Platform Design for Fleet-Level Efficiency

21 October 2013

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Prepared for the Naval Postgraduate School, Monterey, CA 93943.
This research leverages techniques from the fields of multidisciplinary design optimization and operations research into an approach to improve energy efficiency-related defense acquisition decisions. The work focuses on the acquisition of new cargo aircraft for the U.S. Air Force Air Mobility Command (AMC), which is the largest consumer of fuel in the Department of Defense. The approach here extends prior work in fleet-level acquisition decisions from a commercial aviation context into the context of Air Mobility Command. The framework, with the abstractions and assumptions used, successfully considers the design requirements of the new aircraft to meet fleet-level metrics. The framework does this by using the new aircraft design requirements to describe that new aircraft’s characteristics and then uses those characteristics to allocate the new aircraft, along with other existing aircraft, to meet fleet-level metrics. The approach begins to address uncertain cargo demand following scheduling-like constraints to represent typical AMC operations more closely. Fuel efficiency of the resulting fleet provides a metric for comparison of the effect of the new aircraft requirements.
The research presented in this report was supported by the Acquisition Research Program of the Graduate School of Business & Public Policy at the Naval Postgraduate School.

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Abstract

This research leverages techniques from the fields of multidisciplinary design optimization and operations research into an approach to improve energy efficiency-related defense acquisition decisions. The work focuses on the acquisition of new cargo aircraft for the U.S. Air Force Air Mobility Command (AMC), which is the largest consumer of fuel in the Department of Defense. The approach here extends prior work in fleet-level acquisition decisions from a commercial aviation context into the context of Air Mobility Command. The framework, with the abstractions and assumptions used, successfully considers the design requirements of the new aircraft to meet fleet-level metrics. The framework does this by using the new aircraft design requirements to describe that new aircraft’s characteristics and then uses those characteristics to allocate the new aircraft, along with other existing aircraft, to meet fleet-level metrics. The approach begins to address uncertain cargo demand following scheduling-like constraints to represent typical AMC operations more closely. Fuel efficiency of the resulting fleet provides a metric for comparison of the effect of the new aircraft requirements.
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Disclaimer: The views represented in this report are those of the author and do not reflect the official policy position of the Navy, the Department of Defense, or the federal government.
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Executive Summary

The Energy Efficiency Starts With the Acquisition Process factsheet by the Deputy Under Secretary of Defense for Acquisition and Technology (DUSD[A&T], 2012) presents a compelling need for acquisition practices to consider technologies and opportunities to improve energy efficiency. However, few decision support frameworks can deal with the impact that new system acquisitions decisions have on the operations of a fleet of systems that includes the new systems along with existing systems. The research provides a quantitative approach that treats design requirements of new systems or platforms, which will serve, along with other systems, as design variables in an optimization formulation that minimizes or maximizes fleet-level objectives rather than system-level objectives. Results of the approach identify requirements for and design features of promising new systems that work alongside others systems to provide necessary capability with the goal of reduced total fuel use or cost.

This report specifically presents the past year’s effort and contributions focused on introducing a framework to inform the choice of new platform design requirements for those helping determine these requirements for potential acquisition of the new platform. This work considers a new aircraft to support military air cargo transportation, using demand information from the US Air Force Air Mobility Command (AMC) and details the fleet-level objectives and constraints in the form of an optimization problem that is decomposed into related subdomain problems. This report also documents the use of this framework to describe a new cargo aircraft using information about actual operations of AMC.

The decomposition framework separates the aircraft design sub-problem from the allocation sub-problem, which are then coordinated by a small top-level sub-problem. The allocation problem incorporates scheduling-like features to account for time-driven operational constraints and the asymmetry in the demand network observed from the AMC data. Relatively naïve Monte Carlo Sampling techniques address the uncertainty observed in data from AMC in the year 2006.

The decomposition framework provides not only a geometric and performance description of the new platform but also an opportunity for acquisition decision practitioners to assess the impact of the requirements set for the new platform based on how the new platforms integrates into the existing fleet. Optimal solutions obtained from the simulation inform acquisition practitioners of the utilization of the new platform under uncertain operational scenarios.
**List of Acronyms and Abbreviations**

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<tr>
<th>Acronym</th>
<th>Definition</th>
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<tr>
<td>AMC</td>
<td>Air Mobility Command</td>
</tr>
<tr>
<td>APOD</td>
<td>aerial port of debarkation</td>
</tr>
<tr>
<td>APOE</td>
<td>aerial port of embarkation</td>
</tr>
<tr>
<td>BTS</td>
<td>Bureau of Transportation Statistics, Civil Reserve Air Fleet</td>
</tr>
<tr>
<td>CRAF</td>
<td>Civil Reserve Air Fleet</td>
</tr>
<tr>
<td>DUSD</td>
<td>Deputy Under Secretary of Defense for Acquisition and Technology</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GATES</td>
<td>Global Air Transportation Execution System</td>
</tr>
<tr>
<td>MCRS</td>
<td>Mobility Capabilities and Requirement Study</td>
</tr>
<tr>
<td>MCS</td>
<td>Monte Carlo Sampling</td>
</tr>
<tr>
<td>MDO</td>
<td>multidisciplinary design optimization</td>
</tr>
<tr>
<td>MDS</td>
<td>mission distribution system</td>
</tr>
<tr>
<td>MINLP</td>
<td>mixed integer, non-linear problem</td>
</tr>
<tr>
<td>MTOW</td>
<td>maximum takeoff weight (lbs)</td>
</tr>
<tr>
<td>nmi</td>
<td>nautical miles</td>
</tr>
<tr>
<td>SA</td>
<td>simulated annealing</td>
</tr>
<tr>
<td>USTRANSCOM</td>
<td>United States Transportation Command</td>
</tr>
</tbody>
</table>
Nomenclature

\[ AR_X \] = aspect ratio of aircraft type X  
\[ B_p \] = maximum average daily utilization of each aircraft (20 hours)  
\[ BH_{p,k,i,j} \] = number of block hour for \( k^{th} \) trip of aircraft \( p \) from base \( i \) to base \( j \)  
\[ C_{p,k,i,j} \] = cost coefficient for \( k^{th} \) trip of aircraft \( p \) from base \( i \) to base \( j \)  
\[ Cap_{p,k,i,j} \] = number of pallet carrying capacity for \( k^{th} \) trip of aircraft \( p \) from base \( i \) to base \( j \)  
\[ Dem_{i,j} \] = demand from base \( i \) to base \( j \) in number of pallets  
\[ DOC \] = direct operating cost  
\[ MTM/D \] = million ton-miles per day  
\[ O_{p,i} \] = indicates if airport \( i \) is the initial location (e.g., home base) of an aircraft \( p \)  
\[ Pallet_X \] = number of pallets carried by aircraft type X  
\[ S_{TO} \] = take off field length  
\[ (T/W)_X \] = thrust-to-weight ratio of aircraft type X  
\[ UTE \] = utilization rate (number of flying hours per day, 12 hours)  
\[ (W/S)_X \] = wing loading of aircraft type X, in lb/ft\(^2\)  
\[ x_{p,k,i,j} \] = binary (0,1) variable indicating if the \( k^{th} \) trip if flown by aircraft \( p \) from base \( i \) to base \( j \)
Introduction and Motivation

The Energy Efficiency Starts with the Acquisition Process fact sheet (DUSD [A&T] 2008) states, “Neither current requirements or acquisition processes accurately explore tradeoff opportunities using fuel as an independent variable.” The fact sheet also states, “Current processes undervalue technologies with the potential to improve energy efficiency.” Studies conducted by the Institute for Defense Analyses, the Defense Science Board, Energy Security Task Force, and JASON (an independent scientific advisory group) have all alluded to the significant risk and operational constraints that energy-efficiency issues pose on military operational flexibility. The consumption and transport of fuel across a combat theater, throughout the life cycle of operational systems, poses significant operational risk, strategic vulnerability, and increased monetary cost in supporting forward-force assets. Additionally, increasing fuel consumption shifts focus to the acquisition of an increasing number of tail units in maintaining forward-force assets. Aviation fuel contributes the largest percentage of energy consumption in the Department of Defense (DoD), with the Air Mobility Command (AMC) being the single largest consumer (Allardice, 2012). The enormous energy consumption of the AMC in the DoD makes an air mobility-related application relevant for the current research effort. Figure 1 shows the breakdown of fuel consumption in the DoD in 2007.

