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Statistical Methods for UXO Pattern Recognition

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**List of Acronyms**

CSR: complete spatial randomness  
DOD: U.S. Department of Defense  
ESTCP: Environmental Security Technology Certification Program  
UXO: unexploded ordnance
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Executive Summary

This report summarizes the findings of a statistical analysis of the locations of metallic anomalies detected at the Pueblo Precision Bombing Range Number 2 in Otero County, Colorado, and at the Victorville Precision Bombing Range in San Bernardino County, California. The purpose of the study was to explore whether statistical properties of the pattern of anomaly locations can be used to discriminate areas likely to contain unexploded ordnance (UXO) left over from previous bombing practice from those unlikely to contain UXO. Techniques for discriminating areas with and without UXO are needed because historic records have left an incomplete account of previous military training activities, so that locations historically used for target practice are often unknown. This study differs from previous research on metallic anomaly data at former military training ranges in that it analyzes the spatial pattern of the discrete locations of the anomalies, rather than the average number of anomalies per unit area. The results indicate that differences in spatial pattern may be a distinguishing feature between areas that were used for target practice and those that are unlikely to contain UXO, even when a large number of ferrous rocks and other inert metallic anomalies are present. We found that at both former bombing ranges, the anomaly patterns in sample areas that are distant from all known bombing targets are consistent with a complete spatial randomness (CSR) pattern, while those near the target areas fit a radially symmetric, bivariate Gaussian pattern. Furthermore, anomaly location patterns generated by surveys with airborne metal detectors have the same statistical properties as the patterns generated by surveys with on-ground detectors, even though the airborne systems detect only a subset of the anomalies found by the ground-based detectors. Thus, pattern information revealed by airborne surveys with metal detectors can be useful in identifying areas where careful searches for UXO are needed.
1 Introduction and Objectives

This report discusses statistical analyses of the spatial pattern of metallic anomalies, buried and on the ground surface, detected during airborne surveys above two former Air Force bombing ranges: the former Pueblo Precision Bombing Range Number 2 in Otero County, Colorado, and the Victorville Precision Bombing Range in San Bernardino County, California. The main purpose of the analyses is to determine whether statistical properties of anomaly spatial patterns can be used to delineate areas with and without unexploded ordnance (UXO). A second goal is to estimate the expected number of UXO at given locations within former military sites based on the results of airborne surveys. Whereas previous statistical characterization of metallic anomaly data at UXO sites has used geostatistical approaches that convert discrete anomaly locations to "concentrations" of UXO per unit area (McKenna et al. 2002; McKenna and Saito 2003; Saito et al. 2005a,b), this paper focuses on analytical methods that preserve the discrete nature of the pattern information that the data contain.

This research is part of the UXO Wide Area Assessment program run by the Department of Defense (DOD) Environmental Security Technology Certification Program (ESTCP). The goal of this program is to provide the DOD with the tools needed to fully characterize the amount of land in the United States that is contaminated with UXO. Due to the long time period during which the military has conducted live-fire training in some areas of the United States, combined with poor record-keeping over much of the DOD’s history, the amount of land contaminated with UXO and specific geographic boundaries of the contamination are highly uncertain (Defense Science Board 2003). The Defense Science Board estimated that some $40,500 \text{ km}^2$ (10 million acres) in the United States need to be surveyed for UXO, based on information on past military training, but that, once surveyed, only about 20% of that total land is likely to contain UXO (Defense Science Board 2003). The Wide Area Assessment Program goal is to provide technologies that can be used to survey large tracts of land for UXO in relatively short time periods and analytical methods that can be used to assess the resulting data for evidence as to which areas are likely to and unlikely to contain UXO.
This report considers whether areas without UXO have patterns of metallic anomaly locations that differ from the patterns in areas with UXO. It assesses whether statistical properties of these patterns can be used to distinguish areas likely to contain UXO from areas that are unlikely to contain UXO. The report first provides background information about wide-area surveys for UXO and what they reveal. It then discusses previous approaches to statistical analysis of the data from such surveys and how the approach in this paper differs from this previous work. Next, it provides an overview of the data sets used as the basis for this analysis. Then, it analyzes some important statistical properties of metallic anomaly locations in areas unlikely to contain UXO and compares them to areas around former bombing targets at both ranges. Last, it analyzes whether patterns revealed by airborne sensors are consistent with those observed on the ground.

