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### 14. ABSTRACT
Stability analyses of high-speed boundary-layer flow past a 5° half angle sharp cone with the wall-normal injection of air through a porous strip are performed using Navier-Stokes solutions for the mean flow and linear stability theory. The configuration and free-stream parameters are chosen to be similar to the experiments, which were carried out at Caltech’s T5 shock tunnel to investigate the effect of CO2 injection on laminar-turbulent transition. The analysis is focused on pure aerodynamic effects in the framework of perfect gas model. It is shown that the injection leads to destabilization of the Mack second mode in the nearfield relaxation region and its stabilization in the far-field relaxation region. To reduce the destabilization effect it was suggested to decrease the injector surface slope or use suction-blowing of zero net injection. However, the computations showed that these modifications did not improve the injector performance in the near-filed region in general. For special cases of low injection rates in which the N-factors in the near field region are below the critical level, shaping can produce a significant stabilization in the mid- and far-field regions.

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Stability analysis of high-speed boundary-layer flow with gas injection

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Outline

• Background and motivation
• Baseline configuration and numerical approach
• Stability analysis for the baseline configuration
  – Mean flow
  – Acoustic instabilities
  – N-factors
• How to improve the injector performance
• Shaping of injector
  – Conical shapes
  – Cylindrical shape
• Suction-blowing of zero mass injection
• Conclusions
PROBLEM: In hypersonic flight, heating loads are typically a dominant design factor

Turbulent heat transfer rates can be about an order of magnitude higher than laminar rates at hypersonic Mach numbers

A reduction in heating loads by keeping the boundary layer laminar longer means less thermal protection needed and hence less weight to carry, or conversely more payload deliverable for a given thrust.

OBJECTIVE: Delay transition from laminar to turbulent flow in the boundary layer of a slender hypersonic body by using nonequilibrium CO₂

Transition in high Mach numbers occurs through the Mack mode – amplification of acoustic waves traveling in the boundary layer

Molecular vibration and dissociation damp acoustic waves

At relevant conditions, CO₂ absorbs most energy at the frequencies most strongly amplified by 2nd (Mack) mode

Inject CO₂ to delay transition in air flows of interest
Background

- For CO\textsubscript{2} the broad sound absorption curve peak coincides with the amplification peaks
- This coincidence is most pronounced at enthalpies of \(~10\) MJ/kg

Baseline configuration

Free-stream parameters correspond to Run 2540* in GALCIT T5 shock tunnel

\[ M_\infty = 5.3 \]
\[ \rho_\infty^* = 0.05788 \text{ kg/m}^3 \]
\[ T_\infty^* = 1323.77 \text{ K} \]
\[ U_\infty^* = 3866 \text{ m/s} \]
\[ p_\infty^* = 21993 \text{ Pa} \]
\[ T_w^* = 293 \text{ K}, \frac{T_w^*}{T_\infty^*}=0.22 \]
\[ \mu_\infty^* = 4.897 \times 10^{-5} \text{ Pa} \cdot \text{s} \]
\[ L^* = 1 \text{ m} \]

5-deg half-angle sharp cone with the injector

Gas is injected with the total mass flow rate ranging from 3 g/s to 13.5 g/s

*Parameters are determined using \( M_\infty, T_\infty, \) and \( \rho_\infty \) reported by Wagnild, R.M. et al. (AIAA-2010-1244) and perfect-gas model with \( Pr=0.72 \) and \( \gamma=1.4 \)
Numerical approach for mean flow

In-house Navier-Stokes code HSFlow*

- Perfect gas of Pr=0.72, \( \gamma = 1.4 \)
- Sutherland viscosity-temperature dependence
- Implicit second-order finite-volume method
- Shock-capturing scheme
- Third-order WENO for advection terms

597×649 grid with
- 50% clustering in the boundary layer
- Clustering near the injector

Stability analysis

Local-parallel stability computations

- Third-order Runge-Kutta scheme for integration of stability equation
- Gramm-Schmidt orthogonalization procedure
- Eigenvalues are calculated using a shooting/Newton-Raphson procedure

\[
\text{Disturbance~} q(y) \exp(i\alpha x + i\beta z - i\omega t)
\]

For temporal problem \( \omega(\alpha, \beta, x) \) is complex, growth rate = \( \omega_i \)

For spatial problem \( \alpha(\omega, \beta, x) \)'s complex, growth rate = \( \sigma = -\alpha_i \)

\[
N(x, \omega, \beta) = \int_{x_0(\omega, \beta)}^{x} \sigma(\omega, \beta, x) dx
\]

Boundary layer thickness
baseline configuration with injection rate=13.5 g/s
Mean-flow profiles

Baseline configuration with injection rate=13.5 g/s
• Thick region of cold dead flow near wall
• Slow relaxation downstream
We are dealing with acoustic instability

