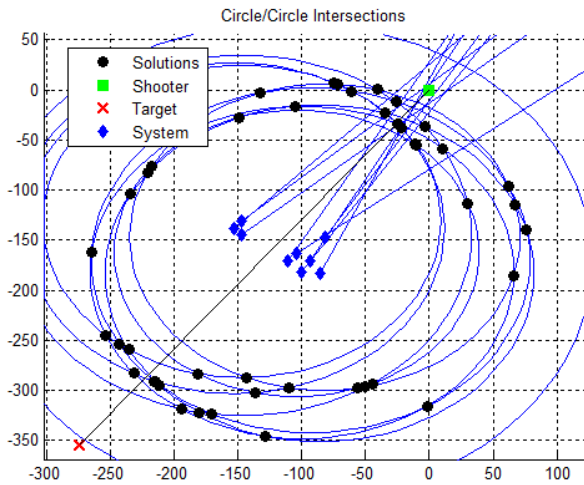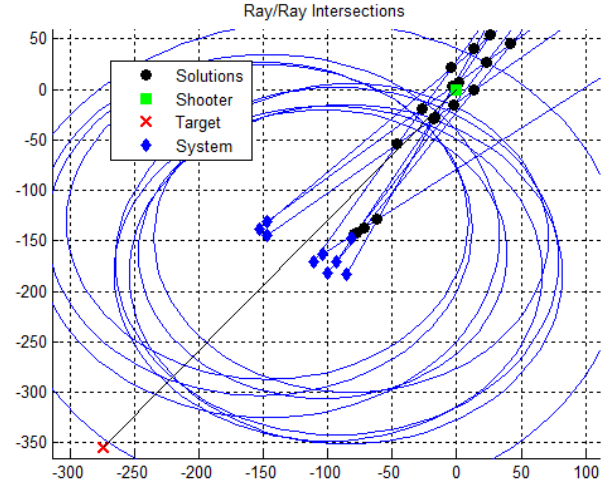


Figure 2b. Example of ray-ray intersections



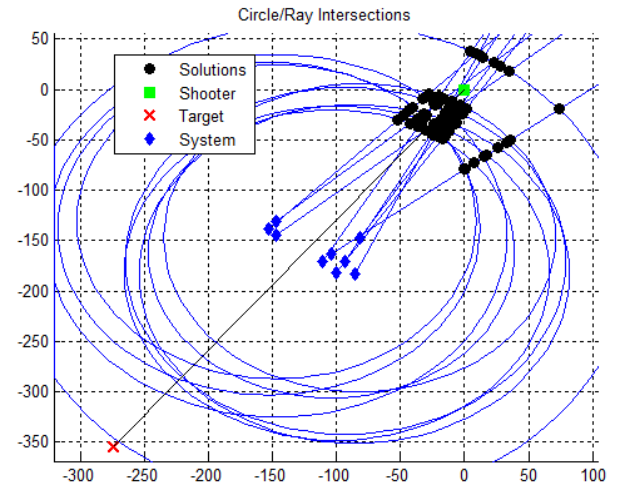Figure 2c. Example of circle-ray intersections



Figure 2a. Example of circle-circle intersections

#### B. Fusion of Simple solutions

The circle-circle intersections depicted in figure 2a are not particularly useful for the given geometry because of the associated dilution of precision. Under different scenarios these may be useful (e.g. If the shooter was much closer to each of the sensors) [3], [4], [5], [7].