2 Background: Wide-Area Surveys for UXO

Most UXO at former military training ranges is buried, due to either the force of the initial impact or to later soil deposition. Currently, crews charged with cleaning up UXO sites must rely on metal detectors to locate UXO. They traverse the ground equipped with either hand-held or cart-mounted detectors, with survey times limited by the walking speed of the operator, terrain, vegetation, and number of metallic anomalies present. As is well known in the UXO community, these metal detectors generally are unable to discriminate between buried UXO and other sources of metal, such as ferrous rocks, shrapnel, hubcaps, coins, soup cans, and other sources of cultural debris. Typically, anywhere from 10 to 1,000 metallic items are excavated for every UXO found, depending on the nature of the site (Defense Science Board 2003; MacDonald et al. 2004).

Surveying potentially contaminated land with hand-held or cart-mounted metal detectors is a slow process. Typically, at most a few acres can be searched per day (Defense Science Board 2003). Over the past several years, with the goal of speeding up the process, organizations funded by the DOD have developed airborne systems for metal
detection. Deployed on helicopters, these systems automatically record the geographic coordinates of the metallic signals they receive. These airborne systems can survey hundreds of acres per day. As an example, at the Pueblo range, the airborne system averaged 456 acres per day (Sky Research 2006).

A limitation of the airborne platforms is that the signal strength of metallic anomalies decreases at a rate of $1/distance^3$, so that the helicopters must fly very low to the ground (1-2 m) to locate anomalies (Tuley and Dieguez 2005; Sky Research 2006). Even when maintaining these very low flight altitudes, the airborne systems do not detect all of the anomalies found by on-ground detectors, which can be held within millimeters of the soil surface. Nonetheless, the DOD has invested in continuing to develop these airborne systems in the hopes that the data they produce will be useful for quickly identifying the areas most likely to contain UXO, thus narrowing the scope of the problem and limiting the amount of searching that must be done by ground crews.

The product of a survey for UXO, whether conducted via helicopter-mounted or on-ground detectors, is a map showing the $x,y$-coordinates of all metallic anomalies found. Virtually every land parcel contains some buried ferrous material (natural and/or anthropogenic). Thus, the detection of metallic anomalies, alone, does not necessarily indicate that UXO are present. In fact, the number of metallic anomalies can be very high in areas with natural deposits of ferrous rock. The challenge is to determine how UXO-containing areas that pose risks to future land users can be segregated from areas that contain inert sources of metal.

3 Previous Approaches to Statistical Analysis of Metallic Anomaly Data

The DOD has for many years used two tools--“SiteStats/GridStats” and “UXO Calculator”--to analyze the spatial distribution of metallic anomalies at potential UXO sites and guide site sampling (QuantiTech 1995; Fanning 1999; Engelhardt et al. 2000). Both are software packages that field crews use in real time, on lap-top computers, to estimate the density of UXO items (i.e., the number of items per unit of land area) based on metallic anomaly data. However, technical reviewers and the Environmental
Protection Agency (EPA) have criticized the statistical model that underlies these tools, and thus DOD has sought to develop improved methods (Fields 1999; Engelhardt et al. 2000). Recent research funded by the Strategic Environmental Research and Development Program has applied geostatistical kriging methods to the spatial modeling of anomalies at potential UXO sites (McKenna et al. 2002; McKenna and Saito 2003; Saito et al. 2005a,b).

3.1 SiteStats/GridStats and UXO Calculator: complete spatial randomness model

SiteStats/GridStats and UXO Calculator both assume that UXO are distributed in two dimensions according to a “complete spatial randomness” (CSR) model. For CSR to hold, the spatial arrangement of UXO must exhibit two properties (Cressie 1993; Diggle 2003):

1. The number of UXO items in a region with area $|A|$ must follow a Poisson distribution with mean $\lambda|A|$, where $\lambda$ represents the average number of UXO items per unit area. In other words, the probability that the number of UXO items in any area of size $|A|$ equals some integer $k$ is given by

\[
\Pr(N = k) = \frac{(\lambda |A|)^k e^{-\lambda |A|}}{k!} \quad (1)
\]

\[k = 0, 1, 2, 3, \ldots\]

2. The specific locations of UXO in planar region $A$ are an independent random sample from the uniform distribution on $A$.

