(DARPA) TOPOLOGICALLY PROTECTED QUANTUM INFORMATION PROCESSING IN SPIN- ORBIT COMPLETED SEMICONDUCTORS

SUMANTA TEWARI

CLEMSON UNIVERSITY

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We proposed and studied in detail the non-Abelian topological order and Majorana fermions (MFs) in a heterostructure consisting of both electron- and hole-doped spin-orbit coupled semiconductors (e.g., InAs, InSb) and an s-wave superconductor (Nb, Al) in the presence of a suitably directed Zeeman field in both 2 and 1 dimensions. The experiments we proposed to reveal the MFs, zero bias conductance peak in local tunneling experiments and the fractional Josephson effect, have now been successfully carried out by a number of groups worldwide. We have analyzed the experimental results in detail, explaining the absence of the gap closing signatures before the emergence of the MFs in the electron doped nanowires, and deducing alternative scenarios for the emergence of the zero bias peaks so as to help rule out alternative interpretations of the experimental results. After an extensive set of analysis and publications, our current outlook is that although the positive experimental results are extremely encouraging, one cannot rule out the alternative interpretations for the experiments. This necessitated proposing a true smoking-gun signature of MFs in semiconductor nanowires which we also accomplished in terms of a charge tunneling interference experiment.
Executive Summary

Because of their special particle exchange statistics, the emergence of non-Abelian topological orders in any experimental system would in itself be an extraordinary phenomenon. Their potential use in fault-tolerant topological quantum computation (TQC) makes their realization in controllable experimental systems even more significant from a long term technological point of view. The broad objective of our proposal was to design and characterize new and realistic platforms using spin-orbit coupled semiconductors on which the basic elements of non-Abelian topological order and a workable TQC architecture could be externally induced. In the three year grant period, we have made the following progress:

1) We proposed and studied in detail the non-Abelian topological order and Majorana fermions (MFs) in heterostructures consisting of electron- or hole-doped spin-orbit coupled semiconductors (e.g., InAs, InSb) and an s-wave superconductor (Nb, Al) in the presence of a suitably directed Zeeman field in both 2 and 1 dimensions. The experiments we proposed to reveal the MFs - zero bias conductance peak in local tunneling and charge transport experiments and the fractional AC Josephson effect - have now been successfully carried out by a number of groups worldwide (TU-Delft, Netherlands; Lund, Sweden; Weizmann Institute, Israel; Harvard, USA; and UIUC, USA).

2) We analyzed the zero bias tunneling peak experiments performed in our proposed semiconductor heterostructures in detail, explaining the absence of the gap closing signatures
before the emergence of the MFs in the electron doped nanowires, and deducing alternative scenarios for the emergence of the zero bias peaks so as to help rule out alternative interpretations of the zero bias peak using non-Majorana physics. Towards this goal, we proposed ways to disentangle the conventional zero bias peaks in semiconductor heterostructures resulting from impurities and boundary effects from the topological zero bias peaks expected from Majorana fermions. This body of work established that although the recent positive zero bias peak experiments indicating the emergence of MFs in semiconductor heterostructures are extremely encouraging, more work is necessary to conclusively identify them with MFs.

3) We proposed a true smoking-gun signature of MFs in spin-orbit coupled semiconductor nanowires in terms of a charge tunneling quantum interference experiment. Although a zero bias conductance peak and a fractional ac-Josephson effect are necessary experimental signatures of MFs, neither of them constitutes a sufficient smoking gun experiment. Since one pair of MFs share a single conventional fermionic degree of freedom, MFs are in a sense fractionalized excitations. The fractionalization leads to an inherent quantum non-locality, using which we devised and proposed a nearly unique signature of MF end states in semiconductor quantum wires allowing one to distinguish MFs from accidental conventional zero energy modes induced by disorder or boundary effects.

