Detection of Fast Moving and Accelerating Targets Compensating Range and Doppler Migration

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DSTO–TR–2978

ABSTRACT

We are reporting on a new signal processing technique for the detection of fast accelerating targets that spread over multiple range bins and Doppler filters in the radar receiver. By compensating for these undesirable effects, improvements in coherent gain of more than 2 dB can be achieved, thus optimizing detection performance. The technique is demonstrated with simulated data, and implementation can be expected to be readily feasible with many existing radar systems.
Detection of Fast Moving and Accelerating Targets
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Executive Summary

In this report, we present a new signal processing algorithm for detecting highly maneuvering targets in pulsed Doppler radars mitigating range walk and Doppler spread effects. The radar received signal from a fast moving target may straddle multiple range bins during a single coherent processing interval, an effect known as range walk. Additionally, if the target is accelerating the target returns may spread across multiple Doppler bins. Thus, such target returns may spread across multiple cells in range-Doppler map, reducing peak signal to noise ratio (SNR) and detection probability of the target.

The proposed method consists of a range bin collapsing technique to compensate for range bin migration of fast moving targets, and a de-chirping technique to compensate for Doppler spread of accelerating targets. The range bin collapsing technique identifies returns from the same target which fall into contiguous range bins during an initial detection stage, and then coherently combines target returns from all the pulses into a collapsed range bin. Next, Doppler processing is applied in combination with the de-chirping technique such that maximum coherent gain could be achieved, in addition to estimating the target acceleration.

The new algorithm was compared with the conventional processing, and detection improvement of the new method was demonstrated. Probabilities of target detection were estimated using Monte-Carlo simulations for a range of target SNR levels, and for a number of different target dynamics, in two L-band and X-band radar systems, at a fixed false alarm rate. The proposed method achieved a detection probability of 0.5 at a lower target SNR compared to the standard processing for each of the maneuvering targets considered. On average, 2.2 dB gain was found at $P_d = 0.5$. Also, importantly, the proposed method achieved identical detection performance as in the standard processing for a slow moving non-accelerating target.
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Glossary

CA-CFAR – Cell Averaging Constant False Alarm Rate
CFAR – Constant False Alarm Rate
CPI – Coherent Processing Interval
FFT – Fast Fourier Transform
i.i.d. – Independent and Identically Distributed
IQ – In-phase Quadrature
ISAR – Inverse Synthetic Aperture Radar
LFM – Linear Frequency Modulation
MACH – Speed of Sound
MIMO – Multiple-input Multiple-output
MTI – Moving Target Indication
PDF – Probability Density Function
PRF – Pulse Repetition Frequency
SAR – Synthetic Aperture Radar
SIR – Signal to Interference Ratio
SNR – Signal to Noise Ratio
1 Introduction

It is well known that, in a pulsed Doppler radar, the received signal from a fast moving target sampled at regular intervals may straddle multiple range bins during a single coherent processing interval (CPI) - an effect also known as ‘range walk’. If the target also accelerates, spectral spreading across multiple Doppler bins may occur. Therefore, such target returns may spread across multiple cells in the range-Doppler map, reducing peak signal to noise ratio (SNR) and detection probability. For optimum detection performance, processing which accommodates the spreading is desirable.

In the open research literature, a number of different techniques have been proposed to overcome range and Doppler migration issues in pulsed Doppler radars. An algorithm for coherent integration while explicitly accounting for range bin migration is introduced by Marzetta [1] in 1993. In this method, two dimensional convolution in fast time (pulse delay) and slow time (pulse index) is used to combine matched filtering and coherent pulse integration, rather than the conventional method where Doppler processing and pulse compression are followed one after the other. As a result it performs velocity dependent matched filtering required for high time-bandwidth product pulses. However, the method does not take into account possible Doppler migration issues.

A Hough transform based pulse integration technique was proposed by Carlson [2] in 1994 for improved detection and tracking in search radars. The method enables non-coherent integration with stationary, moving, or accelerating targets by integrating signals from targets with inter-cell motion, thus avoids range migration issue. However, the integration is performed non-coherently, and therefore for a long CPI with large number of pulses, the optimal integration gain could not be achieved.

A number of methods based on the keystone transform have also been successfully employed to overcome range migration [3–7]. The keystone transform has been first introduced for synthetic aperture radar (SAR) and inverse SAR (ISAR) applications to remove linear range migration, and then later has been applied in the context of pulsed Doppler radars. The keystone transform rescales the slow time axis appropriately to remove coupling between range and Doppler due to linear range migration of targets. The keystone transform is well suited for high pulse repetition frequency (PRF) radars with unambiguous Doppler and for fast moving non-accelerating targets. However, for low or medium PRF radars where Doppler is ambiguous, it fails to decouple the effects of linear range migration. Also, the keystone transform does not compensate for Doppler migration due to target acceleration.

In recent times, the Radon-Fourier transform has been introduced to realize long-term coherent integration of the moving targets with range migration [8, 9]. Radon-Fourier transform uses relationships between a target’s radial velocity, range migration, and Doppler frequency to achieve spatial-temporal decoupling in the target’s range compressed echoes. The method is based on joint searching along range and velocity dimensions, and thus computationally expensive. Additionally, the linear search approach does not take in account effects of target acceleration.

For chirp pulsed radars, a new compression technique has been proposed which is capable of detecting fast moving targets with arbitrary constant velocities [10]. In this method,
the chirp bandwidth is changed from pulse to pulse to realise the range migration compensation. Also, Dai et. al. [11] proposed new adaptive detectors for improved detection of range migrating targets in clutter for chirp waveforms. However, these techniques are developed based on the assumption that linear frequency modulated (LFM) waveforms are used, and hence are not applicable for arbitrary waveforms.