These assumptions imply that (1) the average number of UXO items per unit area is constant across the site and (2) any given location on the site has an equal chance as any other of containing UXO.
3.2 Limitations of the CSR model

While CSR provides a realistic model of some spatial processes, its use for modeling UXO distributions at former training ranges is questionable (Engelhardt et al. 2000). Since soldiers aim their weapons at targets, it is logical to assume that UXO would form clusters around target areas. The CSR assumption, in contrast, implies that soldiers fire at random, without regard to target locations. Thus the CSR model cannot capture the physical reality that target areas will contain more UXO than areas that are distant from targets.

3.3 Geostatistical methods (kriging)

Recent research has sought to develop a stronger foundation for UXO spatial modeling by applying techniques from geostatistics (McKenna 2001; McKenna et al. 2003; McKenna and Saito 2003; Ostrouchov et al. 2003; Saito 2005a,b). Geostatistical methods were developed initially in the mining industry for estimating ore grades from observed samples and are now used in a variety of fields, from soil science to atmospheric science (Cressie 1993). Their primary utility is for predicting the value of a continuous variable (such as the concentration of a particular contaminant in groundwater or soil) from a limited number of environmental samples (Cressie 1993). These approaches make use of the spatial correlation in the variable of interest that may occur for locations that are in close proximity. For example, if the concentration of a groundwater contaminant is high in one location, then one would expect relatively high concentrations in nearby locations, rather than an immediate drop from high concentration to zero concentration. Estimates of the value of the variable of interest at an unsampled location are determined via kriging, in which values of the variable at sampled locations are assigned weights that reflect their spatial correlation to the variable’s value at the unsampled location.

A number of research projects have applied geostatistical methods to extrapolate metallic anomaly information from limited sampling at both simulated and actual UXO sites (McKenna 2001; McKenna et al. 2002; McKenna and Saito 2003; Saito et al.)
Because these methods are designed for analysis of continuous random variables, in order to apply them to UXO sites, the discrete points specifying potential UXO locations must be converted to a continuous random variable. Anomaly pattern information is thus converted prior to geostatistical analyses to “densities” of UXO per unit land area: the sample area is divided into cells of uniform size, the number of suspected UXO items in each cell is counted, and this number is divided by the cell area. Then, this density value is assigned to the spatial coordinates of the entire cell. The density is treated as if it varies continuously in space, so that kriging can be employed.

3.4 Limitations of kriging

The kriging approach is useful for predicting metallic anomaly density on unsampled parcels of land when only a limited amount of the land can be investigated (Diggle 1976, 2003), presuming that the sampling plan is sufficient to detect areas of high anomaly density. However, kriging has limitations as a method for analyzing wide-area survey data. The primary limitation is that the method results in a considerable loss of spatial information. This lost information could be critical in distinguishing UXO signal patterns from signal patterns due to other metal sources, such as metallic rocks. The geostatistical approach destroys pattern information because it requires the transformation of a discrete variable (the locations of potential UXO items) to a continuous variable (the anomaly density). Cressie, in his seminal work *Statistics for Spatial Data*, identifies this problem:

The reduction of complex point patterns to counts of the number of events in random quadrats and to one-dimensional indices results in a considerable loss of information. Only a single scale of pattern can be measured by a sample of random quadrats, and there is no consideration of . . . the relative positions of events within quadrats, so most of the spatial information is lost (Cressie 1993).
The information on locations of suspect items that is erased when geostatistical methods are employed may be valuable in interpreting spatial patterns that could provide evidence for the presence or absence of UXO.

4 Method

4.1 Spatial point pattern analysis

This research employs methods developed for the analysis of spatial point patterns. Diggle, in one of the definitive texts on the topic, defines a spatial point pattern as “a set of locations, irregularly distributed within a designated region and presumed to have been generated by some form of stochastic mechanism” (Diggle 2003). The locations of metallic anomalies detected by airborne sensors can be viewed as a spatial point pattern. In the case where anomalies are UXO at a former firing range, the stochastic mechanism that generated the pattern was the launching of ammunition at targets, which were not hit with perfect accuracy. Examples of other kinds of spatial point patterns include locations of trees in a forest, nests in a bird colony, nuclei in a microscopic tissue sample, and stars in the night sky; scientists studying all of these kinds of patterns have used statistical pattern analysis techniques to make predictions (Cressie 1993; Lawson and Densison 2002; Diggle 2003). Cressie notes that point-pattern analysis differs from geostatistics in that “point patterns arise when the important variable to be analyzed is the location of ‘events.’” He points out that “geostatistical-type problems are distinguished most clearly from . . . point-pattern-type problems by the ability of the spatial index . . . to vary continuously over a subset of \( \mathbb{R}^d \)” (Cressie 1993). Unlike kriging, spatial point-pattern analysis preserves all information about the locations of the events of interest.