![Figure 1. Air Mobility Command Fuel Usage in Relation to the Department of Defense Energy Usage](image)

AMC, a branch of the United States Air Force, is responsible for a wide range of airlift missions that span its global theater of operations. AMC’s mission profile
consists mainly of worldwide cargo and passenger transport, air refueling, and aeromedical evacuation. AMC also provides transports for humanitarian supplies for major natural disaster around the world. Platforms in operation include C-5 Galaxy, C-17 Globemaster III for long-range strategic missions, C-130 Hercules for tactical missions, KC-135 Stratotanker and KC-10 Extender for aerial refueling missions, and various VIP transport platforms, including Air Force One. AMC also charters aircraft from Civil Reserve Air Fleet during peacetime, contractually committed from U.S. airlines (AMC, 2013).

The logistics involved in transporting across the AMC service network requires effective deployment of the cargo aircraft fleet to meet daily cargo delivery requirements. This research adds the additional consideration of minimizing fuel consumption and subsequent operating costs. The choice of aircraft design and individual flight legs flown by the AMC fleet, in meeting cargo obligations within a prescribed schedule timeframe, drive both the fuel consumption and operating costs; therefore, the requirements set for the new aircraft have a crucial impact on fuel and cost. However, the characteristics of aircraft flown dictate the kind of network that the fleet can serve, thus making the consideration of a new aircraft design and the allocation/assignment of the new aircraft, along with existing aircraft, two closely coupled problems. The design of the aircraft itself, operations across routes flown, and manifestation of uncertainty in daily cargo transportation demand creates a highly complex hierarchy of interwoven systems, or a system-of-systems.

The AMC is in the process of modernizing the current strategic fleet, consisting of C-5s and C-17s, by incorporating new materials and engines on existing airframes to operate the current fleet more efficiently. However, the design of new, more fuel-efficient aircraft may potentially provide the biggest cost and fuel consumption savings. The work presented here provides a decision support framework that assists acquisition practitioners in identifying optimal characteristics of new assets (here, aircraft) that can minimize fuel dependency of the entire system architecture in which they serve (here, the fleet of cargo aircraft). The coupled effect that an aircraft design has on fleet operations drives the approach in the framework. The framework can examine how acquisition (and pre-acquisition) decisions describing the requirements for a new aircraft might directly reduce fleet-level fuel usage/cost, considering the operational network and other existing assets along with the potential new (or modified) platform. Consideration of the aircraft design and fleet allocation problems simultaneously presents many decision variables—a condition where the size of the problem rapidly exceeds the mental capability of the designer and a computational approach becomes necessary. Additionally, explicit consideration of uncertainty in operations better informs a new aircraft that improves the fleet-level performance. The research will advance the knowledge on how to perform tradeoffs with fleet-level fuel consumption as one of the quantities of interest.
and will enhance understanding about what features this kind of process should entail.

**Scope and Method of Approach**

**Abstraction**

Previous research at Purdue University addressed the issue of simultaneously designing the “assets” and “operations” of a platform—in this case, the design of yet-to-be-introduced aircraft and the consequent allocation of the fleet (incorporating the new aircraft design along with current aircraft) across a service network. The simultaneous consideration of the design of an asset (here, aircraft), and its operations (here, allocations) as a comprehensive platform has been demonstrated to show potentially significant cost savings for airline, fractional ownership, and air taxi operations (Mane & Crossley, 2006, 2012; Mane, Crossley, & Nusawardhana, 2007). The integrated perspective differs from the traditional perspectives where asset design (the aircraft) and operations (allocation of aircraft) are treated as decoupled facets of a platform. The result is an approach that can maximize or minimize a fleet-level objective function by searching for a set of decision variables that describe the new system design and describe the allocation of the new and existing systems to perform operational missions. Although a single, monolithic problem statement can reflect this kind of problem, solving the resulting mixed integer, non-linear programming (MINLP) problem is difficult, if not impossible.

![Figure 2. Decomposition Strategy of the Monolithic Optimization Problem](image)

The decomposition strategy with an allocation formulation under uncertainty, as notionally depicted in Figure 2, relieves some of the computational challenges by presenting a series of smaller sub-problems controlled by a top-level optimization problem. The decomposition approach addresses the issue of tractability of solving a monolithic, mixed discrete non-linear programming problem and has yielded better design solutions across a set of aviation applications, including commercial airlines,
fractional management companies, and air taxi services (Mane & Crossley, 2006, 2012; Mane et al., 2007). The motivation of these prior works in identifying cost and fuel saving characteristics of a new, yet-to-be-acquired aircraft bears similarity to the U.S. Air Force AMC problem.

To gain representative network resemblance to AMC’s operational network, the Global Air Transportation Execution System (GATES) dataset is used. AMC’s automated air transportation management system is managed by USTRANSCOM, and this system has very detailed information on palletized cargo and personnel transported by the AMC fleet. Cargo transported by the strategic fleet consisting of C-5 and C-17 aircraft, along with chartered Boeing 747 Freighter (747-F) aircraft from the Civil Reserve Air Fleet (CRAF) for long range missions, are considered as a representative measure of typical cargo flow on the AMC service network. Each data entry in “GATES Pallet data” represents transported cargo on a pallet or a pallet-train (i.e., multiple, linked pallets). Each pallet data entry has detailed information of the pallet, such as pallet gross weight, departure date and time, arrival date and time, mission distribution system (MDS), tail number, aerial port of embarkation (APOE), aerial port of debarkation (APOD), pallet volume, pallet configuration, and so forth. These data enable the reconstruction of the route network, pallet demand characteristics, and existing fleet size for our allocation problem.

The following assumptions are made on operations of the fleet, based on the available dataset:

1. The filtered route network from GATES dataset is representative of all AMC cargo operations.
   a. Demand for subset served by C-5, C-17 and 747-F (75% of all pallets in GATES dataset)
   b. Fixed density and dimension of pallet, representing the 463L pallet type

2. Aircraft fleet consists of only the C-5, C-17 and 747-F. The model is indifferent to variants of these aircraft types.

**Determination of Number of New Aircraft Needed**

The number of new aircraft (identified here in “type X”) to be introduced to the existing fleet is unknown before employing the framework because capacity of the new aircraft is one of the new aircraft design requirements that the framework will determine. However, the AMC strategic fleet, by Air Force requirement, must be able to serve the maximum possible demand scenario. The Mobility Capabilities and Requirement Study (MCRS) 2016 (Jackson, 2009) illustrates three different scenarios that capacity of the strategic fleet must always meet. The peak for MCRS Case 1, which represents the highest level of modeled strategic airlift demand, required 32.7 million ton-miles per day (MTM/D). MTM/D values for each type of
Aircraft are calculated using empirical data. A C-5 carries 0.1209 MTM/D, while the newer C-17 carries 0.1245 MTM/D (Kopp, 2004). The 747-F carries 0.1705 MTM/D but is not included in calculating the strategic airlift fleet MTM/D because AMC does not directly operate aircraft in the CRAF. Hence, the availability of the 747-F aircraft for everyday operations does not affect the number of aircraft X required to meet the peak demand.

