Magnonic crystal as a delay line for low-noise auto-oscillator

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Based on the results of the developed analytical theory, the authors propose to use the magnonic crystal patterned on the YIG magnetic film as an efficient delay line in the feedback loop of tunable auto-oscillator. This allows the reduction of the inhomogenity of bias magnetic field and the rasing of the power of input signal in virtue of decreasing the length and increasing the thickness of such delay line as compare to the YIG film with no pattern. In turn, use of this magnonic crystal opens a way to improve noise properties of the auto-oscillator.

I. INTRODUCTION

Spin waves propagating in thin YIG films have useful properties that are suitable for the design of planar ferrite microwave-signal processing devices. In particular, in the microwave-frequency range the group velocity and wavelength of spin waves are smaller by several orders of magnitude than the velocity and wavelength of electromagnetic waves of the same frequency in coaxial cables or dielectric waveguides [1]. These two properties provide a possibility for creation of miniature microwave delay lines within planar design of the device [2]. The group velocity of the spin waves is proportional to the thickness of the YIG film and can be tuned in a wide range of the magnitudes. Such delay lines can achieve large values of delay time, up to 1μs, which is a very promising property for their use in the positive feedback loop of the auto-oscillatory systems as frequency generators. The advantage of an auto-oscillator, based on the YIG thin film feedback loop is the ability to change the generation frequency of the auto-oscillator by tuning the applied bias magnetic field [1].

One of the most important characteristics of an auto-oscillator in technical applications is its ability to generate a pure sinusoidal signal, that can be described as

$$a(t) = A_0 \cos\left(2\pi f_0 t + \varphi_0\right), \qquad (1)$$

where A_0 is an amplitude, f_0 – frequency and φ_0 - initial phase of the generated signal. In reality, noise phenomena leads to amplitude and phase fluctuations. It is known that auto-oscillating systems are mostly sensitive to *phase* fluctuations, and the experimentally observed generation spectrum of a typical auto-oscillator close to the generation frequency, for the most part, is determined by the phase fluctuations[3]$^{-}$[5]. Thus, the phase contains the term $\varphi(t)$ which describes stochastic process:

$$a(t) = A_0 \cos\left[2\pi f_0 t + \varphi_0 + \varphi(t)\right]. \qquad (2)$$

In the frequency domain this stochastic process $\varphi(t)$ can be described by its one-sided power spectral density $S_\varphi(f)$, which has the physical dimension rad^2/Hz. In engineering applications the more common quantity is the "phase noise" – $\mathscr{L}(f)$, which equals (by the definition) $\mathscr{L}(f) = (1/2)S_\varphi(f)$. It's more convenient to measure this quantity in logarithmic scale $\mathscr{L}(f) = 10\lg\left[\frac{1}{2}S_\varphi(f)\right]$, and units of $\mathscr{L}(f)$ in such representation are called "dBc/Hz" [3]. The physical meaning of $\mathscr{L}(f)$ can be defined as the ratio of the noise power in 1Hz bandwidth to the carrier power [3].

As it follows from the [5], [6] the phase noise is inversely proportional to both the power of the signal, circulating in the loop and the square of the delay time. It also should be noted [6], that the delay line should operate in the linear regime to minimize the phase noise, that limits the maximum input power. Such a dependence leads us to the next problem: increasing the film thickness leads to the reduction of the delay time per unit length, while decreasing of the thickness leads to the reduction of the signal power. Thus, one should use long delay lines (usually 1cm) to achieve a satisfactory result. However, the external magnetic field should have high homogeneity along the delay line due to the dependence of the dispersion law on the bias magnetic field. Thus, the usage of longer YIG films requires significant increase in size and complexity of the external magnetic system.