Temporal stability analysis at $x=0.3$

Mode 0, $x=0.298$

Mode 0 = Mack second mode

Mode 1, $x=0.298$

Mode 1 = Mack third mode

Mode 2, $x=0.298$

Mode 2 = Mack fourth mode

Mode 3, $x=0.298$

Mode 3 = Mack fifth mode

Unstable acoustic modes at $x=0.298$, $Me=5$, $Re_\text{\delta^*}=8074$

Length scale = $\delta^*$

Mode 0 = Mack second mode

Mode 1 = Mack third mode

Mode 2 = Mack fourth mode

Mode 3 = Mack fifth mode
Temporal instability in the relaxation region for baseline configuration
(injection rate 13.5 g/s)

- There are seven unstable modes!
- Mode 2 (Mack fourth mode) has maximal local increment
We are dealing with acoustic instability (cont’d)

\[ U(y_a) = c - a(y_a) \]

Dispersion relation from WKB analysis*,**:
\[ \int_0^{y_a} \sqrt{\frac{(\alpha U - \omega)^2 M^2}{T}} \, dy = \frac{\pi}{4} + \pi m \]
\[ m = 0, 1, \ldots \]

\textbf{Acoustic modes are formed in the waveguide between the wall (y=0) and the relative sonic line y=y_a: U(y_a)=c_r-a(y_a)}

Injection affects N-factors of Mack second mode

- Destabilization in near-field relaxation region
- Stabilization in mid-field relaxation region
- Destabilization in far-field relaxation region
Perfect gas model captures basic trends

N-factors including real gas effects (Wagnild et al. AIAA-2010-1244)
How to improve the injector performance?

- Negative slope may compensate the blowing effect on the displacement thickness

  ![Diagram showing air and injected CO₂]

- Injection of zero total mass addition may help to reduce the relaxation region

  ![Diagram showing suction of air and blowing of CO₂]
Injector of conical shape

5-deg half-angle sharp cone with the injector having the slope $\theta = \theta_c - \Delta \theta$

\[
\Delta r \approx b[\tan \theta_c - \tan(\theta_c - \Delta \theta)] \approx b\Delta \theta
\]

\[
\Delta b = \frac{\Delta r}{\tan(\theta_c - \Delta \theta)} \approx \frac{\Delta r}{\tan \theta_c}
\]
Mean-flow pressure for conical injectors
(injection rate 13.5 g/s)

Baseline ($\Delta \theta = 0^\circ$)

$\Delta \theta = 1^\circ$

$\Delta \theta = 3^\circ$

Injection region
Mean-flow boundary layer thickness for conical injectors
(injection rate 13.5 g/s)

Boundary-layer thickness increases with $\Delta \theta$ in the relaxation region
Shaping $\Delta\theta=1^\circ$ without injection

Effect of shaping is local and small
Shaping $\Delta \theta = 1^\circ$ with injection of 13.5 g/s

Conical injector with $\Delta \theta = 1^\circ$
- Slightly destabilizes flow in the near-field relaxation region
- Slightly stabilizes flow in the far-field relaxation region
Shaping $\Delta \theta = 1^\circ$ and injection 6.75 g/s

Shaping of $\Delta \theta = 1^\circ$ produces small effect on stability of flow at the injection rate 6.75 g/s
Shaping of $\Delta \theta = 3^\circ$ without injection

Effect of shaping is local and small
Summary plot of N-factors for conical injectors (injection rate 13.5 g/s)

Shaping with $\Delta \theta = 1^\circ$ and $3^\circ$
- Slightly destabilizes flow in the near-field relaxation region
- Stabilizes flow in the far-field relaxation region
Injectors of cylindrical shape

Baseline shape – sharp cone

New shape

Injector surface

\[ \theta_c = 5^\circ \]

\[ \Delta \theta = 5^\circ \]
Mean flow for cylindrical injector

(injection rate 13.5 g/s)

Pressure

Injection region

Axial velocity near injector
Mean flow for cylindrical injector
(various injection rates)
N-factors for cylindrical injector

This shaping
- Slightly destabilizes flow in the near-field relaxation region
- Stabilizes flow in the mid- and far-field relaxation region
Summary plot of N-factors for cylindrical injector
(various injection rates)
Estimates of the injection effect on the transition onset

Cylindrical injector, $m_0 = 13.5 \text{ g/s}$

- Destabilization in the near-filed relaxation region
- Stabilization in the mid-filed relaxation region
- Destabilization in the far-filed relaxation region
Suction-blowing of zero mass addition

\[-l/2 \leq x - x_0 \leq l/2\]

\[z = (x - x_0) \frac{2\pi}{l}, \quad -\pi \leq z \leq \pi\]

\[q(z) = q_0 \frac{l}{2\pi} \sin z\]

\[\dot{m}_+ = q_0 \frac{l}{2\pi} \int_0^\pi \sin zdz = q_0 l / \pi\]

\[\dot{m}_- = q_0 \frac{l}{2\pi} \int_{-\pi}^0 \sin zdz = -q_0 l / \pi\]
Mean flow for suction-blowing system