The MTM/D of the new aircraft X is calculated using the following equation.

\[
\frac{MTM}{D} = \frac{(Block Speed) \times (Avg. Payload) \times (UTE Rate) \times (Productivity Factor)}{1,000,000}
\]  

(1)

AMC force structure programmers use MTM/D when funding out-year aircraft purchases, and many civilian agencies visualize the AMC strategic fleet capability in terms of MTM/D (Air Force Pamphlet, 2003). This work assumes utilization rate (UTE rate) of the new aircraft as 12 hr/day and the productivity factor of 4.8, which are within the typical range of the strategic airlift fleet average values. However, the simple three-base problem, which appears later, uses a limitation that only three new aircraft are introduced to the fleet, because the small size of that example problem does not permit MTM/D calculation.

**Aircraft Sizing**

Aircraft sizing is the process of determining an aircraft's size, weight, and performance. The allocation formulation used to estimate fleet-level metrics requires estimates of the cost, block time, and fuel consumed by each aircraft type in the fleet to determine the appropriate allocation of aircraft to the various routes in the network. Therefore, the framework requires a sub-problem to size aircraft X to meet different values of design requirements for payload and range. A Purdue in-house aircraft sizing code, written in MATLAB, provides these estimates in the aircraft sizing subspace shown in Figure 2. Jane's All the World's Aircraft 2001–2004 (Jackson, Peacock, & Munson, 2009) provided the input parameters for the three existing aircraft types (C-5, C-17, 747-F) used in this study, as shown in Table 1. The MATLAB sizing code's predictions of the existing aircraft size, weight, and performance are acceptably close to published values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>C-5</th>
<th>C-17</th>
<th>747-F</th>
</tr>
</thead>
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<tr>
<td>Range (nmi)</td>
<td>2,982</td>
<td>2,420</td>
<td>4,445</td>
</tr>
<tr>
<td>Pallet Capacity</td>
<td>36</td>
<td>18</td>
<td>29</td>
</tr>
<tr>
<td>W/S (lb/ft²)</td>
<td>135.48</td>
<td>161.84</td>
<td>137.34</td>
</tr>
<tr>
<td>T/W</td>
<td>0.205</td>
<td>0.263</td>
<td>0.286</td>
</tr>
<tr>
<td>AR</td>
<td>7.75</td>
<td>7.2</td>
<td>7.7</td>
</tr>
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</table>

Table 1. Existing Aircraft Characteristics
Direct operating cost (DOC) estimates for commercial aircraft include fuel costs, crew costs, maintenance, depreciation, and insurance. DOC estimates also depend on the payload, route distance, empty weight, landing weight, and takeoff gross weight. Although AMC does not have the same operating cost structure, the problem formulation here considers total fleet operating cost as the objective function. Because cost-estimating relationships exist for commercial aircraft and suitable operating cost predictors for AMC were not readily available, the initial AMC problem formulations here use DOC estimators based on commercial airline operations, recognizing that they may not directly match the costs for AMC operations. This appears sufficient to demonstrate the framework but may limit the quality of the design requirements recommended for the AMC-related problems.

Figure 3 shows a typical mission profile used for the aircraft sizing and operating missions. The aircraft sizing code computes and aggregates the fuel required for each mission segment to estimate the fuel weight necessary for flying the route distance. The fuel weight fractions for the different mission segments such as warm-up and takeoff, climb, landing and taxi, and reserves are based on empirical data presented in Raymer’s (2006) aircraft design textbook. The Breguet range and endurance equations predict the fuel weight fractions for the cruise and loiter mission segments. The descent segment uses a no-range credit assumption. The aircraft sizing code assumes a reserve fuel fraction of 6%, which also accounts for a small amount of trapped and unusable fuel.

![Figure 3. Mission Flight Profile](image)

The payload-range curves for the existing aircraft fleet, depicted in Figure 4, indicate the maximum payload carrying capacity of the aircraft as a function of the distance flown by the aircraft. The payload-range curves for the existing fleet are constructed by using piecewise linear interpolation between specified points from published charts in (Baker, Morton, Rosenthal, & Williams, 2002). The sizing code used to predict performance and costs for the new aircraft type X on various
operating mission combinations of payload and range also provides predictions for these existing aircraft.

\[
\text{Figure 4. Payload-Range Curves for Existing Fleet}
\]

**Monolithic Problem Formulation**

**Traditional Aircraft Allocation Problem**

The objective for the allocation problem seeks to minimize fleet level DOC by allocating the available fleet to the three routes, using the information provided on the aircraft flight costs (including fuel costs). The formulation of the following mathematical programming problem uses cost coefficients from the aircraft sizing code. Mathematical programs have two important aspects of formulation: the **objective function** that reflects the metric being minimized/maximized, and **constraints** that reflect resources constraints to the problem. The **decision variables** are the variables of interest that can be manipulated to optimize the objective. The traditional allocation problem statement, considering only existing aircraft, is as follows:

Minimize \[ \text{Fleet DOC} = \sum_{i=1}^{3} \left( \sum_{A=C-5,C-17,747-F} C_{Ai} x_{Ai} \right) \] (2)

Subject to \[ \sum_{i=1}^{3} x_{Ai} \leq B_{Ai} \quad A = C-5, C-17, 747-F \quad \text{(trip limits / aircraft count)} \] (3)
In the case of the traditional aircraft allocation problem, the objective function in Equation 2 seeks to minimize the fleet DOC. The decision variable is given by $x_{Ai}$ (with subscripts for aircraft type and route) and is an integer, making the allocation problem an integer programming problem. The total fleet DOC is the sum of costs associated with the number of round trips an aircraft of type $A$ flies on route $i$. The constraints expressed in Equations 3 and 4 are the aircraft trip limit and cargo capacity limits on each route $i$. The trip limit constraints account for the number of aircraft available; the limiting values for number of trips operated by a given aircraft type in one year are based on information from the GATES data.

**AMC Fleet Allocation Including Design of New Aircraft**

Here, the authors extend the AMC aircraft allocation problem to consider the potential addition of a new, yet-to-be-designed aircraft and its impact on fleet wide operating costs and fuel consumption. The optimization problem now needs to consider the aircraft costs of the new aircraft as a function of the variables describing the new aircraft. The monolithic optimization problem simultaneously considers the aircraft design and allocation of the fleet’s aircraft to meet demand obligations and appears as the following equations.

Minimize

$$\text{Fleet DOC} = \sum_{i=1}^{3} \left[ \sum_{A \in \{C-5, C-17, 747-F\}} C_{Ai} x_{Ai} \right] + C_{xi} \left( \text{Pallet}_x, (AR)_x, (W/S)_x, (T/W)_x \right)$$

Subject to

$$\sum_{i=1}^{3} x_{Ai} \leq B_{Ai} \quad A = \{C-5, C-17, 747-F\}$$

(capacity)

$$S_{70} \left( \text{Pallet}_x, (AR)_x, (W/S)_x, (T/W)_x \right) \leq D$$

(aircraft takeoff distance)

6 \leq \text{Pallet}_x \leq 36

6.0 \leq (AR)_x \leq 9.5

65 \leq (W/S)_x \leq 161
\[ 0.18 \leq (T/W)_x \leq 0.35 \quad (13) \]
\[ x_{ai}, Pallet_x \in \text{int}, \quad x_{ai} \geq 0 \quad (14) \]

Equation 6 is the objective function that seeks to minimize fleet DOC. Changing this equation can reflect different studies, such as directly minimizing fuel consumption, maximizing fleet productivity, and so forth. During the period of this research, the implementation has only minimized fleet-level direct operating cost.