One of the ways to achieve a low level of phase noise in an auto-oscillator is to use a a resonance element with a large time delay in a positive feedback loop of the device [3]. The use of delay lines containing ferrite (usually, low-damping yttrium-iron garnet (YIG)) films in auto-oscillators contributes additional advantages of the auto-oscillator frequency tuneability by changing the bias magnetic field, and the possibility of miniaturiza-

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tion of the device due to the low velocity (and, therefore, low wavelength) of the spin waves propagating in the delay line. It has been shown in [6] that there is an optimum delay time of a YIG film delay line corresponding to the minimum phase noise of an auto-oscillator based on this line. This optimum delay can be achieved either by increasing the length of the YIG film or by decreasing its thickness, but the increase of the YIG film length increases the device size, while the reduction of the YIG film thickness significantly reduces the maximum microwave power that can be handled by the device. The reduction of the carrier power increases the phase noise of the auto-oscillator [3–5], while the increase of the film length increases spatial inhomogeneities of bias magnetic film and temperature.

Here we propose to use a relatively thick YIG film with periodically modulated thickness (or one-dimensional "magnonic crystal"[7]–[13]) in the delay line placed in the positive feedback loop of an auto-oscillator to achieve the optimum delay time (and minimum possible phase noise), while keeping a reasonably small size and high power-handling capability of the device. To obtain a large delay in a relatively thick YIG film we choose the generated frequency f to be in the region near one of the band-gap [7] in the dispersion relation $f(k)$ (where k is the wave number) of a magnonic crystal where the spin wave group velocity $v = \partial f/\partial k$ is low.

II. MODEL

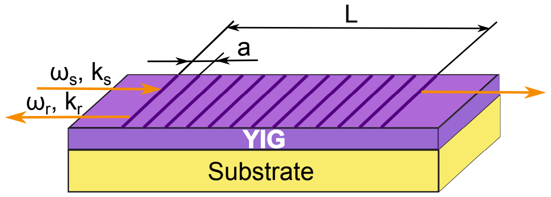

FIG. 1. The model of the magnonic crystal. The propagating wave with frequency ω_s interacts with the magnonic crystal and creates a reflected wave with the frequency ω_r.

In our model, we consider a one-dimensional (1D) magnonic crystal of length L, made of an YIG film with periodically modulated (with the spatial period a) thickness and the ferromagnetic resonance linewidth (or damping factor) ΔH. The spatial period a of the magnonic crystal determines the resonance wave number k_a and the resonance frequency ω_a (frequency of the band gap center) of the crystal through the condition $\omega(k_a) = \omega_a$. A forward-propagating spin wave with a frequency $\omega_s(k_s) \simeq \omega_a$ and group velocity v moving in the periodically modulated YIG film (magnonic crystal) is coupled to another wave with the frequency $\omega_r(k_r) \simeq \omega_a$ propagating in the opposite direction (reflected wave) [13].

One can write the following laws of energy and momentum conservation:

$$\begin{aligned} 2\omega_a &= \omega_s + \omega_r, \\ 2k_a &= k_s - k_r = 2\pi/a, \end{aligned} \tag{3}$$

where a is the lattice size of the magnonic crystal and a/L should be an integer. Since this process is efficient only over a narrow range of wave vectors k_s and k_r, the dispersion relation can be approximated using a first order Taylor series:

$$\begin{aligned} \omega_s &= \omega_a + v(k_s - k_a), \\ \omega_r &= \omega_a - v(k_r + k_a), \end{aligned} \tag{4}$$

where v is the wave group velocity. Taking into account the above equations one can express the dynamics of the complex slow amplitudes a_s and a_r near the frequency ω_a of the incident and the reflected wave by the pair of coupled equations (see e.g. Eq.(1a,b)[13]):

$$\begin{aligned} da_s(z,t)/dt + v da_s(z,t)/dz + \Gamma a_s(z,t) = U(z) a_r(z,t), \\ da_r(z,t)/dt - v da_r(z,t)/dz + \Gamma a_s(z,t) = U(z) a_s(z,t), \end{aligned} \tag{5}$$