In previous research, we used spatial point-pattern analysis to assess anomaly distribution patterns at two former Army ground artillery ranges: Fort Ord, California, and Tobyhanna State Park, Pennsylvania (MacDonald and Small 2006). We concluded that metallic anomaly locations at both ranges were consistent with a spatial point-
process model known as the Poisson cluster process. This model accounts for the clustering of objects of interest around cluster centers, whose locations may not be known in advance. Locations of firing targets changed frequently over the course of operation of these two ranges, and data on target locations were not maintained. Due to heavy vegetation at both sites and the long time periods over which they were used, target locations could not be determined from physical evidence. The Poisson cluster process model was able to provide statistically significant predictions of the patterns of UXO locations at these ranges, despite the fact that target locations were unknown, because it accounted for the random nature of target placement, as well as for clustering of UXO around the targets.

4.2 Data sources

The data analyzed in this paper are from surveys of the Pueblo Precision Bombing Range Number Two and the Victorville Precision Bombing Range. The data were gathered as part of an ongoing ESTCP demonstration of airborne metal detectors. The Pueblo range served as a U.S. Army Air Forces target practice site between 1942 and 1945 (U. S. Army Corps of Engineers 1995). Ammunition dropped on this range included 100-lb. bombs and practice bombs, 4-lb. incendiary bombs, and 75-mm cannon rounds (Sky Research 2006). The total area of the range is 272 km$^2$ (105 square miles). The Victorville range was used for bombing practice during the same time period as the Pueblo range. The historical record indicates that the primary (and perhaps only) munitions used were 100-lb. practice bombs (Hathaway et al. 2007). The total site acreage is approximately 22 km$^2$ (8.6 square miles).

A 30-km$^2$ land parcel at the Pueblo range has been designated for testing wide-area UXO detection technologies. Within this demonstration area, a swath of land of about 3 km x 9 km was surveyed with the airborne system. Portions of this area also were surveyed with an array of metal detectors towed on the ground with a motorized vehicle. Figure 1 shows all of the metallic anomalies detected in the airborne survey. Also shown are the locations of known bombing targets. The bombing target spatial
**Figure 1** Locations of metallic anomalies identified in airborne survey of the study area at Pueblo Precision Bombing Range Number Two, Otero County, Colorado. The crosses show locations of targets at which pilots aimed during bombing practice.
coordinates were provided by ESTCP, based on historical information and aerial photography (Sky Research 2006).

The Pueblo airborne data are the result of an 11-day survey conducted by Sky Research, Inc., in September 2005 (Sky Research 2006). The metal detection system employed in this survey consisted of seven Geometrics 822 cesium vapor magnetometers mounted on a 9-m Kevlar boom at the front of a helicopter. An on-board computer collected the metallic signal data as well as helicopter location coordinates in real time. After each flight, the data were processed to remove geologic background noise, estimate the spatial coordinates of the signals, and identify which anomalies should be investigated as possible UXO. Sky Research generated the list of anomalies of interest with an automated signal-processing routine (Sky Research 2006). We were provided with this post-processed data, consisting of \( x, y \) locations of the anomalies of interest as well as information about their estimated size, depth, orientation, and signal strength.

The Victorville airborne data also were gathered by Sky Research, Inc., using the same equipment as at the Pueblo site. Figure 2 shows the anomalies detected in the airborne surveys and the location of Target 15, which is the focus of this study. Sky Research used an automated signal-processing routine, as at the Pueblo site, to identify the locations of anomalies of interest based on the signals from the airborne detectors. The data set provided was similar to that provided for the Pueblo site.