Recently, Deudon et. al. [12] proposed a new algorithm for wideband low PRF radars to counter range migration phenomenon. The method was developed in the framework of high range resolution moving target indication (MTI) radars. These algorithms are based on iterative versions of wideband-Capon and wideband amplitude and phase estimation (APES) adaptive filtering techniques [13], and utilize the range migration information to solve velocity ambiguities using a single low PRF wideband waveform. However, these algorithms are computationally expensive, and better suited for wideband low PRF waveforms with MTI processing.

When Doppler processing is performed with the conventional Fourier method, accelerating targets induce a Doppler spread in the range-Doppler map, reducing coherent gain and detection performance. The limitations of the conventional Fourier method in detecting accelerating targets by pulsed Doppler radars are analysed in [14]. It shows that when the number of pulses in a CPI is increased, the output SNR of an accelerating target varies as a concave function, increasing to a maximum and then decreasing before completely failing. In order to overcome the implied performance loss and to compensate for Doppler smearing of accelerating targets, methods such as de-chirping, time-frequency distributions, and fractional Fourier transform based methods have been proposed in the literature.

For a pulsed Doppler radar, the signal return from a constant accelerating target takes the form of a discrete-time chirp signal. The instantaneous frequency of a chirp signal varies with time, and therefore time-frequency distributions have been proposed for detecting such signals. Wigner-Ville distribution based methods have been proposed for concentrating radar returns from accelerating targets in the time-frequency plane [15,16]. The Wigner-Ville distribution of a chirp signal is highly concentrated at the instantaneous frequency in the time-frequency plane. However, due to their quadratic nature, cross-terms are present, and have many undesired effects when detecting multiple targets. The S-method, which was introduced by Stanković [17], has also been proposed to detect accelerating targets in pulsed Doppler radars [18]. It has the advantage of relatively low computational cost compared to the Wigner-Ville distribution.

Recently, fractional Fourier transform has been proposed as a viable tool for detecting accelerating targets in clutter environments [18,20]. With the fractional Fourier transform, chirp signals will appear as pure tone signals in ‘fractional time’ when appropriately rotated in the time-frequency domain. Another advantage of using the fractional Fourier transform is the possible defocussing or suppression of clutter in the rotated time-frequency plane when focused to the desired target signal [18]. However, the fractional Fourier transform is more computationally expensive compared to the previously described methods.

A simpler de-chirping technique can also be used to detect chirp signals induced by accelerating targets. It involves multiplying the received signal by a de-chirping factor, followed by traditional Fourier transform [21]. Similar to the application of fractional Fourier transform, chirp signals can be detected as efficiently as pure tone signals, and
extra processing gain may be achieved by defocussing the clutter spectrum. The de-chirping method slightly reduces the computational complexity of the detection algorithm, compared to the fractional Fourier transform based method. The de-chirping algorithm can be extended to include the CLEAN technique \cite{22} to detect multiple accelerating targets present in the same range bin.

Multiple-input multiple-output (MIMO) radars and passive bistatic radars are of great interest within the current radar research community. Range and Doppler migration become more problematic for coherent MIMO radars as longer integration times are required to compensate for reduced antenna gain in orthogonal waveform modes in MIMO radars \cite{23}. Passive bistatic radars also encounter range and Doppler migration problems, and CPI length becomes limited by bistatic velocity and acceleration. A hybrid integration technique where non-coherent addition of several coherently processed intervals has been proposed in the context of passive bistatic radars \cite{24}. A stretch processing technique \cite{25,26} which alters the time scale of the reference signal of the bistatic system has also been proposed to achieve long integration times while overcoming range migration of moving targets in passive bistatic radars.

In this report, we propose a new algorithm to simultaneously compensate for both the Doppler spread effect due to target acceleration and range walk problem due to high target velocities. The proposed algorithm uses a de-chirping technique \cite{21} to integrate target power spread across multiple Doppler bins, and a range bin collapsing process to integrate power spread across multiple range bins. This improves the target SNR, and hence its probability of detection. Areas of application may include detection of airborne targets with high closing velocities or missiles.

This report is organized as follows: In Section 2, we introduce the signal model used and how range and Doppler migration could occur in a pulsed Doppler radar for some target dynamics. In Section 3, we describe the new algorithm for compensating range and Doppler migration and improving detectability. In Section 4, we present preliminary results for two simulated radar systems and number of target motion scenarios. Conclusions are given in Section 5.

2 The Range-Doppler Migration Problem

In this Section, we set up a signal model to be used for the investigation, and describe how the problem of range and Doppler migration may occur in a pulsed Doppler radar for a high velocity target which may accelerate as well. Pulsed Doppler radars transmit a sequence of pulses and coherently integrate receiving echoes to maximize target SNR hence target detection. They usually assume the target’s velocity remains relatively constant during each CPI and all samples of a CPI are collected in a single range bin. However, for a highly maneuvering target, this assumption may not be valid, and both Doppler spread and range walk may occur.
2.1 Signal Model

The transmit signal of a pulsed Doppler radar consisting of M pulses during a CPI can be represented as

\[ s_{tx}(t) = \sum_{m=0}^{M-1} p(t - t_m) \exp\{-j2\pi f_c(t - t_m)\}, \]  

where \( p(\cdot) \) is the pulse waveform at baseband,

\[ t_m = m T_{pri} \]  

are the slow time points, \( m \) is the pulse index, \( T_{pri} \) is the pulse repetition interval, and \( f_c \) is the carrier frequency. The waveform \( p(t) \) will be time limited to \( T_p \), where \( T_p \) is the pulse width. For a typical 10\% duty cycle, \( T_p = 0.1T_{pri} \).

Assume that a point scatterer is present at range \( R_m \) for the \( m^{th} \) pulse. The demodulated received signal, ignoring the carrier phase term, will then take the form

\[ s_{rx}(t) = \sum_{m=0}^{M-1} A(t_m)p(t - 2R_m c) \exp\{j2\pi f_c \left( t_m + \frac{2R_m c}{c}\right)\}, \]  

where \( A(t_m) \) is the amplitude of the target return for pulse \( m \). In the model, the distance traveled by the target from pulse to pulse is taken into account, but target motion within a single pulse is assumed to be negligible. The target range at the sampling time of the \( m^{th} \) pulse is approximately equal to

\[ R_m \approx R_0 + v_0(t_m) + \frac{1}{2} a(t_m)^2, \]  

where \( R_0 \) and \( v_0 \) are respectively the target range and radial velocity at the beginning of the CPI, and \( a \) is the constant radial acceleration.

