BRL
112
c.2A

BRL R 112

BRL

AD PB68634

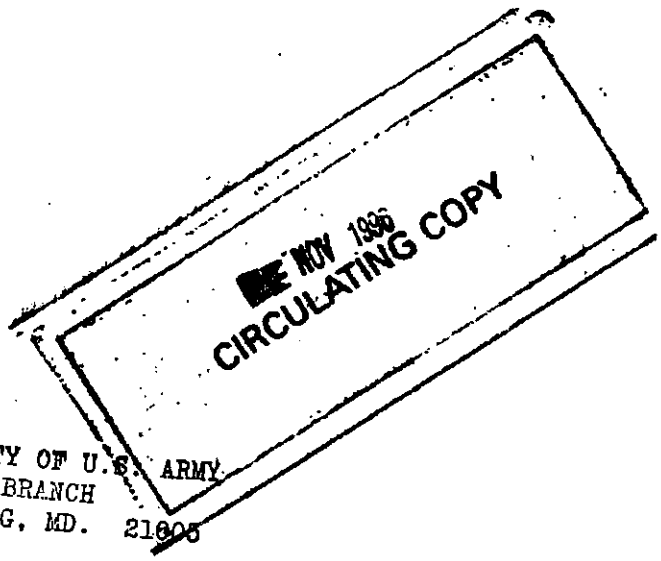
REPORT NO. 112

TARGET ERRORS DUE TO ERRORS IN ALTITUDE AND AIR SPEED

by

R. H. Kent

July 1938



This document has been approved for public release and sale; its distribution is unlimited.

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
21 Jul 1938		

4. TITLE AND SUBTITLE

Target Errors Due to Errors in Altitude and Air Speed

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

R. H. Kent

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Ballistic Research Laboratories, Aberdeen Proving Ground, MD 21005

8. PERFORMING ORGANIZATION REPORT NUMBER

BRL-112

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Ballistic Research Laboratories, Aberdeen Proving Ground, MD 21005

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

Distribution Statement A. Approved for Public Release.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Expressions are obtained for the errors in range and deflection due to errors in altitude and air speed. An illustrative example is given.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	UU	7	**19b. TELEPHONE NUMBER** *(include area code)*
U	U	U			

Ballistic Laboratory
Report No. 112

RHK/emh
Aberdeen Proving Ground, Md.
July 21, 1938

TARGET ERRORS DUE TO ERRORS IN ALTITUDE AND AIR SPEED

Abstract

Expressions are obtained for the errors in range and deflection due to errors in altitude and air speed. An illustrative example is given.

The condition for hitting of a bomb in range with a stationary target and horizontal flight (see the chapter on Bombing from Airplanes in Hayes' 'Elements of Ordnance' page 501) is

$$\tan \beta_o = \tan \beta' - \tan \tau \cos \varphi \qquad (1)$$

where β_o is the value of the position angle β at the instant of release, β' is the travel angle, τ is the trail angle and φ is the azimuth of the trail.

If the correct value of β_o is computed and the bomb dropped when $\beta = \beta_o$, the range component of the target error* will be zero.

Suppose that there is an error $\Delta \tan \beta'$ in the estimate of $\tan \beta'$ and an error $\Delta \tan \tau$ in the estimate of $\tan \tau$, then if the error in $\tan \beta_o$ is $\Delta \tan \beta_o$, we have

$$\tan \beta_o + \Delta \tan \beta_o = \tan \beta' + \Delta \tan \beta' - (\tan \tau + \Delta \tan \tau) \cos \varphi. \qquad (2)$$

* The target error is the distance between the point of fall and the target.

PROPERTY OF U.S. ARMY
STINFO BRANCH
BRL, APG, MD. 21005

From (1) and (2) it follows that

$$\Delta \tan\beta_o = \Delta\tan\beta' - \Delta\tan\tau \cos\varphi. \qquad (3)$$

The value of $\tan\beta'$ is obtained by multiplying

$\frac{d(\tan\beta)}{dt}$ (measured by the sight) by t_ω , or

$$\tan\beta' = \frac{d(\tan\beta)}{dt}\, t_\omega.$$

If it is assumed that $\frac{d(\tan\beta)}{dt}$ is correctly measured, then the error, $\Delta\tan\beta'$, in $\tan\beta'$ is given by

$$\tan\beta' + \Delta\tan\beta' = \frac{d(\tan\beta)}{dt}(t_\omega + \Delta t_\omega)$$

where Δt_ω is the error in the time of flight. From this

$$\Delta\tan\beta' = \frac{d(\tan\beta)}{dt}\Delta t_\omega .$$

Inserting this value of $\Delta\tan\beta'$ in (3) we get

$$\Delta\tan\beta_o = \frac{d(\tan\beta)}{dt}\Delta t_\omega - \cos\varphi\,\Delta\tan\tau .$$

The range component of the target error corresponding to $\Delta\tan\beta_o$ is $y_o\Delta\tan\beta_o$. Hence the target error in range of the bomb will be

$$y_o\Delta\tan\beta_o = y_o\frac{d(\tan\beta)}{dt}\Delta t_\omega - y_o\cos\varphi\,\Delta\tan\tau .$$

$y_o\frac{d(\tan\beta)}{dt}$ is the ground speed, v_g. Hence the target error in range is

$$v_g\Delta t_\omega - y_o\cos\varphi\,\Delta\tan\tau .$$

If the errors in t_ω and $\tan\tau$ are due solely to an error Δy_o in y_o,

$$\Delta t_\omega \equiv \frac{\partial t_\omega}{\partial y_o} \Delta y_o,$$

$$\Delta \tan\tau = \frac{\partial(\tan\tau)}{\partial y_o} \Delta y_o.$$

Thus the target error in range as depending upon the error in altitude Δy_o is

$$v_g \frac{\partial t_\omega}{\partial y_o} \Delta y_o + y_o \cos\varphi \frac{\partial(\tan\tau)}{\partial y_o} \Delta y_o . \qquad (4)$$

If there is also an error, Δu_h, in the horizontal air speed, u_h, it may readily be shown that the resultant target error in range due to an error Δy_o in y_o and an error Δu_h in u_h is

$$v_g \left(\frac{\partial t_\omega}{\partial y_o} \Delta y_o + \frac{\partial t_\omega}{\partial u_h} \Delta u_h\right) - y_o \cos\varphi \left(\frac{\partial \tan\tau}{\partial y_o} \Delta y_o + \frac{\partial \tan\tau}{\partial u_h} \Delta u_h\right) =$$

$$v_g \left(\frac{\partial t_\omega}{\partial y_o} \Delta y_o + \frac{\partial t_\omega}{\partial u_h} \Delta u_h\right) - y_o \cos\varphi \sec^2\tau \left(\frac{\partial \tau}{\partial y_o} \Delta y_o + \frac{\partial \tau}{\partial u_h} \Delta u_h\right) .* \qquad (5)$$

The above expression give the amount by which the bomb will fall short of the target in <u>range</u>. In using (5), due attention should be paid to signs of Δy_o and Δu_h. If the estimated altitude is greater than the true altitude, Δy_o is positive; if less, Δy_o is negative. If (5) is negative, the bomb will overshoot the target. For a stationary target and horizontal flight, the target error in <u>deflection</u> may be computed from the condition for hitting in deflection,

* *It is assumed that the trail is measured in radians. A mil is equal to .001 radians.*

$$\tan \gamma_o = \frac{\tan \bar{\tau} \sin \varphi}{\left[1 + (\tan \tau \cos \varphi - \tan \beta')^2\right]^{\frac{1}{2}}} = \frac{\tan \bar{\tau} \sin \varphi}{(1 + \tan^2 \beta_o)^{\frac{1}{2}}} = \frac{\tan \tau \sin \varphi}{\sec \beta_o},$$

where γ_o is the position angle of the target for deflection at the instant of release.

$$\frac{y_o \sin \varphi \sec^2 \tau}{\sec^3 \beta} (\sec^2 \beta_o + \cos \varphi \tan \tau \tan \beta_o)\left(\frac{\partial \tau}{\partial y_o} \Delta y_o + \frac{\partial \tau}{\partial u_h} \Delta u_h\right)$$

$$- \frac{\tan \tau \sin \varphi \tan \beta_o v_g}{\sec^3 \beta_o} \left(\frac{\partial t_\omega}{\partial y_o} \Delta y_o + \frac{\partial t_\omega}{\partial u_h} \Delta u_h\right) \qquad (6)$$

If the above expression is positive, the bomb will fall short of the target in deflection, which is measured to the leeward.

ILLUSTRATIVE EXAMPLE

Suppose a bomb is dropped from an airplane having an air speed of 150 miles/hour at an altitude of 10,000 ft. with the azimuth of trail, φ, equal to 10°. If the ballistic coefficient is 2, what are the target errors in range and deflection due to an error of 10 miles/hour in air speed and of 100 ft. in altitude?

To obtain the error in range, expression (5) is used. First the derivatives,

$$\frac{\partial t_\omega}{\partial y_o}, \frac{\partial t_\omega}{\partial u_h}, \frac{\partial \tau}{\partial y_o} \text{ and } \frac{\partial \tau}{\partial u_h}$$

are computed approximately by the aid of figures 9 and 10 of 'Bombing from Airplanes'.

From figure 10, it is seen that, at an air speed of 100 miles/hour, a change of altitude from 7500 to 12,500 changes t_ω from 22 sec. to 29 sec. The change is 7 sec. while at 200 miles/hour, t_ω changes by almost the same amount.

Hence approximately, for an airspeed of 150 miles/hour and an altitude of 10,000 ft.,

$$\frac{\partial t_\omega}{\partial y_o} = \frac{7}{5000} = .0014.$$

From the two curves of Fig. 10 the value of $\frac{\partial t_\omega}{\partial u_h}$ is estimated to be approximately $\frac{.1}{100} = .001$.*

Similarly from plot 9, it is found that $\frac{\partial \tau}{\partial y_o}$ is negligible and that $\frac{\partial \tau}{\partial u_h}$ is about $\frac{47.5 - 39.5}{20} = \frac{8}{20} = .4$ mils/miles/hr. $= .0004$ radians/mile/hr.

For the given conditions $\tau \approx$ 43 mils $= .043$ radians, whence $\sec^2 \tau = 1.00$.

If then we take $y_o = 10{,}000$, $\Delta y_o = 100$ and $\Delta u_h = 10$, we find from (5) that the target error in range is

$$v_g(.0014 \times 100 + .001 \times 10) - 10{,}000(.00040 \times 10)\cos\varphi = v_g \times (.15) - 40\cos\varphi.$$

If φ is 10°, then $\cos\varphi = .984$; hence if v_g is 250 ft/sec. the target error is

$$37 - 39 = -2 \text{ ft.}$$

The bomb will overshoot the target by about two feet.

* *The value of .001 was really obtained from the Exterior Ballistic Tables; the curves would indicate a value of about .002. The discrepancy is due to the facts that (1), the linear interpolation is inaccurate and (2), the scale of Fig. 10 is small.*

If $\Delta u_h = 0$, and $\Delta y_o = 100$, the bomb will fall short by 35 ft. If $\Delta y_o = 0$ and $\Delta u_h = 10$ mi/hr. the bomb will overshoot by 37 ft.

To compute the error in deflection we need to know sin φ, tan τ, tan β_o, and sec β_o.

For these, the following values are obtained,

$$\sin \varphi = .174$$

$$\tan \tau = .043$$

$$\tan \beta_o = \frac{v_\beta}{y_o} t_\omega - \tan \tau \cos \varphi = \frac{250 \times 26}{10{,}000} - .043 \times .984 = .61$$

$$\sec \beta_o = \sqrt{1 + .61^2} = 1.17.$$

If these values and the values of $\frac{\partial t_\omega}{\partial y_o}$, etc. previously obtained are inserted in (6), the result is

$$\frac{10{,}000 \times .174 \times 1}{1.17^3} \times (1.17^2 + .984 \times .043 \times .61) \times (.0040)$$

$$- \frac{.043 \times .174 \times .61 \times 250}{1.17^3} (.15) =$$

$$1{,}085 \times (1.39) \times .0040 - .71 \times .15 =$$

$$6.0 - 0.1 = 5.9 \text{ ft.}$$

Thus the bomb will fall about 6 ft. to the right in deflection.

If the calculations are repeated with $\Delta y = 0$ and $\Delta u_h = 10$ mi/hr. almost the same result will be obtained, indicating that an error in altitude will produce only a negligibly small error in deflection.

R. H. Kent.
R. H. Kent

PROPERTY OF U.S. ARMY
STINFO BRANCH
DRL, APG, MD. 21005