5 Results

5.1 Spatial patterns of anomalies in areas with and without UXO

5.1.1 Pueblo Range

As can be seen from Figure 1, the area surveyed by helicopter at the Pueblo range included land that is more than 2 km on all sides from any known bombing targets. We chose a 1,000-m square of this land (labeled in Figure 1 as “study area”) as representative of acreage unlikely to contain UXO. Figure 3 shows a close-up of the anomaly locations
Figure 2 Locations of metallic anomalies identified in an airborne survey of the Victorville Precision Bombing Range, San Bernardino County, California.
Figure 3 Metallic anomalies detected in airborne survey of central area of the Pueblo range, more than 2 km from bombing targets.
in the study area. We compared the spatial point pattern of the anomalies in this area
with that of the anomalies surrounding the northern-most target (known as Target 3 and
labeled in Figure 1 as “north target study area”). Figure 4 shows a detail of the
anomalies surrounding this target. The purpose of comparing these to areas was to
evaluate the hypothesis that the metallic anomaly patterns differ in areas likely and not
likely to contain UXO from target practice. In theory, the anomalies in target areas
should form a pattern that clusters around the target, while the anomalies in areas distant
from targets should not be clustered. Although the anomaly density in Figure 3 is an
order of magnitude lower than that in Figure 4, one cannot conclude based on density
alone that this area is not associated with a former bombing target. It is possible, for
example, that the area could contain UXO from a target on which fewer bombs were
dropped than on Target 3. Likewise, it is not possible to determine based on visual
inspection alone whether or not the anomalies shown in either of these figures form a
pattern can be misleading.” Statistical analyses are needed to detect true cluster patterns
from random patterns that appear to have clusters.

The spatial point patterns of the two sample areas can be compared using the “$K$
function,” which is commonly used to summarize information about spatial point patterns
(Diggle 2003). The $K$ function is a property of a spatial point process that represents how
the spatial dependence among locations of events varies through space (Cressie 1993;
Kaluzny et al. 1998; Diggle 2003). It is defined for specified distance $h$ as

$$K(h) = \frac{\text{Expected number of extra events within distance } h \text{ of given event}}{\lambda}$$

1 The sample area distant from known targets has boundaries $616,100 \text{ m} \leq X \leq 617,100 \text{ m}$ and $4,172,500 \text{ m} \leq Y \leq 4,173,500 \text{ m}$ (in Universal Transverse Mercator system coordinates); the helicopter-mounted detectors
located 241 anomalies in this area. The sample area around the northern target (Target 3) has boundaries
$616,666 \text{ m} \leq X \leq 617,666 \text{ m}$ and $4,177,077 \text{ m} \leq Y \leq 4,178,077 \text{ m}$; the airborne detectors found 3,227 anomalies
in this area.
Figure 4 Metallic anomalies located during airborne survey of the area surrounding Pueblo Target 3, the northernmost target in Figure 1.
where $\lambda$ is the intensity, which is defined as the average number of events (in this case, metallic anomalies) per unit area. For the CSR model, the $K$ function is given by

$$\hat{K}(h) = \frac{\pi h^2 \times \lambda}{\lambda} = \pi h^2$$

(3)

That is, the expected number of extra events increases linearly with the area of the circular region of radius $h$ around a particular event. $K(h)$ is expected to be larger than for the CSR model (equation 3) when events are clustered and smaller for regularly spaced events.

A common method of evaluating whether a given type of parametric model fits a spatial point pattern is to estimate $K(h)$ for the data set and compare it with $K(h)$ for the model (Cressie 1993; Kaluzny et al. 1998; Diggle 2003). We compared the anomaly patterns in the two sample areas to the CSR model using $K(h)$, estimating $K(h)$ from the anomaly location data as described in Kaluzny et al. (1998). Figures 5 and 6 show the results. The figures show the mean and upper and lower 98% confidence intervals for $K(h)$ for the CSR model and $K(h)$ estimated from the data for the two sample areas. Figure 5 shows that the CSR model of $K(h)$ provides a virtual perfect fit to the data for the area unlikely to contain UXO. In contrast, Figure 6 shows that the CSR model of $K(h)$ does not fit the estimated $K(h)$ for the area around the bombing target. In this latter sample area, the values of $K(h)$ for the data are consistently above $K(h)$ for the CSR model, showing a larger number of points around any given data point than would be expected under CSR and thus suggesting clustering.

We also compared $K$-function values for the Pueblo target area anomalies with those for a radially symmetric, bivariate Gaussian model having the target location (617,166 m; 4,177,577 m) as its mean and a standard deviation of 250 m.² Our previous analyses of data from the two Army artillery ranges demonstrated that a point-process model that assumes UXO items are distributed around targets as a radially symmetric Gaussian distribution fit the data (MacDonald and Small 2006). Analysis of the Pueblo anomalies surrounding it.