For convenience, we write the received signal in the baseband as a two-variable function of fast time \( t \) and slow time \( t_m \) corresponding to delay of the target return and pulse index respectively,

\[ \tilde{s}_{rx}(t, t_m) = A(t_m)p(t - t_m - \frac{2R_m}{c}) \exp\{j2\pi f_c \left( t_m + \frac{2R_m c}{c}\right)\}. \]  

For each pulse, we assume perfect pulse compression where all the pulse energy in the target return for that pulse is compressed into a single range bin. However, across consecutive pulses, that compressed range bin is not necessarily the same range bin index due to target motion. We write

\[ s_{rc}(n, t_m) = C(t_m) \delta(n - n_m) \exp\left(\frac{j4\pi f_c R_m}{c}\right), \]  

where \( C(t_m) \) is the range-compressed amplitude, \( n \) is range bin index, \( n_m \) is the compressed range bin index for pulse \( m \), and \( \delta(\cdot) \) is the Dirac delta function. Depending on radar parameters and target dynamics, migration in either or both Doppler and range can thus be captured by the model expressed by [6].

The above signal model with added noise and/or clutter components will be used in this work to simulate target returns in the pulse-range domain.
2.2 The Range-Doppler Migration Problem

In this Section, we show the criteria for range walk or Doppler spread to occur, and show numerical examples of target SNR reduction due to range and Doppler migration.

First, the velocity of an accelerating target may change during a CPI such that its Doppler frequency change is greater than the Doppler bin size. The instantaneous Doppler frequency $f_d$ of a target moving at a radial velocity $v$ is given by

$$f_d = \frac{2v}{\lambda},$$

where $\lambda$ is the carrier wavelength. If the target is moving at a constant radial acceleration $a$, then the change in Doppler frequency $\Delta f_d$ during the CPI is given by

$$\Delta f_d = \frac{2a T_{cpi}}{\lambda},$$

where $T_{cpi}$ is the CPI time.

The standard Doppler bin size $\delta f_d$ of the radar is given by

$$\delta f_d = \frac{1}{T_{cpi}},$$

and in order for the target to remain in a single Doppler bin during the CPI, the condition

$$\Delta f_d \leq \delta f_d$$

must hold, which leads to

$$T_{cpi} \leq \sqrt{\frac{\lambda}{2a}}.$$  

Thus, if no further processing is applied, to avoid Doppler migration, a CPI time must be limited by the target acceleration and cannot be arbitrarily increased for more coherent integration gain. For example, an L-band radar (carrier frequency of 1.3 GHz) which intends to detect targets accelerating at $6g$ ($g = 9.81 \text{ ms}^{-2}$) would be limited to using CPIs of 44.3 ms. If longer CPIs were used, Doppler migration would degrade the coherent processing gain.

Fast moving targets may appear in contiguous range bins, rather than a single range bin, during a single CPI when the distance travelled by the target during the CPI exceeds the range bin size. This problem is most likely to be the case in radars employing fine range bins, and needs to be corrected. The distance travelled by a target with an initial radial velocity $v_0$ and radial acceleration $a$ during a CPI is

$$\Delta r = v_0 T_{cpi} + \frac{1}{2}a T_{cpi}^2.$$  

Range resolution $\delta r$ of the radar is given by

$$\delta r = \frac{c}{2B}.$$
where \( B \) is the radar bandwidth and \( c \) is the speed of light. Suppose range bin size is equal to range resolution, then in order for target to remain in a single range bin during the CPI, the condition
\[
\Delta r \leq \delta r
\]  
must hold. Assuming range migration due to the acceleration component is negligible, this leads to
\[
T_{\text{cpi}} \leq \frac{c}{2v_0 B}.
\] (15)

For an example, a radar with a bandwidth of 5 MHz which is designed to detect targets moving at MACH-2 speed will be limited to 43.7 ms CPIs. If longer CPIs were used, range migration will degrade the coherent processing gain. Thus, in conventional pulsed Doppler radars, to detect fast accelerating moving targets, the number of pulses which can be coherently integrated must be limited. However, this requirement conflicts with the requirement for detecting small targets at a long range in which longer CPIs may be necessary.

In this report, we present an algorithm for improving the detection of fast accelerating targets using long CPIs compensating for possible Doppler and range migration problems as described above. Note that even when target dynamics and CPI time satisfy Equations \([1]\) and \([15]\), target signal energy could still be present in multiple range-Doppler bins due to target straddling across bins, due to oversampling, or due to imperfect pulse compression. However, those issues are well understood, and the focus in this report is solely on range and Doppler migration due to target motion.

### 2.2.1 Examples

The range and Doppler migration problem is illustrated here. Figure 1 shows range-Doppler maps for range and/or Doppler migrating targets as well as for a target which does not migrate in range or Doppler. For the simulations, an X-band radar (\( \lambda = 3 \text{ cm} \)) with a 10 MHz bandwidth and a CPI of 102 ms was used. A non-fluctuating target with an SNR of 20 dB was injected at different closing velocities and acceleration values.

Figure 1(a) shows the range-Doppler map of a non-accelerating target moving at 100 knots. The target returns are concentrated into a single range-Doppler cell and a peak SNR of 19.6 dB was generated. Figure 1(b) is for the same target but accelerating at 1g. At this acceleration, the condition \([1]\) does not hold and the target returns are spread across multiple Doppler bins. The peak target SNR has been reduced to 13.45 dB, which would result in a significant detection loss.