Various combinations of the ray-ray and circle-ray intersections were taken and different measures of central tendency were used to estimate the shooter location. In general, the fusion approaches with best performance were achieved by methods robust to outliers (e.g. the centroid was typically a bad choice because of its sensitivity to outliers). A much more robust estimate is provided by the geometric median. Whereas the centroid fusion approach minimized the sum of the squared distances from each data point, the geometric median is the point that minimizes the sum of the distances from each data point (1) [8].

$$\arg min_{y \in \mathbb{R}^n} \sum_{i=1}^{m} \|x_i - y\|_2 \qquad (1)$$

The geometric median is commonly calculated via Weiszfeld's algorithm which is an iteratively reweight least squares approach [10].. Figure 3 provides an illustration of the robustness of the geometric median as opposed to the centroid. Three points are randomly placed around the origin and a single outlier is placed significantly further off. The geometric median provides a much better estimate of the center of the cluster than the centroid approach, which is heavily skewed by the single outlier
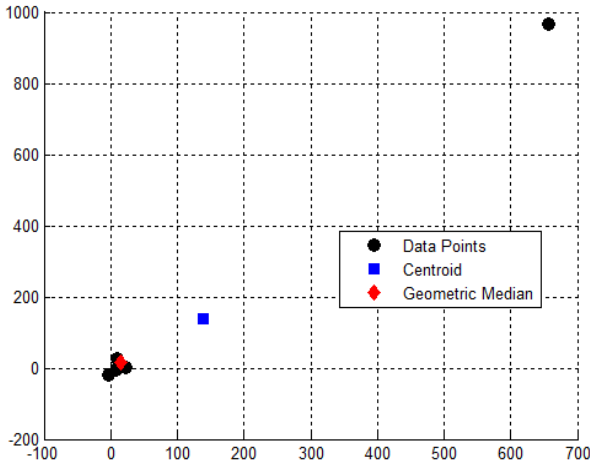


Figure 3. Geometeric Median vs. Centroid

Illustrated in Figure 4 is the distribution of system solutions and the corresponding estimates obtained by utilizing the circle-ray intersections shown in figure 2c and computing the geometric median of the system solution.
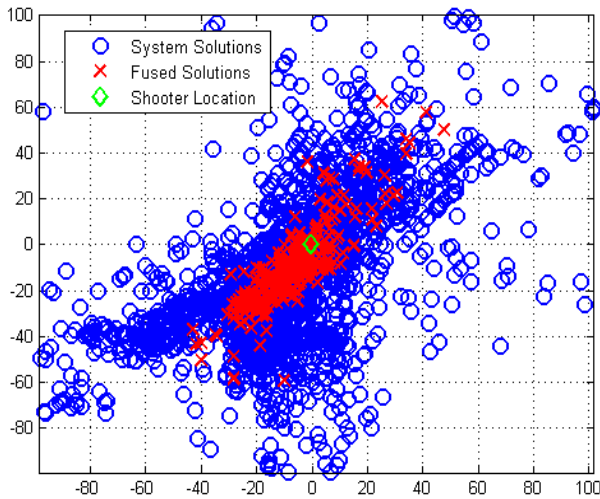


Figure 4. Distribution of system solutions

## IV. MODELING SYSTEM ERRORS AND ML ESTIMATION

### A. Modeling Solution Azimuth Errors

Using the collected data, statistics on system errors were extracted. The direction of arrival of the muzzle blast will always point back to the shooter. This effect provides the basis upon which gunfire localization systems determine azimuth. Even though at further ranges the accuracy of solutions could be expected to diminish, a simple model was desired and errors in azimuth were assumed independent of range. The model used for error is azimuth was a von-mises distribution (2)[8], [9].

$$f(x|\mu, \kappa) = \frac{e^{\kappa \cos (x-\mu)}}{2\pi I_0(\kappa)} \qquad (2)$$