² The 250-m standard deviation was estimated from the mean distances from the northern target to the anomalies surrounding it.
Figure 5 Evaluation of fit of the CSR model to the anomaly data in Figure 3 using the function $K(h)$. As shown, the estimated $K(h)$ for the anomaly data is within the 98% confidence interval for the CSR model.
Figure 6 Evaluation of fit of the CSR model to the anomaly data around Pueblo Target 3 (Figure 4) using the function $K(h)$. As shown, the estimated $K(h)$ for the anomaly data is above the 98% confidence interval for the CSR model, indicating that the anomalies are clustered.
anomaly data indicates that this assumption also appears reasonable for the target area data. Figure 7 shows quantile-quantile probability plots comparing the $X$ and $Y$ coordinates of the anomalies around the target area with the standard normal distribution. The linearity of the plots shows that the marginal distributions of $X$ and $Y$ are consistent with a normal distribution. Further, the correlation between $X$ and $Y$ is statistically not significantly different from 0 at the 95% confidence level; the estimated correlation coefficient between $X$ and $Y$ is 0.03, with 95% confidence interval (0.00, 0.06). It is thus reasonable to assume that a Gaussian model with zero correlation will be representative of the data.

The $K$ function for a radially symmetric Gaussian model with the target center as its mean and a standard deviation of 250 m matches the $K$ function for the data from the target area. Figure 8 shows this result: the 98% confidence interval for the $K$ function for the Gaussian distribution, mean value for this $K$ function, and empirical $K$ function for the data are nearly indistinguishable. The bivariate Gaussian model in this case has probability density function

$$f(x, y) = \frac{1}{2\pi(250)^2} \exp \left[ -\frac{1}{2} \left( \frac{(x - 617,166)^2 + (y - 4,177,577)^2}{(250)^2} \right) \right]$$  \hspace{1cm} (4)$$

In this case, spatial point-pattern analysis reveals distinctly different spatial characteristics of anomaly locations in the area distant from all known targets, compared to the anomaly locations in a known target area. The anomaly pattern in areas distant from targets follows the CSR model, while the pattern around the target is consistent with a bivariate, radially symmetric Gaussian model. This provides evidence that spatial point-pattern analysis may be helpful in screening areas free of UXO from known target areas.

5.1.2 Victorville range

The results from Victorville lend further evidence to the hypothesis that anomaly patterns are distinctly different in areas with and without UXO. The Victorville range
Figure 7  Normal-based quantile-quantile plots of the $x$- and $y$-coordinates of anomaly locations in the target areas at the Pueblo and Victorville ranges. The linearity of each plot suggests that both the $x$- and $y$-coordinates at both ranges are normally distributed.
**Figure 8** Evaluation of the fit of a radially symmetric Gaussian model to the anomaly locations in the sample area around Pueblo Target 3 using the $K$-function. As shown, $K(h)$ estimated for the data follows the expected $K(h)$ for the Gaussian model. The narrow confidence interval for the Gaussian model is due to the large number of points in the sample area around Target 3 (see Figure 4).
contains a number of areas with high anomaly density that are not known to be associated with former bombing targets. Our analysis focused on three of these areas (shown on Figure 2), and on the anomalies around Target 15 (Figure 2). In each case, we selected a square sample area of 1000 m on a side. We conducted the same type of analysis as for the Pueblo data to determine whether or not the anomaly patterns in these areas are consistent with a CSR distribution. Figures 9-11 show that the patterns in the three areas not associated with the target fit a CSR model, even though these areas contain high anomaly densities. Figure 12 shows that the pattern around the target is not consistent with a CSR pattern. The empirical $K$-function values are consistently above the 98% confidence interval for a CSR model up through distances of about 600 m, providing evidence of clustering.