Equation 7 preserves the aircraft trip limits for a typical year from values calculated from existing flight data; this represents utilization rate. Equation 8 ensures sufficient pallet capacity for cargo traveling on route \( i \). Equation 9–14 limits the aircraft design based on maximum takeoff distance to ensure that the new aircraft can operate at bases in the network. The continuous design variables describing the new aircraft area limited to remain near the range of values associated with current cargo aircraft. As in the traditional allocation problem, the number of trips of each aircraft type, \( x_{ai} \), are integers. The coupling of the fleet allocation (integer programming) with the aircraft design (non-linear programming) makes the resource allocation problem a mixed-integer, non-linear (MINLP) problem. MINLPs are sometimes impossible to solve for even moderate-sized problems. However, the work here adopts a multidisciplinary design optimization (MDO; inspired subspace decomposition approach from prior literature; Mane et al., 2007) that breaks the monolithic MINLP problem of Equations 6–14 into a coordinated sequence of more tractable problems, as depicted in Figure 2.

**Demand Asymmetry in AMC Network**

Initial investigations have treated demand as symmetric, due to the inherent nature of the observed demand in previous work (e.g., airline passenger demand served from airport A to airport B nearly equals the demand from airport B to airport A for a given time period, as data obtain from the Bureau of Transportation Statistics (BTS) demonstrates). However, the GATES data appears to show asymmetric demand (cargo demand served from base A to base B does not equal the cargo demand served from base B to base A), so the metric shown in Equation 15 quantifies the asymmetry of the GATES dataset.

\[
\text{Demand asymmetry} = \frac{\sum_{D=1}^{N} \sum_{D=1}^{N} |\text{Demand}_{D,D} - \text{Demand}_{D,D}|}{\sum_{D=1}^{N} \sum_{D=1}^{N} \max(\text{Demand}_{D,D}, \text{Demand}_{D,D})} \quad (15)
\]

The full network reconstructed from the GATES dataset shows 65.15% demand asymmetry on this scale, where 0% is a fully symmetric network (i.e., even demand each direction on all base pairs) and 100% is a fully asymmetric network.
(i.e., all demand is in only one direction for all base pairs). The inherent demand asymmetry in the AMC network indicated by the high asymmetry measure warrants a problem formulation that tracks the aircraft and the flight legs in the network to handle asymmetric demand. A scheduling-like formulation, which differs from the previous airline allocation formulation, also eliminates the round-trip assumption.

**Monolithic Optimization With Scheduling-Like Allocation**

The system-of-system level representation involves the confluence of resource allocation (under uncertainty) and aircraft design perspectives that make up the monolithic problem; this encompasses the resource allocation problem under uncertainty (stochastic integer programming) and the aircraft design problem (non-linear programming), resulting in a stochastic mixed integer non-linear programming problem, which is typically very difficult to solve. The following equations represent the resulting optimization problem:

Minimize

$$
E \left[ \sum_{p=1}^{P} \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} x_{p,k,i,j} \cdot C_{p,k,i,j} + \left( x_{p,k,i,j} \cdot C_{p,k,i,j} \left( Pallet_x, (AR)_x, (W/S)_x, (T/W)_x \right) \right)_x \right]
$$

(DOC or Fleet fuel cost)  \hspace{1cm} (16)

Subject to

$$
\sum_{i=1}^{N} x_{p,k,i,j} \geq \sum_{i=1}^{N} x_{p,k+1,i,j} \hspace{0.5cm} \forall k = 1,2,3...K, \forall p = 1,2,3...P, \forall j = 1,2,3...N \hspace{1cm} (Node balance constraints) \hspace{1cm} (17)
$$

$$
\sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} x_{p,k,i,j} \cdot BH_{p,k,i,j} \leq B_p \hspace{0.5cm} \forall p = 1,2,3...P \hspace{1cm} (Trip constraints) \hspace{1cm} (18)
$$

$$
\sum_{p=1}^{P} \sum_{k=1}^{K} Cap_{p,k,i,j} \cdot x_{p,k,i,j} \geq dem_{i,j} \hspace{0.5cm} \forall i = 1,2,3...N \hspace{1cm} (Demand constraint) \hspace{1cm} (19)
$$

$$
\sum_{j=1}^{J} x_{p,i,j,k} \geq O_{p,j} \hspace{0.5cm} \forall p = 1,2,3...P, \forall i = 1,2,3...N \hspace{1cm} (Home base constraints) \hspace{1cm} (20)
$$

$$
S_{ro} \left( Pallet_x, (AR)_x, (W/S)_x, (T/W)_x \right) \leq D \hspace{1cm} (Aircraft takeoff distance) \hspace{1cm} (21)
$$

$$
10 \leq Pallet_x \leq 38 \hspace{1cm} (Design pallet capacity bounds) \hspace{2cm} (22)
$$

$$
2400 \leq Range_x \leq 3800 \hspace{1cm} (Range at design capacity bounds) \hspace{2cm} (23)
$$

$$
6.0 \leq (AR)_x \leq 9.5 \hspace{1cm} (Wing aspect ratio bounds) \hspace{2cm} (24)
$$
\[ 65 \leq (W/S)_X \leq 161 \]  
\[ 0.18 \leq (T/W)_X \leq 0.35 \]  
\[ x_{p,k,i,j} \in \{0,1\} \]  
\[ (AR)_X, (W/S)_X, (T/W)_X \]  

(Wing loading bounds, lb/ft\(^2\)) \hspace{1cm} (25)  
(Thrust-to-weight ratio bounds) \hspace{1cm} (26)  
(Binary assignment variable) \hspace{1cm} (27)  
(Continuous aircraft design variables) \hspace{1cm} (28)

Equation 16 is the objective function that seeks to minimize the expected fleet-level DOC by altering pallet capacity and maximum payload range of aircraft X, where \( C_{p,k,i,j} \) indicates the cost coefficient (or fuel-cost coefficient) of the \( k \)th trip for aircraft \( p \) from base \( i \) to base \( j \). The constraint Equation 17 is the balance and sequencing constraint that ensures that the \((k+1)\)th trip of an aircraft out of a base occurs only after a preceding \( k \)th trip into that base. Equation 18 limits flights to a daily utilization limit (20 hours) of the aircraft. In this equation, \( BH_{p,k,i,j} \) indicates the block hour for the \( k \)th trip for aircraft \( p \) from base \( i \) to base \( j \). Equation 19 ensures that carrying capacity of combined trip meets the demand, where \( Cap_{p,k,i,j} \) indicates the pallet carrying capacity of the \( k \)th trip for aircraft \( p \) from base \( i \) to base \( j \). Equation 20 ensures that the first trip of each aircraft originates at the initial location (home base), which is randomly generated. Equation 21 limits the aircraft design based on maximum takeoff distance to ensure that the new aircraft can operate at bases in the network. Equations 22–23 describe limits on the payload and range (in nautical miles) capabilities of the new aircraft; the limiting values are within ranges exhibited by current military cargo aircraft. The continuous design variables, aspect ratio \((AR)_X\), thrust-to-weight ratio \((T/W)_X\), and wing loading \((W/S)_X\) (here, \( W/S \) uses lb/ft\(^2\) units) describing the new aircraft are bounded within the range of values associated with current cargo aircraft; the bounds appear in Equations 24–26.

This monolithic formulation would require solving the aircraft sizing problem “in-line” with the allocation, so that the \((C_{p,k,i,j})_X\) and \((S_{TO})_X\) are non-linear functions of some of the decision variables; this results in the MINLP formulation.

**Subspace Decomposition Strategy**

The subspace decomposition strategy, as shown in Figure 5, decomposes the MINLP problem into smaller optimization problems—each sub-problem follows the natural boundaries of disciplines involved in formulating the original problem. The top-level problem helps explore the requirements space for the new yet-to-be-introduced aircraft based on fleet-level metrics. The top-level problem seeks to minimize the expected fleet level DOC using pallet capacity and range of the new, yet-to-be-introduced aircraft type \( X \); this is a small MINLP. A simple enumeration scheme solves the top-level optimization problem for the small sample problems to follow. For larger problems, this top-level problem requires an approach that can
address this smaller MINLP. The use of Monte Carlo Simulation (or Sampling) to address the uncertain demand discussed previously also appears in Figure 5.