Equation (2) the x represents direction, $\mu$ is the concentration direction, $\kappa$ is the concentration parameter, and $I_0(\kappa)$ is the modified bessel function of the first kind [x]. The choice of the von-mises distribution is a very natural one as the domain of this distribution is the unit circle [7], [8], [9]. A random sampling of this distribution for $\kappa = 5$ is illustrated in Figure 5.
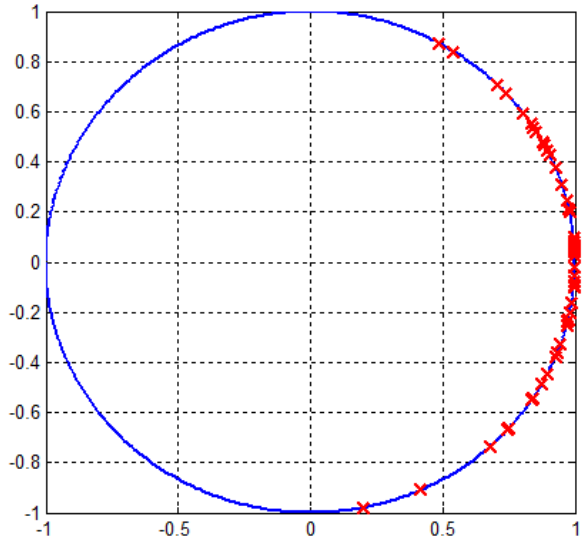


Figure 5. Random distribution of $\kappa = 5$ of unit 1

The concentration parameter can be estimated because both the actual sensor system and shooter positions are available for the calculations. An empirical fit of the parameter $\kappa$ to the observed data is shown in Figure 6 with a blue trace. The system errors in azimuth have been binned and normalized to form a PDF and the model PDF is superimposed.
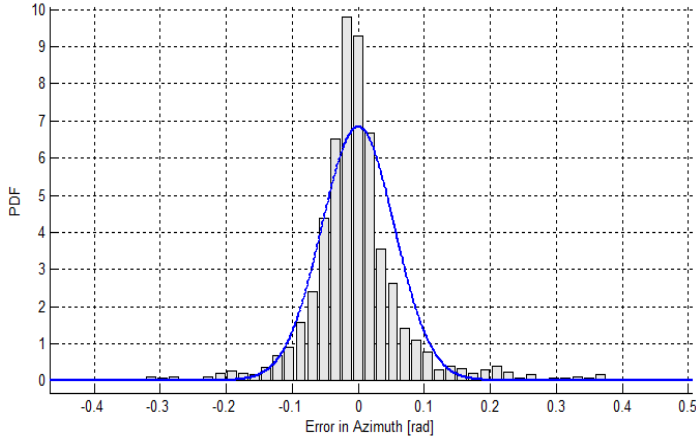
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Figure 6. System errors in azimuth



Figure 7. Distribution of relative error in range

Although the distribution does not fit the empirical data precisely (e.g. the empirical data has longer tails) the results appear reasonable.

## B. Modeling Solution Range Errors

A simple model for errors in range was desired. Some of the factors that affect range estimates include: errors in the estimated time between the shockwave and muzzle blast, errors in their corresponding angles, and errors resulting from choosing an incorrect ballistic table. If the values of these errors are small, the resulting range estimate will be scaled by an amount proportional to each error. Additionally, the errors in range are typically proportional to the actual range. For these reasons, a plausible distribution of the relative errors in range might be the log-normal distribution. If we denote the actual range as r, and the estimated (i.e. system estimate) range as r-hat we have the following model:

$$r' = \frac{\hat{r}}{r} \qquad (3)$$

$$f(r'|\sigma) = \frac{1}{r'\sqrt{2\pi\sigma^2}} e^{\frac{-(\ln(r'))^2}{2\sigma^2}} \qquad (4)$$

The parameter $\sigma$ is estimated from the available data. The following plot shows the PDF of the relative errors in range as a histogram superimposed with the distribution with the estimated parameter $\sigma$ [7], [8], [9].
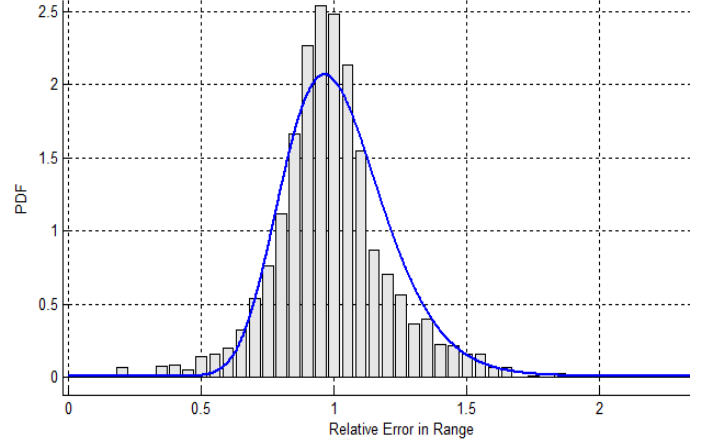
## C. Additional Modeling Concerns