We fitted the anomaly locations around Target 15 to a bivariate Gaussian model using the same process as for the Pueblo data. As for the Pueblo data, the $X$ and $Y$ coordinates appear on a quantile-quantile plot (Figure 7) to be normally distributed. The correlation between $X$ and $Y$ is -0.04, with 95% confidence interval (-0.09, 0.006), and the standard deviations of both $X$ and $Y$ are 147 m, indicating that a radially symmetric Gaussian model is consistent with the anomaly locations around the target. Figure 13 compares the empirical $K$-function for the anomalies in the target area to the theoretical $K$-function for a radially symmetric Gaussian model with the target as its mean and a standard deviation of 147 m. As shown, this model fits the data well, providing further evidence that the anomaly pattern around the target is a cluster pattern centered at the target.

5.2 Predicting ground-based survey results from airborne data

Due to the drop-off of magnetic signal strength with distance, airborne magnetometers detect fewer anomalies than ground-based systems. This raises two questions. First, are the patterns revealed by airborne surveys consistent with those produced by ground surveys? Second, how can the number of anomalies in the survey area be estimated, given the limitations in detection ability of the airborne system?
**Figure 9** Evaluation of fit of the CSR model to anomaly data from Area 1 of the Victorville range using the function $K(h)$. As shown, the empirical $K(h)$ for the anomaly data is within the 98% confidence interval for the CSR model.

**Figure 10** Evaluation of fit of the CSR model to anomaly data from Area 2 of the Victorville range using the function $K(h)$. As shown, the empirical $K(h)$ for the anomaly data is within the 98% confidence interval for the CSR model.
Figure 11 Evaluation of fit of the CSR model to anomaly data from Area 3 of the Victorville range using the function $K(h)$. The empirical $K(h)$ for the anomaly data is within the 98% confidence interval for the CSR model.

Figure 12 Evaluation of fit of the CSR model to anomaly data from Target 15 of the Victorville range using the function $K(h)$. As shown, the empirical $K(h)$ for the anomaly data is above the 98% confidence interval for the CSR model for distances up to about 600 m, indicating that the anomalies are clustered.
Figure 13  Evaluation of fit of a radially symmetric Gaussian model to anomaly data from Target 15 of the Victorville range using the function $K(h)$. As shown, the empirical $K(h)$ for the anomaly data is within the 98% confidence interval for the Gaussian model.
5.2.1 Airborne versus ground-based anomaly patterns

To assess whether the anomaly patterns are similar for the data sets from the airborne and ground-based surveys, we compared the $K$-function values for the data sets in areas of the Pueblo site that were surveyed with both types of sensor platforms. Figure 14 shows the results of such an analysis (for a sample area designated as 3C in the data set): it plots $K(h)$ for the ground-based data against $K(h)$ for the airborne data. The result is a straight line with slope 1.05. This provides evidence that the airborne anomalies represent a thinned subset of the spatial point-process that generated the ground-based anomalies. The results for the other three areas that were surveyed with both airborne and on-ground detectors were similar. The implication is that the spatial pattern revealed by the airborne survey is the same as that revealed by a ground-based survey. Thus, any inferences made about the spatial point pattern observed from airborne metal detectors also should apply to the anomaly set observed by ground-based detectors, with the air-generated pattern representing a thinned subset of the ground-generated pattern.

5.2.2 Airborne versus ground-based estimates of expected number of UXO

Since the anomaly data from airborne surveys provide only a partial representation of the set of anomalies located by ground-based surveys, an important question is how many anomalies the airborne surveys miss. If reasonable estimates for the probability that an airborne system will detect anomalies resembling UXO can be made, then airborne survey results can be converted directly to estimates of the expected number of anomalies in any given region. This can help DOD to further define the scope of the UXO problem and develop more accurate estimates of remediation costs based on airborne wide-area surveys, before ground surveys are completed.

We used the data from all four of the areas that were fully surveyed by both ground-based and airborne magnetometers at the Pueblo range to assess the probability that the airborne detectors will find anomalies located by ground-based detectors. Table 1 and Figure 15 summarize probabilities of detection for the four sample areas for different detection radii. We define the detection radius as the distance from a given
Figure 14 $K(h)$ for the ground-based anomalies in Pueblo Area 3C compared to $K(h)$ for the anomalies identified by the airborne system. As shown, the $K(h)$ values are nearly equivalent. This provides evidence that the two anomaly patterns are drawn from the same type of spatial point-pattern distribution. The anomalies identified by the airborne system thus may represent a thinned version of the spatial point-process of the anomalies identified by the ground-based system.
Table 1  Effect of Search Radius on Probability of Detection