Figure 1(c) shows the range-Doppler map of a non-accelerating target, but moving at a closing velocity of 850 knots. Due to the high radial velocity, it travels across three range bins during a single CPI. Note that in this case, the condition \([15]\) does not hold, and the peak SNR has been reduced to 12.13 dB. Finally, range-Doppler map for a fast moving accelerating target is shown in Figure 1(d). The target is moving at a velocity of 850 knots, and also accelerating at 1g. Both range and Doppler migration is visible, and target peak SNR has been further reduced to 11.22 dB. These examples illustrate how range and Doppler migration occur due to different target dynamics during a single CPI, and how they typically manifest in a range-Doppler map. Due to the loss in target peak
Figure 1: Examples of range and Doppler migration for: (a) a target with no range or Doppler migration, with \( v = 100 \) knots, \( a = 0 \); (b) a target with only Doppler spread, with \( v = 100 \) knots, \( a = 1g \); (c) a target with only range walk, with \( v = 850 \) knots, \( a = 0 \); and (d) a target with both Doppler spread and range walk, with \( v = 850 \) knots, \( a = 1g \).
SNR, the subsequent detection would be considerably degraded unless cell migration is compensated.

3 The Proposed Solution

Here, we describe the building blocks of our proposed solution to the range-Doppler migration problem.

3.1 Overview of the New Algorithm

First, received IQ data is processed as usual using standard signal processing techniques such as beam forming, pulse compression, and Doppler processing to generate a range-Doppler map. At this point, standard processing would usually involve applying a constant false alarm rate (CFAR) detection algorithm to generate a hit matrix. The threshold multiplier for the CFAR detection is set by an acceptable probability of false alarm rate. For cell averaging CFAR (CA-CFAR) [27], and for the case of square law detector and i.i.d. exponential distributed interference, this threshold multiplier $\mu$ is given by

$$
\mu = N \left( P_{fa}^{-1/N} - 1 \right), \quad (16)
$$

where $P_{fa}$ is the acceptable probability of false alarms, and $N$ is the ‘sliding window’ length for calculating CFAR noise estimates. For any other CFAR detection algorithm, $\mu$ may be computed analytically or estimated empirically.

The proposed algorithm involves running the detection algorithm with a lower threshold to generate an initial hit matrix. Then, a new methodology is used to process range and Doppler migration generating a new range-Doppler map. Finally, a second-pass CFAR detection is applied with the original threshold. The new algorithm is visualised in flowchart form in Figure 2. How much the threshold should be lowered in the first-pass detection stage is dependent on CPI length, range bin size, and maximum target velocity intended to be detected by the radar. The initial threshold multiplier $\mu^*$ is given by

$$
\mu^* = \frac{\mu}{[v_{max} T_{cpi}/\delta r]^2}, \quad (17)
$$

where $v_{max}$ is the maximum target velocity intended to be detected, and $\lceil \cdot \rceil$ is the lowest integer greater than the argument. This initial threshold is computed such that targets which migrate multiple range bins are detected in the initial hit matrix. In this work, the maximum target velocity intended to be detected was set to 2000 knots. Note that the second-pass detection stage involves running the detection algorithm with the original threshold multiplier $\mu$ to ensure that the same false alarm rate will be reached as in the standard processing.

The new methodology to process range and Doppler migration consists of the following 3 steps:

1. Range-walk processing: to identify contiguous range bins containing migrating hits,
2. **Range-bin collapse processing**: to combine time samples that belong to a single target,

3. **De-chirp processing**: to correct for Doppler spreads due to target acceleration.

These processing steps are described in detail in the following sub-sections.

### 3.2 Range-Walk Processing

The initial hit matrix is used to identify possible range migrating targets. If range migration occurs, standard processing may fail to detect such targets as energy from the target return is ‘spilled’ across multiple contiguous range bins. However, with a lower threshold level $\mu^*$ used in the first pass, those hits can be retained in the initial hit matrix, along with a higher rates of false hits as well. The false hits will be subject to further filtering and elimination. As an initial processing step, this hit matrix is centroided in Doppler frequency to obtain more accurate Doppler estimates for subsequent analysis.

Hits in two or more contiguous range bins are searched which would indicate a possible range migrating target. If hits are found in contiguous range bins, that range bin group is further tested to eliminate possible false hits, while retaining genuine multiple targets.
in contiguous range bins. These tests which help isolating only the targets with range migration are described below.

**Test 1:** If the detected hits in contiguous range bins are from the same target, their velocity variation would be compatible to a realistic acceleration. Otherwise, those hits are rejected for the purpose of range migration processing. In our current work, a maximum acceptable acceleration is set at \( a_{\text{max}} = 10g \), which could be varied according to the types of targets expected. For two hits with corresponding velocities \( v_1 \) and \( v_2 \), they would only be considered for further range migration processing only if
\[
\frac{|v_1 - v_2|}{T_{\text{cpi}}} \leq a_{\text{max}}.
\]
As the inequality is based on difference in velocity, Doppler ambiguities need not be considered.

**Test 2:** Next, a test whether the target velocity is high enough to cause range migration is carried out. For that purpose, we take the average velocity \( (v_m) \) of hits in the contiguous range bin group and compute the distance travelled by a target during the CPI. Range migration would occur only if
\[
v_m T_{\text{cpi}} > \delta r.
\]
Note that this test is only used for high PRF radars where Doppler frequencies are unambiguous.

**Test 3:** The proposed range migration processing can improve the target coherent SNR by combining range bins which contain returns from a single target; it however can also increase the noise floor. Hence, consideration must be given to establish a criteria for the combination processing to achieve a net SNR improvement for a target signal.

