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DAMPING MODELS FOR SHEAR-DEFORMABLE BEAM WITH APPLICATIONS TO SPACECRAFT WIRING HARNESS

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Final Report

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Cabling attached to a spacecraft bus structure can represent a significant fraction of the dry mass. While current models can accurately predict vibration frequencies of the coupled system, typical damping models are inadequate. Instead, a viscous damping model that produces approximately frequency-independent modal damping in Euler-Bernoulli and shear beams is considered. The relevant viscous damping terms are extended and modified for application to Timoshenko beams. The inclusion of rotary inertia does add some frequency-dependence; however, careful selection of damping coefficients can produce a large range of approximately frequency-independent modal damping. This effort will extend recent basic research to explore damping models for use with shear-deformable beams, and will begin to address fundamental physical damping mechanisms and potential non-linear dissipative sources in spacecraft cables. The desired outcome is a time domain beam model that represents with physical consistency the damping behavior of wiring harnesses. The emphasis in this project will be on the extension of the shear-beam damping model to the Timoshenko beam, a beam model that includes the effects of rotatory inertia. Including this sometimes-significant effect is likely to change the damping behavior at higher mode numbers.

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1 SUMMARY

Power and data cabling are attached to a spacecraft bus structure at many points and can account for a significant fraction of a spacecraft's dry mass. This combination leads to coupled spacecraft and cable dynamics that require a model to predict the effects of this interaction. While current models can accurately predict vibration frequencies, typical proportional damping models are inadequate. Instead, a viscous damping model that produces approximately frequency-independent modal damping in Euler-Bernoulli and shear beams is considered. The relevant viscous damping terms (as well as those commonly employed in proportional damping approaches) are extended and modified for application to Timoshenko beams. The inclusion of rotary inertia does add some frequency-dependence; however, careful selection of damping coefficients can produce a large range of approximately frequency-independent modal damping.

This paper will specifically consider a careful selection of coefficients for the rotation-based viscous damping models to provide modal damping that is approximately independent of modal frequency, at least over a significant range of vibration modes. Guidance in this parameter selection and the resulting modal damping will be presented for several combinations of canonical boundary conditions, including simply-supported and clamped-clamped Timoshenko beams. Furthermore, a series expansion of the mathematical form of "square-root" damping will be explored to provide an increased range of frequency-independent modal damping and a corresponding explanation of the underlying dissipative physical mechanisms.

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2 INTRODUCTION

Recent advances in spacecraft observation, data handling, and vibration control capabilities and requirements have led to large amounts of power and data cabling on spacecraft. Coupled with materials advances that lead to structural materials with decreased density, these cables can account for a large proportion of the spacecraft mass. In fact, this cabling mass ratio can reach 30% (of the dry mass) for some spacecraft [1,2].The cabling is attached to the spacecraft structure at a large number of points, leading to coupled spacecraft dynamics. While this coupling is not typically modeled in the design process, it is commonly observed during ground system-level testing. In most cases, this testing provides the data required to model the spacecraft structure and cabling interaction; however, an accurate model during the initial design phases is significantly more desirable.

Spacecraft cabling typically consists of wrapped, stitched, and twisted wires, so previous research toward such a model treated the cabling as shear beams with effective parameters [1-4]. While individual cable bundles can be long and slender, they are tightly secured at numerous tie down points. As such, previous research treated each bundle as a series of short beams with a free span between those tie down points. Initial research efforts employed structural damping through a loss factor in a NASTRAN finite element model. While providing adequate results for frequency-domain modeling, a time-domain model was desired. Furthermore, experimental tests indicated the cable bundles displayed modal damping that was approximately constant across a range of vibration modes. As such, a viscous damping model developed by Lesieutre that produced frequency-independent modal damping in Euler-Bernoulli beams was extended to and modified for the shear beam [5,6]. This model yielded modal damping that was approximately frequency-independent and agreed well with experimental data. On that basis, this paper will further extend and modify the model for application to Timoshenko beams by including the effects of rotary inertia.

3 METHODS, ASSUMPTIONS, AND PROCEDURES

3.1 Damping Model Background

There are several common approaches to incorporate a damping model for Timoshenko beams. Perhaps the most common is the structural damping model. Taken to its extreme, it accommodates a complex form for every parameter of the system. As an example, Lunden and Akesson showed this approach results in 14 damping terms, although simultaneously pointing out that in practical terms all but a few of these terms would be identically zero [7]. More typical is a complex form of the Young's and shear moduli, resulting in at most two damping terms [7-11]. While these methods are ill-suited for time-domain modeling, they can provide valuable insight into potential damping models.

The second common approach is through the inclusion of proportional damping terms. Here damping terms result as some combination of terms proportional to the mass and stiffness terms [12-19]. This technique is mathematically convenient; however, it fails to provide modal damping that is physically realistic. Indeed, a high degree of frequency-dependence is seen with proportional damping, whether formulated for an Euler-Bernoulli or Timoshenko beam. Lesieutre provided a solution for this unrealistic behavior, developing a “rotation-based" (also called “geometric-based") viscous damping model that results in approximately frequency-independent modal damping for Euler-Bernoulli beams [5]. Subsequent research extended and modified this model for application to shear beams; that model serves as the starting point for this study of frequency-independent modal damping for Timoshenko beams [6].

3.2 Governing Equations of Motion

Including rotary inertia and _rst-order transverse shear deformation, the governing equations of motion for a Timoshenko beam without damping and subject to transverse loading are

$$
\begin{gathered}
-\frac{\partial}{\partial t}\left(\rho A \frac{\partial w}{\partial t}\right)+\frac{\partial}{\partial x}\left[\kappa A G\left(\frac{\partial w}{\partial x}-\varphi\right)\right]=-q \\
-\frac{\partial}{\partial t}\left(\rho I \frac{\partial \varphi}{\partial t}\right)+\frac{\partial}{\partial x}\left(E I \frac{\partial \varphi}{\partial x}\right)+\kappa A G\left(\frac{\partial w}{\partial x}-\varphi\right)=0
\end{gathered}
\tag{1}
$$

where the shear deformation is described by

$$
\beta=\frac{\partial w}{\partial x}-\varphi \tag{2}
$$

Note that this approach results in three coordinates to describe the beam, yet only two are independent. The following approach continues in terms of the beam displacement (w) and the slope of the beam centerline associated with bending (φ) only. Analysis in terms of the shear deformation (β) yields identical results.

For simplicity, restrict the beam to have constant properties along its length. Denoting spatial partial derivatives as ( )' and temporal partial derivatives as (˙), the equations of motion are

$$\begin{aligned} -\rho A\ddot{w} + \kappa AG\left(w'' - \varphi'\right) &= -q \\ -\rho I\ddot{\varphi} + EI\varphi'' + \kappa AG\left(w' - \varphi\right) &= 0 \end{aligned} \tag{3}$$

These equations of motion can be written in terms of a single coordinate; however, the current effort will incorporate damping terms that precludes such a simplification.

This initial analysis treats a simply-supported Timoshenko beam; the associated boundary conditions require both the displacement and bending moment be zero at the beam endpoints:

$$w(0,t) = w(L,t) = EI\varphi'(0,t) = EI\varphi'(L,t) = 0 \tag{4}$$

Subsequent analysis may consider alternative boundary conditions. For example, a clamped-clamped Timoshenko beam requires the displacement and slope associated with bending to be zero at the beam endpoints:

$$w(0,t) = w(L,t) = \varphi(0,t) = \varphi(L,t) = 0 \tag{5}$$

Viscous Damping Models

Several viscous damping models are considered here. They can largely be classified as motion-, rotation- (or geometric-), and strain-based models. Two motion-based terms are proportional to the inertia of the beam and represent loads that oppose the beam velocity (α_{Mw}) and rotation rate ($\alpha_{M\varphi}$). Two strain-based terms are proportional to the rigidity of the beam and represent loads that oppose the temporal rates of change of beam centerline curvature ($\alpha_{K\varphi}$) and shear curvature ($\alpha_{K\beta}$). Finally, rotation- (geometric-) based terms are associated with loads that oppose the temporal rates of change of the beam centerline angle (α_{φ}) and shear angle (α_{β}). The resulting equations of motion are

$$\begin{aligned} -\rho A\ddot{w} + \kappa AG\left(w'' - \varphi'\right) &= -q + \alpha_{Mw}\dot{w} - \alpha_{K\beta}\dot{\beta}' - \alpha_{\varphi}\dot{\varphi}' \\ -\rho I\ddot{\varphi} + EI\varphi'' + \kappa AG\left(w' - \varphi\right) &= -\alpha_{EI}\dot{\varphi}'' + \alpha_{M\varphi}\dot{\varphi} - \alpha_{\beta}\dot{\beta} \end{aligned} \tag{6}$$

Note that due to the use of all three coordinates, some of these damping terms will necessarily overlap; in fact, some terms may actually offset the damping introduced by other terms. For example, the equations of motion were presented in terms of the displacement (w) and slope of the beam centerline associated with bending (φ) only; substituting Equation (2) into Equation (6) yields:

$$\begin{aligned} -\rho A\ddot{w} + \kappa AG\left(w'' - \varphi'\right) &= -q + \alpha_{Mw}\dot{w} - \alpha_{K\beta}\dot{w}'' - \left(\alpha_{\varphi} - \alpha_{K\beta}\right)\dot{\varphi}' \\ -\rho I\ddot{\varphi} + EI\varphi'' + \kappa AG\left(w' - \varphi\right) &= -\alpha_{EI}\dot{\varphi}'' + \left(\alpha_{M\varphi} + \alpha_{\beta}\right)\dot{\varphi} - \alpha_{\beta}\dot{w}' \end{aligned} \tag{7}$$

In the first equation, the $\dot{\varphi}'$ term is multiplied by the difference between α_{φ} and $\alpha_{\kappa\beta}$. That is, independent selection of positive damping coefficients can result in negative damping. As such, the damping coefficients are coupled and must be selected together, just as the beam displacement (w), centerline bending angle (φ), and shear deformation (β) are coupled and must be analyzed jointly.

3.3 Finite Element Model

Unlike the Euler-Bernoulli and shear beams, the damped Timoshenko beam equations of motion are, in general, not readily treated analytically through consideration of modal motion. While some limited combinations of viscous damping models and boundary conditions admit such an approach, numerical techniques are required for a more general analysis. As such, a finite element formulation is developed from the weak form of the damped equations of motion.

Discretization of Equation of Motion with Damping Terms

Beginning with a weak formulation of the unforced Timoshenko beam equations of motion, application of the Galerkin method of weighted residuals and integration by parts results in

$$\begin{aligned} \int_0^L \left(w\rho A\ddot{w} + w'\kappa AGw' + w\kappa AG\varphi' + w\alpha_{Mw}\dot{w} + w'\alpha_{K\beta}\dot{w}' + w\alpha_{K\beta}\dot{\varphi}' - w\alpha_{\varphi}\dot{\varphi}'\right)\mathrm{d}x &= 0 \\ \int_0^L \left(\varphi\rho I\ddot{\varphi} + \varphi' EI\varphi' - \varphi\kappa AGw' + \varphi\kappa AG\varphi + \varphi'\alpha_{K\varphi}\dot{\varphi}' - \varphi\alpha_{M\varphi}\dot{\varphi} - \varphi\alpha_{\beta}\dot{w}' + \varphi\alpha_{\beta}\dot{\varphi}\right)\mathrm{d}x &= 0 \end{aligned} \tag{8}$$

The next step is the introduction of a set of shape functions for the coordinates, w and φ, taking care to avoid the shear locking problem. Several such approaches exist; the solution employed here is to interpolate the displacement, w, using cubic shape functions and the centerline slope associated with bending, φ, using quadratic shape functions [20]. The coordinates can thus be written in terms of the shape functions

$$
\begin{aligned}
w &= [N_w]\{\delta\} \\
\varphi &= [N_\varphi]\{\delta\}
\end{aligned}
\tag{9}
$$

where

$$
\{\delta\} = \begin{bmatrix} w_A & \varphi_A & w_B & \varphi_B \end{bmatrix}^t
\tag{10}
$$