The parameters of the previous error models for azimuth and range have been estimated on data which has had the outliers removed. In order to incorporate outliers into this model another parameter $\alpha$ is introduced. This parameter represents that a given estimate is drawn from one of the probability distributions described above or is drawn from a uniform distribution over the range of possible values (with probability $(1-\alpha)$).

The purpose of modeling the errors in azimuth and elevation is to devise an algorithm for estimating shooter location. The data fusion problem must be able to contend with incomplete data (e.g. a range may not be available at each system node). In the previous section the error probabilities in range and azimuth have been assumed to be independent of each other. This is certainly not true; however it substantially simplifies the analysis.

## D. Maximum Likelihood Estimation

To estimate shooter locations for Maximum Likelihood Estimation we first find the probability of the given sets of observations (azimuths and ranges) given a presumed shooter position. Let $\beta$ denote the set of available observations:

$$\beta = [\vec{v}_1, \vec{v}_2 \dots \vec{v}_N, \vec{p}_1, \vec{p}_2, \dots \vec{p}_N, r_1, r_2 \dots r_M, \vec{q}_1, \vec{q}_2, \dots \vec{q}_M] \qquad (5)$$

Where $v_1, v_2, \dots v_n$ are the N available sighting vectors pointing towards the shooter from sensor positions $p_1$, $p_2$, $p_N$, and $r_1$, $r_2$, $\dots$ $r_M$ are the M estimated ranges from sensors at positions $q_1$, $q_2$, $\dots$ $q_M$. The log likelihood that the shooter is at a given location is given by (6).

$$LL = log\big(p(\beta|\vec{x})\big) = A + B \qquad (6)$$

$$A = \sum_{n=1}^{N} log \left( \frac{\alpha}{2\pi} + (1 - \alpha) \frac{exp\left(\kappa \vec{v}_n \cdot \frac{(\vec{x} - \vec{p}_n)}{\|(\vec{x} - \vec{p}_n)\|}\right)}{2\pi I_0(\kappa)} \right) \quad (6a)$$

$$B = \sum_{m=1}^{M} log \left( \frac{\alpha}{r_{max}} + (1 - \alpha) \frac{e^{\frac{-\left(\ln\left(\frac{r_m}{\|(\vec{x} - \vec{q}_m)\|}\right)\right)^2}{2\sigma^2}}}{\frac{r_m}{\|(\vec{x} - \vec{q}_m)\|}\sqrt{2\pi\sigma^2}} \right) \quad (6b)$$

The value of **x** that yields the maximum value for the Log-Likelihood is taken as the shooter position. An example of the Log-Likelihood of shooter position given a set of actual observations is illustrated in figure 8. The actual shooter position is at the origin.
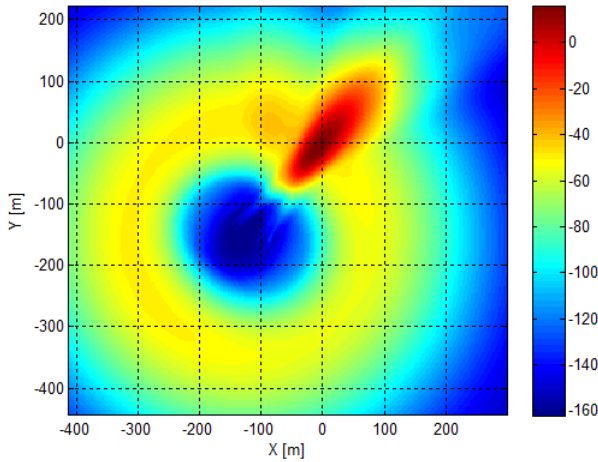