In addition we studied the fundamental robustness of topologically protected quantum coherence against disorder effects in the considered solid state heterostructures by considering (in self-consistent Born approximation) the interplay of broken time-reversal symmetry and disorder on electron and hole doped semiconductor-superconductor heterostructures. For understanding the topological robustness of MFs in semiconductor heterostructures we deduced the minimum gap above the MF excitations (so-called minigap) for various values of the chemical potential and Zeeman field. Towards developing a topological quantum computation architecture we studied the techniques for braiding and fusion of MFs through T and Y junctions in semiconductor wire networks to implement universal topological quantum computation. Among alternative platforms for MFs and TQC we proposed carbon nanotubes, organic superconductors, and Li_{0.9}Mo_{0.1}O_{17} as robust platforms for realizing MFs. For fundamental theoretical understanding of topological superconducting states and MFs we studied the topological invariants of semiconductor-superconductor heterostructures and discovered a hidden chiral symmetry allowing the realization of multiple topologically protected MFs localized at the same end. We also deduced the conditions necessary for realizing topological superconducting states and MFs in non-centrosymmetric superconductors and a class of organic superconductors called Bechgaard salts.

Supported by this grant, we have published 36 papers in peer-reviewed journals, including 8 in Physical Review Letters, one topic review in J. Phys. B. Some of our papers have been selected as Editor Choices in Phys. Rev. Journals.

II Detailed Research Summary

In the three year grant period, we have focused on the possibility of topological quantum computation using MF excitations on spin-orbit coupled electron and hole doped semiconductors. The following are our principal theoretical results which helped reveal the first experimental signatures of MFs in condensed matter systems.
1) *Devising a practical way to externally induce non-Abelian topological order in a solid state heterostructure*: We proposed and studied in detail the non-Abelian topological order and Majorana fermions in heterostructures consisting of electron-doped semiconductors and a s-wave superconductor in the presence of a suitably directed Zeeman field in both 2 and 1 dimension. The experiments we proposed, zero bias conductance peak in local tunneling and global charge transport experiments, as well as the fractional AC Josephson effect, to reveal the MFs, have now been successfully carried out by a number of groups worldwide. This is our principle achievement in this project and has ushered in a new era in the search for MFs in condensed matter systems by proposing experimentally realistic platforms able to support MFs and topological quantum computation. The transformative idea proposed (and claimed to be realized in a number of recent landmark experiments) is that ordinary garden-variety experimental effects such as spin-orbit coupling, Zeeman field, and s-wave superconductivity can conspire to produce exotic topologically ordered superconducting states supporting Majorana fermions, with a fundamentally new (non-Abelian) quantum statistics and potential to support to topological quantum computation. (“Non-Abelian quantum order in spin-orbit-coupled semiconductors: Search for topological Majorana particles in solid-state systems”, Phys. Rev. B 82, 214509, 2010; Chunlei Qu, Yongping Zhang, Li Mao, Chuanwei Zhang, Signature of Majorana Fermions in Charge Transport in Semiconductor Nanowires, arXiv:1109.4108;)

2) *Quantitative understanding of the fundamental robustness of topologically protected quantum coherence against disorder effects in the considered solid state heterostructure*: By considering (in self-consistent Born approximation) the interplay of broken time-reversal symmetry and disorder on heterostructures of electron and hole doped semiconductors (InAs, InSb etc) and s-wave superconductors (Nb, Al), we derived an expression for the disorder suppression of the superconducting quasiparticle gap in the topological superconducting states carrying MFs. Our principle conclusions guiding the experimental efforts are that for a robust topological state with well localized MFs we need (1) strong spin-orbit coupling, (2) high effective mass, (3) low Zeeman field and (4) reasonable band filing (within the topological regime). (“Experimental and materials considerations for the topological superconducting state in electron- and hole-doped semiconductors: Searching for non-Abelian Majorana modes in 1D nanowires and 2D heterostructures”, Phys. Rev. B 85, 064512 (2012); Li Mao, Chuanwei Zhang, Robustness of Majorana Modes and Minigaps in a Spin-Orbit-Coupled Semiconductor-Superconductor Heterostructure, Phys. Rev. B 82, 174506 (2010).)

3) *Devising realistic methods to experimentally create the topological qubits and implement the appropriate braiding, fusion, and read-out operations (TQC architecture) for scalable, universal TQC in the proposed solid state heterostructure*: We have studied the techniques for braiding and fusion of MFs through T and Y junctions in semiconductor wire networks to implement universal topological quantum computation. Our principle result is that the semiconductor quantum wire network allows us to create two-qubit entanglement in a topologically protected manner (without requiring error correction). Moreover, all the single qubit
gates except one (the $\pi/8$ phase gate) can also be implemented topologically, even though the $(\pi/8)$ gate needs some minimal error correction. The implementation of TQC using the MFs and the semiconductor wire network is then potentially feasible once the non-Abelian particles are experimentally realized. With the recent experiments claiming the observation of MFs in semiconductor heterostructures the prospect of TQC in spin-orbit coupled semiconductor platforms appears to be bright. (“Controlling non-Abelian statistics of Majorana fermions in semiconductor nanowires”, Phys. Rev. B 84, 094505 (2011); “Majorana fermion exchange in quasi-one-dimensional networks”, Phys. Rev. B 84, 035120 (2011); “Universal quantum computation on a semiconductor quantum wire network”, Phys. Rev. A 82, 052322 (2010))

4) **Experimental signatures of Majorana fermions in semiconductors: Zero bias peak:**
We carried out detailed analytical and numerical studies of experimental signatures of MF modes in semiconductor nanowires. Our studies consisted of both proposing and predicting new experiments as signatures of MFs and analyzing the existing experiments for confirming the realization of MFs. Our principle conclusions are that although the recent experiments conform to all the selection rules expected for the zero bias peaks from Majorana fermions (and thus are extremely encouraging for the potential realization of TQC), there are also alternative explanations of the experiments in terms of conventional zero bias peaks produced from, say, impurity effect and/or the effects due to the wire boundaries. For conclusive demonstration of the existence of MFs, therefore, one needs experiments exploiting the MF statistics, requiring quantum interference experiments as in the fractional quantum Hall effect. (“To close or not to close: the fate of the superconducting gap across the topological quantum phase transition in Majorana-carrying semiconductor nanowires”, Phys. Rev. Lett. 109, 266402 (2012); “Probing a topological quantum critical point in semiconductor-superconductor heterostructures”, Phys. Rev. B 85, 155302 (2012); “Topologically trivial zero bias conductance peak in semiconductor Majorana wires from boundary effects ”,Phys. Rev. B 88, 020502 (Rapid) (2013); “Non-locality in zero bias anomaly in the topologically trivial phase of Majorana wires”, arXiv: 1310.4175 (PRL, under review); “Disentangling Majorana fermions from conventional zero energy states in semiconductor quantum wires”, Phys. Rev. B (Rapid) 87, 140504 (2013)

5) **Unambiguous determination of Majorana fermions: Charge tunneling interference in semiconductor nanowires:**
Using the fractionalization property of MFs we proposed a true smoking-gun signature of MFs using charge tunneling interference experiments in semiconductor nanowires. We devised a non-local correlation experiment in terms of tunneling which is able to provide a direct verification of the MFs with no remaining ambiguities as in the zero bias conductance peak and the fractional Josephson effect experiments. The physical picture of the experiment is such that an external superconducting electrode is connected to the two ends $a$ and $b$ of the semiconductor wire through tunnel barriers. In the presence of finite Majorana-assisted electron transfer amplitude (derived from the fractionalization property of the MFs) the quasiparticle tunneling from $a$ to $b$ through the
superconducting electrode can complete the circuit by tunneling back from $b$ to $a$ through the topological nanowire. The Berry phase associated with such a tunneling around the loop is sensitive to the flux $\Phi$ through the loop with a periodicity $2\Phi_0 = h\Phi/e$. Therefore, similar to the Aharonov-Bohm effect in mesoscopic rings, the energy-level of such a quasiparticle excitation spectrum $\epsilon(\Phi)$ in the ring is expected to develop a $2\Phi_0$ periodic dependence on $\Phi$ for systems where there is a finite electron transfer amplitude via MFs (and only a $\Phi_0$ periodicity in the absence of the MFs). This is a true smoking gun experiments for MFs which cannot be mimicked by accidental zero energy modes such as those that can be induced by disorder effects. We are happy to note that already several experimental groups are beginning to perform this experiment for a conclusive demonstration of MFs in semiconductor quantum wires. (“A proposal to probe quantum non-locality of Majorana fermions in tunneling experiments”, arXiv: 1210.5514 (PRL, under review))