The shape functions are

$$
[N_w] = \begin{Bmatrix}
\frac{1}{1+\phi}\left(2\left(\frac{x}{L_{\mathrm{el}}}\right)^3 - 3\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - \phi\frac{x}{L_{\mathrm{el}}} + 1 + \phi\right) \\
\frac{L_{\mathrm{el}}}{1+\phi}\left(\left(\frac{x}{L_{\mathrm{el}}}\right)^3 - \left(2+\frac{\phi}{2}\right)\left(\frac{x}{L_{\mathrm{el}}}\right)^2 + \left(1+\frac{\phi}{2}\right)\frac{x}{L_{\mathrm{el}}}\right) \\
-\frac{1}{1_\phi}\left(2\left(\frac{x}{L_{\mathrm{el}}}\right)^3 - 3\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - \phi\frac{x}{L_{\mathrm{el}}}\right) \\
\frac{L_{\mathrm{el}}}{1+\phi}\left(\left(\frac{x}{L_{\mathrm{el}}}\right)^3 - \left(1-\frac{\phi}{2}\right)\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - \frac{\phi}{2}\frac{x}{L_{\mathrm{el}}}\right)
\end{Bmatrix}^t
\tag{11}
$$

$$
[N_\varphi] = \begin{Bmatrix}
\frac{6}{L_{\mathrm{el}}(1+\phi)}\left(\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - \frac{x}{L_{\mathrm{el}}}\right) \\
\frac{1}{1+\phi}\left(3\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - (4+\phi)\frac{x}{L_{\mathrm{el}}} + 1 + \phi\right) \\
-\frac{6}{L_{\mathrm{el}}(1+\phi)}\left(\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - \frac{x}{L_{\mathrm{el}}}\right) \\
\frac{1}{1+\phi}\left(3\left(\frac{x}{L_{\mathrm{el}}}\right)^2 - (2-\phi)\frac{x}{L_{\mathrm{el}}}\right)
\end{Bmatrix}^t
\tag{12}
$$

Where ϕ is an elemental shear stiffness parameter:

$$
\phi = \frac{EI}{\kappa AG}\frac{12}{L_{\mathrm{el}}^2}
\tag{13}
$$

Inserting these shape functions in Equation (8) ultimately leads to the mass, damping, and stiffness matrices for each element. The elemental mass matrix comes from terms involving the second time derivative of the displacements and rotations:

$$[M] = \int_0^{L_{\mathrm{el}}} \rho A \left([N_w]^t [N_w] + R [N_\varphi]^t [N_\varphi] \right) \mathrm{d}x \tag{14}$$

where $R = \rho I / \rho A$ is a parameter describing the effect of rotary inertia.

The elemental stiffness matrix comes from terms involving no time derivatives:

$$[K] = \int_0^{L_{\mathrm{el}}} EI \left([N'_\varphi]^t [N'_\varphi] + S \left([N'_w]^t [N'_w] + [N_w]^t [N'_\varphi] - [N_\varphi]^t [N'_w] + [N_\varphi]^t [N_\varphi] \right) \right) \mathrm{d}x \tag{15}$$

where $S = kAG/EI$ is a parameter describing the effect of transverse shear. Note that, in contrast to R, large values of S indicate that transverse shear can largely be neglected (since the beam is very stiff in shear compared to bending).

Finally, the elemental damping matrix comes from terms involving a single time derivative:

$$\begin{aligned}[C] = \int_0^{L_{\mathrm{el}}} \big(& \alpha_{Mw} [N_w]^t [N_w] - \alpha_{M\varphi} [N_\varphi]^t [N_\varphi] + \alpha_{K\varphi} [N'_\varphi]^t [N'_\varphi] \\ & + \alpha_{K\beta} [N'_w]^t [N'_w] - \alpha_\varphi [N_w]^t [N'_\varphi] - \alpha_\beta [N_\varphi]^t [N'_w] + \alpha_\beta [N_\varphi]^t [N_\varphi] \big) \mathrm{d}x \end{aligned} \tag{16}$$

4 RESULTS AND DISCUSSIONS

4.1 Modal Damping Values for Several Viscous Damping Models

The key result is that the rotation-based viscous damping models can provide modal damping values with less frequency dependence than those produced by common motion- and strain-based viscous damping models. However, the effects of shear deformation and rotary inertia re-introduce the frequency-dependence commonly seen with motion- and strain-based models, although to a lesser extent.