Figure 8. Shooter position given set of actual observations

## V. COMPARISON OF RESULTS

Both the Maximum Likelihood and geometric based fusion approaches improve the estimates of shooter position. In general, the performance of the Maximum Likelihood method performed better than geometric based methods. It should be noted however that the geometric median based method showed an increased robustness due to the geometric medians robustness to outliers. This can be seen from figure 9. The following plot shows a Cumulative Probability Distribution (CDF) of the distance of the estimated shooter position from the true position for the different data fusion methods.
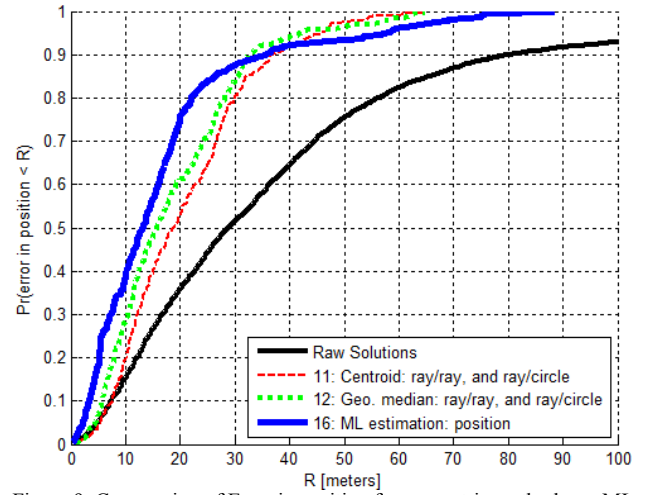

Figure 9. Comparsion of Error in position for geometric methods vs. ML

A comparison of Maximum Likelihood estimates and the original system estimates is shown below in Figure 10 with the system solutions represented by circles, and the fused solutions represented by Xs   It is clear that though the Maximum Likelihood solution will not be better than the best system solution in general the fused solution will be superior.
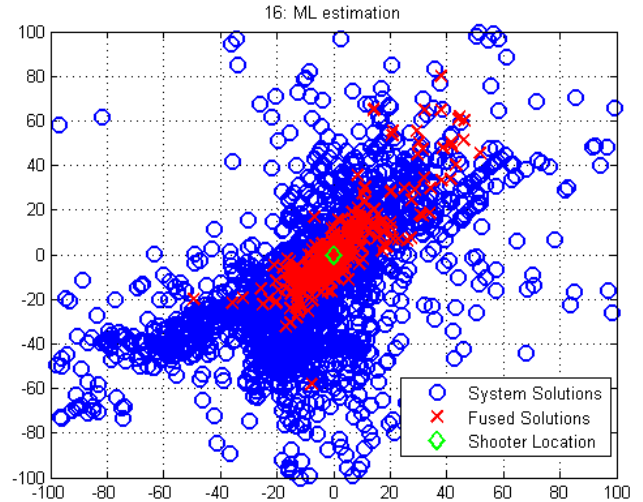

Figure 10. Comparsion of ML estimates against system solutions

## VI. CONCLUSIONS AND DIRECTION FORWARD

Presented in this paper are models for errors in small arms localization systems and their demonstrated utility in improving estimates from several sensors.  The focus has been on utilizing higher-level solutions (i.e. each system calculates a solution from its available data and the results from each system are combined to improve the estimate of shooter location).

A better approach would be to utilize partial solutions from each sensor (e.g. use the muzzle blast and shockwave directions and times of arrival at each system). If this were the

case addition errors to be modeled would be time synchronization accuracy between systems. Several aspects could be reused in this new framework. Since azimuth estimates are directly tied to the muzzle blast direction of arrival, the existing model could be reused.

The current solution level fusion method has the advantage that each system only has to return an azimuth, and range, and the fusion algorithm does not have to know or implement any additional details. The parameters modeling systems could be tuned for each individual sensor.

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