6) **Topological invariants and chirality symmetry of semiconductor Majorana wires:** We investigated the possibility of realizing multiple MF modes in semiconductor nanowires with spin-orbit coupling and Zeeman fields and showed that this was possible because the system followed an additional symmetry, called chirality symmetry, in addition to the particle-hole (PH) symmetry (“Topological invariants of spin-orbit coupled superconductor nanowires” Phys. Rev. Lett. 109, 150408 (2012). In this work we also established an underlying connection between the PH symmetric BdG Hamiltonians (in class D) and chiral Hamiltonians (in class BDI), which has since been profitably applied (by the PI and collaborators) in the evaluation of topological mini-gaps in the Rashba-coupled semiconductor-superconductor heterostructures (the so-called “Semiconductor Majorana Wire”) (“Topological minigap in quasi-one-dimensional spin-orbit-coupled semiconductor Majorana wires”, Phys. Rev. B 86, 024504 (2012)), and by others, in studying the effects of inter-tube tunneling in the topological superfluid phases of Rashba-coupled quasi-1D Fermi gases and, more recently, in the prediction of a new class of 3D topological superconducting systems protected by the magnetic group symmetries.

7) **Hole-doped nanowires as a platform for MFs:** One disadvantage of using electron-doped semiconductors as the platform for TQC is the low electron density, owing to their small effective mass and spin-orbit coupling strength. In contrast, hole-doped semiconductors have larger effective mass and spin-orbit coupling, leading to larger hole densities. We investigated the heterostructure composed of a hole-doped semiconductor thin film, an s-wave superconductor and a magnetic insulator and showed that a novel topological order, a chiral $f + i f$-wave superconducting pairing, can be induced in this heterostructure. We found that there exists a Majorana zero energy state in the vortex core of the semiconductor-superconductor heterostructure in some parameter regions. Furthermore, we showed that MFs may also be realized in the hole-doped semiconductor nanowire. We found that the required parameters for observing the MFs are within current experimentally feasible region. Thus this system can be a potential breakthrough facilitating solid-state demonstration of MFs which are more robust against disorder (Li Mao, Junren Shi, Qian Niu, Chuanwei

III Management Summary

One graduate student (Eugene Dumitrescu, partial support) and one post-doctoral scholar (Kangjun Seo) were supported by this grant at Clemson University. PI Tewari has supervised them on a daily basis to accomplish the projects. The PI’s summer salary support enabled him to mentor three undergraduate students, Benjamin Schroeder, Brenden Roberts, and Aaron Allen in senior thesis work on TQC. With the travel money the PI has given invited talks to APS March Meeting, 2011 and APS DAMOP meeting, 2011.

One postdoc (Li Mao) was supported by this grant at WSU and UTDallas. Co-PI Zhang has supervised him on a daily basis to accomplish the projects.

IV Invited Talks

By Sumanta Tewari


(5) “Topological quantum criticality in spin-orbit coupled fermions”, American Physical Society, Division of Atomic, Molecular, and Optical Physics (DAMOP), Annual Meeting (2012), Annaheim, CA, USA.

(6) “Majorana fermions and TQC using semiconductor nanowires and Carbon nanotubes”, DARPA QuEST Meeting (2012), Charleston, SC, USA

(7) “Topological superconducting states and protected qubit manipulations” – American Physical Society (APS), March Meeting (2012), Boston, MA, USA.
(8) “Majorana fermions and topological quantum computing using semiconductor-supercollider heterostructures”, - Tata Institute of Fundamental Research, Mumbai, India (December, 2011)