<table>
<thead>
<tr>
<th>Search Radius (m)</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area 1A</td>
<td>0.077</td>
<td>0.189</td>
<td>0.272</td>
<td>0.302</td>
<td>0.320</td>
<td>0.331</td>
<td>0.349</td>
<td>0.355</td>
</tr>
<tr>
<td>Area 3B</td>
<td>0.073</td>
<td>0.190</td>
<td>0.242</td>
<td>0.265</td>
<td>0.283</td>
<td>0.300</td>
<td>0.317</td>
<td>0.335</td>
</tr>
<tr>
<td>Area 3C</td>
<td>0.072</td>
<td>0.174</td>
<td>0.222</td>
<td>0.266</td>
<td>0.275</td>
<td>0.285</td>
<td>0.295</td>
<td>0.319</td>
</tr>
<tr>
<td>Simmons</td>
<td>0.111</td>
<td>0.208</td>
<td>0.250</td>
<td>0.292</td>
<td>0.319</td>
<td>0.319</td>
<td>0.319</td>
<td>0.319</td>
</tr>
</tbody>
</table>
Figure 15 Probability that the airborne magnetometers will locate anomalies found by the ground-based magnetometers as a function of detection radius. Detection radius is the distance around an anomaly found by the ground-based magnetometer within which the airborne detector must generate a signal for the anomaly to have been counted as a detected item.
ground anomaly to the corresponding airborne signal. If the airborne sensors locate an anomaly within the given detection radius of an anomaly found in on-ground surveys, then we credit the airborne sensors as having successfully detected that anomaly. As can be seen in Table 1 and Figure 15, the detection probability increases with detection radius. The maximum detection probability is 35.5%, at the very large search radius of 4 m. For a more realistic detection radius of 1.5 m, the probability of detection is 22-27%. As a point of comparison, an analysis in 2005 of anomaly data collected at a former practice bombing range used by Kirtland Air Force Base found that, with a detection radius of 1.5 m, a helicopter-mounted detection system found fewer than 25% of 60-mm mortars and 75% of 105-mm rounds; on average, this system found 56% of UXOs emplaced in a controlled field test (Tuley and Dieguez 2005). The lower detection probabilities observed in our research may be a result of differences in site conditions, such as amount of geologic interference, or in differences in the algorithms used to identify which signals generated by the airborne system represented anomalies worthy of investigating. In the Kirtland Air Force Base study, geophysicists hand-picked anomalies for investigation based on the signal information, while in the data we used, anomalies lists were generated by a computer search algorithm.

6 Conclusions

Analysis of metallic anomalies detected in airborne and on-ground surveys of the Pueblo and Victorville bombing ranges suggest the following conclusions:

- Differences in the spatial point patterns of metallic anomalies may be a distinguishing feature between areas that were used for weapons training and areas not used for such training. At both bombing ranges, anomalies in sample areas that are distant from all known bombing targets are completely spatially randomly distributed, while those near the targets do not fit this CSR model.
• A symmetric, bivariate Gaussian model appears to represent the anomaly pattern around targets at both the Victorville and Pueblo ranges. This model may be useful in identifying how far from the target searches for UXO should be conducted. As an example, 95% of the anomalies are expected to be within two standard deviations of the target center, while 99.7% are expected to lie within three standard deviations of the target center.

• Based on analyses of the Pueblo data, the airborne anomaly locations appear to be drawn from a spatial point process that is a thinned version of the process that generated the anomalies detected by the ground-based system. In other words, the airborne system is able to reveal the same type of pattern information as the ground-based system.

• The probability that helicopter-mounted metal detectors will locate anomalies detected by ground-based systems depends on how close the airborne anomaly must be to an anomaly found by the ground-based system to be declared a successful detection. For a detection radius of 1.5 m, the probability of detection of the areas surveyed in the Pueblo site ranged from 22-27%.

As the DOD moves forward with development of airborne systems for UXO detection, the kinds of spatial point-pattern analysis demonstrated in this report can help to document the geographic boundaries of areas likely to contain UXO. Where the spatial pattern of anomalies is not consistent with that expected at UXO sites, statistical analysis of the pattern information can provide evidence that the area is not a high-priority for exhaustive ground-based searches for UXO. Where the pattern analysis suggests that UXO is present, pattern information can be used to predict the boundaries of former target areas and thus reduce the amount of acreage that must be searched.

References


Appendix. List of Technical Publications