Suppose a particular CPI is comprised of \( M \) pulses, of which \( M_1 \) pulses are compressed to a range bin with index \( n_k \) while another \( M_2 \) pulses fall in the next range bin with index \( n_{k+1} \) due to target motion \( (M_1 + M_2 \leq M) \). Without loss of generality, assume that \( M_1 \geq M_2 \). Let the expected value of the amplitude of the target return for pulse \( m \) after compression be \( E(A_m) = A \), and noise variance be \( \sigma_n^2 \). Then, the SNR in linear units for range bin \( n_k \) is
\[
SNR_{n_k} = \frac{A^2 M_1^2}{\sigma_n^2},
\]
and similarly for range bin \( n_{k+1} \). When the two range bins are combined with the phase correction corresponding to the range bin size, the SNR in the combined range bin \( (n_c) \) is given by
\[
SNR_{n_c} = \frac{A^2(M_1 + M_2)^2}{2\sigma_n^2},
\]
based on the fact that the signal is coherent while noise is incoherent. Now, a net gain in SNR is achieved when \( SNR_{n_c} > SNR_{n_k} \), which yields
\[
\frac{M_2}{M_1} > \sqrt{2} - 1 \approx 0.414.
\]
The target power in each range bin as calculated by the first-pass CFAR detection algorithm is used to check whether the above criteria is met. This criteria is transformed to power levels in each range bin as follows. If hits are found in two contiguous range bin indices \( n_k \) and \( n_{k+1} \) with power levels \( P_k \) and \( P_{k+1} \) respectively, two hits are considered for further range walk processing only if

\[
\rho_{\min} < \frac{P_{k+1}}{P_k} < \rho_{\max},
\]

where \( \rho_{\min} = 3 - 2\sqrt{2} \approx 0.172 \), and \( \rho_{\max} = 3 + 2\sqrt{2} \approx 5.828 \).

When target returns are equally distributed among two range bins, i.e., \( M_1 = M_2 \), then \( SNR_{n_{k}} = 2 SNR_{n_k} \). Thus, a maximum gain of 3dB is achieved by the range bin combination process over single range bin processing for the particular case. Note that above analysis could be extended to combining three or more range bins as well, and the proposed method does not have any restriction on how many range bins can be combined.

### 3.3 Range-Bin Collapse Processing

Once a group of contiguous range bins containing a potential range-walking target are identified, the range bin group is combined to improve the SNR and hence its probability of detection. In order to preserve the coherency of the signal, the phase difference between range bins is taken into account. For a target with Doppler frequency \( f_d \), the phase change between two contiguous range bins is given by

\[
\Delta \phi = 2\pi f_d t_s,
\]

where \( f_d = 2v/\lambda \), \( v \) is the target radial velocity, \( \lambda \) is the carrier wavelength, and \( t_s \) is the sampling interval. If significant change in \( f_d \) is observed for the contiguous range bins, then an average \( f_d \) is used for the estimate of \( \Delta \phi \).

Next, for a target which migrates across \( l \) range bins starting from the range bin index \( n_k \), the combined range bin can be computed by

\[
s'_{rc}(n_c,m) = \frac{1}{\sqrt{l}} \sum_{q=k}^{k+l-1} s_{rc}(n_q,m)e^{-jq\Delta \phi},
\]

where \( s_{rc}(\cdot) \) is given by signal model (6). The normalisation factor \( (1/\sqrt{l}) \) is used to achieve the same mean noise power level before and after the range bin combination process, thus the new range-Doppler map will have the same mean noise power as the original. \( s'_{rc}(n_c,m) \) will contain target returns from all the pulses with correct phase shifts which would result in maximum coherent gain during subsequent Doppler processing. The set of range bins \( \{s_{rc}(n_q,m)\} \) in the original range-pulse data matrix is replaced by \( s'_{rc}(n_c,m) \) in order to obtain the new data matrix. This process is repeated for each possible range migrating target identified previously.
3.4 De-chirp Processing

After range bin combination processing described above to compensate for range bin migration, we have \( s'_{rc}(n_c, m) \) which contains target returns from all the pulses in a collapsed range bin \( n_c \). Doppler processing can then be applied to coherently integrate target returns from all the pulses. Conventional Doppler processing consists of applying a windowed Fourier transform in slow time assuming linearly changing phase across the pulses due to target motion. But if the target is accelerating, phase change across pulses is non-linear, and thus maximum coherent gain can not be achieved by conventional Fourier processing as target returns will be spread across multiple Doppler bins. Therefore, here we apply a de-chirping technique to compensate for the Doppler spread and achieve maximum coherent gain. Assuming the target’s acceleration is constant, we apply a de-chirping factor to the signal to achieve a linear phase change.

The de-chirping factor takes the form of \( \exp\{-2\pi j (a/\lambda)m^2\} \), where \( m \) is the pulse index, \( \lambda \) is the carrier wavelength, and \( a \) is the target acceleration. The de-chirp rate is defined as \( a/\lambda \). The signal after applying the de-chirping factor is given by

\[
s'_{rc}(n_c, m, a) = s'_{rc}(n_c, m) \exp\left(-2\pi j \frac{a}{\lambda}m^2\right). \quad (26)
\]

In this work, a range of possible target acceleration are considered from \( 0 \) to \( 10 \) g, and \( a \) is incremented by \( 0.1 \) g steps. Next, conventional Fourier transform is applied to transform above signal to the Doppler frequency,

\[
S'_{rc}(n_c, f_d, a) = \sum_m s'_{rc}(n_c, m) \exp\left(-2\pi j \frac{a}{\lambda}m^2\right) \exp(-2\pi j f_d m), \quad (27)
\]

where \( m \) is the pulse index. The result is a set of Doppler profiles for the collapsed range bin \( n_c \) for a range of possible target accelerations. The true target acceleration is where the coherent gain is maximum. Thus we can estimate the target acceleration \( \hat{a} \) by

\[
(\hat{f}_d, \hat{a}) = \arg \max_{f_d, a} \{S'_{rc}(n_c, f_d, a)\}. \quad (28)
\]

\( S'_{rc}(n_c, f_d, \hat{a}) \) is the Doppler profile for the collapsed range bin \( n_c \) at estimated target acceleration \( \hat{a} \). At this point, the target velocity and acceleration are estimated.