The combination of pallet capacity \( (\text{Pallet}_x) \) and design range \( (\text{Range}_R) \) examined in the top-level problem then becomes an input to the aircraft sizing problem. Here, the aircraft sizing problem seeks to minimize the direct operating cost of the new yet-to-be-introduced aircraft, subject to performance constraints on takeoff distance. The outputs of the aircraft sizing problem and top-level optimization problem, namely the cost of operating the yet-to-be-introduced aircraft \( X \) on individual routes and pallet capacity, also become inputs in the aircraft allocation problem. Here, the objective is to minimize the fleet-level direct operating costs using characteristics of the yet-to-be-introduced aircraft (cost, pallet capacity), subject to capacity and aircraft trip limits.

**Aircraft Sizing Subspace**

With the pallet capacity and design range of the yet-to-be-introduced aircraft from the top-level problem, the aircraft sizing problem seeks to minimize the direct operating cost of the new yet-to-be-introduced aircraft, subject to constraints on minimum takeoff distance. Other forms of the objective function could minimize fuel burn on the design mission, minimize gross weight of the aircraft, and so forth, and the problem can contain to additional constraints as required. The design variables are the wing aspect ratio \( (AR)_X \), thrust-to-weight ratio \( (T/W)_X \), and wing loading.

**Figure 5. Subspace Decomposition of Monolithic Optimization Problem With Monte Carlo Sampling**
There are many other design variables, but these three have significant impact on the size, weight, and performance of the aircraft. The problem here appears sufficient to demonstrate the framework but may lack desired detail. Equations 29–35 describe the nonlinear programming aircraft sizing problem.

Minimize

$$f = (DOC_{pallet\_range})_X$$  \hspace{1cm} (29)$$

Subject to

$$10 \leq Pallet_X \leq 38$$ \hspace{1cm} (Design pallet capacity bounds) \hspace{1cm} (30)$$

$$2400 \leq Range_X \leq 3800$$ \hspace{1cm} (Range at design capacity bounds) \hspace{1cm} (31)$$

$$S_{ro} \left( Pallet_X, (AR)_X, (W/S)_X, (T/W)_X \right) \leq D$$ \hspace{1cm} (Aircraft takeoff distance) \hspace{1cm} (32)$$

$$6.0 \leq (AR)_X \leq 9.5$$ \hspace{1cm} (Wing aspect ratio bounds) \hspace{1cm} (33)$$

$$65 \leq (W/S)_X \leq 161$$ \hspace{1cm} (Wing loading bounds) \hspace{1cm} (34)$$

$$0.18 \leq (T/W)_X \leq 0.35$$ \hspace{1cm} (Thrust-to-weight ratio bounds) \hspace{1cm} (35)$$

Equation 29 is the objective function that seeks to minimize DOC or fuel cost by altering of the fleet. The aircraft X design input variables are pallet carrying capacity of the aircraft design maximum range at maximum loading condition as described in Equations 30 and 31; these echo Equations 22 and 23 above. Equation 32 limits the aircraft design based on maximum takeoff distance to ensure that the new aircraft can operate at bases in the network within the bounds of current cargo aircraft shown in Equations 33 to 35. In the implementation used for this research, a sequential quadratic programming algorithm solves the aircraft-sizing sub-problem.

**Scheduling-Like AMC Allocation Subspace**

**Monte Carlo Sampling Technique**

The cost of operating a fleet depends on the trip demand characteristics because the routes flown and payload carried—which are typically uncertain—determine the duration of the trip and the amount of fuel required to complete the trip. Although the routes served on a given day remain relatively constant for a typical quarterly schedule for a commercial airline, the same cannot be said for AMC operations, which typically experience high levels of variation in the origin and destination of demanded trips and cargo size/weight carried (Air Force Pamphlet, 2003). The GATES dataset reveals the variation in pallet demand (number of pallets transported on a route) over a year reflecting the uncertainty associated with pallet demand in AMC operations. Thus, it becomes imperative for any systems designer/planner to consider the uncertainty in the network as part of the decision-making framework. Figure 6 shows the extent of fluctuation of the pallets transported daily between two popular bases in the GATES dataset. Figure 7, showing the
histogram of the number of pallets transported per aircraft per day, reveals that the aircraft are very lightly loaded on many days. Also in contrast with commercial airline service, the AMC aircraft fleet do not always begin and end their operating day at the same locations. Where aircraft are located at the beginning of a day of operations provides an additional source of uncertainty.

The effort described in this report addressed the issue of uncertainty through a Monte Carlo Sampling (MCS) approach developed for on-demand air transportation services like fractional aircraft management and air taxi (Mane & Crossley, 2012). The MCS technique solves an allocation problem for a number of different demand instances sampled from a historical demand data distribution. The MCS technique is computationally expensive with increasing sample sizes because of the difficulty of solving an integer program for each sample of cargo trip demand and starting location for each aircraft in the fleet. This approach assumes that when using the MTM/D calculations to determine fleet size, the resulting fleet has feasible allocations for all realizations of demand instances sampled from distributions. With this approach to address uncertainty, the expected fleet direct operating cost used as the top-level problem objective function is the average fleet cost across the entire set of solved allocation problems using different samples of cargo trip demand and starting aircraft locations.
Figure 6. Distribution of Number of Pallets Transported by Date on a Sample Route From GATES Dataset

Figure 7. Histogram of Number of Pallets Transported Daily on a Sample Route From GATES Dataset
Results

Three-Base Network Problem

A very simple baseline problem reflective of AMC operations consisting of six directional routes and single period of demand between three bases provides an initial study. The motivation here is to illustrate decomposition approach to introducing a yet-to-be-designed aircraft that minimizes fleet-level operating costs. The GATES dataset provided the airbase locations and the route data. Figure 8 depicts the demand structure of the network extracted from the GATES data set using the bases ETAR, LTAG, and OKBK (International Civil Aviation Organization airport codes), which are among the most flown routes. The shortest distances between the routes are calculated using the International Civil Aviation Organization (ICAO) coordinate system. The maximum distance of the three chosen routes is 2,193 nautical miles, which allows all three types of current strategic airlift aircraft to provide service on these routes without refueling. The intent is to allocate aircraft to the three routes to satisfy all cargo demand. The problem formulation assumes an average pallet weight of 5000 lbs each; this is an important assumption because actual pallet weight varies based on the density of the cargo carried. The demand on the route originating from LTAG to OKBK has no pallet demand, which resembles the asymmetric route.

![Figure 8. Location of Bases (left) and Schematic Describing Distances and Pallet Demand (right) of the Three-Base Allocation Problem](image)

Baseline Scenario Allocation

The baseline scenario describes the current fleet operation without the introduction of the new aircraft type X. In the baseline scenario, reduced fleet size consists of five of each aircraft types: type A representing the C-5s, type B aircraft representing the C-17s, and type C aircraft representing the 747-Fs, which is
assumed to be operated as a chartered aircraft. The intent is to allocate aircraft to
the three routes to satisfy all cargo demand. The allocation problem result with all
existing aircraft provides a baseline to measure the effectiveness introducing the yet-
to-be-designed aircraft into the AMC fleet.

**Introduction of New Aircraft**

This scenario introduces three of the new aircraft type X to the existing fleet. The number of new aircraft is pre-determined because the demand network size is
too small to calculate MTM/D of the fleet. The subspace decomposition approach of
Figure 5 using range and pallet capacity as the top-level design variables for the
new, yet-to-be-designed aircraft X generates a solution. In this particular scenario,
the demand is deterministic, and the simulation allocates aircraft for various routes in
the network once for each top-level iteration. Because the problem is small, the
solution uses partial enumeration of cargo capacity and design range of the new
aircraft. The description of the best aircraft X and the DOC and fuel cost savings
compared to the baseline scenario appears in Table 2. Here, the top-level objective
seeks to minimize operating cost, and the fuel savings results from minimizing
operating cost.