Figure 1 shows the modal damping values produced by the various damping models when the shear deformation and rotary inertia effects are present but small. As is expected from the analysis of Euler-Bernoulli beams, the motion-based model provides modal damping that very rapidly decreases with mode number, while the damping values produced by the strain-based model increase unrealistically with mode number. The two rotation-based models as developed for the shear beam provide modal damping that is approximately constant with mode number. Note that the finite element analysis used 50 elements, so numerical artifacts may have some effect for the higher mode numbers.

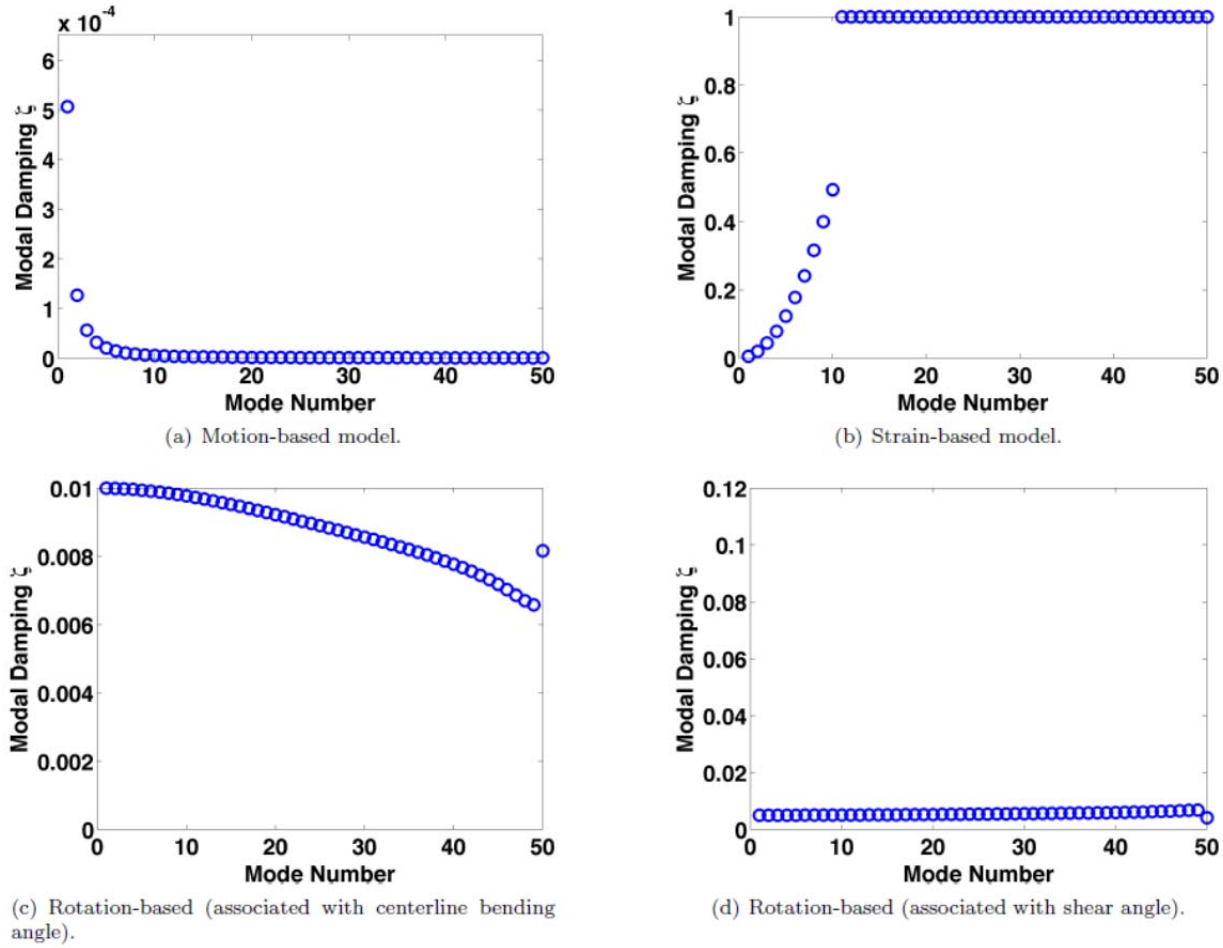
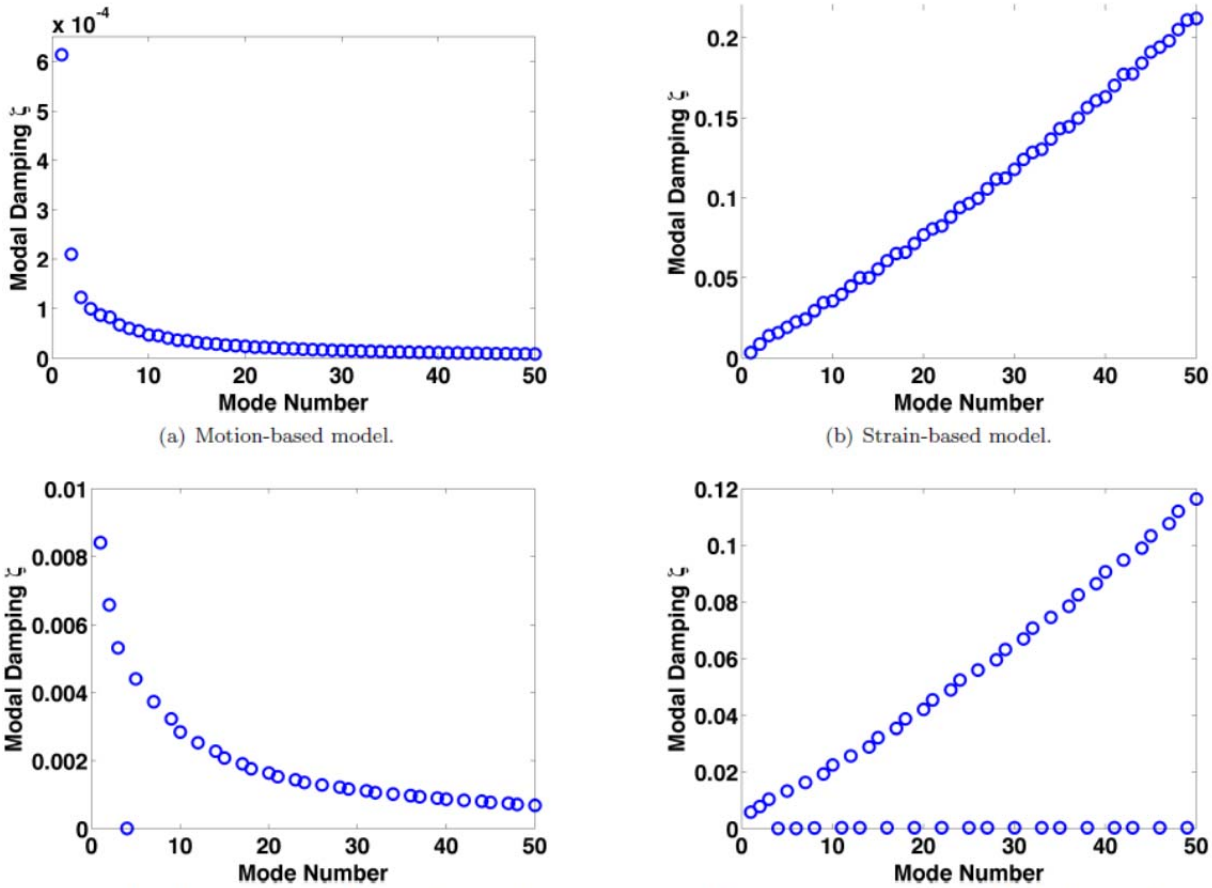