3.5 Impact on Target, Noise, and Clutter Statistics

In this Section, we analyse the impact of the proposed processing on target, noise, and clutter statistics. In the proposed technique, if a target migrates \( l \) range bins from the range bin index \( n_k \) to \( n_{k+l-1} \) during a CPI, those \( l \) range bins are collapsed, i.e., added together across the range bins, with a phase correction. Assuming that interference is noise limited, and noise samples at range bin indices \( n_k \) to \( n_{k+l-1} \) are complex random variables where each I and Q component is i.i.d Gaussian with zero mean and variance \( \sigma_n^2/2 \), then noise at the collapsed range bin \( n_c \) will also be Gaussian (I and Q component wise) with zero mean and variance \( l\sigma_n^2/2 \). The summation process increases the noise variance by a factor of \( l \). The normalisation factor \( 1/\sqrt{l} \) used in (25) brings the noise variance back to
the original value of $\sigma_n^2$, thus noise statistics is unchanged after the range bin collapsing processing.

The target statistics also remains unchanged by the range bin collapsing processing. The method does not explicitly process target returns; rather it aligns them to a single range bin, when target returns are spread across multiple range bins, to increase the number of samples of the target returns in that range bin for improved coherent integration, but does not alter target statistics.

The impact of the range bin collapsing on clutter statistics is such that the probability density function (PDF) of the summed clutter is the convolution of the two identical PDFs of the individual clutter samples; the exact form of which depends on the type of clutter in the received signal. For K-distributed or Weibull distributed clutter in particular, Dong [28] has reported approximation formulas for distributions of summation of multiple clutter samples. In general, the effective average power increase in clutter is also approximately $l$, as in the case of noise. Thus, the range bin collapsing processing in (25) is equally valid for the case of clutter. But, as usual, threshold for CFAR detection process needs to be appropriately altered in the case of clutter compared to noise limited case. Thus, an effective increase in signal to interference ratio (SIR) of up to $l$ (in linear units) can be expected by the proposed range bin collapsing processing.

4 Preliminary Results

In this Section, we demonstrate the improvement in the detection of fast moving and accelerating targets by applying the proposed algorithm on two sets of simulated data. The radar data are generated using the signal model described in Section 2.1 simulating highly maneuvering targets with different closing velocities and accelerations. The results are shown for radars operating in L-band and X-band, to show how range and Doppler migration effects differ on the different systems, for similar CPIs and dwell times. We show that in each case the proposed algorithm is capable of improving the detection probability compared to the conventional processing.

But first, we need a means to quantify any improvement this proposed processing may bring about, through comparisons of the probability of target detection.

4.1 Probability of Target Detection

The probability of target detection was computed by applying standard CA-CFAR detector [27] on the final range-Doppler map for the standard processing and for the proposed algorithm. The threshold multiplier $\mu$ used in the CFAR detector was computed to give a probability of false alarm of $10^{-6}$. The CFAR noise estimate for each cell under test was computed as the mean power of 32 neighbouring cells (i.e., 8 cells in each direction unless cell under test is at an edge of the range-Doppler map), with 4 guard cells in each direction. A hit was declared if power of the cell under test exceeds the threshold multiplier times the CFAR noise estimate for that cell.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency ($f_c$)</td>
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</tr>
<tr>
<td>Pulse Repetition Frequency (PRF)</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Bandwidth ($B$)</td>
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<tr>
<td>Number of Pulses in CPI</td>
<td>1024</td>
</tr>
<tr>
<td>Reference Target (closing velocity, acceleration)</td>
<td>200 knots, 0g</td>
</tr>
<tr>
<td>Target 1 (closing velocity, acceleration)</td>
<td>1100 knots, 0g</td>
</tr>
<tr>
<td>Target 2 (closing velocity, acceleration)</td>
<td>1100 knots, 9g</td>
</tr>
<tr>
<td>Target 3 (closing velocity, acceleration)</td>
<td>1700 knots, 0g</td>
</tr>
<tr>
<td>Target 4 (closing velocity, acceleration)</td>
<td>1700 knots, 9g</td>
</tr>
</tbody>
</table>

**Table 1:** High-PRF L-band radar system parameters and different target dynamics used in the simulations.

The target returns were generated using the signal model described in [6], with added Rayleigh noise at different SNR levels to simulate the target returns in the radar receiver noise. If a hit as described above was found in at least one range-Doppler cell where target was expected to be present, then the target was declared to be detected. Monte-Carlo simulations were used to determine probability of target detection estimates for a range of target SNR levels for different target motion scenarios.

### 4.2 A High-PRF L-band Radar

First, we apply the proposed method to a high-PRF L-band radar system looking at four maneuvering targets as described in Table 1, which could be representative of maneuvering fighter aircrafts or missiles. The radar parameters result in a CPI length of 102.4 ms, and a range bin size of 30 m. The algorithm was tested on these four targets with high closing velocities with and without acceleration.

A reference target was chosen with a velocity of 200 knots and no acceleration to compare how much degradation in probability of detection occurs due to cell migration of maneuvering targets, and how much the new algorithm regain some of the lost detection performance. Targets 1 and 2 have a closing velocity of 1100 knots which results in target returns spread across approximately two range bins. Targets 3 and 4 have a closing velocity of 1700 knots, thus spread across three range bins. Additionally, targets 2 and 4 are accelerating at 9g, thus Doppler migration is also observable during the CPI.

Figure 3 shows the Doppler spectral for single range bins using standard processing, and also for the collapsed de-chirped range bin when processed using the proposed algorithm. Figure 3(a) shows the Doppler spectrum for target 2 which approximately migrates across two range bins. In this particular case, the target SNR is set at 30 dB. But due to target power being spread in two range bins as well as due to the Doppler migration due to target acceleration, a peak power of 23.8 dB and 23.5 dB was found in each of the two range bins processed conventionally. The proposed algorithm combines the two range bins with the correct phase factor, thus coherently combining the target power into a single range bin. However, the target power is still spread across multiple Doppler bins due to acceleration effects as shown by the solid thin line in Figure 3(a).
technique corrects Doppler migration by performing the Doppler processing correcting for the target acceleration. Thus, the algorithm coherently integrates most of the target power into a single range-Doppler cell. The SNR has increased to 27.5 dB after range bin combination and de-chirping processing. Note that all the Doppler spectra shown in the Figures are normalised to have a 0 dB noise floor, thus adjusted to counter the noise floor increase in the range bin combination process for fair comparison.