\( m_+ = 6.75 \text{ g/s} \)
Wall pressure distribution for suction-blowing system \((m_+=6.75 \text{ g/s})\)
N-factors for suction-blowing system

\( (m_+=6.75 \text{ g/s}) \)

Run 2540

- **baseline**
- 6.75 g/s
- 6.75 g/s, suction-blowing

**Effect is negative**
Conclusions

- Injection induces a cold dead-flow layer in the downstream relaxation region
- The near-wall flow behaves as a wave guide which can support several unstable modes of acoustic type
  - The most unstable is the Mack second mode
  - The phase speeds of instability are close to those of slow acoustic waves in the free-stream
  - Instability frequencies are several times smaller than in the no injection case
  - This may lead to dramatic increase of receptivity to free-stream noise
Conclusions (cont’d)

• The $e^N$ computations for baseline configuration showed
  – Injection leads to destabilization of the near-field region, stabilization of the mid-field relaxation region, and destabilization of the far-field relaxation region
  – The level of these effects essentially depend on the injected mass flow rate

• The injector shaping considered
  – Does not stabilize the near-field flow at sufficiently large injection rates

• For relatively small injection rates the shaping produces a significant stabilization effect in the mid- and far-field relaxation regions

• The suction-blowing of zero mass addition destabilizes the flow in the whole relaxation region
Backup
Stability analysis (cont’d)

With small correction of $\gamma$, N-factors predicted by the perfect-gas model are close to N-factors predicted by STABL*

*Wagnild, R.M. et al. AIAA-2010-1244
Mean flow for baseline configuration
(injection rate=13.5 g/s)

- **Axial velocity**
- **Pressure**
- **Temperature**

Injection region
Mean flow near injection
baseline configuration
injection rate = 13.5 g/s
It is not easy to convert temporal growth rates to spatial ones

\[ \omega_r \]

\[ \alpha_i = -\frac{\omega_i}{V_g} \]

\[ V_g = \frac{d\omega_r}{d\alpha} \]

does not work here
Spatial stability analysis in the relaxation region
(injection rate=13.5 g/s, x=0.3)

• Mode 0 (Mack second mode) is most unstable
• Its instability is observed at low phase speeds
Injector of conical shape

5-deg half-angle sharp cone with the injector having the slope $\theta = \theta_c - \Delta \theta$

\[
\Delta r \approx b\left[\tan \theta_c - \tan(\theta_c - \Delta \theta)\right] \approx b\Delta \theta
\]

\[
\Delta b = \frac{\Delta r}{\tan(\theta_c - \Delta \theta)} \approx \frac{\Delta r}{\tan \theta_c}
\]
Mean-flow axial velocity for conical injectors
(injection rate 13.5 g/s)
Shaping of $\Delta \theta = 3^\circ$ with injection 6.75 g/s

Conical injector with $\Delta \theta = 3^\circ$ shaping

- Slightly stabilizes flow in the near-field relaxation region
- Almost zero effect in the far-field relaxation region
Shaping of $\Delta \theta = 3^\circ$ with injection 13.5 g/s

Conical injector with $\Delta \theta = 3^\circ$ shaping
- Slightly destabilizes flow in the near-field relaxation region
- Stabilizes flow in the far-field relaxation region
We are dealing with acoustic instability (cont’d)

- Phase speeds of unstable modes are close to those of slow acoustic waves
- Resonant interaction can enhance receptivity to slow free-stream noise
- Instability is observed at low frequencies where free-stream noise is higher
Maximal growth rates in the relaxation region
(injection with 13.5 g/s)

• The most unstable mode is Mack second mode (mode 0)
• Unstable x-region decreases with the mode number

Further analysis is focused on the Mack second mode

\[ \sigma_{\text{max}} = -\alpha_{i,\text{max}} = \max_{\omega} \left[ -\alpha_i(\omega) \right] \]

length scale=δ*M

0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.04 0.08 0.12 0.16 0.20 0.24

\[ \sigma_{\text{max}} \]

\[ \chi \]
Maximal growth rates of Mack second mode

In the relaxation region $x>0.2$ m
- High maximal growth rates
- Low frequencies
- Low phase speeds
Injection affects growth rates and frequencies in the relaxation region

Maximal growth rates are increased
Unstable region is
  • narrowed down for x<0.6
  • widened for x>0.6

Frequencies are decreased
Maximal growth rates for cylindrical injector (various injection rates)

\[ \sigma_{\text{max}} \]

\[ X \]

length-scale = \( \sqrt{\nu^*_e x^* / U^*_e} \)