**Table 2. Solution to Three-Base Fleet Allocation Problem**

<table>
<thead>
<tr>
<th>Variables, Parameters</th>
<th>Three-Base Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Aircraft X</td>
<td>3</td>
</tr>
<tr>
<td>Design Range (nmi)</td>
<td>2,400</td>
</tr>
<tr>
<td>Pallet Capacity</td>
<td>10</td>
</tr>
<tr>
<td>$W/S$, Wing Loading (lb/ft^2)</td>
<td>137.37</td>
</tr>
<tr>
<td>$T/W$, Thrust-to-Weight Ratio</td>
<td>0.27</td>
</tr>
<tr>
<td>AR, Wing Aspect Ratio</td>
<td>7.10</td>
</tr>
<tr>
<td>DOC Savings</td>
<td>0.61%</td>
</tr>
<tr>
<td>Fuel Cost Savings</td>
<td>3.14%</td>
</tr>
</tbody>
</table>

The result suggests introduction of three aircraft type X with a design range of
2,400 nautical miles (nmi) and pallet capacity of 10. The addition of three aircraft X
to the three-base network will save 0.61% in fleet-level DOC and 3.14% of fleet-level
fuel cost compared to the baseline scenario. The optimal solution suggests a small
pallet capacity aircraft that takes advantage of the low pallet demand in the network.
In this example, the smaller pallet capacity aircraft operates with a much higher load
factor compared to existing aircraft, resulting in a lower cost per pallet transported.
The enumerated design space appears in Figure 9.
The three-base problem provides a simplified example network to illustrate the decomposition approach and demonstrate its ability to generate plausible solutions. Increasing the size of the network to investigate the ability to solve larger and more complex network system using decomposition is appropriate.

**Larger Network Problem With 22 Bases**

**Solutions Without Uncertainty in Demand for 22-Base Network Problem**

This problem of increased size draws from one day of operation from the GATES dataset. The resulting 22-base network connected this day transports 310 pallets amongst these bases. The very sparse nature of the AMC network results in only 23 routes between 22 bases. The longest route in the network is 5,711 nmi, which only type A aircraft can service at its full payload weight capacity, and the mean distance is 1,947 nmi. The weight of each pallet on a given route uses the average weight of the pallets transported on that route as an approximation. The average weight of a pallet from the network is 4338.8 lbs, which is very small compared to the 10,000-lb maximum weight capacity of the 463L pallet. Figure 10 depicts the 22-base network used in this scenario.
The size of the actual strategic airlift fleet dedicated to cargo transport is obtained from the GATES dataset by accumulating unique tail numbers resulting in a fleet composition of 92 C-5s, 145 C-17s, and 69 747-Fs. In this 22-base problem, fleet size is reduced in proportion to the amount of cargo carried on this given day relative to the cargo carried in the GATES dataset. This enables the combined capacity of the existing fleet to meet the demand on this extracted 22-base network. The reduced existing fleet consists of six type A aircraft representing the C-5s, nine type B aircraft representing the C-17s, and five type C aircraft representing the 747-Fs. Figure 11 depicts the top-level optimization problem design space as a function of pallet capacity and design range generated through partial enumeration.
Figure 11. Enumeration Result From 22-Base Demand Problem

The result from this enumeration suggests introduction of nine new aircraft type X to the existing fleet with maximum pallet capacity of 10, using the design pallet weight of 7,500 pounds, and design range at maximum takeoff weight (MTOW) of 2,400 nmi. The wing loading of the aircraft X is 137.37 lb/ft$^2$, the thrust-to-weight ratio is 0.274, and aspect ratio is 7.10. The introduction of the new aircraft will result in 1.70% DOC savings and 1.26% fuel cost savings compared to the baseline allocation of only existing aircraft.

In this problem, and the preceding three-base example, a partial enumeration approach handled the mixed integer (number of pallets) and continuous (aircraft design range) variables; however, this is computationally expensive. Heuristic optimization techniques, such as the genetic algorithm (GA) and simulated annealing (SA) are suitable candidate methodologies for solving the top-level optimization problem. Using the 22-base example problem, the work explored the computational efficiency and tractability of solving the top-level problem using GA and SA schemes. The GA employed here is a “Gray-coded” genetic algorithm, in which all variables are discretized. In the GA, the design range variable representation has resolution of 200 nmi, while the pallet capacity has a discretization of one pallet. As implemented here, SA will find a result in the continuous domain, possibly resulting in a design with fractional pallet capacity.
Table 3 compares the results from these two candidate top-level optimization techniques with the enumeration technique; this includes computational run time in addition to the aircraft design requirement and aircraft sizing variable values, the fleet-level direct operating costs, and the associated reduction in fleet-level fuel costs.

The aircraft X description obtained via GA are identical to that of the enumeration; not only are the top-level variables the same, but the aircraft sizing input parameters are the same. The allocation result obtained through GA matches the enumeration solution resulting in 1.70% DOC savings and 1.26% fuel cost savings compared to the baseline scenario. The small demand size of the 22-base network is the primary reason for the modest DOC and fuel cost savings.

The result from the simulated annealing technique suggests eight aircraft type X with design requirements for a maximum pallet capacity of 10.03 and design range at MTOW of 2,467 nmi. The aircraft sizing variable values for aircraft type X includes wing loading of 132.59 lb/ft², thrust-to-weight ratio of 0.265, and aspect ratio of 6.87, which very closely matches the description of aircraft X from the enumeration result. However, optimizing the variables in the continuous domain, with the algorithm parameters used here, SA required additional computational expense to reach the optimal solution. In addition, SA converged to an optimal pallet capacity value of 10.03, which is not suitable for the aircraft description. Rounding the pallet capacity to the nearest integer is not a reasonable option given the discrete nature of the allocation problem; however, rounding up might be a viable approach. The allocation of the aircraft in the network could differ significantly for a unit change in pallet capacity of the new aircraft. Hence, with the effort to date, the GA appears to be the better choice as the top-level optimization technique.
Table 3. Solution to 22-Base Fleet-Allocation Problem

<table>
<thead>
<tr>
<th>Variables, Parameters</th>
<th>Enumeration</th>
<th>GA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computation Time</td>
<td>3 hr 42 min</td>
<td>1 hr 11 min</td>
<td>2 hr 22 min</td>
</tr>
<tr>
<td># of Aircraft X</td>
<td>9</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>Design Range (nmi)</td>
<td>2,400</td>
<td>2,400</td>
<td>2,467</td>
</tr>
<tr>
<td>Pallet Capacity</td>
<td>10</td>
<td>10</td>
<td>10.03</td>
</tr>
<tr>
<td>W/S, Wing Loading (lb/ft²)</td>
<td>137.37</td>
<td>137.37</td>
<td>132.59</td>
</tr>
<tr>
<td>T/W, Thrust-to-Weight Ratio</td>
<td>0.274</td>
<td>0.274</td>
<td>0.265</td>
</tr>
<tr>
<td>AR, Wing Aspect Ratio</td>
<td>7.10</td>
<td>7.10</td>
<td>6.87</td>
</tr>
<tr>
<td>Baseline DOC</td>
<td>$2,193,400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Fuel Cost</td>
<td>$997,100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Allocation With Aircraft X</td>
<td>$2,156,100</td>
<td>$2,156,100</td>
<td>$2,159,200</td>
</tr>
<tr>
<td>DOC</td>
<td>$ 984,560</td>
<td>$ 984,560</td>
<td>$ 985,670</td>
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<tr>
<td>Allocation With Aircraft X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel Cost</td>
<td>1.70 %</td>
<td>1.70 %</td>
<td>1.56 %</td>
</tr>
<tr>
<td>DOC Savings</td>
<td>1.26 %</td>
<td>0.68 %</td>
<td>1.07 %</td>
</tr>
</tbody>
</table>