![Doppler Spectral](image)

**Figure 3:** Doppler spectral for single range bins (standard processing), and combined and de-chirped range bin (new algorithm). (a) Target 2 (1100 knots, 9g) migrates across 2 range bins, and (b) Target 4 (1700 knots, 9g) migrates across 3 range bins. Both targets show Doppler migration due to acceleration. Simulations were performed for L-band radar parameters described in Table 7. Note that all signals have same mean noise power level.

Figure 3(b) shows Doppler spectral for target 4 which migrates across three range bins during the CPI. Maximum integration gain after Doppler processing was found to be 20.4 dB when individual range bins are processed as in the standard case. But com-
Bining the range bins and de-chirping results in an improved gain of 25.8 dB. Thus, over 5 dB of SNR gain is achieved by the proposed algorithm compared to the single range bin processing with conventional Fourier based processing. It is evident that range bin collapsing process combines target returns into a single range bin, and the de-chirping process compresses target returns into a single Doppler filter.

The probability of detection estimates for the reference target and other maneuvering targets are shown in Figure 4. Note that the SNR values shown in the plots are for the entire CPI length (i.e., not for a single pulse). The $P_d$ plot for the reference or ‘normal’ target (200 knots, 0g) which does not migrate in range or Doppler is shown in each plot for comparison. This $P_d$ curve is identical for both standard processing and the proposed algorithm, and it will be used to quantify the detection performance of the proposed algorithm on other fast moving and accelerating targets.

![Figure 4](image)

**Figure 4:** Probability of detection estimates of four targets with different closing velocities and accelerations for the high-PRF L-band radar. $P_d$ curve for the reference target is shown in all the plots for comparison.

Figure 4(a) shows the probability of detection estimates for target 1. For the reference target with no cell migration, a $P_d$ of 0.5 is achieved with an SNR of 14.2 dB. For target
1, returns are spread into two range bins degrading the detection in each range bin. Thus, in order to achieve the same $P_d$ of 0.5, an SNR of 17.4 dB is required. Note that when estimating probability of detection in the case of standard processing a hit in at least one of the range bins where a target is expected to be present is considered as a detection. By applying the proposed cell migration compensation algorithm, the same detection performance could be achieved at an SNR of 15.6 dB, thus reducing the detection loss by 56% in terms of the SNR. When a target is accelerating as well, the detection performance further degrades due to Doppler spread effects as shown in Figure 4(b) for target 2. In order to achieve a $P_d$ of 0.5, an SNR of 18.3 dB required, which is further 0.9 dB loss due to the target power being spread across multiple Doppler filters. Again, the proposed method compensates for the cell migration, and a detection probability of 0.5 is achieved with an SNR of 16.2 dB recovering just over 50% of the loss.

Figures 4(c) and 4(d) show the probability of detection estimates for targets 3 and 4, respectively. These targets migrate across three range bins, thus detection performance is further degraded compared to the reference target. However, the proposed method is capable of recovering more than 2 dB of lost gain during its cell migration compensation process in these two cases when comparison is made at $P_d = 0.5$.

### 4.3 A Medium-PRF X-band Radar

Next, we analyse the performance of the proposed cell migration compensation algorithm on an X-band radar. The radar parameters and different target dynamics used in the simulations are given in Table 2. Due to the smaller wavelength, Doppler shifts are considerably higher, thus the effects of the target acceleration are expected to be more profound compared to the case of the L-band radar. Also, we analyse the performance of the proposed method when detecting Doppler ambiguous targets as medium-PRF waveforms are used in this case.

For the X-band radar, a typical 10 MHz bandwidth would result in a finer 15 m range bin resolution, compared to 30 m resolution used in L-band. Though PRF was doubled to 20 kHz compared to the L-band case, unambiguous velocity is limited to 583 knots, thus targets 7 and 8 will be Doppler ambiguous. Note that 10 kHz PRF in L-band yield to an

<table>
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<td>Bandwidth ($B$)</td>
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<tr>
<td>Number of Pulses in CPI</td>
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<tr>
<td>Reference Target (closing velocity, acceleration)</td>
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</tr>
<tr>
<td>Target 5 (closing velocity, acceleration)</td>
<td>570 knots, 0g</td>
</tr>
<tr>
<td>Target 6 (closing velocity, acceleration)</td>
<td>570 knots, 3g</td>
</tr>
<tr>
<td>Target 7 (closing velocity, acceleration)</td>
<td>850 knots, 0g</td>
</tr>
<tr>
<td>Target 8 (closing velocity, acceleration)</td>
<td>850 knots, 3g</td>
</tr>
</tbody>
</table>

**Table 2:** Medium-PRF X-band radar system parameters and different target dynamics used in the simulations.
unambiguous velocity of 2330 knots, thus targets 1-4 were unambiguous in Doppler. Same CPI length was kept as in the case of L-band by increasing number of pulses in the CPI to 2048. Similar to previous simulation, targets 5 and 6 would migrate across two range bins, and targets 7 and 8 would migrate across three range bins. Additionally, targets 6 and 8 are accelerating at 3\textit{g}.

Figure 5 shows Doppler spectral after processing single range bins as well as combined and de-chirped range bin processing using the proposed algorithm. For target 6 shown in 5(a) the SNR has dropped from 30 dB to 19.5 dB due to the effects of range and Doppler migration. However, after compensating for cell migration, an SNR of 27.2 dB was achieved. Note that compared to the L-band case, Doppler migration is more significant even though target acceleration is small. Significant Doppler migration is caused by the small Doppler bin size due to reduced wavelength in X-band. Figure 5(b) shows Doppler spectral for target 8 which migrates three range bins. In this case, the target SNR has been increased from 19.3 dB to 24.7 dB by applying the range bin collapsing and de-chirping algorithm.