where Γ is the damping factor, $U(z,t)$ characterizes the potential barrier and can be modeled by the rectangular shape in $z-$direction $U(z) = \kappa\Theta(z)\Theta(L-z)$, where $\Theta(z)$ means Heaviside $\Theta-$function and κ is the coupling parameter. The coupling strength κ between the waves can be characterized by half of the band gap width $\Delta\omega_a$ of a magnonic crystal – $\kappa \simeq \Delta\omega_a$, and typically is in the range 5-100MHz [8]. In the above equations (5) we have assumed that the damping factor Γ and group velocities are equal for incident and reflected waves. The above system of equations can be augmented with the initial and boundary conditions:

$$\begin{aligned} & a_s(0,t) = a_z^0(t), \\ & a_r(L,t) = 0, \\ & a_s(z,0) = a_t^0 = a_z^0(z/v)\exp(-\Gamma z/v), \\ & a_r(z,0) = 0; \end{aligned} \tag{6}$$

III. RESULTS

Now, it is straightforward to find the transfer function of the magnonic crystal in the frequency domain, by solving Eqs. (5) with the boundary conditions given in Eqs. (6):

$$\begin{aligned} L_s(\omega) = vG_s(s,1,0) = \Omega/\left[s\sinh\Omega + \Omega\cosh\Omega\right] \\ L_r(\omega) = vG_r(s,0,0) = i\lambda/\left[s + \Omega\coth\Omega\right]. \end{aligned} \tag{7}$$

Here $\Omega = \sqrt{s^2 + \lambda^2}$ and $s = \tau_L(i\Delta\omega + \Gamma)$, where $\tau_L = L/v$ and $\Delta\omega$ is the detuning frequency from the band gap center of the magnonic crystal ω_a.

Usually, the optimum delay time, which minimizes the phase noise of an auto-oscillator based on the YIG-film delay line, is of the order of a few hundred nanoseconds [6], depending on the damping factor ΔH of the YIG film. With the known transfer functions of the magnonic crystal it is easy to obtain the effective delay time, which the magnonic crystal contributes to the incident or the reflected wave, and compare this result with the delay time of a usual non-patterned YIG film of the equal length. This effective delay time can be expressed as:

$$T_{eff} = -i\frac{\partial L(\omega)}{\partial \omega} \tag{8}$$

An example of the Amplitude-Frequency Response (AFR) characteristic curve and the dependence of the spin wave propagation (delay) time on the frequency detuning $\Delta\omega = \omega_s - \omega_a$ for a incident wave is shown in Fig. 2.

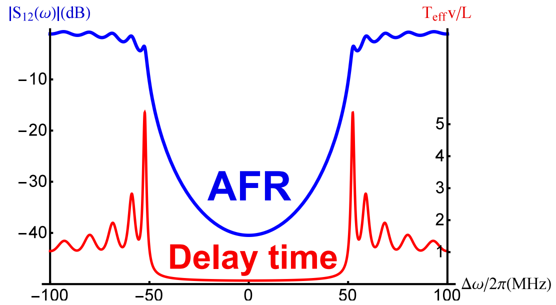

FIG. 2. Amplitude-frequency response (AFR) characteristic (upper dotted curve) and effective delay time (lower solid line) of the forward-propagating spin wave for $\kappa = 50$MHz, $L/v = 200$ns and $\Delta H = 0.2$Oe. Delay time of the non-patterned YIG film of the equal length $T_{eff} = 1$.