The payload-range diagram of the aircraft X with design range of 2,400 nmi and capacity of 10 pallets is shown in Figure 12 compared to the existing aircraft in the fleet. From the design result, it is evident that the new aircraft will serve shorter, low demand routes in the network, but at a higher efficiency than the larger existing aircraft in the fleet. This highlights an interesting point for a potential acquisition decision; given that the day-to-day operations of the AMC fleet consumes a significant amount of fuel, introducing an aircraft that improves the day-to-day operations of the fleet, but may be less useful in “extreme” scenarios, could be one mechanism to reduce the fleet-level fuel consumption.
Solutions With Uncertainty in Demand and Home Base Location for 22-Base Network Problem

With the GA serving as the top-level optimization technique, the approach generates the simultaneous design requirement and sizing variable description of the new type X aircraft using the same 22-base network but now considering uncertainty in demand. The top-level aircraft design requirement variables have a resolution of 200 nmi for range and one pallet for design capacity at MTOW. To address uncertainty, a MCS approach samples the “home base” for the aircraft from a uniform distribution (i.e., the aircraft has an equal chance of starting the day at any one of the 22 bases in the network) and also samples the uncertainty in pallet demand from the historical distributions for each route (see, for example, Figure 7). The AMC allocation subspace samples 30 times (due to computational time constraints) and computes the average value of the objective function from the 30 solutions, which is fleet DOC, for each description of the new aircraft from the aircraft sizing subspace. The intent is to obtain an aircraft description that is more robust to the uncertain demand network and the random home base, because fluctuation in daily cargo demand is high in the AMC network, as shown in Figures 6 and 7. When sampling the demand, the MCS technique is set to calculate the probability of the number of pallets carried on an airplane on each route. Then a random number generated between 0 and 1 will select the number of pallets carried on a route based on the historic distribution of cargo demand from the GATES data.
in a manner akin to a weighted roulette wheel. The use on only 30 samples greatly inhibits the accuracy of the predicted mean values; however, this does demonstrate that the approach can incorporate uncertainty, but at a high computational cost. An improved approach to uncertainty quantification here is an avenue for further investigation.

Table 4 shows the GA optimized description of the design requirement variables and sizing variables for aircraft X in the 22-base fleet allocation problem using the Monte Carlo sampling approach to address uncertainty and its savings based upon comparing expected costs with Aircraft X to the expected costs of baseline solution using only existing aircraft. With uncertainty, both the baseline operating cost and fuel cost are expectations (mean values) based upon 30 samples using the MCS approach. The computation time listed here, which is nearly 1.5 days, uses serial computation. This indicates why the MCS approach only uses 30 samples, and this suggests the potential for improvement using distributed or parallel computation.

### Table 4. Solution to 22-Base Fleet Allocation Problem With Uncertainty in Demand

<table>
<thead>
<tr>
<th>Variables, Parameters, Objectives</th>
<th>GA</th>
</tr>
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<tbody>
<tr>
<td>Computation Time</td>
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</tr>
<tr>
<td># of Aircraft X</td>
<td>4</td>
</tr>
<tr>
<td>Design Range (nmi)</td>
<td>3,000</td>
</tr>
<tr>
<td>Pallet Capacity</td>
<td>27</td>
</tr>
<tr>
<td>W/S, Wing Loading (lb/ft²)</td>
<td>130.75</td>
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<td>T/W, Thrust-to-Weight Ratio</td>
<td>0.261</td>
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<tr>
<td>AR, Wing Aspect Ratio</td>
<td>6.89</td>
</tr>
<tr>
<td>Expected Baseline DOC</td>
<td>$2,223,800</td>
</tr>
<tr>
<td>Expected Baseline Fuel Cost</td>
<td>$1,026,100</td>
</tr>
<tr>
<td>Allocation With Aircraft X DOC</td>
<td>$2,211,000</td>
</tr>
<tr>
<td>Allocation With Aircraft X Fuel Cost</td>
<td>$1,003,300</td>
</tr>
<tr>
<td>DOC Savings</td>
<td>0.58%</td>
</tr>
<tr>
<td>Fuel Cost Savings</td>
<td>2.22%</td>
</tr>
</tbody>
</table>

The approach using the GA to drive the top-level problem results in $2,211,000 expected fleet DOC and $1,003,300 expected fleet fuel cost when introducing the new aircraft on the 22-base network. This results in a saving of 0.58% in fleet DOC compared to the expected baseline result of $2,223,800 and 2.22% saving in fuel cost from baseline result of $1,026,100 with introduction of six
aircraft X. The aircraft X description results in a design range of 3,000 nmi, capacity of 27 pallets, wing loading value of 130.75 lb/ft², thrust-to-weight ratio of 0.261, and aspect ratio of 6.89. The description of the aircraft X when addressing demand and home base uncertainty suggests introduction of larger, longer-range aircraft compared to the deterministic scenario with the aircraft X (see Table 3) description of 1,000 nmi design range and capacity of 11 pallets. Figure 13 superimposes the payload range diagram of aircraft X from Table 4 along with the payload range diagrams of the existing aircraft.

With addition of uncertainty in demand and random home base generation, the simulation result suggests a design that accounts for the variations in demand when compared to a design that ignores uncertainty in demand. However, the current formulation is very expensive computationally even for a network consisting of only 22 bases, and 30 Monte Carlo samples. The simulation tool will need improvements to make it computationally less expensive before extending the framework for the full-scale AMC network with 170+ bases described in the GATES dataset.

![Payload-Range Diagram Result for 22-base Network Problem](image)

**Figure 13. Payload Range Curves for Existing Fleet and the Aircraft X From 22-Base Network With Uncertain Demand**

A very coarse design space with resolution of four pallets and 200 nmi was enumerated to investigate the impact of uncertain demand and uncertain home base on the function space. Figure 14 plots the expected fleet DOC as the objective function. As with the optimization study, the Monte Carlo sampling uses only 30 samples because of the high computational expense. The aircraft X description
result from this coarse enumeration suggests the design range of 3,000 nmi, a
capacity of 18 pallets. Also identified in Figure 14 is the location of the result from
Table 4. The design ranges of these two solutions coincide, but the design pallet
capacities do not. Given that the coarse partial enumeration uses a resolution that
would not find 27 pallets and that the estimates of expected DOC are low accuracy,
the discrepancy is not unexpected. Figure 14 also illustrates that with the approach
used here, incorporating uncertainty leads to a less smooth design space when
considering design range and design payload capacity as decision variables.

![Figure 14. Enumerated Surface of Expected Fleet-Level Direct Operating
Cost With Individual Sample Results From 22-Base Network With
Uncertain Demand and Home Base](image)

**Conclusions**

The work presented here demonstrates the viability of the decomposition
approach in better informing acquisition decisions for an application motivated by the
US Air Force Air Mobility Command. The AMC operations typically involve uncertain
and asymmetric cargo demand operations, in contrast to the commercial or
passenger airline operations where routes and cargos are reasonably consistent.
The round trip assumption, though valid for the studies with the symmetric demand
route network, is poor for the AMC application. Subsequent versions of the
decomposition framework incorporated scheduling-like formulations for the resource
allocation problem by implementing node balance constraints to address the flow of individual aircraft. The scheduling-like formulation using node balance constraints, more accurately models AMC operations, allowing for directional pallet cargo and aircraft tail number tracking.

The studies presented here also use direct operating cost as the objective function. This follows from the previous work for commercial airline related investigations, where cost and profit are primary motivators. In the context of the AMC, both cost and fuel are of concern. Using the current approach to represent the AMC fleet as the C-5 and C-17 aircraft along with chartered Boeing 747-F aircraft, the Boeing 747-F cost uses a cost-per-hour approach to reflect a typical contractual agreement. At this point in the effort, simply minimizing fuel used might lead to carrying all cargo on the chartered 747-F aircraft because there is no explicit fuel cost in the chartered cost model. As demonstrated above in the result section, fleet-level fuel values are readily available, and minimizing DOC has a strong relationship to minimizing fuel consumption.