Next, we perform probability of detection analysis to demonstrate the detection improvement achieved by the proposed cell migration compensation algorithm. Identical detection probabilities were found for standard processing and proposed algorithm for a slow moving non-accelerating target, confirming that proposed algorithm does not degrade the detection performance of a ‘normal’ target. However, for fast moving and accelerating targets, improvement in detection was found when the new cell migration compensation algorithm was used. For target 5 which migrates two range bins, the standard processing requires an SNR of 17.3 dB for a detection probability of 0.5, compared to 14.7 dB required for the reference target. But the proposed algorithm achieves the same detection probability at 15.8 dB, which is a recovery of 58% of SNR loss. For target 5 with 3\textit{g} acceleration, the probability of detection degrades significantly due to Doppler migration effects, further demonstrating the sensitivity of X-band radar to Doppler spreading effects. But the proposed method is capable of reducing the required SNR by 3.4 dB to achieve the same detection probability of 0.5.

For targets 7 and 8 shown in Figures 6(c) and 6(d) the required SNR for \( P_d \) of 0.5 is dropped from 20.2 dB to 18.2 dB, and from 21.2 dB to 19.1 dB, respectively due to the proposed cell migration compensation algorithm. Note that regardless of the Doppler ambiguities in targets 7 and 8, a gain of 2 dB is achieved compared to the standard processing.

5 Discussion

In this report, we have introduced a new range bin collapsing technique for combining range migrating target returns into a single range bin. This technique, coupled with the de-chirping technique to process accelerating targets, provides an useful algorithm for coherent integration of fast moving and accelerating targets, compensating for both range and Doppler migration.

In terms of a comparison with other known techniques, the keystone transform is probably the most widely used technique for correcting linear range bin migration in high
Figure 5: Doppler spectral for single range bins (standard processing), and combined and de-chirped range bin (new algorithm). (a) Target 6 (570 knots, 3g) migrates across 2 range bins, and (b) Target 8 (850 knots, 3g) migrates across 3 range bins. Both targets show Doppler migration due to acceleration. Simulations were performed for X-band radar parameters described in Table 2. Note that all signals have same mean noise power level.
Figure 6: Probability of detection estimates of four targets with different closing velocities and accelerations for the medium-PRF X-band radar. $P_d$ curve for the reference target is shown in all the plots for comparison.
range-resolution radars, hence our choice as a benchmark technique to discuss the relative usefulness and performance of the current proposed technique.

The keystone transform involves rescaling the slow time axis appropriately to decouple the target’s range and velocity during a CPI in the process of forming range-Doppler maps for target detection. However, unlike the proposed technique, it does not account for target acceleration resulting in Doppler spread effects and hence sub-optimal integration gain. In other words, it solves the range-walk problem, leaving the target acceleration problem unsolved. In its solution for range-walk, the advantage over the proposed technique is that it does not involve collapsing multiple range bins, thus does not incur extra noise or clutter power in its processing.

Another drawback of the keystone transform is in the case of Doppler ambiguity, i.e., where the Doppler frequency of the target is greater than the PRF. In such cases, the keystone transform requires the degree of Doppler ambiguity to be known. In many real scenarios, the degree of Doppler ambiguity is often unknown and must be hypothesized. The proposed method does not require hypothesizing the Doppler ambiguity except for Test 2 discussed in Section 3.2. But even without this test, the proposed technique can still achieve good detection performance as we demonstrated in Section 4.3 using two simulated Doppler ambiguous targets.

In short, the two techniques differ primarily in their applicability. If the target velocity is high (in magnitude) but not ambiguous in Doppler frequency and no significant acceleration, the keystone transform technique is best suited. If the target exhibits large acceleration as well as high velocity, the proposed technique is best suited. It remains an interesting open research topic to investigate the performance of the keystone transform combined with the de-chirping technique; this is currently planned to be the topic of a future report.

6 Conclusion

In this report, we presented a new algorithm for detecting highly maneuvering targets in pulse-Doppler radars mitigating range migration and Doppler spread effects. ‘Range-walk’ of fast moving targets was compensated by introducing a contiguous hit detection and range bin collapsing technique where returns from the same target in multiple range bins are coherently combined. Additionally, a de-chirping technique was used to compensate for Doppler spread effects due to target acceleration. The overall algorithm is capable of overcoming both range and Doppler migration effects, thereby improving target detection.

The new algorithm has been compared with the conventional processing, and the detection improvement of the new method was demonstrated. Probabilities of target detection were estimated using Monte-Carlo simulations for a range of target SNR levels, and for a number of different target dynamics, in two L-band and X-band radar systems, at a fixed false alarm rate. The proposed method achieved a detection probability of 0.5 at a lower target SNR compared to the standard processing for each of the maneuvering targets considered. When all eight maneuvering targets applied in the two radar systems are considered, on average, 2.2 dB gain was found at $P_d = 0.5$. The proposed method
preserves the same detection performance for slow moving non-accelerating targets as the standard processing.

Acknowledgements

The authors would like to thank Dr. Luke Rosenberg and Dr. Andrew Shaw for critically reviewing the report and providing useful comments, and Mr. Scott Capon for his valuable support in this work.

References


**Detection of Fast Moving and Accelerating Targets Compensating Range and Doppler Migration**

We are reporting on a new signal processing technique for the detection of fast accelerating targets that spread over multiple range bins and Doppler filters in the radar receiver. By compensating for these undesirable effects, improvements in coherent gain of more than 2 dB can be achieved, thus optimizing detection performance. The technique is demonstrated with simulated data, and implementation can be expected to be readily feasible with many existing radar systems.