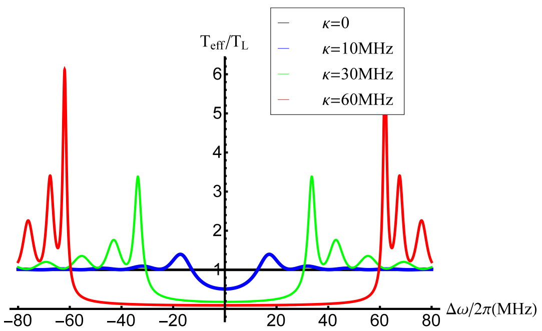

FIG. 3. Effective delay time T_{eff} for different values of coupling parameter κ (width of the magnonic crystal band gap) for $L/v = 200$ns and $\Delta H = 0.2$Oe.

As one can see from Fig. 2 the characteristic AFR of the magnonic crystal near the bandgap has a series of maxima corresponding to the so-called "magnonic crystal modes" [13] caused by the periodic energy transfer between the incident and reflected spin waves. Due to this effect, the maximum delay time of a spin wave in a magnonic crystal is achieved at a certain value of the frequency detuning $\Delta\omega$, as it is shown in Fig. 2. The

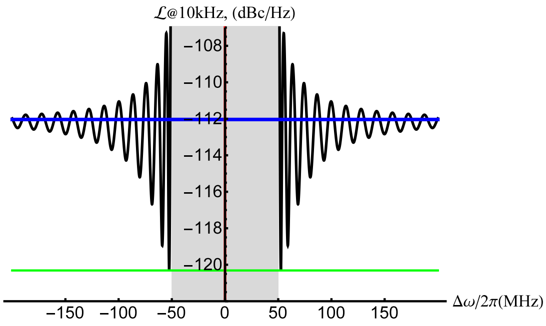

FIG. 4. Dependence of the phase noise figure (see Leeson Eq. in [3]) calculated at 10kHz Fourier frequency on the frequency detuning $\Delta\omega$ of the forward wave from the center of the magnonic crystal band-gap. Input power - 0dBm, the other parameters are the same as in the Fig. 2.

maximum of the effective delay time increases with the increase in the magnonic crystal band gap, as one can see from Fig. 3. Thus, by using a correct detuning it is possible to substantially increase the delay time in a magnonic crystal having a sufficiently large bandgap. Fig. 4 shows well-pronounced minima of the auto-oscillator phase noise take place at the detuning frequencies $\Delta\omega$ corresponding to the maxima of the delay time. In particular, the noise level below -110 dB/Hz can be achieved for the detuning of $\Delta\omega/2\pi \simeq 50$ MHz at the mode that is the nearest the band gap. It should be noted, that depending on the parameters of the magnonic crystal, the value of the phase noise can achieve a minimum at the higher mode of the magnonic crystal.

We also calculated the expected value of the phase noise of an auto-oscillator, based on delay line with magnonic crystal, used in [13]. It turned out, that the minimum of the phase noise could be achieved, when the frequency of the carrier lies in the center of the magnonic crystal band gap. In this case, the magnonic crystal acts as a reflector of the spin waves and doubles the power of the spin wave in the delay line. Therefore, such insertion of the magnonic crystal in the delay line has no advantages as compare to the simple reflective edge of the magnetic film. If the frequency of the carrier lies outside the band gap of the magnonic crystal, the value of the phase noise is close to the non patterned delay line of the same length between transducers. The influence of "magnonic crystal modes" near the band gap does not exceed the value of 1dBc/Hz at the phase noise figure. In this case, the effective delay time of magnonic crystal is small due to the low value of the band gap width, that is equal to 10 MHz, and is comparable to the delay time of non patterned film of the same length.

IV. CONCLUSIONS

We have considered the dynamics of the spin waves interacting with a magnonic crystal. It was determined, that the AFR characteristic curves for both transmitted and reflected waves have a series of maxima and minima near the bandgap cause strong non-monotonic behavior of the effective delay time as a function of the detuning frequency. As the result, the effective delay time introduced into the transmitted wave can exceed a few times that of a non-patterned YIG film of equal length. Thus, the use of a magnonic crystal in a delay line of an auto-oscillator can substantially improve its stability and reduce its phase noise value.

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