In studies, using a 22-base subset of the AMC network served in 2006 with a deterministic representation of demand led to a new aircraft with design requirements that suggest a smaller aircraft than those existing in the current AMC strategic fleet. This solution appears to exploit the fact that on day-to-day operations, the existing large-size aircraft generally carry only a fraction of their maximum payload weight capacity and often at a fraction of their design range. Although the fidelity of the aircraft modeling and the representation of AMC operations in the allocation problem, this illustrates a potentially interesting result for acquisition decisions. If fleet level cost (and/or fuel) consumption were a driving factor, perhaps acquiring a smaller aircraft for day-to-day operations would substantially improve cost (and fuel use). A challenge with this is to ensure that the AMC fleet could still meet extreme demand scenarios, such as in wartime or in large-scale humanitarian relief, when high payload and range capabilities become more important.

Recognizing the uncertainties in the cargo demand structure of the AMC fleet led the research to consider uncertainty via a comparatively naïve Monte Carlo Sampling technique. By minimizing an expected fleet-level operating cost in the presence of non-deterministic demand and aircraft starting locations, the approach determined design requirement values and aircraft sizing variable values for a new cargo aircraft that accounts for the uncertainty. The computational cost associated with MCS in a serial computation environment hampered the quality of the resulting solutions, because of the high error associated with using only 30 samples to compute mean values of fleet-level cost. However, the ability to conduct this kind of study with the decomposition approach under uncertainty is possible. Improved approaches to address uncertainty and improved computational approaches will
improve the quality of the results, and the framework should readily accommodate these improvements.

**Potential Extensions of Framework**

An acquisition support issue is the selection of the top-level design variables that represent some of the requirements for a new platform. Payload capacity, design cruise velocity, and range are common aircraft design and are logical choices for these top- or system-level variables. Our current investigations have considered design range and the maximum number of pallets as top-level variables. Although palletized cargo has well-defined geometric dimensions (particularly length and width), the pallet density (weight per pallet) of cargo carried has a wide variation. Further, outsized or unusually dimensioned payload often set cargo bay dimensions for new aircraft; for instance, the large size of the C-5’s cargo bay allowed air transport of the 74-ton mobile scissors bridge that is seldom carried but was part of the original requirements to allow for an extreme scenario. To improve the credibility of the aircraft design portion of the decomposition approach, the payload capacity requirements must incorporate both weight and volume (or dimension) as two distinct, but not wholly independent, aspects. One potential approach to this is to select a discrete set of potential outsized payloads to set the dimensions, recognizing that the aircraft will most often carry palletized cargo, and then use maximum payload weight as one of the top-level design variables/new aircraft requirements. The resulting values for these requirement variables can inform acquisition decisions about what new platform requirements will lead to a more successful fleet. The decomposition framework also informs how the new platform needs to be used to improve the fleet-level objective(s).

The authors would like to improve upon the fidelity of capturing AMC operations through considering the time-sensitive nature of cargo. Cargo is tiered according to urgency of delivery, and thus poses implicit constraints on the routes traveled on (relating to the range of the aircraft used), and the capability (here, speed) of the aircraft. The researchers are currently exploring adaptations, based on block hour allocation per aircraft type within the fleet, as a means of keeping track of time-related constraints for aircraft trips within the allocation problem.

Fleet-fuel and fleet-operating costs have provided the performance metrics for the current work where the objective function of the allocation problem seeks to minimize the total amount of fuel burned or the fleet operating cost, resulting from cargo-carrying trips across the AMC network of operations. However, “fleet-productivity,” as referred to in prior studies (Mane et al., 2007) is a metric that combines speed and weight of cargo transported into a single metric that serves as the problem objective. Our proposed future work seeks to provide a metric that adequately captures salient measures of productivity for the allocation sub-problem.
to ensure a balanced representation of tradeoffs between fleet performance and fuel consumption. To illustrate tradeoffs, this potential future task could conduct multi-objective studies. Under the multi-objective formulation, the two objectives examined are maximizing productivity and minimizing fuel consumed. Employing an epsilon constraint approach allows the use of a single objective formulation while incorporating the second objective function (in this case, fuel consumed) as a constraint. The single objective is to maximize productivity, and constraints restrict fleet-level fuel consumption to different levels. Maximizing productivity under different fuel consumption limits will lead to a Pareto frontier of optimal solutions representing the best possible tradeoffs between the two objectives.

Further investigating the concept of using smaller aircraft than the current strategic fleet, as suggested by some of the studies documented above, follows a multi-focus approach. First, the focus will be on fuel alone. This will examine what fleet is operated and how much fuel savings are possible with "smaller" aircraft. The second focus will incorporate a "super scenario" that includes specific fleet requirements for strategic lift capability needed in wartime or other urgent, but uncommon, scenarios. The third focus will consider the cost of ownership to determine whether fuel savings can offset ownership of a mixed aircraft fleet. For example, the framework developed in this research could help identify whether the fuel savings (and cost savings associated with fuel savings) of using new, smaller aircraft for day-to-day operations might offset the cost of having larger aircraft in the fleet that are not frequently used but are available for extreme situations. This approach will also allow studies to be conducted that change the price of fuel to see how fluctuations in fuel prices might impact requirements of new aircraft; perhaps under very high fuel prices, the aircraft more customized for day-to-day operations may be a better overall choice.

The authors would also seek to explore the possibility of upgrading or modifying aircraft (e.g., re-engining or addition of winglets) in the fleet; in recent history, this approach is not uncommon to prolong service life and improve fleet performance among military aircraft. The addition of these discrete upgrade actions in the design of the aircraft, in concert with the decomposition-based approach, can potentially yield strategically more beneficial design solutions for energy efficiency as well. This can be done by representing these aspects in the aircraft sizing sub-problem and then restricting the design variables to values associated with the current un-modified aircraft. For example, a notional re-engined C-5 aircraft (perhaps like the C-5M) would retain the same aspect ratio and likely the same wing loading, because the aircraft geometry would remain unchanged. The thrust-to-weight ratio may remain the same, if the intent is to use more efficient engines with the same amount of installed thrust. In this case, the specific fuel consumption of the aircraft would be changed, and resulting costs and fuel consumption on the various
operating routes would be computed for use in the allocation problem. By modeling costs associated with a re-engining of an existing airframe, the approach can reveal the impact on fleet-level fuel consumption relative to cost.

**Contributions of Research**

The research performed in this report has illustrated the application of a framework that accounts for determining new system design requirements so that the resulting system design has a desired impact on a fleet-level metric. In this case, the approach determines the design requirements of aircraft range and payload capacity of a new cargo aircraft by coordinating the sizing of this new aircraft and the allocation of the new aircraft along with existing aircraft to meet cargo trip demand. The research then proceeded to leverage analytical tools and techniques from operations research in providing the means to objectively identify acquisition relevant *requirements* that in turn directly drive quantitative measures of metrics (in this case cost and fuel usage). The research work has led to the following advances for supporting acquisition decisions:

1. A computational tool that treats the design requirements for the new platform as decision variables in an optimization problem. This approach then suggests or recommends the best new platform requirements to optimize metrics associated with an entire fleet of platforms.

2. The approach also demonstrates how the problem of identifying the design requirements, design variables, and allocation strategy can follow a decomposition approach that enables solution of what would be difficult, if not impossible, to solve as a single monolithic problem statement.

3. The approach is amenable to addressing uncertainty in modeling the operations of the new system along with existing systems.

4. The decomposition approach can employ different models in each sub-problem without requiring a change in the overall approach.
References