Figure 1 Modal damping for several viscous damping models with small shear deformation and rotary inertia effects.

Figure 2 shows the modal damping values produced when the shear deformation and rotary inertia effects are significant (well beyond that typically observed in spacecraft wiring harnesses). Again, the motion-based model provides modal damping that very rapidly decreases with mode number, while the modal damping produced by the strain-based model increases very rapidly with mode number. In this case, the two rotation based models now also provide modal damping that decreases or increases with mode number, though not nearly as quickly as that of the motion- and strain-based models. As such, a combination of these models (with some modification) is expected to provide approximately frequency-independent modal damping over a significant range of vibration modes. Certainly motion- and strain-based viscous damping models can also be modified to provide approximate frequency-independence over a small range of modes; similar use of the rotation-based damping models should significantly extend that range.

Finally, note that Figures 2(c) and 2(d) show significant damping for only some of the modes. Figure 2(d) makes this point most clearly, with approximately linearly increasing modal damping for one set of modes and approximately constant (but zero!) modal damping for another

set of modes. In fact, the modes with approximately linearly increasing modal damping have deflection shapes that primarily reflect shear deformation. Conversely, the modes with nearly zero modal damping exhibit deflection shapes that primarily involve bending. Indeed, this particular model involves only damping associated with the shear angle, so it makes sense that modal damping is significant in those modes most heavily associated with shear deformation. This behavior also provides insight into the damping model effects and may further guide the selection of damping coefficients to produce frequency-independent modal damping across both sets of modes.



(a) Motion-based model.

(b) Strain-based model.

(c) Rotation-based (associated with centerline bending angle).

(d) Rotation-based (associated with shear angle).

Figure 2 Modal damping for several viscous damping models with large shear deformation and rotary inertia effects.

4.2 Mathematical Aside

Previous research has indicated that so-called \square-root" damping results in modal damping that is independent of frequency [21]. This approach involves representing the governing equation of motion in terms of differential operators:

$$\mathcal{M}\ddot{\gamma} + \mathcal{C}\dot{\gamma} + \mathcal{K}\gamma = f \tag{17}$$

where γ includes the coordinate(s) describing the system configuration and f includes the external load(s) acting on the system. A damping differential operator that is twice the square root of the product of the mass and stiffness differential operators can be scaled to produce a desired modal damping ς_0 that is independent of frequency [6]:

$$\mathcal{C} = 2\zeta_0\sqrt{\mathcal{K}\mathcal{M}} \tag{18}$$

In general, the square root of these operators depends on the system boundary conditions; however, approximate results can be found that provide modal damping that is approximately constant, at least over a large range of mode numbers.

Comparing Equation (17) with Equation (3), the mass and stiffness operators for a system with constant properties are, in matrix form,

$$[\mathcal{M}] = \begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix} \qquad \text{and} \qquad [\mathcal{K}] = -\begin{bmatrix} \kappa AG\,(\,)'' & -\kappa AG\,(\,)' \\ \kappa AG\,(\,)' & EI\,(\,)'' - \kappa AG \end{bmatrix} \tag{19}$$

While the square root of the mass operator is straight-forward, the square root of the stiffness operator is much more involved. Yet even for the shear beam (in which the differential operators are scalars instead of matrices), this approach did not yield terms that were easily explained in terms of physical dissipative mechanisms. A series expansion of the square-root damping terms, however, did lead to viscous damping terms that had clear physical underpinnings and provided approximately frequency-independent modal damping. As such, a similar series expansion will be explored for the Timoshenko beam in terms of physical dissipative mechanisms.

$$[\mathcal{K}] = \frac{\sqrt{\kappa AG}}{\sqrt{\left(1+\sqrt{\frac{EI}{\kappa AG}}\right)^2 (\,)'' - (\,)}} \begin{bmatrix} \left(1+\sqrt{\frac{EI}{\kappa AG}}\right)(\,)'' & -(\,)' \\ (\,)' & \left(\frac{EI}{\kappa AG} + \sqrt{\frac{EI}{\kappa AG}}(\,)'' - (\,)\right) \end{bmatrix} \tag{20}$$

5 CONCLUSIONS

Previous research developed viscous damping models that resulted in approximately “frequency-independent" modal damping for both Euler-Bernoulli and shear beams. This research extends those models to a Timoshenko beam and examines their behavior (as well as that of more common proportional damping models) when rotary inertia effects are significant. In that case, the rotation-based model associated with the temporal rate of change of the centerline bending angle ' provides modal damping that decreases with mode number, though to much less degree than that of typical motion-based (mass-proportional) models. Similarly, the rotation-based model associated with the temporal rate of change of the shear angle _ provides modal damping that increases with mode number, though to much less degree than that of typical strain-based (stiffness-proportional) models.

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