By Chuanwei Zhang

(9) Search for Majorana fermions in spin-orbit coupled degenerate Fermi gases, American Physical Society March Meeting invited talk, March 2013, Baltimore, Maryland.
(10) Search for Majorana fermions in spin-orbit coupled superconductors and superfluids, Condensed Matter seminar, Department of Physics, Purdue University, March 2013 (Forthcoming), West Lafayette, IN.
(11) Search for Majorana fermions in spin-orbit coupled superconductors and superfluids, Physics Colloquium, Department of Physics, University of Washington, October 2012, Seattle, Washington.
(12) Topological Superfluids in Spin-Orbit Coupled Cold Fermi Gases: a Roadmap to Majorana Fermions, AMO Seminar, Department of Physics, Rice University September 2012, Houston, Texas
(13) Topological Superfluids in Spin-Orbit Coupled Cold Fermi Gases: a Roadmap to Majorana Fermions, Physics Colloquium, Temple University, March 2012, Philadelphia, PA
(14) Topological Quantum Materials at Nanoscale: a Roadmap to Majorana Fermions, Physics Colloquium, The University of Texas at Dallas, February 2012, Richardson, TX
(15) Spin-Orbit Coupled Bose-Einstein Condensates and Degenerate Fermi Gases, Physics Colloquium, Department of Physics, Indiana University Purdue University Indianapolis, October 2011, Indianapolis, Indiana
(16) Search for Majorana Fermions in p-type Semiconductor Thin Films and Nanowires, Condensed Matter Seminar, Center for Advanced Study, Tshinghua University, July 2011, Beijing, China.
(17) Search for Majorana Fermions in p-type Semiconductor Thin Films and Nanowires, Condensed Matter Seminar, Institute of Physics, Chinese Academy of Science, July 2011, Beijing, China.
(18) Search for Majorana Fermions in p-type Semiconductor Thin Films and Nanowires, Colloquium, Institute of Computational Physics and Applied Mathematics, June 2011, Beijing, China.
(19) Search for Majorana Fermions in p-type Semiconductor Thin Films and Nanowires, Colloquium, Institute of Quantum Information, Tshinghua University, June 2011, Beijing, China.
(20) Search for Majorana Fermions in p-type Semiconductor Thin Films and Nanowires, Condensed Matter Seminar, International Center for Quantum Materials, Beijing University, June 2011, Beijing, China.
(22) Search for Majorana Fermions in p-type Semiconductor Thin Films and Nanowires, Condensed Matter Seminar, Department of Physics, the University of Washington, May 2011, Seattle, Washington.
1) K. Seo, Chuanwei Zhang, Sumanta Tewari, “Topological uniform superfluid and FFLO phases in 3D to 1D crossover of spin-orbit coupled Fermi gases”,


2) D. Roy, N. Bondyopadhaya, Sumanta Tewari, “Topologically trivial zero bias conductance peak in semiconductor Majorana wires from boundary effects”,


3) T. D. Stanescu, Sumanta Tewari, “Majorana fermions in semiconductor nanowires: Fundamentals, modeling, and experiment”,


4) K. Seo, Chuanwei Zhang, Sumanta Tewari, “Thermodynamic signatures for topological phase transitions to Majorana and Weyl superfluids in ultracold fermionic gases”,


5) P. Goswami, Sumanta Tewari, “Axion field theory and anomalous non-dissipative transport properties of (3+1)-dimensional Weyl semi-metals and Lorentz violating spinor electrodynamics”,

   PRB (in press; selected as “Editor’s Choice”), arXiv: 1210.6352


7) Sumanta Tewari, J. D. Sau, “Topological invariants for spin-orbit coupled semiconductor nanowires”,


8) E. Dumitrescu, Chuanwei Zhang, D. C. Marinescu, Sumanta Tewari, “Topological thermoelectric effects in spin-orbit coupled electron and hole doped semiconductors”,


9) T. D. Stanescu, Sumanta Tewari, “Disentangling Majorana fermions from conventional zero energy states in semiconductor quantum wires”,

   Phys. Rev. B (Rapid) 87, 140504 (2013)

10) J. D. Sau, I. Mandal, Sumanta Tewari, S. Chakravarty, “Collective modes of the d-density wave state and its relevance to high-Tc cuprates”,


11) T. D. Stanescu, Sumanta Tewari, J. D. Sau, S. Das Sarma, “To close or not to close: the fate of the superconducting gap across the topological quantum phase transition in Majorana-carrying semiconductor nanowires”,


12) M. Gong, L. Mao, Sumanta Tewari, Chuanwei Zhang, “Majorana fermions under stress”,

    Phys. Rev. B (Rapid) 87, 060502 (2013)

13) J. D. Sau, Sumanta Tewari, “Majorana fermions in carbon nanotubes”,
14) J. D. Sau, Sumanta Tewari, S. Das Sarma, “Experimental and materials considerations for the topological superconducting state in electron and hole doped semiconductors: Searching for non-Abelian Majorana modes in 1D nanowires and 2D heterostructures”,


15) J. D. Sau, Sumanta Tewari, “Topologically protected surface Majorana arcs and bulk Weyl fermions in ferromagnetic superconductors”,


16) Sumanta Tewari, J. D. Sau, V. W. Scarola, Chuanwei Zhang, S. Das Sarma, “Probing topological quantum critical points in semiconductor-superconductor heterostructures”,


17) L. Mao, M. Gong, E. Dimitrescu, Sumanta Tewari, Chuanwei Zhang, Hole-doped semiconductor nanowire: A new and experimentally accessible system for Majorana fermions,


18) M. Gong, Sumanta Tewari, Chuanwei Zhang, BCS-BEC crossover and topological phase transition in spin-orbit coupled degenerate Fermi gases,

   Physical Review Letters 107, 195303 (2011)

19) D. J. Clarke, J. D. Sau, Sumanta Tewari, Majorana fermion exchange in quasi-one-dimensional networks,


20) J. D. Sau, D. J. Clarke, Sumanta Tewari, Controlling non-Abelian statistics of Majorana fermions on Majorana dimer lattices,


21) Sumanta Tewari, T. D. Stanescu, J. D. Sau, S. Das Sarma, Non-Abelian s-wave superconductivity in spin-orbit coupled systems: Bulk phases and quantum phase transitions,


   Nanotechnology 22, 095703 (2011)

23) Q. Li, P. Ghosh, J. D. Sau, Sumanta Tewari, S. Das Sarma, Anisotropic surface transport in topological insulators in proximity to a helical spin-density wave,


24) J. D. Sau, Sumanta Tewari, Diamagnetism from the 6-vertex model and implications for the high temperature cuprate superconductors,

   Physical Review Letters 107, 177006 (2011)

25) J. D. Sau, Sumanta Tewari, S. Das Sarma, Universal quantum computation on a semiconductor quantum wire network,


26) Jay D. Sau, Roman M. Lutchyn, Sumanta Tewari, S. Das Sarma, Robustness of Majorana fermions in 2D topological superconductors,
27) Jay D. Sau, Sumanta Tewari, S. Das Sarma, Probing non-Abelian statistics with Majorana fermion interferometry in spin-orbit-coupled semiconductors, 


30) Li Mao, Chuanwei Zhang, Robustness of Majorana Modes and Minigaps in a Spin-Orbit-Coupled Semiconductor-Superconductor Heterostructure,


31) Li Mao, Junren Shi, Qian Niu, Chuanwei Zhang, Superconducting phase with a chiral f-wave pairing symmetry and Majorana fermions induced in a hole-doped semiconductor,


32) Yongping Zhang, Chuanwei Zhang, Quantized Anomalous Hall Insulator in a Nanopatterned Two-Dimensional Electron Gas,


33) Chunlei Qu, Yongping Zhang, Li Mao, Chuanwei Zhang, Signature of Majorana Fermions in Charge Transport in Semiconductor Nanowires

arXiv:1109.4108

34) Yongping Zhang, Li Mao, and Chuanwei Zhang, Mean-field dynamics of spin-orbit coupled Bose-Einstein condensates,


35) Sanfeng Wu, Li Mao, Aaron M. Jones, Wang Yao, Chuanwei Zhang, Xiaodong Xu, Quantum-Enhanced Tunable Second-Order Optical Nonlinearity in Bilayer Graphene,

Nano Letters, 12, 2032 (2012),

36) Ming Gong, Gang Chen, Suotang Jia, Chuanwei Zhang, Searching for Majorana Fermions in 2D Spin-orbit Coupled Fermi Superfluids at Finite Temperature