The Effect of Stochastic Noise on Predictability

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LONG-TERM GOAL

Our long-term goal is to improve the accuracy of numerical prediction models of weather and climate. We shall concentrate on sources of prediction error involving interactions between physical phenomena having different timescales.

OBJECTIVES

We examine how inadequate representation of rapidly-varying (i.e., stochastic) atmospheric effects in General Circulation Models (GCMs) can systematically affect prediction on a variety of scales, and how errors arising from this inadequacy may be ameliorated. Concurrently, we develop useful, efficient and accurate methods of accounting for these systematic effects of stochastic forcing, and also provide methods for estimating the spread of ensemble predictions taking into account multiplicative stochastic effects.

APPROACH

We approach the problem both analytically and numerically. Linear empirical models and barotropic models of the atmosphere present cases that are meteorologically relevant, yet simple enough to solve analytically, and we use these models to identify and solve some of the numerical problems that will
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arise in realistic, more complex models. We intend to apply our results to studying the impact of stochastic forcing in more complete models, culminating with long runs of stochastically forced, full-fledged climate GCMs and numerical weather prediction (NWP) models.

The theory of stochastic white noise is well known and analytic solutions are available, at least in principle. We have extended these results to the more realistic problem of stochastic colored noise forcing, and are currently investigating how best to implement these results in a real-time forecasting system.

Investigations into the numerical implementation of stochastic theory is still a major effort. We are in communication with numerical analysts who, at our request, consider the errors incurred by including stochastic forcing into existing deterministic models, most of which use a leap-frog integration scheme. Further, we consider recent developments in solving numerically the Fokker-Planck equation for the forecast distribution.

**WORK COMPLETED**

We have completed numerical work on the steadily forced barotropic vorticity equation with stochastic variations in the basic state. We also completed analysis of a similar system with stochastically varying friction on the forced waves.

We derived analytic expressions for the expected mean response to colored-noise forcing with widely different timescales.

We produced linear inverse models for both winter and summer. These models produce forecasts of weekly averaged global atmospheric circulation and diabatic heating anomalies whose skill is competitive with NCEP’s MRF ensemble at week 2, and possibly better at weeks 3 and 4. We have analyzed the models and determined predictability limits of the models.

**RESULTS**

We have obtained two principal results this year:

The fact that stochastic variations often do not decay on timescales much faster than those of the weather systems to be predicted required us to consider how robust are the white noise approximations used in our previous year’s research. We developed a general formula for the drift in the mean response of a steadily forced linear system due to stochastic perturbations with timescales ranging from much faster than the deterministic timescales to much slower than the deterministic timescales. We showed that this general formula is rigorously correct, and numerical simulations verified this. This was an important advance upon analytic results last year, which only exactly agreed with numerical simulations at asymptotically long and short decorrelation time scales of the stochastic variations. We found that whether stochastic variations in the basic state of a steadily forced barotropic vorticity model act to stabilize or destabilize Rossby waves depends subtly upon their impact on the wave properties of the system.

Secondly, we constructed a linear inverse model (LIM) suitable for studies of atmospheric extratropical predictability on weekly time scales using global observations of the past 30 years. Notably, the LIM includes tropical diabatic heating as an evolving model variable rather than as an externally specified
forcing, and also includes, in effect, the feedback of the extratropical weather systems on the more slowly varying circulation. We have shown that both of these features are important contributors to the realism of this empirical-dynamical model.

**Predictability of Weekly Averages during Winter**

1. **Forecast skill and predictability of weekly averages during winter.**

   **Top:** Correlation of observed and 3-week forecasts of upper troposphere streamfunction anomalies, averages over 52 forecast cases in the winters of 1985/86—1988/89. (a) LIM. (b) GCM (NCEP Reanalysis model).

   **Bottom:** Potential predictability limit: forecast lead at which skill (i.e., the correlation of observed and predicted anomalies) drops below 0.5. (a) Determined from the full LIM. (b) Determined from a version of the LIM in which the effects of tropical forcing are removed.

At week 2 the LIM’s forecast skill is competitive with that of the global nonlinear medium-range forecast (MRF) model with $O(10^6)$ degrees of freedom in use at the National Centers for Environmental Prediction (NCEP). Figure 1 (a and b) shows such a comparison for Week 3 forecasts made throughout
the winters of 1985/86-1988/89. Analysis of the LIM suggests that without inclusion of tropical heating, weekly averages in the extratropics may be predictable only about two weeks ahead, but with tropical heating included, they may be predictable as far as seven weeks ahead. This difference is displayed in Figs. 1c and 1d. This shows that accurate prediction of tropical diabatic heating is key to enhancing extratropical predictability.

**IMPACT/APPLICATION**

Our research has a direct impact on ensemble weather and climate prediction. We have found that we needed to use numerical methods and criteria for choice of time step and integration time that are generally not used in the numerical modeling community. The methods currently used in stochastic numerical models are likely to be highly inaccurate, and our work helps to 1) quantify how inaccurate those methods may be and 2) offer alternatives to them in order to improve the accuracy of numerical prediction models.

Our LIM results mark the first time that a successful linear model of atmospheric variability on weekly time scales has been constructed. If such variability is *effectively* linear then a *detailed* knowledge of nonlinearities may not result in significant further improvement in predictability on these time scales.

The LIM should also prove useful in our planned research this year. Because it produces realistic atmospheric variability yet has only 37 degrees of freedom, it is particularly suited to analyzing the effects of realistic multiplicative noise. We have already shown (see last year’s report) that accurately determining the effects of multiplicative noise takes very lengthy computer runs, and that additionally there may be unexpected numerical modeling pitfalls in the correct treatment of the noise. Although we have learned much about the effects of multiplicative noise in the barotropic context, it may be quite a leap between that and a full GCM, where very long integrations could be impractical without additional guidance. Thus, for our multiplicative noise experiments the LIM may be a very important intermediate model between the barotropic model we used next year and a full GCM we will use next year.

**TRANSITIONS**

The analytical work we have done has proven to be interesting to the general stochastic modeling community. We discussed our work at a workshop hosted by the Centre Européen de Calcul Atomique et Moléculaire, in Lyon, France, last summer and have been invited to participate in a workshop on stochastic climate modeling hosted next June by the Bereich Stochastik, Institut fuer Mathematik der Humboldt Universitaet in Chorin, Germany. Dr. Sardeshmukh has presented these results in a series of invited talks at the EGS meeting in Nice, the Geophysical Fluid Dynamics Laboratory (GFDL) in Princeton, and NCAR in Boulder.

We have also interested the numerical analysis community and Dr. Penland will give a talk in October at the Dept. of Applied Mathematics, University of Indiana, Bloomington.

We have used the LIM to produce realtime forecasts during both winter and summer. These forecasts are made available worldwide at the CDC web site (http://www.cdc.noaa.gov/~crw/lim).
RELATED PROJECTS

The methods developed in this project are also being applied to examine feedbacks between the ocean and atmosphere in the extratropics on submonthly time scales. Dr. Newman is collaborating with Michael Alexander (CDC) and Gabriel Lau (GFDL) on this project.

Dr. Sardeshmukh and Dr. Penland are collaborating with Gilbert Compo (CDC) on a NOAA-funded project involving the National Centers for Environmental Prediction’s Medium-Range Forecast Model. Using a distributional approach, they evaluate that model’s ability to predict extreme temperature and precipitation events during El Niño, La Niña and normal epochs.

PUBLICATIONS

The following are all refereed publications:

