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THE DIFFERENTIAL ANALYZER

Albert A. Bennett

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ABERDEEN PROVING GROUND, MARYLAND

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THE DIFFERENTIAL ANALYZER

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PREFACE

This report on the Differential Analyzer is included in historical survey of the machine and expounds a theory of its operation. The report seeks to present for the first time in easily available published form a discussion of this whole problem. However, most of this material here developed has been long familiar (in slightly differing dress) in the Ballistic Research Laboratory, due to the painstakingly thorough study of the machine undertaken by Dr. L. S. Dederick and his assistants. An extensive system of forms and accompanying notes has been in use in the Laboratory and has been modified from time to time to keep up with improvements. On account of a revision in the notation here adopted, and some consequent simplification in practical procedure, it has seemed unwise to devote the space that would be required to explaining in detail the several computation sheets and other forms which have been employed steadily hitherto but which may be hoped in some cases, at least, to have completed their period of usefulness.

No attempt has been made to provide a practical handbook for the mechanical operator and for the repairman. There is a large amount of accumulated technical experience covering matters of mechanical adjustment and repair, which has been and probably should be transmitted under immediate operating conditions from one mechanic to the next and from one operator to the next. It is hoped, however, that this report may (without need for personal instruction) explain the theory to any mathematically trained observer of the machine.

The author's indebtedness to the generous and timely assistance and friendly constructive criticism of numerous local veterans in this field of applied mathematical engineering has been obvious to the many who have patiently answered questions and labored over hardly legible manuscript copy. It seems hardly feasible to try to name them all.

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Ballistic Research
Laboratory Report No. 319

AAB/dik
Aberdeen Proving Ground, Md.
December 23, 1942

REPORT ON THE DIFFERENTIAL ANALYZER AT ABERDEEN PROVING GROUND, MARYLAND

(As in use July, 1942)

Part I

A. THE PURPOSE AND SCOPE OF THE ANALYZER:

The Differential Analyzer, so named in 1930 by its inventor, designer and builder, Dr. Vannevar Bush, (then at the Massachusetts Institute of Technology), is a power driven machine for obtaining mechanically and printing in numerical tabular form the solution of any one of a wide variety of total differential equations. The machine at Aberdeen Proving Ground, adapted from the original (now obsolete) Differential Analyzer at Cambridge, Massachusetts, is officially "The Bush Differential Analyzer".

The name "Differential Analyzer" was doubtlessly chosen because of the analogy which this name suggests, with the various well-known types of harmonic analyzers. However, this name has been thought by many to be somewhat of a misnomer. The machine parts perform as indicated below, beyond certain few restricted algebraic operations, only quadratures. Although this machine has the essential advantage over early types of planimeters and integraphs of being devised to accept as input for one part of the machine the output of another part, it does not "analyze differentials". It cannot even obtain the derivative of an arbitrarily given smoothly varying function. Save at

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most in one case, it will not (as now set up) multiply a given variable by a given constant unless this constant is, save for sign, the product of a small number of "simple ratios". Because of the extraordinary flexibility of interconnections in the machine, and the theoretical possibility of adjoining one or more further "tables" if needed, other types of problems might be handled quite differently from those which now employ the full time of the machine.

For theoretical purposes, the Differential Analyzer may be described as a group of independently operable sub-machines (each with fixed and movable parts) interconnected by rigid shafts or cable drives, the entire mechanical group being housed on a rigid steel frame. There are also electrical control elements with an elaborate separate panel board. As a further feature of the practical running of the machine may be mentioned an air compressor.

The sub-machines in current use and required in explanation of the theory, which serve to drive or adjust other parts of the Analyzer may be listed as follows. They will be described in Part III of this report under the following headings:

a. 1 Calibrating drum, fixed setting (does not deal with a sub-machine)

b. 4 Calibrating drums and counters (hand set for trajectory parameters).

c. 3 Cranks, each hand driven (used only for reduction runs).

d. 1 Prime motor (requiring no mechanical input from other parts of the machine).

e. 8 Integrator units, each with separate power drive for the output, and involving two inputs.

f. 3 (Input) Tables, each with separate power drive for the output; two with two inputs each, and one with a single input.

g. 6 Adders, each with a single output and two inputs.

h. 39 Couplings, each consisting either of a single pair of enmeshed spur gears, or of a train of such pairs.

65 TOTAL: Sub-machines

The twenty-six sub-machines which are not couplings, together with parts of the recorder, called printers, which latter receive but do not yield torque, constitute a list further sub-divided into machine elements. The first twenty-six machine elements are the output elements of the respective sub-machine and are designated correspondingly. There remain to be mentioned (as currently in use) thirty-nine input machine elements corresponding to the inputs of the given twenty-six sub-machines, (not couplings). In particular, we have for machine elements the following output elements:

a. - h. 26 Output elements.

i. 16 Integrator inputs, consisting of eight differential inputs (each designated by $\longleftrightarrow$, indicating abscissa) and eight integrand inputs (each designated by $\updownarrow$, indicating ordinate.)

j. 5 Table inputs, consisting of three abscissa inputs (each designated by $\longleftrightarrow$) and two ordinate inputs (each designated by $\updownarrow$).

k. 12 Adder inputs, two to each adder.

l. 6 Printer inputs, one to each printer unit of the recorder.

65 TOTAL: Machine elements (exclusive of couplings).

Exclusive of the five calibrating drums and counters, each of the remaining sixty machine elements when connected with a given coupling is so connected by means of a (nominal) bus shaft. These sixty bus shafts are similarly numbered (1' - 3', 0 - 56).

Any adequate description of the mechanical structure of the Differential Analyzer would mention also many other features and in particular, cross shafts and spiral gears (some right handed, some left handed) which serve to convert rotation from one shaft to rotation on another at right angles to it, with no change in absolute rate of rotation.

Of prime importance to the practical success of any mechanical contrivance of the general character of the Differential Analyzer are the many ingenious devices for the supply of fresh power, the reduction of friction and of injurious vibration, the elimination of back lash and free play between connecting elements, and the stops, buzzers, and controls for safety of operation. While it is true that without these the machine would be valueless, they need not be emphasized in discussing the theory of the problems handled by the machine.

In handling a physical problem by means of the Differential Analyzer, one has initially

(i) Certain selected primary variables and parameters to each of which is assigned a specific symbol. Each of these primary variables may involve physical units such as feet, pounds, seconds, feet per second, etc., or it may happen that one or more are intrinsically dimensionless, either ordinarily expressed as pure numbers, or again as angles. The selected list of primary variables is sufficiently extensive to permit expressing every variable supposedly concerned in the formulated problem in terms of these given primary variables.

(ii) Numerical values assigned to the parameters used in any one solution.

(iii) Certain algebraic and differential equations interrelating the primary variables and parameters.

Despite its extraordinary flexibility in many regards, the Differential Analyzer in the form now in use is equipped to handle combinations of certain mathematical steps only.

For theoretical purposes we may speak of one independent and a large but yet limited number of dependent primary variables each of these latter related directly or indirectly to the independent variable through fixed preassigned functional relations. The machine does only the following (the lettering corresponding to the notation of machine elements given previously)

(a) Accepting a given additive absolute constant to be set by a calibrating drum.

(b) Accepting as many as four given additive parameters depending upon the given initial conditions to be set by respective calibrating drums one or more having an auxiliary counter.

(c) Accepting as many as three numerically tabulated one-valued functions each of some primary one variable, for more or less continuous hand-feeding through a crank (equipped with a setting lock).

(d) Accepting for automatic operation one independent variable.

(e) Integrating one given variable with respect to the differential of another given variable by means of an integrator unit. (At present eight such integrations are continuously performed. The machine is equipped with a total of ten integrator units, two of which remain currently idle.)

(f) Performing continuously (on any one run) one or more of the following three operations (but with not more than a single use of any one of them)

(i) Obtaining (with the Template Table) the variable ordinate of a single one-valued function for a given interval for the abscissa by means of an arc previously traced to specifications and cut upon a template. (The template, like the gear train, may be changed between runs.)

(ii) Obtaining (with the Vector Table) the distance from the origin to a variable point, given the variable ordinate and variable abscissa of the point within a fixed circle in the squared cartesian plane.

(iii) Obtaining (with the Division Table) continuously the variable value (within certain bounds) of the quotient, given the variable numerator and the variable denominator (not near zero).

(g) Forming (with an Adder) the negative of the sum of two given variables.

(h) Multiplying a variable by one or several simple ratios, each contained in a fixed short list (at present consisting essentially of -4, -2, -3/2, -5/4, -1, and their reciprocals only).

(i) Using any previously (or simultaneously) obtained variable as differential or as integrand in a quadrature by an integrator unit.

(j) Using any previously (or simultaneously) obtained variable as independent abscissa or as independent ordinate in any one of the three operations described under (f).

(k) Using any previously obtained variable as term in a sum whose negative is obtained as in (g).

(l) Maintaining automatically (with the Recorder) a printed table of values of as many as six of the variables as these change during a run, for convenient assigned equal intervals of a selected one of these variables. Also printing the values of the selected six record variables at any desired preassigned value for any one of the variables, through direct observation by the operator of the changing values of the chosen variable, who slows the machine near the critical position, and operates the printer by hand-punch when this position is reached.

A variable is represented on the machine by the total turn (angular displacement) of an appropriate shaft. Angular speeds enter in the theory only through their ratios, although for mechanical efficiency there are certain restrictions (discussed later) upon the maximum speed of some of the moving parts.

Part II

HISTORY OF THE MACHINE

A machine (now superseded) called by its inventor and builder, Dr. Vannevar Bush, (then at the Massachusetts Institute of Technology), the "Differential Analyzer", was in operation in 1930, although according to its inventor, was "not yet completed" in December, 1931. It incorporated in more flexible form the same basic idea of interconnection of integrating units as did an earlier model by the same inventor referred to as the "M. I. T. Integraph" designed for solving second order ordinary differential equations, and which was described in several articles in 1927 and 1928 to which references may be found in the article next named. The first authoritative description of the Differential Analyzer is the following: V. Bush: "The Differential Analyzer. A new machine for solving differential equations." Journal of the Franklin Institute (212) 447-488 (1931). It has references to previous work and other related devices.

At the Christmas meetings, 1929, of the American Mathematical organizations, held in Bethlehem, Pennsylvania, Dr. Bush was one of several speakers invited to present expositions of various phases of applied mathematics. Plans and objectives of the Differential Analyzer were discussed. Professor Barker, Major, Ordnance Reserve, then newly appointed head of the Electrical Engineering Department at Lehigh University, discussed with Dr. L. S. Dederick of the Proving Ground and with Dr. Bush the possible use of the Analyzer for computation of trajectories. The accuracy of the machine was expected to fall so far below the normal standard being demanded for hand-computed trajectories that the project seemed hardly feasible at that time. In 1931, Captain Phillip L. Alger of the General Electric Company, Schenectady, and associated with M. I. T., while on summer duty as reserve officer, also became interested in the possibility of replacing the laborious hand computation by the mechanical use of the M. I. T. Differential Analyzer whose encouraging performance ("considerably better than 1/10 of one percent") he had witnessed. In September 1931, he

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wrote letters to Major Somers at M. I. T. and to Captain Guion at the Proving Ground and to Professor Bennett at Brown University, formerly in charge of ballistic computations, and urged the staff at M. I. T. to adopt steps to transfer some of the trajectory computation to the Differential Analyzer. On September 11, 1931, Dr. L. S. Dederick in accordance with suggestions from Major Somers and Captain Alger forwarded to Dr. Bush data sufficient for setting up a particular ballistic problem for trial runs on the machine. A brief demonstration was conducted on September 29, 19 before a small group of interested persons including among others General Hamilton (retired), Captain Guion, Dr. Dederick, Mr. Kent, and Professor Bennett. A more extended report was prepared by Dr. Dederick for blue printing and distribution entitled: "The Application of Dr. Bush's Integraph to Ballistic Problems". An informal report of this demonstration was prepared by Professor Bennett and sent to General Tchappat in anticipation of a general conference at M. I. T. on the subject of the Analyzer at which General Hamilton, then retired, was present. Dr. Dederick, on November 6, 1931, wrote expressing the hope "that the Ordnance Office would see fit to authorize a systematic program of trajectory computations for the M. I. T. Analyzer."

As an outcome of this demonstration, authority was granted (O.O 471.9/642 and A.P.G. 413.6/330) to conduct during the month of June, 1932 a test at Massachusetts Institute of Technology in Cambridge, Massachusetts "of the utility of Dr. V. Bush's Differential Analyzer as applied to work in Ballistics." This was carried out by a collaboration of personnel of the Institute and of Aberdeen Proving Ground. Dr. L. S. Dederick with Mr. F. Atkins, a computer, spent the month of June in Cambridge working long hours daily with Professor Samuel H. Caldwell, Director of the Research Hall in Electrical Engineering, and numerous assistants, in particular with Mr. Frost, machinist, as well as enjoying occasional interviews with Dr. Bush himself.

As Dr. Dederick states in his official Report on the Test:

"It was planned to execute systematically by means of this machine all the basic integrations necessary for the construction of a complete range table and to use whatever time remains on short tests covering as wide a variety of problems as possible. The range table selected was for the 16" Howitzer, Model 1920, 2100 lb. A. P. Projectile, Muzzle Velocity 1950 f/s. This required angles of departure from 0°

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to 65° and large values of the range, time of flight, maximum ordinate, and ballistic coefficient. The remaining time was devoted to three small groups of trajectories, one for the 16" Gun, Model 1919, 2340 lb. A. P. Projectile, Muzzle Velocity 2650 f/s, at maximum range, one for the 0.30 caliber rifle, M1, near 17°, and one consisting of trajectories of Interior Ballistics. In order to test the performance of the machine most effectively, all the problems were selected from among those which had previously been solved by numerical integration or other approved methods. The general purpose of the test was to determine whether or not it was desirable to build a similar machine at Aberdeen Proving Ground. ....

"Plan for Range Table. The methods used in preparing a range table by means of the Analyzer must be materially modified from those regularly in use at present [i.e. at date of report July, 1932]. In the latter case every effort is made to cut down the number of trajectories necessary to compute and to use every possible device to replace them by other methods of computation. This is because of the great time and labor necessary to compute trajectories by numerical integrations. The Analyzer, on the other hand, will work trajectories forty or fifty times as fast as a computer; but it will do nothing else. [This estimate must now be considerably reduced if the time spent in setting up new gear trains and repairing minor failures is to be counted.] The plan of work for the test of the Analyzer therefore was to use a large number of trajectories wherever that would reduce the labor of other forms of computation. In any case the work of preparing a range table falls into two parts, the reduction of firings, and the range table proper.

"Reduction of Firings. ....All the usual corrections being provided for, the final values of C (the ballistic coefficient) for the range firing angles were obtained in the usual way. For purposes of comparison these values were plotted on the sheet on which the short arc C had previously been drawn. It was found in general that the new points lay closer to the curve than the old ones. Thus whatever error had been introduced by the machine was less than the accidental error originally attributed to the range-firing in smoothing out the C-curve. ....

"Characteristics to be considered. The characteristics of the analyzer may be considered from the point of view of speed, accuracy, reliability, simplicity of method, field of utility, possible improvements, personal requirements, ease of repair, and total expense. [These are then treated seriatim.] ...... It seems very conservative to say that the machine will work trajectories fifty times as fast as the average computer. ...... In any case we may say that accuracy sufficient for all practical purposes either has been obtained or can in all probability be obtained by simple changes in procedure. ...... In fact we may say that the month's test of the Analyzer was attended by a somewhat remarkable chapter of accidents. ....

"Summary. It becomes evident from the foregoing that a Differential Analyzer could be built at the Proving Ground which would perform various kinds of ballistic computations with all needful accuracy, that its operation and maintenance would require no unusual skill, that its speed is such as to insure greatly increased output of work by the Ballistic Section, in any emergency, and the performance of routine work at a smaller total expenditure of funds, that its use in the preparation of range tables would replace numerous complicated and indirect methods now in use, by relatively simple, uniform, and direct processes, that its use would permit the investigation of various problems whose solution is necessary to any advance in ballistic theory or practice, but whose study by present methods involves a prohibitive amount of labor of computation.

"Recommendations. It is recommended, therefore, that the building of a Differential Analyzer be authorized, similar to that in operation at the Massachusetts Institute of Technology, but having ten or more integrating units, and some or all of the improvements already described."

The test runs not only proved as a whole satisfactory--they even brought to light two instances of errors in the printed tables and one error in an exterior ballistics and one in an interior ballistics trajectory.

On July 8, 1932, Dr. Dederick writing to Dr. Bush remarked, "My own opinion is that the Government certainly ought to build a machine essentially the same as yours and I shall do all I can to forward such a project. Major Rehm, the new officer in charge of the Ballistic Section, seems very favorably impressed. I fear, however, that there are serious difficulties. ...."

No further official steps were taken, however, for more than a year. Captain Alger sought to have active measures initiated, and he visited and corresponded with responsible authorities at Aberdeen and at M. I. T. In November, 1933, Colonel Shinkle made plans to travel to Cambridge to inspect the Differential Analyzer and discuss it with Dr. Bush with a view to possible construction of a machine at the Proving Ground. The travel order was never utilized however.

Early in December, 1933, Captain Alger visited Professor Caldwell and shortly thereafter wrote to him urging the prompt securing of bids by manufacturers for building at Aberdeen an Analyzer for the Ordnance Department.

Finally in mid-December, 1933, the Proving Ground was authorized to procure a Differential Analyzer to be installed at the Proving Ground. Since the original Differential Analyzer was the progressive outgrowth of much continuing experimental trial, and since most of the parts were custom-made and often prepared in the university laboratories, and since several basic improvements had already suggested themselves, the blueprinting of suitable standard specifications involved the intelligent work of a designing draftsman familiar with the principles and practical problems of the machine. At the request and with the cooperation of the Moore School of Electrical Engineering of the University of Pennsylvania, working drawings and specifications were drawn up for the construction by C. W. A. labor of a Differential Analyzer in Philadelphia. The Army arranged to purchase for $500.00, as they were being made, a copy of these plans and soon began receiving competitive bids for the several parts involved in the erection of a similar machine at the Aberdeen Proving Ground. Eventually the sum of $800.00 was paid to the University.

The construction of the two machines, one in Philadelphia, the other at Aberdeen, while carried on by different means and differing in details, were roughly parallel and based upon essentially the same specifications as these were successively drawn up.

A three-cornered interchange of correspondence and official visits was maintained involving the Electrical Engineering Department of M. I. T. (chiefly Professor Samuel H. Caldwell and Mr. Frost, machinist,), the Moore School of Electrical Engineering at the University of Pennsylvania (chiefly Professors C. D. Fawcett and Irven Travis and Mr. Nelson, machinist,), and the Aberdeen Proving Ground (chiefly Major N. W. Rehm, on contracts, and Dr. L. S. Dederick and Mr. L. E. Bauer, machinist,). Some notion of the further chronological sequence will be given by the following list of selected dates:

February 21, 1934, Working drawings for substructure received from University of Pennsylvania.

April 2, 1934, Specifications for integrator units (194 drawings, some 340 parts) completed. (Units for delivery before June 30, 1934.)

April 23, 1934, Specifications for Adders and Front Lash Units completed.

May, 1934, First bids received for integrator unit parts.

August 22, 1934, Specifications for 2 rectangular input tables (87 drawings) completed.

October 21, 1934, Newspaper report, "Three-ton Thinking Machine is just 75,000 parts now."

February, 1935, Input tables inspected at M. I. T. and shipped to Aberdeen Proving Ground.

March 15, 1935, Specifications for one Recorder (52 drawings) completed.

May, 1935, Major N. W. Rehm traveled to Cambridge to observe and confer on electrical equipment, etc.

June, 1935, Specifications for Control, Protective and Auxiliary Electrical Equipment, completed.

September 30, 1935, First complete trajectory run off at Aberdeen.

January, 1936, Mr. L. E. Bauer, machinist of the Ballistics Laboratory, Aberdeen Proving Ground, went to M. I. T. to discuss numerous mechanical changes in details and to compare performance records.

Ordered parts arrived from time to time, corrections and modifications were made in working drawings, and elaborate tests were instituted to discover the most effective distribution of limits for loads, speeds, travel, etc., and to recognize and correct where possible mechanical defects. Incidentally, Dr. Dederick noted that the resistance to turning of sequences of the spur-gear connected shafts seemed to vary approximately as the (3/2) power of the angular velocity of the drive. Since December, 1935, when the Aberdeen machine started its life of practical service, experimentation has continued, numerous modifications, some of them fairly fundamental, have been introduced, and further experience has justified alterations in limits formerly specified. Among the improvements originating at Aberdeen (many of them during the first experimental years) are the following:

(1) Auxiliary large damper wheels (used to damp out fluctuations in angular velocity) attached by friction contact (with springs) to shafts bearing output of integrators (suggested by Dr. L. S. Dederick) 1935.

(2) A drying chamber incorporated in the air compressor unit which latter drives the impression pistons of the recorder (suggested by Dr. L. S. Dederick).

(3) A follower with blunt tapered truncated "point" to follow a ridge in a template in an input table (suggested by Captain Elmer Goebert) 1935. (To replace simple pointer and reading glass.)

(4) A second synchronized lead screw for the carriage of the input table to avoid accidental deflections of the carriage (suggested by Dr. L. S. Dederick) 1935.

(5) Automatic cam contact ("punch") for recorder ("printer") (designed by Mr. L. E. Bauer) 1938.

(6) Present design (after several intermediate models) of automatic, power driven follower using magnetic clutch drive (suggested by Mr. L. E. Bauer) 1939.

(7) Vector Input Tables (designed by Dr. L. S. Dederick) 1941.

Most of the numerous minor improvements in mechanical adjustment originating in Aberdeen are due to Mr. L. E. Bauer, who also among his other duties supervised the construction and installation of the machine and its later associated parts.

Part III

DESCRIPTION OF THE DIFFERENTIAL ANALYZER AT ABERDEEN

A. General Diagram

It will be unnecessary to describe in minute detail

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the appearance of the Differential Analyzer at the Aberdeen Proving Ground since this Report is designed only for those readers who will have immediate access to the machine itself. In mechanical appearance, the Differential Analyzer may be briefly described as a three-ton machine composed of two mutually perpendicular systems of parallel shafts and interconnecting gears (with some flexible drive cables) bound in a massive rectangular steel frame, 25 feet long and 38 inches wide, together with adjacent "integrators", "tables" for input and output, and other features described later. During a run, the operator remains at the front. For computing standard trajectories the operation of the machine is almost wholly automatic save for the need of control over the variable speed. However in reducing observed data to identify the appropriate value of the ballistic coefficient, recorded deviations of observed meteorological variations from standard conditions are fed in by the operator by hand cranks attached to appropriate bus shafts.

A schematic diagram of the machine is as follows:

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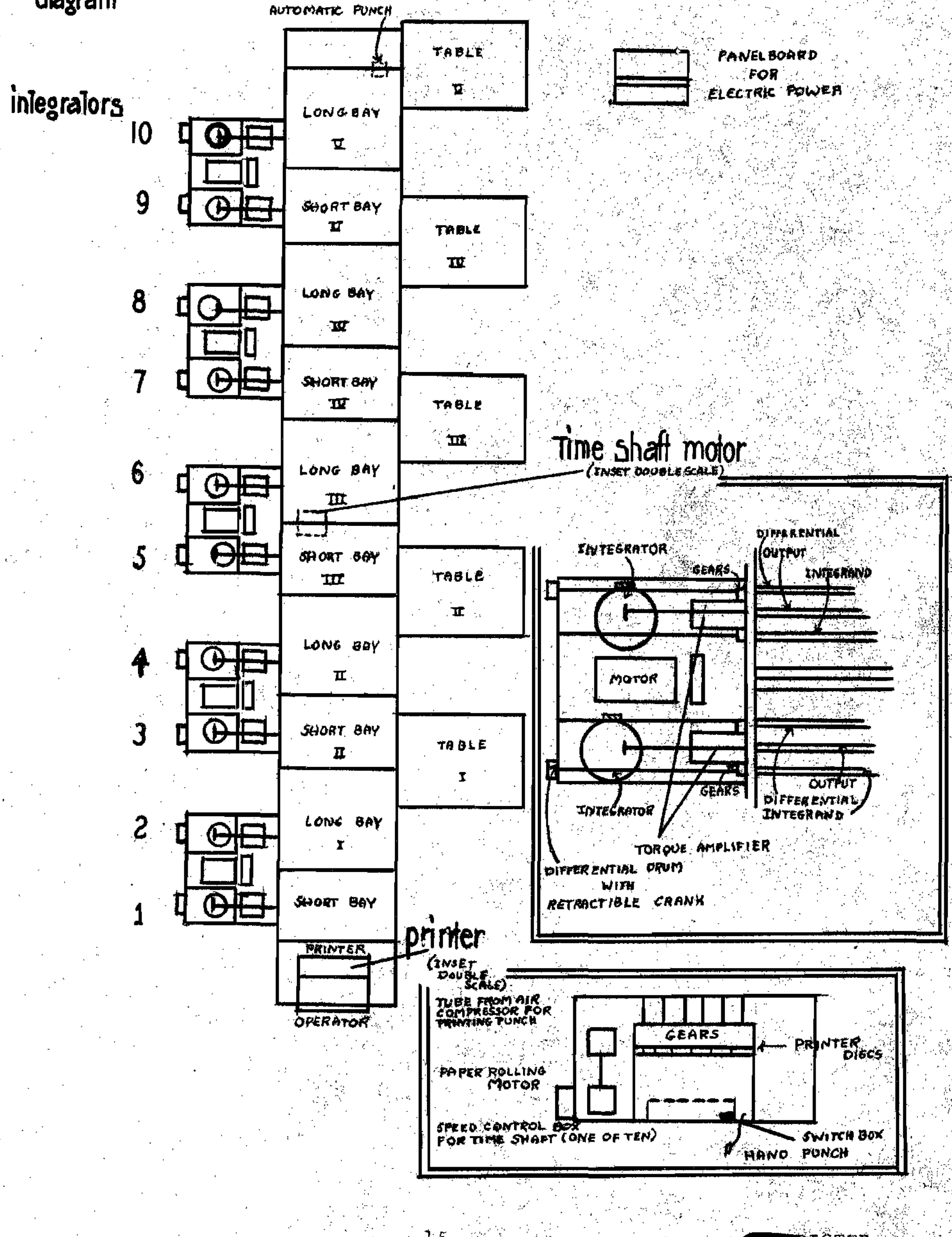
DIFFERENTIAL ANALYZER
diagram



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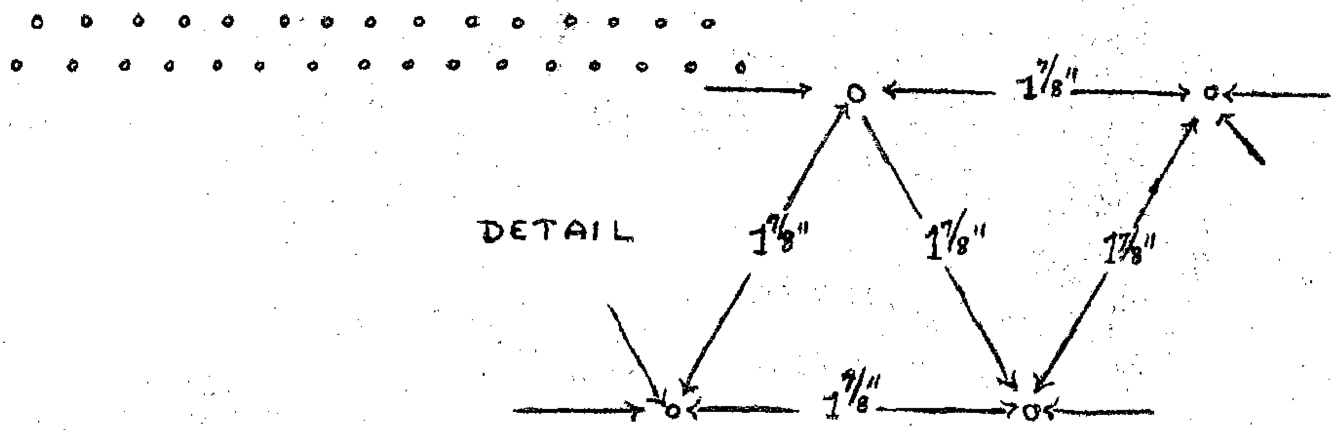
Various parts of the machine proper are firmly bolted to a single welded substructure which is a frame-work of structural steel elements on a concrete base. The body of the machine consists of ten separate "bays" alternately "short" and "long", separated by cross tie plates. These bays are of the same width (38 inches) and general design but differ in length. A short bay is 22½ inches long with three cross-shafts to accommodate a single integrator. A long bay is 31½ inches long with place for six cross-shafts, the first three (counting from the front) for the second integrator of a pair, and the other three for a "Table".

In each bay the border cross-beams are penetrated by 31 housings, 15 in an upper row, 16 in a lower row, with 1 7/8" between adjacent centers, both horizontally and diagonally, at 60°, thus



Fitted permanently in each housing and penetrating the cross tie plates separating adjoining bays is a short ½ inch "bearing shaft" or "fixed bus shaft" protruding 4-7/16 inches from the housing equally into both bays. Available for joining two longitudinally adjacent collinear bearing shafts are "bus shaft sections", 11½ inches for use within a short bay, 20½ inches for use within a long bay. Many steel sleeve couplings, each with two set screws, are on hand and many are used to join a bus shaft section rigidly with a bearing shaft. The other end of the bus shaft section is then either coupled similarly with the next longitudinally adjacent collinear bearing shaft, or is supported while revolving freely in a bronze housing, called an "end coupling", attached by a single set screw to the next collinear bearing shaft which may or may not be rotating under separate drive. A rigid longitudinal sequence, composed alternately of bus shaft sections and bearing shafts, constitutes a single complete bus shaft 1/2" in diameter, which may or may not extend the entire length of the machine. We shall later refer to gear couplings also.

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The machine is equipped also with auxiliary detachable overpasses or "viaducts", composed of one or more housings, for transmitting the angular rotation of a shaft through a train of spur gears at a level above that of the normal upper row of bearing shaft housings already provided in the regular cross tie plates.

As Dr. Bush remarks: "Any cross shaft can be readily connected to any bus shaft by inserting a spiral gear box between them, and one will then drive the other. Right and left-hand boxes are supplied in order to secure correctly related directions of rotation. One bus shaft, assigned permanently to the independent variable, is driven by a variable speed motor." (This motor is controlled by any one of ten control boxes located at various places on the structural frame as explained later.)

B. <u>The Calibrating Drums and Counters</u>

For the bus shafts and their interconnecting couplings, there is little occasion to consider the choice of an origin. The gearing determines relative rates of shaft rotation without regard to any additive constant. However the integrand for an integrating unit, and each of the variables on any table, as well as the recorder would be affected by the introduction of an additive constant. A calibrating drum with retractible operating crank is provided for the integrand with each integrator unit. It will be mentioned again in describing the integrators. For each of the input variables in each table, and for each of the six printer units arrangements are provided for making a correct initial setting for any desired run. The total number of revolutions of a lead screw for a table is registered by a counter, with a calibrating drum to indicate fractions of a complete revolution. In the case of the printer the printing discs can be set individually by hand, and there is also a knurled knob which may be pulled out to disengage all printer discs, and then turned to set all thirty of these discs simultaneously at zero.

C. <u>The Cranks</u>

There are three hand cranks, operated only for reduction of observed range firings, each of which transmits through a gear train and adder an imposed correction upon an otherwise automatically regulated dependent variable. Each

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crank is equipped with a locking device to avoid accidental input. When used, the cranks deal with slowly varying corrections, and more than one may be easily operated by a single person, who keeps one crank locked while feeding in a small change by another crank. It has been found convenient to introduce small finite increments in a regular staggered order, rather than to attempt to follow all three continuously. The adders associated with these gears are left in connection even for standard runs, in order that the loads may not be changed unnecessarily.

D. <u>The Prime Motor</u>

The prime motor is located under the main frame beneath Short Bay III. It is a direct current, one-half horse power, variable speed motor connected by chain and sprocket wheels (with a speed reduction to one-third) to the main drive bus shaft of the entire Differential Analyzer. The maximum rated operating speed of the motor is 1725 r.p.m., so that unless spur gears are introduced the main driving bus shaft should not attain more than 575 r.p.m. but in fact with current available will run at nearly 800 r.p.m. The speed is indicated by a tachometer visible to the operator. The normal running speed is often not more than 400 r.p.m. or less according to limits on the speeds of certain integrator units or to limits imposed by the automatic follower in maintaining contact during a steep part of the retardation coefficient curve. Speed control boxes, one at the operator's left hand and others on various parts of the machine, as well as automatic cut-outs, will be discussed later.

E. <u>The Integrators</u>

Quoting from the official Specifications, April 2,1934, we read

"Description of the Integrator

(a) The integrator consists of a massive carriage which is moved horizontally by means of an accurate lead screw. This carries a disc in a horizontal plane, which can be revolved independently of the carriage position. Resting on this disc and pivoted in jewelled bearings is a wheel or "roller" with its axis parallel to the ways and [with the roller] lying in a vertical plane through the center of the disc. The outer bearing of the roller shaft is located directly over the point of contact. End play in roller-shaft and carriage pivots is removable by an adjustment. There are also provided adjustments by which the roller-shaft can be brought accurately to its correct position.

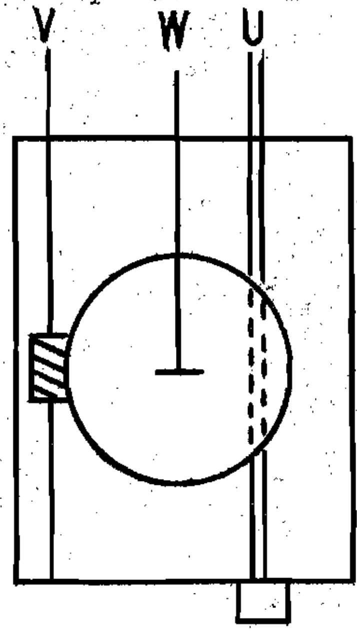
(b) The torque transmitted to the roller-shaft is transferred to the input shaft of a torque amplifier, a given rotation of the input shaft producing an equal rotation of the output shaft but with a greatly increased torque. The energy is supplied by an independent and constantly running (half-horse power) motor which drives a pair of drums in opposite directions.

(c) The ten (10) integrators are mounted in five (5) pairs on a single base, with one motor supplying the energy for both torque amplifiers........"

The several integrators although applied to different variables coincide in construction and manner of operation. At present (July, 1942) the first four pairs only are in active use. The roller is of hardened steel, ground and lapped, its edge is given a radius of about .002 inch. The jewel of the bearings is artificial sapphire rather than hardened steel. The integrator discs are of plate glass .250 ± .005 inches in thickness, with surface true to within .0005 inch. They are attached to their mounts by Dupont Household Cement. Each integrator unit is housed in a recessed but removable glass case provided with lifting handles and with removable metal cover for a small opening in the top sloping glass face. This opening permits the insertion of an operator's fingers for raising or lowering the small "fly" lever which disengages the hinge-connected roller from contact with the disc. Each integrator is connected with input and output shafts through spur gear trains and universal joints, the main translational input being through a magnetic clutch, and the output is stabilized through friction held dampers. The magnetic clutch may be released in each case by pressing a push button set in the steel structural frame below and to the left of the integrator concerned. The displacement of the disc may be hand-set at the beginning of any run by means of a retractible hand crank in the face of a rotating centesimally scaled six-inch brass drum placed on an extension of the main displacement lead screw. One complete rotation of the scaled drum is accompanied by a rotation through about a fifth of an inch peripherally of a secondary brass drum scaled (in partial arc) for 80 equal units extending from 40 units in one direction through zero to 40 units in the other direction. The shaft regulating the rotation of the disc (in a horizontal plane) is slotted, and bears a sliding helical gear working in a gear box which is attached to the carriage.

The theory of the operation of the integrator unit is essentially that of an ordinary planimeter, save that here the center and axis of the roller remain stationary while the horizontal plane (usually fixed but here represented by the disc) is given rotation and translation. Unlike the case of some uses of a planimeter, instead of using always a single closed circuit about a center, the integration here continues over an arbitrary succession of complete or partial rotations.

For a discussion of the theory the following schematic diagram of the top view of an integrating unit will suffice.



Integrating Unit
schematic diagram
Top view

Rotation of the shaft marked "u" imparts (by lead screw) translation to the (square) carriage and to the (circular) disc. Rotation of the shaft marked "v" imparts rotation (in the horizontal plane) to the disc. Resting on this disc is the circular roller (of fixed position in space) whose vertical projection only is shown. This roller acquires rotation through friction contact with the disc, this rotation being transmitted back into the main machine through its axial shaft marked "w" as the output of the integrator. For a small angle of the main drive of the whole machine, u (calibrated to

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measure displacement of the roller from the center of the disc) remains approximately constant, the disc turns through an angle proportional to dv, and the output shaft turns through an angle dw, proportional to udv since the peripheral travel of the roller (this being of fixed radius) varies jointly with the angular displacement, dv, and the radial distance, u, of the point of contact. Thus $Kdw = udv$ or $Kw = \int udv + C$. The calibration may be adjusted so that C is either zero or some other desired constant. The scales are so fixed that with u set as constant and equal to 32, each complete rotation of the v shaft is accompanied by one complete rotation of the w shaft. Hence $K = 32$.

The lead screw on the integrand shaft (u-shaft) which regulates the travel of the disc carriage is threaded to give one inch of travel to ten revolutions. Its threaded portion permits travel from -40 to 40 revolutions from the central position, or a radial displacement of 4 inches.

Experiment with the Analyzer in its present form seems to show that an integrator unit should not be run at a speed which will cause the output shaft to rotate faster than 600 revolutions per minute, due to limitations of the torque amplifier

As to the arrangement of integrator units, experience with undesired vibrations suggests that it would be wise, other things being equal, to place close together any two integrator units, if the output of one is to feed in as differential input of the other. This is the principle observed in numbering the integrator units in this Report.

We may remark for those unfamiliar with the machinery that the lead screw is a shaft threaded to constitute a long screw. This screw passes through a nut which in turn is attached rigidly to the carriage which would otherwise be free to slide on ways parallel to the lead screw. Rotation of the shaft causes the nut (and carriage) to move along the ways a distance proportional to the rotation of the shaft. In the Differential Analyzer many of the lead screws are threaded to give one inch of translation to twenty revolutions, while others give one inch of translation to ten revolutions.

F. The Tables

There are five Tables situated along the right side of the main frame. These tables differ among themselves according to their purpose. They are rectangular in form.

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All except the third are (now) kept tilted desk-wise, and are easily observable by the operator of the main machine. From the original specifications we may quote the following:

"The ...... Table has a lead screw which moves the carriage (horizontally) in the direction of abscissas. The carriage extends perpendicularly across the table in the direction of ordinates, and on it moves a slider carrying a pointer. A second lead screw located on this carriage moves this slider and is driven through a pair of spiral gears by another cross-shaft of the device. This second device is also controlled by a crank. Thus if the first shaft is connected to the machine so as to revolve proportionally to a certain variable and if the crank is turned so that the pointer always registers with a curve placed on the table, the second cross-shaft may be connected to pass out into the analyzer a function represented by this curve and with this variable as argument."

The pointer with its auxiliary reading glass is now replaced in the three input tables by an automatic follower to be described later. At Aberdeen each Table was also equipped with a knurled lock collar for engaging and disengaging the shaft controlling the motion of the slider in the carriage. The motion of the carriage is in most cases no longer controlled by a single lead screw, but more rigidly by a pair of synchronous lead screws (at twenty revolutions per inch), one along the near edge and one along the far edge of the Table. In the Vector and Division Tables this pair of lead screws also is controlled by a crank on the far lead screw with attached drum and counter, the drum to show partial revolutions, the counter to show the number of complete revolutions of the lead screw and thus the travel of the carriage; in each of these tables also there is a corresponding counter and drum for showing travel of the slider. The hand crank for the slider has an adjustable gear ratio box with ratios respectively (from the outside) of 1/1, 2/1, 0/1, and 1/2. Adequate means of lubrication are of course now provided at all necessary points. The range of the abscissa is 480 revolutions, of the ordinate 360 revolutions.

The "Division Table" and the "Vector Table" are essentially modifications for use with automatic followers of the "Polar Input Table" designed by Dr. Bush for many purposes and described by him in part as follows:

"An important addition which greatly increases the convenience and flexibility of the machine is a combined polar

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input table and multiplier. The polar table consists essentially of a large circular plate which can be placed in position on the platen of an input table. This plate can be turned by means of an additional cross shaft by a worm and gear, so that its angular movement is proportional to the revolutions of the cross shaft. The connection as a multiplier will be treated below. There is a handle which is geared to this third cross shaft so that it may, when desired, be turned manually."

"The worm gear drive may be disconnected and the second form of drive mentioned above used in its stead. The circular plate is now replaced by a bar having on its face an accurately scribed line passing through the center. ...... In this arrangement there is a lead screw placed parallel to the axis of abscissas and driven by the cross shaft previously connected to the worm and gear. On this lead screw travels a carriage with a swivelled bearing, and through this bearing passes a rod firmly attached to an extension to the shaft carrying the bar, and located perpendicular to the axis about which the bar revolves. Evidently with this arrangement the revolutions of the third cross shaft will be proportional to the tangent of the angle turned through by the bar. Call the revolutions of the third cross shaft from the position in which the rod is perpendicular to the lead screw z, and the revolutions of the other cross shafts from the positions in which the pointer lies at the center of the plate x and y. If the manipulation is such that the pointer remains always on the diametrical straight line, we have then

$$y = xz$$

with proper proportionality factors not now considered. There is thus available a multiplier. If x and z are driven from the machine, and y is controlled manually, a product of two variables is obtained and passed out to the machine. Either x or z can go through zero and take on negative values. If we control x and y by drive from the machine, and control z manually, a quotient can be obtained. Of course in this case x can not go through zero unless y does simultaneously. The choice of scales, and scale changes during a solution when necessary, will maintain the pointer normally at a considerable distance from the center of the plate. This matter of scales always needs to be considered in connection with the question of precision in the use of an input table or multiplier. The maximum angle with the axis of abscissas through which the bar can turn when the multiplier arrangement is in use is a little over 45 degrees. When this is

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inconvenient the bar may be unclamped from the rest of the mechanism, rotated through 90 degrees and then clamped again so that the straight line occupies a position perpendicular to the lead screw when $z = 0$. We have then an interchange of variables, so that

$$x = yz."$$

The "Division Table" remains essentially as thus described by Dr. Bush. However to prevent destructive jamming due to passing reasonable limits of travel, an automatic cut off switch breaks the current to the main drive motor when the swivel nut reaches an end position. A further modification was required for the Vector Table as designed by Dr. Dederick. The grooved bar which directs the follower in the Division Table is now replaced by a slotted guide bar placed as in the Division Table on a rotating pedestal turning in ball bearings in the face of the Table. The slotted guide bar carries a sliding nut (or cross head block) capable of moving radially from the center to the end of the guide bar but controlled by a lead screw, itself driven by a flexible cable drive. This cable drive is turned by the magnetic clutch motor regulated by the follower. There is recessed in the sliding nut a raised edge on which the follower rides. In the Vector Table the abscissa is the horizontal component of projectile velocity with respect to the air and the ordinate is the vertical velocity component. The radial displacement fed back to the machine is the velocity with respect to the air, (distinguished from velocity with respect to the ground only in the case of reduction of observations, or to take account of the earth's rotation).

It may be of interest to contrast briefly the two tables, the Division Table and the Vector Table. In each case there is an abscissa (fed by the fifth cross shaft of a long bay), an ordinate (fed by the fourth cross shaft), an angle of orientation, and a radial distance. In the Division Table orientation is controlled directly by a lead screw acting on an arm below the face of the Table, the arm being rigidly connected with the bar of variable orientation. For this table it is the radial distance which is not recorded and remains unimportant. In the Vector Table the radial distance is controlled directly by a lead screw from the follower, while the orientation is not recorded and remains unimportant. In each of these tables the output of the table is fed back into the main machine through the sixth cross shaft of a long bay.

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In the Division Table, only the first quadrant is utilized. The maximum displacement of abscissa or ordinate (from the origin in the center of the table) is for 45° inclination. Hence the range for both the abscissa and ordinate is restricted to the interval from zero to 180 revolutions. The quotient lead screw is threaded at ten revolutions per inch of travel and is limited to little more than nine inches. The output is, however, geared at 4 to 1 so that the output shaft has a total range of 360 revolutions.

In the Vector Table, two quadrants are in customary use, since the vertical velocity component may be of either sign, while the horizontal component remains essentially positive. The horizontal component is in practice restricted to approximately from 0 to + 180 (in revolutions), the vertical component may range from -180 to + 180 (in revolutions). The radial distance is subject to the same limitation in absolute value, with maximum at 180 revolutions.

The third table to be mentioned is the third and last input table. It is the "Template Table". It carries securely fitted a 3/32 aluminum template (changed when needed) upon a 3/16" aluminum base plate, and so cut out that its edge ridge represents (save for 1/8" clearance) the appropriate drag function $B_1(u)$ where u is the "adjusted speed", (discussed later), and $vB_1(u)$ is the air resistance function appropriate to the projectile considered. This table has a single input, u, and has $B_1(u)$ as output. The automatic follower on this table, on reaching the position of maximum ordinate , closes a buzzer switch. This buzzer serves to direct the attention of the operator to the fact that a steep part of the B-curve is being reached, and that the operating speed of the whole machine should be lowered in order that the follower may not fail to function.

For this table the total horizontal and vertical span of the table may be used, 480 revolutions (or 24 inches) horizontally and 360 revolutions (or 18 inches) vertically.

Another table is the Output Table, relatively little used at present. It is described in the official Specifications as follows:

"Description of the Output Table.

The output table is very similar to the Input Table except that there are on the carriage two sliders each driven by its own cross shaft and each carrying a recording

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pencil. The cross shaft which traverses the carriage is ordinarily connected to revolve proportionately to the independent variable, but may be connected otherwise. Any two chosen quantities may be simultaneously recorded by properly connecting the cross shafts. The two pencils are placed so that they record in the same vertical line, provision being made to allow one pencil to pass the other without interference."

When both pointers are used the independent variable is the same for both, the dependent variables serve to describe two distinct functions of the independent variable. Ordinarily when used at all, if the independent variable is T, the dependent variables might be X and X' or again Y and Y'. For X as independent variable one may plot the ordinate Y and the velocity V as functions of X.

As with other Tables, the fourth cross-shaft of the long bay moves a slider and pointer along the carriage in the direction of ordinates. The fifth cross-shaft moves the carriage in the direction of abscissas. The sixth cross-shaft may be used to move the second slider and pointer on the same carriage in the directions of ordinates for a second dependent variable.

The last table to be mentioned is now used for laying out templates. The abscissa is controlled by a hand crank on the far edge of the table and turns the fifth cross-shaft of the long bay. The ordinate is turned by separate hand crank at the near edge and transmits rotation to the fourth cross-shaft. The sixth cross-shaft is removed or remains idle. This Table is not interconnected with other shafts of the main machine. Counters and drums are attached to the two cross-shafts used, and with their aid, the pointer locates accurately points to be pricked on a blank template sheet, and to serve as guide points in marking a curve along which the template is to be cut. The cut is made with 1/8 inch clearance from the guide points.

The large circular plate for polar input, which came as part of the initial equipment of the Differential Analyzer, is at present detached, as are the reading glasses designed to follow the pointers on input tables when these are operated manually.

G. The Adders

By an "adder", indicated in diagrams by the symbol Σ, is meant, as suggested previously, any sort of what is

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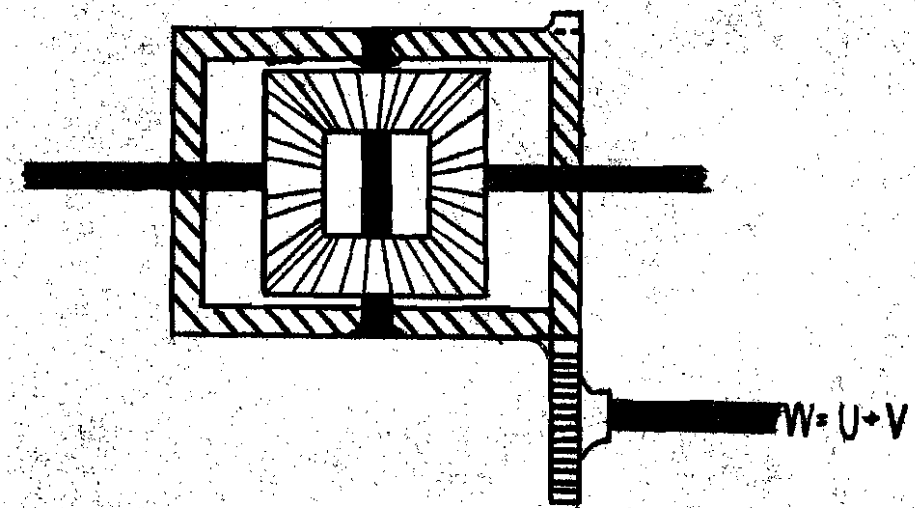
commercially known as a "differential gear" which can be connected to three shafts so that the number of revolutions of one will be the sum of the numbers of revolutions of the other two. To avoid possible confusion with a differential input "dv", we shall regularly use the word "adder". In the Differential Analyzer at Aberdeen, the adders are all of a common pattern-- a beveled gear type. The two terms to be added are fed as rotations into the adder housing by collinear coterminal bus-shafts, the sum being given (with reverse sense of rotation) by a bus-shaft parallel to and coterminal with the shaft extending beyond that of one of the two given terms. A schematic diagram is as follows:



The housing carries two 45° beveled transmission gears meshing with congruent gears on the abutting ends of the input shafts. It thus acquires a rotation equal to the arithmetic mean of the rotations of the two collinear but oppositely extending input shafts. The rotation of the housing is transmitted through an external gear by a $-\frac{1}{2}$ ratio to a parallel shaft which thus rotates (in reverse sense) as the sum of the rotations of the input shafts.

It may be noted that the algebraic relation is entirely symmetrical in the three variables. One has $u + v + w = 0$, when sense of rotation is considered. Associating the externally geared variable with the output, $-w$, is a matter of mechanical procedure.

H. The Gear Couplings.

As Dr. Bush remarks, "There are provided also sets

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of spur gears which can be connected between adjacent shafts so that one will drive the other with a chosen ratio of speed. ..... These last units are inserted as needed in the body of the Table."

There is a normal direction of forward drive for the prime motor which is said to give a positive rotation for the initial bus shaft chain-geared to it. Any other bus shafts will be regarded as rotating in a positive sense when it rotates in the same direction as the initial shaft. It will be recalled that all bus shafts are parallel so that sameness of rotational direction has an unambiguous meaning. Two rotating bus shafts spur-geared together by a single pair of gear wheels will necessarily rotate in opposite directions since enmeshed gear teeth are instantaneously moving in a common tangential linear direction. If one is rotating positively, the other is rotating negatively. The ratio of rotations is then negative.

In technical notation the spur gears are $14\frac{1}{2}^{\circ}$ involute, 24 diametral pitch, $\frac{1}{4}$ inch face, .5000 inch bore with $\frac{1}{2}$ inch hub on one side of 1" diameter. Each has one $\frac{1}{4}$ inch set-screw 28 Tap, located $\frac{1}{4}$ inch from the hub end of gear. These spur gears come in intermeshed pairs with 1 7/8 inches between centers throughout, and hence (with 24 diametral pitch) have a total of 90 teeth for the matched gears of a pair.

The reference to "involute" concerns the shape of the teeth. The diametral pitch indicates the number of teeth per inch of pitch diameter, where pitch diameter is the diameter of the pitch circle passing through the contact points on the line of centers between two enmeshed gear wheels.

The gears hitherto used have been practically confined to the following list:

Teeth on "larger" wheel	45	50	54	60	72
Teeth on "smaller" wheel	45	40	36	30	18
Coupling Coefficient	-1	-5/4	-3/2	-2	-4

There are also a very few "special" gear pairs, namely some with coupling coefficients -32/13, -11/4, -7/2. In Dr. Bush's article one reads, "The binary system has been adhered to in nearly all gear ratios so that spur gears which may be introduced to connect adjacent shafts are supplied in ratios -1:1, -1:2, and -1:4." Considerations of mechanical strength limit the choice to gears of not less than 18 teeth. To make available a reasonable measure of the permissable

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ranges and loads, it was necessary to drop the rigid adherence to ratios in the binary scale and to introduce coupling coefficients, -5/4 and -3/2 freely. A further study directed toward possible utilization of other gear ratios within the same general setup of a total of 90 teeth per pair has recently led to the ordering for regular use of the following combinations also:

Teeth on larger wheel	46	48	55
Teeth on small wheel	44	42	35
Coupling coefficient	-23/22	-8/7	-11/7

A table of the fractions, decimals lying between .5 and 2.0, which may be secured as coupling coefficients by use of at most three gear pairs (with a change of sign) appears appended to this Report. For decimals outside of this range, a power of 4 may be introduced.

I. The Recorder

The recorder is described as follows in the original Specifications (March 15, 1935):

"Description of Recorder

(a) ...."Provision is made for tabulating successive values of several variables as functions of one of these variables (for equal intervals of the variable). The recorder provides the means by which this tabulation is accomplished.

(b) "The recorder consists of a battery of six (6) five digit counters each of which is connected by a flexible shaft to that counter shaft of the analyzer which is generating continuously the values of the variable whose values it is desired to tabulate. The counter wheels are provided with raised numerals. Opposite the counter wheels and operated independently of the counters is a battery of air operated pistons. ..... By means of these pistons an inked impression is taken of the counter readings on a continuous roll of paper.

(c) "The charge and discharge of the compressed air which actuates this printing device is controlled by a Master Printing Cylinder. A master piston is held against a compressed spring by means of a latch stud. .... There are three valves operated by a cam shaft so that when

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two valves are open the third is closed. When these two valves are open, the valve in the right (when reading the drawing) permits a charge of air at approximately 45 pounds per square inch to fill the upper air chamber, while the middle valve releases the back pressure in the piston. Simultaneously with the opening of the two valves, the electrical control sends current through the solenoid which releases the latch stud and the piston is driven to the left by the release of the spring. One half revolution further of the cam shaft causes the third valve to open, thus admitting compressed air to the head of the master piston, and restores it to its cocked position in readiness for the next cycle.

(d) "Provision is made for moving the paper and inked ribbon after each printing operation."

The electrical equipment installed on the recording counter is so designed that the recording action of the unit becomes automatic after a recording impulse is furnished either by an operator or automatically by the cam contact located at the far end of the Differential Analyzer.

Continuing from the Specifications, we read:

"A system of relays is provided for this purpose so arranged that upon receiving the impulse, the relay system acts first to exclude the effect of an additional impulse before the operating cycle is completed, then to actuate the trip solenoid of the recorder, then to actuate the motor and clutch operating the resetting mechanism, and then to restore the relay system to neutral so as to be ready for the next impulse.

"The power supply for the recording counter will be obtained from an individual transformer and rectifier in order to prevent polar operation of the main control circuit due to surge set up by the recorder. The transformer and rectifier tube are mounted on the main control panel. A suitable time delay relay is mounted to prevent operation until a rectifier filament is heated.

"An outlet panel is mounted directly on the recorder together with the relays controlling the recorder. This panel has a main power switch and its associated indicating lamp and three groups of contact positions, each of which contains a switch, indicating lamp, and two prong-outlet. Each of these three groups provide positions from which the recorder can be actuated. It is only necessary to plug into the outlet of a

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group any contacting arrangement which will close a circuit when the recorder is to be actuated."

The counter wheels and the punch although functioning together as a recorder are mechanically essentially independent. The counter wheels of the printer continue their rotation yielding predetermined intervals of either X or T as may have been previously decided. This action is entirely mechanical. On the other hand the punch with gear-operated cam transmits electrical impulse to operate the impression pistons, themselves driven by compressed air. The printer has shown remarkable efficiency. It makes a clear impression even from a type wheel moving as fast as 400 r.p.m.

The operator has a hand punch (like an electric call button) and uses this (at present) to print a further special record of all the variables at three positions: at the start, at the summit, and at the end of a trajectory. The summit occurs when $Y' = 0$, and the operator watches the printing wheels for the moment this position is reached. Similarly, the end (level point) is found when the $Y = 0$ position is reached. Automatic devices for activating the punch at these positions were made but have been discarded.

An important part of the operator's general task is to regulate the speed of the main drive of the machine according to the varying output of the tables or the performance of the printer. This control is exercised through a variable speed control box located at the left of the printer.

The compressed air is furnished through flexible tubing from the machine shop on the floor below. Early experience with rust indicated the need of an air dryer in connection with the air compression. The compressed air was formerly furnished through an air compressor of capacity 1.2 cubic feet per minute, driven by a quarter horse power motor. The motor was controlled by an automatic switch which opens at 150 pounds per square inch and closes at 130 pounds per square inch. The guage indicated pressure up to 200 pounds per square inch with graduations every 5 pounds per square inch. At present this special condenser is relegated to emergency use, since in the new Ballistic Research Laboratory building, there is a "house" compressor furnishing compressed air by fixed tubing throughout the machine shop, and in particular to the dryer for the recorder.

A reducing valve with guage to indicate pressures at every 5 pounds per square inch up to 100 pounds per square inch furnishes pressure to the recorder at approximately 45 pounds per square inch constantly.

At present the roll of paper is placed below and beyond the printing counters. It is kept slack by hand. The slack sheet is fed by motor through the printer and over a brass tray toward the operator. A hand-pressed cutter serves to cut the paper at a convenient place after a trajectory is completed. A hand knob is used to start the new sheet at a convenient distance from the bottom of the sheet. Each printed line is placed above that last printed. At present all printed results are in "machine units" and must be multiplied by appropriate "conversion factors" to obtain results in standard units, feet, seconds, etc. To aid the operator, the printing wheels are provided not only with raised numerals for printing but also with a set of easily read impressed numerals placed at a 90° angle of rotation for the raised numerals, so that numbers about to be printed match with those which show in large numerals in front.

The variables (in machine units) now printed are T, X, Y, V, Y', X', each to five digits. For each variable the right-most digit is printed by a continuously rotating disc (as distinguished from the other four digits each of which remains stationary except for instants when it is engaged and is turning to the next number), and so usually appears out of alignment, and with trace of an adjacent number showing. Hence one can estimate roughly a sixth digit by visual interpolation.

J. <u>Some auxiliary mechanical devices to eliminate accidental errors</u>

As Dr. Bush remarks concerning the Analyzer, "The attainment of precision in a device of this sort granted sound mechanical design and accurate construction is largely a struggle with the problems of back lash and integrator slip." The use of ball bearings and appropriate bushings, suitable lubrication arrangements, accurate rigid alignment of bus shaft housings, universal joints between approximately aligned shafts, free fly-wheels ("damper wheels") attached by slight friction (applied by springs or felt) to reduce fluctuation in angular velocity, the use of synchronized lead-screws,-- these and many other mechanical devices for securing smooth running will be passed over without comment. Nor shall we discuss the operation of motor or magnetic clutches. Mention may be made however in particular of (i) back lash elimination, (ii) the torque amplifier, and (iii) the automatic follower with magnetic clutch motor drive.

(i) Back lash elimination. As Dr. Bush remarks: "Back lash in the lead screw drive is almost completely removed by using two nuts on the screw, with spring-backed wedges between them forcing the nuts apart, the wedge supports being rigidly fastened to the carriage. By proper choice of the wedge angle this scheme gives a positive drive in either direction while allowing the slight irregularities of the screw to be taken up without binding." This device is called a "lashlock". ..... "The same idea is utilized in the disc drive. On the disc shaft (of the integrator) is a spiral mesh gearing with two spiral pinions located in the carriage and driven by two symmetrically disposed horizontal splined shafts. ...... These shafts are coupled by a pair of spur gears, and at this point is introduced the lashlock. The lashlock-wedge tends to separate the parts against which it presses in such a manner as to bring gear teeth into contact all the way around the double drive to the disc. ......

"A serious matter in any precision device, it (backlash) becomes especially important in this, because the interconnection of units often renders cumulative the error caused by its presence. The problem is acute when it is desired to produce complete flexibility, for the way in which shafts are geared together is then continually altered.

"The general scheme of attack has been as follows: Within the drives of integrating units themselves backlash has been substantially eliminated by the use of lashlocks. Elsewhere it has been held down to a small amount by careful construction, although a certain amount of clearance is imperative for the free running of any extensive system of shafts and gearing. In any specific problem the effect of a given backlash angle in a particular drive can then be reduced by specifying scales so that the shafts of that drive will make a large total number of revolutions in the course of a problem. In many situations this is sufficient. Finally, for important drives where the effect of backlash might be especially serious, there has been developed a unit which can be conveniently inserted in any such drive and which reduces the influence of backlash at that point to a second-order effect. This is, in reality, a unit having a negative backlash which can be adjusted to compensate for the positive backlash present in the drive into which it is inserted. It has been aptly termed a "frontlash" unit.

"The underlying idea is as follows: The unit, when connected into a line of shafting ordinarily furnishes simply a rigid driving connection from the in-going to the out-going shaft. When the direction of rotation changes, however, it immediately steps the outgoing shaft ahead in the new direction of rotation by an adjustable amount. This it does during part of the first revolution in the new direction. Hence for the balance of the revolutions up to the next reversal the net backlash in the total drive is brought to zero. This action is repeated at each reversal, so that there is always zero backlash in the shaft, except during the brief periods at each reversal of direction. The effect of backlash itself being small, the residual effect due to this transition period is negligible.

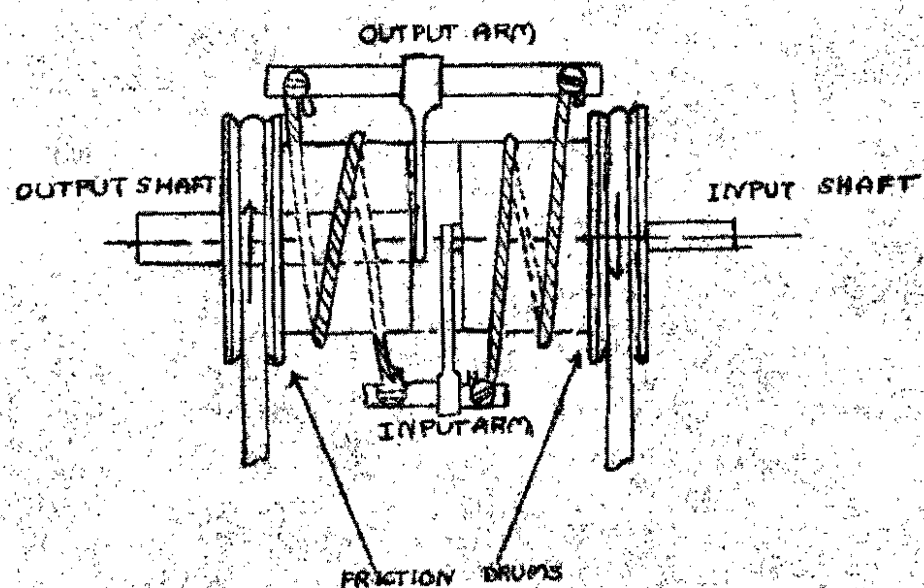
"When the drum is up against one of the stops, the whole mechanism rotates as a unit, slipping the friction band. The drive is then direct and none of the gears are rotating with respect to one another, all of course having a common rotation about the main axis. If the direction of rotation of the in-going shaft is now reversed, there is a period before the opposite stop is engaged during which the drum is held stationary by the friction band. During this period the out-going shaft is being driven through the planetary train in such manner that the out-going shaft revolves about ten per cent. faster than the in-going shaft. This continues until the opposite stop engages, when the drive is again one to one. Evidently the actual angular separation of the stops will be about ten times the angular backlash which it is desired to cancel, and hence can readily be set with precision. The friction torque on the drum need be but little more than ten per cent. of the transmitted torque. Hence the load imposed on the drive is not seriously increased by the presence of the frontlash unit.

"The usual manner of adjusting a frontlash unit is as follows: It is inserted in a drive, or singly-connected system of shafts and gearing, in which it is important that backlash should be removed, and usually near the output end of the drive. The drive is marked at each end so that angles may be accurately noted. The input end is then turned through several revolutions in one direction, brought to an accurate mark and the output position noted. This is repeated in the opposite direction. The stops are then adjusted until there is no detectable difference in the position of the output end when a definite position of the input end is thus approached from the two directions of rotation. Gear ratios, if present in the drive, can readily be taken into account. Usually the input end of the drive will be at the roller of an

[redacted]

integrating unit. The frontlash unit is then set to cancel the backlash of the torque amplifier as well as that of the balance of the drive. It is of course not always necessary to cancel backlash in this manner. Where it occurs in a drive connected to an input table, for example, it may if necessary be compensated for by adjustment of the plot. Six frontlash units are available, and this appears to be a sufficient number for the present."

(ii) <u>Torque-amplifiers</u>. The problem of elimination of integrator slip, that is, of slippage of a sharp-edged rotating roller upon a horizontal rotating disc is largely one of reducing the load borne by the roller, while necessarily maintaining at high level the output of power on a shaft synchronously related to the roller shaft. The solution of this problem was achieved through the use of a torque-amplifier recently developed by the Bethlehem Steel Corporation. As Dr. Bush explains, "This is, in brief, a device having an input and an output shaft, and so arranged that, when the input shaft is turned, the output shaft will turn an equal amount, but with a greatly increased torque, so that a small torque applied to the input shaft is magnified and applied to a load. The energy is supplied by an independent and constantly-running motor which drives a pair of drums in opposite directions. The central idea involved is the use of friction bands wrapped around the rotating drums. A small force applied at one end of such a band produces a much amplified force at the other end, the ratio of these forces varying exponentially with the angle of wrap. The two drums provide for rotation in both directions. Figure 10 shows the basic idea somewhat diagrammatically.



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"A study showed that about a pound-foot of output torque from integrators would probably be ample, and that, in order that errors due to load might be negligible within the limits of precision aimed at, the torque supplied by the roller could not well be more than a ten-thousandth part of this. It was therefore necessary to develop the torque amplifier along the lines of very high torque-ratio and very low input-torque. A two-stage device was found necessary in order that the bands on the first stage might be exceedingly flexible and thus properly controlled by small forces, while the bands of the second stage might be sufficiently rugged to carry the load. The output of the first stage simply moves a pivoted lever which in turn supplies the input to the second stage. The final model is shown in Fig. 11. The rotating drums are belt-driven in opposite directions, and are stepped to furnish two small-diameter friction surfaces for the first-stage bands, and larger ones for the second-stage bands. There are two concentric shafts. The internal shaft is the roller shaft, held in a light bearing mounted in the outer shaft, and furnished with an arm which projects through a hole in the latter. This arm carries the input ends of the first-stage bands, which are actually pieces of silk-braided fishline, the output ends of these bands being connected to the pivoted lever mentioned above. The second-stage bands are heavy cord, with their input ends connected to this same lever, and their output ends attached to a ring carried rigidly on the output shaft. One band in each stage is wound clockwise and the other counterclockwise. Every band is given two and one-half turns wrap. There is a very slight initial tension in all the bands, and the effect of a movement of the input shaft is to loosen one pair and tighten the other. The relative angular motion between input and output shafts is only a very few degrees even when carrying full load. This angle has been made very small indeed in certain designs produced by Nieman, but in the design for use with the integrator, where the problem of extremely small input torque is paramount, it has been found advisable to allow it to be several degrees. The error which might be caused by this small play is avoided by" the use of the vibration damper. This is "a relatively massive fly-wheel on the output shaft. This is loose on the shaft and coupled thereto only through a slight friction introduced by pressure on a felt washer. During uniform rotation the fly-wheel simply rotates with the shaft. If there are oscillations, it slips and introduces a damping force. ...... Its presence does not simply limit the amplitude of oscillations; rather it prevents them from starting at all. This scheme long used for limiting the torsional vibration of machinery here performs a new function in preventing the initiation of self-oscillations."

[redacted]

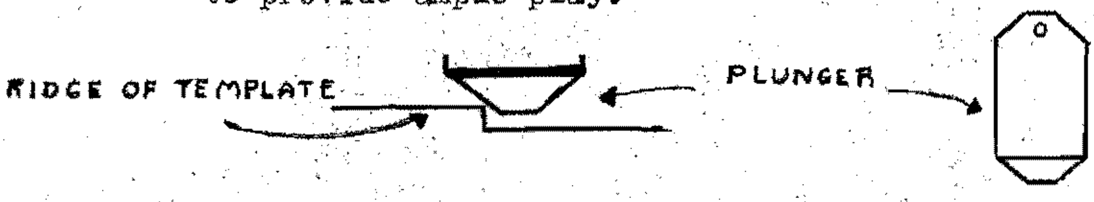
One may remark that the replacing of broken or worn straps in the torque-amplifier is one of the more common unavoidable causes of shut down of the machine. The proper insertion of a new fish line involving overpassing and underpassing and the tying of end knots in the interior of the mechanism is one of the small fussy jobs consuming much time in the aggregate.

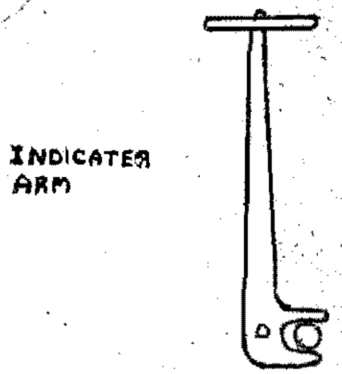
(iii) <u>The automatic follower with magnetic clutch drive.</u> The automatic follower with magnetic clutch drive consists of (i) the follower proper to be described below, (ii) of a constantly running motor, which drives on one shaft (iii) two cylindrical insulated wire spools (with brush connections) with each of which as an electro-magnet is associated (iv) a cast iron disc as contact armature, (with thin interposed demagnetizing copper disc) carried freely on the main shaft and free to move laterally through a slight displacement. In the other face of the disc are set four pins which engage in holes in (v) a 27-toothed brass sprocket wheel carried on ball bearings on the shaft and held laterally in place by a sleeve on the shaft. Ordinarily both sprocket wheels remain idle. When the follower makes either one of its two contacts, the corresponding one of the spools is magnetized, the cast iron disc (with interposed demagnetizing copper disc) is drawn into contact, and through the pins, drives the associated sprocket wheel until the magnetic contact is released. The sprocket wheels are connected by chain drive to a common lead screw, one chain giving direct drive, the other through a half-twist in the chain, carrying reverse drive. Thus when either sprocket wheel is being driven the other is carried without load at the same rate in the opposite direction and is not directly connected with the motor.

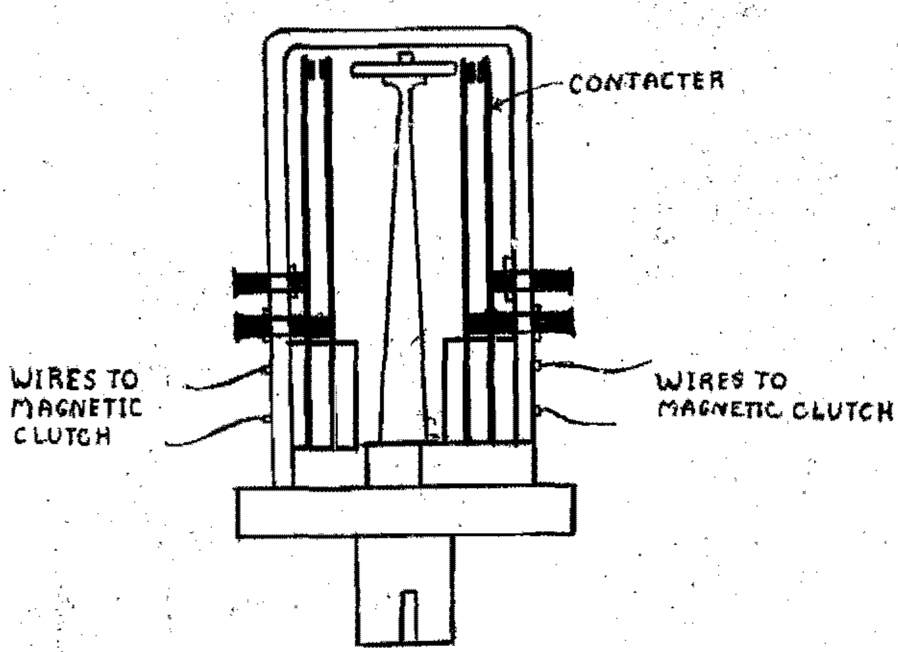
The follower proper which with its bent steel guard stands some 6 inches high on a slider which latter moves on a "vertical" carriage along parallel guide bars by action of a lead screw. The carriage as a whole moves horizontally by synchronized lead screws, one at its near edge ("bottom") and the other at its far edge ("top"). A cylindrical brass base contains a thick cylindrical plunger whose lower end terminates in a blunt truncated cone (of some 120° vertical angle). The plunger working under gravity alone is pivoted upon an indicator arm which has a lateral fork engaging with a fixed pivot bar. The other end of the indicator arm carries a bakelite disc. As the plunger rises or falls beyond narrow limits contact is made on one side or the other by the bakelite disc pressing a thin vertical conducting metal strip with copper knob against another such strip. Adjustment screws with insulated tips regulate the position of the conductors and hence govern the sensitiveness of the follower.

[redacted]

In operation the cone of the plunger rides on the ridge of a template (or platen) and the plunger center traces a path 1/8" from this ridge. The template is 3/32" thick. This is found to provide ample play.







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K. The Control

The Specifications (of June5, 1935) state(with slight verbal modifications) the following:

The main control unit consists of a set of panels as described below mounted on a standard steel relay rack, designed for floor mounting at any convenient location.

A power and indicator panel contains the rectifiers and transformers used to supply direct current for motor drive, relay and contactor operation, clutch operation, recording counter operation, and motor field current. This panel contains also the necessary time delay relay and circuit breaker for rectifier protection, and a group of colored indicator lamps for showing the operating status of the Differential Analyzer and also for indicating false operation and circuit faults. These indications are all automatic and require no attention except to replace burned out lamps.

The relay panel contains a completely interlocked system of relays through which control is established. The interlocks are designed to permit proper sequences of operations and to exclude all others so that no accidental or deliberate mismanagement can cause improper operation or damage to the control or to the Analyzer. Furthermore the design is such that in case two or more operating signals are given, the relays will choose the more conservative action. That is, the speed will decrease rather than increase and the machine will come to a full stop in all situations which demand it. The relays are arranged so that external limit switches will actuate them.

The contactor panel contains all power contactors which operate the circuits directly to the motors. These contactors are actuated by the relays.

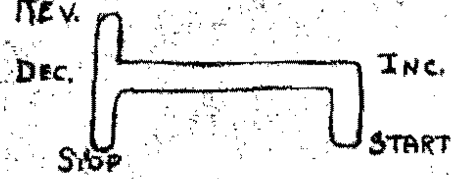
A speed control panel contains a motor driven variable voltage transformer to obtain smooth speed change without the undesirable jumps which are inherent in the push-button method of control. This motor-driven unit includes a magnetic clutch to prevent overshoot and a contactor drum which is interlocked with the relays to prevent excessive speed, sudden starts, and improper control sequence.

Main power switches are mounted on a separate panel.

[redacted]

The relay rack contains the equipment required for automatic speed control.

The control boxes provide for complete control at the following locations, at each table, at the recording counter, and between each two pairs of integrators. The design of the control boxes is such that all control operations are secured by moving a single handle into various easily remembered positions with natural motions. They are mounted by lugs directly on the structural steel of the Analyzer. The design of the slot in which the handle operates is as follows:



The abbreviations stand for "Increase", "Decrease", and "Reverse". The "Start" position puts torque amplifier motors in operation; the "Increase" position starts the main (Time) drive, and is used to increase up to its fixed maximum, the speed of this main drive.

The torque amplifier motors are further provided with thermal cut-outs. These are equipped with special switches so arranged that in case the fuze burns out and stops the motors, the main drive of the machine stops automatically also.

Two tachometers are attached to the main frame, one is geared to the main drive, and thus registers increase in T; the other is geared to the X variable. These are clearly visible to the operator who watches them and who when necessary reduces the speed of the machine through a hand-operated control box. The maximum rated speed of the time shaft, connected by chain drive with the main variable speed motor is 575 r.p.m. but as noted earlier a speed of nearly 800 r.p.m. is obtainable. The X shaft tachometer is chiefly of interest in helping to maintain approximately a suitable constant speed in this shaft for accurate printing of equal X-intervals for even integral values (of X) through the manual operation of the control box to vary the rate of T as may be demanded by the particular part of the particular trajectory concerned.

As auxiliary equipment of the Differential Analyzer is a stroboscope, little used currently. It may be used to compare rotational speed of an integrator disc with the speed of the time shaft. To accommodate this equipment is a small fibre spool with a small strip of brass set longitudinally in its surface. This spool is geared to the time shaft. Contact points resting upon the spool form a switch which is closed momentarily with each passage of the brass strip under them.

Part IV

THE DIFFERENTIAL EQUATIONS

A. Equations in terms of physical variables.

1. Physical Theory for Standard Conditions.

This chapter will deal with the problem of obtaining the equations for the trajectory in a form suitable for numerical computation.

Briefly described, the trajectory computation in the preparation of firing tables for a given combination of gun, mount, and ammunition involves two parts: Reduction of observed range firings, and computation of standard trajectories. Range firings are conducted under reasonably favorable conditions (approximately standard conditions) with elaborate precautions to avoid unnecessary deviations and with extensive observations of those deviations from standard which it is not possible or practicable to avoid. These include in particular meteorological deviations. Groups of rounds are fired, the several rounds for a given group being at a common elevation (neither vertical nor horizontal) and with ammunition as nearly uniform as conveniently possible. Furthermore to reduce effects of changing weather conditions, the rounds of a program are fired in as rapid succession as is judged consistent with accurate observation. Recordings of meteorological conditions at the ground (located in Aberdeen, essentially at sea level) and in the air aloft are made during the time of firing. An average, based on the rounds of a group, is obtained at the level point, both for total horizontal range and for total time of flight. Proper allowance is made for effects on muzzle velocity required to reach stable firing conditions by use of preliminary warming shots.

The primary objective of achieving a firing table, which will give substantially predictable total horizontal range and total time of flight, requires, at least at certain selected angles of elevation, very little intervention of ballistic theory, especially if substantially standard conditions prevail during the range firing program. One need do little more than observe, record, and average. Ballistic theory plays its important role for the field service in enabling the battery commander to predict the magnitude of the effects due to changes from standard, and reciprocally in enabling the computing staff at the Proving Ground to eliminate the effects of the observed deviations from standard conditions reported for the current range firings. The effects due to nonstandard conditions are small compared to the two empirically reproducible major items: total horizontal range and total time of flight, both measured at the level point. Hence it seems entirely satisfactory to incorporate in the theory approximations which might produce intolerable errors in these two major items (were not the ballistic coefficient and an additive constant for the retardation coefficient adjusted empirically). Since also the magnitude of some of the effects is impracticable to measure directly with convincing accuracy, one is content to secure theoretical estimates.

The actual trajectory is the path of the center of gravity of the projectile from the instant of departure to the terminal point, ordinarily chosen as the "level point", which is at the same altitude as the gun muzzle. In dealing with the trajectory, one is interested not only in the geometric arc traced by the center of gravity but also in the position of this centroid on this arc at all times from the instant of fire to the instant of impact, assuming that impact occurs at the level point.

There are many known complicating features which are conveniently treated as perturbations superimposed upon a simple idealized trajectory. A few of these complications and corresponding practical simplifications of theory adopted are as follows:

(1) Due to the action of the muzzle blast, the actual projectile continues to acquire energy from the powder gases for a short time after leaving the muzzle. For the idealized trajectory there is a nominal muzzle velocity adopted, somewhat in excess of the maximum actual velocity exhibited. Its magnitude is chosen so that the motion of the projectile after it leaves the muzzle blast would be reproduced if the projectile were regarded as endowed with this nominal velocity at the position of the gun muzzle and were thenceforth free of the muzzle blast.

(2) The actual projectile exhibits at the level point a noticeable drift out of the vertical plane containing the initial element of the trajectory (the plane of departure). The idealized trajectory lies in the plane of departure and may be regarded as approximating the projection on this plane of the actual trajectory. In setting up the differential equations of motion of the idealized trajectory, the ignoring of all lateral forces introduces an obvious but yet negligible error in the components of projected motion within this plane.

(3) The actual projectile is a rigid body with a geometric (and dynamic) axis of revolution. The projectile is spinning in an imperfect gas which resists its motion. The initial yaw, the gyroscopic reaction, the downward curving of the trajectory, the partial loss of spin, together produce a complicated motion starting off with significant precession and often with perceptible nutation, and probably some helical motion of the centroid. Often also, especially for relatively high angle fire, some of these phenomena recur near the summit of the trajectory. For the idealized trajectory (for range firing) the projectile is treated as a massive spherical particle without phenomena of gyroscopic reaction, tracing a curve in a vertical plane, and with smoothly turning tangent. This assumption incidentally covers the previous assumption of no drift, but not conversely.

(4) The actual trajectory, when the range to the level point is not too short, involves for the determination of the level point acknowledgment of a noticeable curvature in the earth's surface. This curvature is accompanied by a divergence of vertical lines, a decrease of gravitational attraction with increase in altitude, and in general the situation suggests either the use of curvilinear coordinates or else the drawing of an explicit distinction between the plane horizontal at the gun, and the surface of constant altitude above sea level. Computation of the idealized trajectory makes use of rectangular coordinates, with parallel vertical lines, and treats the earth's surface as if it were a plane. Gravitational attraction is treated as constant independent not only of altitude above sea level, but also of geographical latitude on the earth. This assumption simplifies the form of the differential equations to be used. In practice, the procedure is not unreasonable. The data used in the equations to determine the values of numerical parameters are in fact obtained on the curved earth but the results of the completed firing table are to be used under entirely similar conditions. The discrepancy between simplified theory and fact is largely compensated for in the adjustment of numerical parameters. The desired (curved) ranges and times of flight are reproduced in any case. The errors in the differential corrections due to curved ranges are negligible. Perhaps for a large caliber gun the maximum ordinate as computed would be found to differ noticeably from the actual maximum ordinate if this could be measured. But such a deviation introduces in turn no serious first or second order corrections at the level point.

(5) During the firing of the actual trajectory, the earth rotates, producing earth rotation effects which depend not

only upon the muzzle velocity, ballistic coefficient, and quadrant angle of departure but also upon the geographic latitude of the gun and the azimuth of the plane of fire. Whenever these earth rotation effects are not negligible, they are taken into account explicitly and continuously in reducing the observed data on range firing. However once the character of the resistance law and the ballistic coefficient have been determined, such as to reproduce the observed total range and total time of flight, the further work on standard trajectories treats the earth rotation effects as a separable perturbation. A standard trajectory is computed and published as though the earth were stationary, and special computations provide subsequent corrections to be imposed for given latitude and given azimuth of plane of fire.

(6) The actual projectile experiences resisting force fields and force couples varying in direction and magnitude along the trajectory. The more penetrating studies of ballistic phenomena take explicit account of the varying yaw and of the rotating half-plane of yaw. The resulting air resistance tables are based upon measurements of observed results making use of various plausible (but not always experimentally established) assumptions. For the calibers of artillery ammunition involved, it is assumed that geometrically and dynamically similar projectiles for standard air density and air temperatures and no wind, and with the same velocity, yaw, orientation of half-plane of yaw, and trajectory inclination (to the horizontal) will experience proportional resistance. The factor of proportionality is theoretically a constant for the projectiles as such, independent of all conditions of firing. Actually due to some discrepancy the ballistic coefficient may depend slightly upon the firing conditions. Save for this constant, interpreted as the appropriate ballistic coefficient of each projectile concerned, an air resistance law is determined (for a given type of projectile) as a function of the velocity alone. The standard trajectory is computed using as nearly as feasible the appropriate resistance law as determined in the manner mentioned above. However the theory for practical procedure is somewhat inconsistent with that under which the resistance law was determined. In ignoring the yaw, an empirical overall average is found for the ballistic coefficient of the given projectile for a given muzzle velocity and quadrant angle of departure, and (except in rare cases) this ballistic coefficient is then treated as constant along any one trajectory. A new average determination is obtained for each separate group of firings under like initial conditions. Thus the resistance law used was derived for varying yaw and corre-

sponding varying resistance, but with constant ballistic coefficient independent of velocity. Yet in practice when we adopt the law so obtained, we retain a constant ballistic coefficient along the trajectory but ignore all changes in resistance due to actual change in angle of presentation. We are consequently compelled to accept the possibility of a change in the inferred ballistic coefficient not only for changes in muzzle velocity but also for changes in quadrant angle of departure and consequent changes in length of trajectory. Furthermore on any one trajectory a discrepancy may arise between total horizontal range and total time of flight. While we may select a ballistic coefficient to secure agreement for one of these two quantities, such a choice may not reconcile the other. To handle such a break between theory and observation we introduce an empirical additive constant in one of the factors of the resistance law, which constant, together with the ballistic coefficient are then determined so as to harmonize both computed total horizontal range and computed total time of flight with the corrected results of observed range firings.

(7) The actual trajectory is influenced by wind of varying magnitude and intensity at varying altitude layers, even after effects of momentary gusts and local geographic variations are ignored. Also ignored are the effects of vertical wind often considerable but usually localized. The wind at any one altitude level is assumed to be moving uniformly (at constant level) over the entire portion of the territory involved and throughout the duration of the projectile's flight. The wind velocity vector is analyzed at each altitude into longitudinal and lateral components. Although not entirely accurate, one assumes for the reduction of firing that the longitudinal component only need be considered for effects within the plane of fire, and the lateral component only, for effects perpendicular to this vertical plane. Furthermore in place of using a wind varying continuously with altitude, the assumption is made for reduction of observed range firings that the wind is uniform throughout each of a sequence of vertically adjacent altitude zones. The standard trajectory, as might be expected, ignores cross wind effects and assumes zero range wind component.

(8) The actual trajectory is influenced by a large number of known variables and doubtlessly by an indefinite host of unacknowledged factors. The idealized trajectory is obtained as the solution of a formulated (finite) system of simultaneous algebraic and differential equations with

given boundary conditions. There is good reason to believe that the quantitatively important influences are taken into account. Time, reckoned from the instant of fire, is accepted as sole independent variable.

In setting up the formal system of simultaneous algebraic and differential equations, the theory involves beside familiar principles of rational mechanics (with numerical parameters to be empirically determined) only three special empirical physical functions, (a) the air resistance (at sea level) under stipulated standard conditions of air density and temperature; (b) the velocity of sound in air aloft as a function of the altitude under standard sea level conditions and with standard relative temperature structure for atmosphere aloft; (c) the density of air aloft as a function of the altitude, under standard sea level conditions and with standard relative density structure for atmosphere aloft.

(9) As to the air resistance, endeavor is made to use the most accurate available information concerning the air resistance function concerned. This is particularly important not only because of the magnitude of the effects produced but also because of the difficulty of introducing any observational corrections except as these enter through changes in velocity of sound or in air density. So far as the latter two are concerned, it is not important for the ballistician to revise his standards for velocity of sound in air aloft, or for air density aloft, with each improvement in meteorological information. It is desirable that the standards adopted serve as not unreasonable approximations to average conditions. For reduction of range firings, observations are made aloft and corrections for actual non-standard conditions are incorporated in the trial runs for finding suitable estimates of the ballistic coefficient and of the additive correction to the retardation factor. Furthermore in using the standard trajectory as eventually published similar corrections are called for. A practical essential is that the standards adopted for sound velocity aloft and air density aloft be of a simple form suited for automatic continuous insertion in operation of the Differential Analyzer.

(10) For the standard trajectory the ratio of the velocity of sound in air at altitude y feet (above sea level)/ the corresponding velocity at sea level, is assumed to be given (under standard meteorological conditions) by a formula of the form $a(y) = 1 - a_1 y$, where a_1 (in reciprocal

feet) is an empirical constant. This formula is intended for use only up to moderate altitudes but no explicit procedure has been adopted as standard for use on arcs within the stratosphere. There is excellent theoretical reason and experimental evidence for believing the velocity of sound in air to depend directly upon the temperature of the air, and indeed, to be proportional to the square root of the absolute temperature. Thus indirectly the assumption of a standard formula for the velocity of sound implies the assumption for a standard structure for air temperature aloft. Now the temperature of the air affects separately (for given air pressure) the air density and the speed of sound. For present purposes it is unnecessary to adopt formal assumptions concerning pressure or temperature of the air aloft. We assume merely that the adopted linear law for speed of sound (through moderate altitudes) provides adequate approximation for current ballistic use.

(11) The ratio of air density at altitude y feet (above sea level) to the corresponding density at sea level is assumed to be given (under standard meteorological conditions) by a formula of the form $h(y) = e^{-h_1 y}$, where h_1 (in reciprocal feet) is an empirical constant. For if $h(y)$ is to be dimensionless, as desired, then $h_1 y$ is dimensionless and hence h is in units reciprocal to those for y.

(12) The actual trajectory starts off with mean direction (indicated by quadrant angle of departure) not necessarily equal to the angle at which the bore was laid (indicated by quadrant angle of elevation). This discrepancy due to jump of the carriage and possibly also in part to droop of the bore is ignored in the standard trajectory whose initial data utilize the quadrant angle of departure but not the quadrant angle of elevation when these differ from each other.

2. <u>The Values of the Empirical Constants and Parameters for Standard Meteorological Conditions.</u>

A continuous range of empirical constants is involved in tabulating the function $b_s(u)$ giving the (standard) retardation coefficient as a function of u, the adjusted velocity of the projectile with respect to the air. The term "adjusted" refers to the fact that the actual velocity of the projectile with respect to the air is compared with the velocity of sound in air at the given place which, at the actual altitude involved, may not be the velocity of sound under standard conditions at sea level.

This matter will be discussed in detail later.

The empirical constants to be dealt with individually will be considered now in sequence.

(1) g, the gravitational attraction, (in f/s^2). A constant value for g, independent of altitude and of geographic latitude, is accepted for ballistic purposes. The value 9.80 m/s^2 is taken as exact. (At sea level the actual value ranges at sea level from 9.8319 at the pole to 9.7800 at the equator and is 9.80095 at Washington, D. C.) In units of f/s^2, the numerical equivalent value is (to seven digits), 32.15217, since the meter is exactly 39.37 U. S. inches.

(2) h_1, the exponential coefficient (in reciprocal feet) in the standard air density ratio formula, $h(y) = e^{-h_1 y}$. The accepted ballistic standard formula yields exactly $10^{-.0000453\tilde{y}}$, where $\tilde{y}$ is the altitude in meters. On conversion to the natural base, this yields approximately $e^{-.0001036\tilde{y}}$. On direct conversion to feet, the formula yields (to six significant digits) $h_1 = .0000315823$, (1/f).

(3) a_1, the coefficient (in reciprocal feet) in the standard sound speed ratio formula, $a(y) = 1 - a_1 y$. The accepted ballistic standard formula (for firing table computations on the Differential Analyzer) yields, $a_1 = .000003$ (1/f) regarded as accurate.

(4) The rotational velocity of the earth, in radians per second, computed to be $\Omega = .00007292$ (rad./sec.). This is based on a year of 365.24 days (mean solar time) or 366.24 days (siderial time). This rotational velocity is used in computing the effect of the earth's rotation. Its computation will be discussed presently.

(5) The geographic latitude of the (main front) firing range at Aberdeen Proving Ground, taken as 39° 26.8' North, (although g is then locally, 9.801 m/s^2).

There remain to be mentioned certain empirical parameters treated as fixed on any one trajectory under standard meteorological conditions but not constant as between trajectories.

(1) The ballistic coefficient, "C", (discussed later).

(2) The (nominal) muzzle velocity, "M.V.".

(3) The quadrant angle of departure, "$\angle$ (Dep)".

(4) The azimuth of the plane of fire reckoned from north through east, "α".

(5) The empirical additive constant, "γ", yielding the "actual" retardation coefficient $b(u) = b_s(u)+\gamma$, from the standard retardation coefficient, $b_s(u)$.

Approximate numerical values for the more useful further combinations of the physical parameters g, a_1, h_1, Ω, and L = Latitude (for Aberdeen Proving Ground) are as follows:

$$\lambda = 2\,\Omega \cos L \sin\alpha = 0.00011262 \sin\alpha \text{, (1/sec.)}$$

where as discussed later, α is the azimuth of the plane of fire reckoned from north through east.

$$h_1/a_1 = 10.5274 \quad \text{(dimensionless)}$$

$$\sqrt{gh_1} = .0318660 \quad \text{(1/sec.)}$$

$$\sqrt{h_1/g} = .000991098 \quad \text{(sec./ft.)}$$

$$\sqrt{gh_1}/a_1 = 10622.00 \quad \text{(ft./sec.)}$$

$$1/h_1 = 31663.3 \quad \text{(ft.)}$$

$$a_1/h_1 = .0949899 \quad \text{(dimensionless)}$$

$$1/\sqrt{gh_1} = 31.3814 \quad \text{(sec.)}$$

$$\sqrt{g/h_1} = 1008.98 \quad \text{(ft./sec.)}$$

$$a_1/\sqrt{gh_1} = .0000941442 \quad \text{(sec./ft.)}$$

$$\Lambda = \lambda/\sqrt{gh_1} = .0035342 \sin\alpha \quad \text{(dimensionless)}$$

3. Geometry of the Standard Trajectory.

In the vertical plane of departure, the projectile treated as a particle traces a smoothly turning arc starting from the point of departure, rising to a summit, and descending to the level point (which is at the same level as the point of departure), and with maximum curvature near the summit. Let us introduce rectangular coordinates with the point of departure (muzzle of the gun) as origin. Let x, the abscissa, be measured (in feet) horizontally forward in this vertical plane, and let y, the ordinate, be measured (in feet) vertically upward. The trajectory is then given by a curve in the first quadrant of this (x,y) plane, with y a one-valued, continuous function of x on the interval from $x_0 = 0$ to x_*, the horizontal range in feet, which is the abscissa of the level point. Here the ordinate starts with $y_0 = 0$ and after reaching a maximum, at the summit, decreases again to $y_* = 0$. In general the subscript zero will be used for values at the origin, and a subscript star (*) for values at the level point.

Let p designate the slope of the trajectory at any one of its points, and θ the inclination, where θ is restricted to values between -90° and $+90^\circ$. Then $p = \tan\theta$. Let s represent arc length, in feet, along the trajectory. Then in the notation of the calculus $p = \tan\theta = dy/dx$, $ds^2 = dx^2 + dy^2$, $dx = ds\cos\theta$, $dy = ds\sin\theta$. The initial and terminal values are given by

θ_0 = angle of departure
p_0 = tangent of angle of departure
s_0 = 0
θ_* = negative of angle of fall
p_* = negative of tangent of angle of fall
s_* = length of trajectory (to level point)

Both p and θ decrease smoothly along the standard trajectory, starting with positive values p_0, θ_0, both passing through zero at the summit and finally both reaching minimum negative values, p_* and θ_* respectively. Furthermore dp/dx is negative throughout the trajectory, reaching a minimum a little past the summit of the trajectory and rising to a negative terminal value.

Each point of the trajectory has an associated ordinate y measured in feet, and through y has an associated standard air density ratio $h(y)$, given by the formula $h(y) = e^{-h_1 y}$, as mentioned earlier. For want of a better name let the "<u>ballistic arc length</u>", $j(s)$, be used to designate the weighted arc length expressed by the integral $(1/C)\int_0 h(y)ds$ evaluated along the trajectory, where $h(y)$ is as given above and C is the (constant) ballistic coefficient associated with the trajectory. The total ballistic arc length of the trajectory may be expressed as $\bar{h}/C$ times the geometric arc length, where $\bar{h}$ is a suitably chosen mean value of $h(y)$ for the trajectory.

[redacted]

4. The Equations of Motion.

In accordance with well established principles of rational mechanics we assume

(1) that forces, linear velocities, and linear accelerations share with displacements the character of vectors, resolvable into independent components in the horizontal and vertical directions, respectively.

(2) that force = mass x (linear) acceleration in the direction of force; where the linear acceleration is the time derivative of the magnitude of the velocity in the given direction and the velocity (in the given direction) is the time derivative of the displacement in this direction.

(3) that for the standard trajectory (ignoring earth rotation effects) the forces at any instant consist only of gravity acting vertically downward and the "drag" due to air resistance acting along the tangent to the trajectory in the direction opposite to that in which motion occurs.

Let t designate the variable time measured in seconds from the instant of departure, so that $t_o = 0$, $t_* =$ time of flight to the level point. In the absence of wind, let v designate the magnitude of the velocity. Then $v = ds/dt$, v_o = nominal muzzle velocity, and v_* = final velocity (at level point). Let the horizontal and vertical components of velocity be designated by x' and y', respectively. Then $x' = dx/dt = v \cos \theta$, $y' = dy/dt = v \sin \theta$.

If D designates the drag [redacted], we have as differential equations of motion for the standard trajectory (ignoring earth rotation effects)

$$m\, dx'/dt = -D \cos \theta,$$

$$m\, dy'/dt = -D \sin \theta - mg,$$

where m represents the mass of the projectile (constant on the trajectory) and g represents the gravitational attraction assumed constant as equal to 9.80 m/s^2 or 32.15217 f/s^2 (approx.). The drag [redacted] D depends not only upon the velocity of the projectile but upon (i) the density of the air which varies with altitude, (ii) the velocity of sound which varies with temperature, itself in turn varying with altitude, (iii) the size, shape, mass distribution, surface character, spin, yaw, etc., of the actual projectile.

[redacted]

[redacted]

Let E stand for $D/(mv)$. The first of the two given equations may then be written as

$$dx'/dt = -Ev\cos\theta = -Ex' = -E\,dx/dt,$$

thence as

$$dx'/dx = -E.$$

We may eliminate D and incidentally m also from the original two equations and obtain

$$dy'/dt = (dx'/dt)\tan\theta - g,$$

and this may be cast into the form

$$dp/dx = -g/(x')^2,$$

where p, as before, is dy/dx. Since dp/dx is determined as a geometrical characteristic of the trajectory, it follows that $x'/\sqrt{g}$ is also geometrically determined, and hence save for the intervention of the experimental physical constant g (which does not depend upon the gun, ammunition, or meteorological conditions) that E which determines the drag also is determinable from the trajectory as a geometric arc.

Another slightly variant form for the equations of motion, emphasizing x, y, and t, is as follows:

$$d^2x/dt^2 = -E\,dx/dt,$$

$$d^2y/dt^2 = -E\,dy/dt - g.$$

5. Effects of the Rotation of the Earth.

The effects of the rotation of the earth upon the projection of the trajectory on the plane of departure (and as heretofore we continue to ignore deflections out of this plane) are to replace the pair of equations

$$x'' = -Ex', \quad y'' = -Ey' - g,$$

by the following

$$x'' = -Ex' - \lambda y',$$

$$y'' = -Ey' - g + \lambda x',$$

[redacted]

[redacted]

where $\lambda = 2\,\Omega \cos L \sin\alpha$. Here Ω is the rotational velocity of the earth, L is the latitude of the gun, and α is the azimuth of fire reckoned from the north through the east. We shall not pause to derive the equations given above. This derivation is explained in texts on ballistics. Now the number of seconds (mean solar time) in one solar day is, $60^2 \times 24 = 86400$. But for the rotation of the earth we wish to consider the angular velocity of the earth's rotation, measured not for motion with respect to the sun but absolutely, that is with respect to the stars. In one year a meridian on the earth passes under the sun 365.24 times, but it passes under a given star one more time a year, namely 366.24 times. Hence in radian measure,

$$2\,\Omega = \frac{2 \times 2\pi \times 366.24}{60^2 \times 24 \times 365.24} = .00014584 \text{ (rad./sec.)}$$

Using $L = 39^\circ\ 26.8'$ for the Aberdeen Proving Ground, we find the value of λ to be used is given by

$$\lambda = 0.00011262 \sin\alpha$$

where λ is in units of reciprocal (mean solar) seconds (1/sec.)

6. Analysis of the Drag.

It is regularly assumed that the retarding force, or "drag" for a massive spherical particle is proportional to the air density. Although this assumption may not have been established by experimental test for the range of velocities, sizes, and masses involved in ballistic theory, the assumption is not seriously questioned. For standard conditions, the ratio of air density aloft to air density at the ground at least up to the tropopause (which marks the separation between troposphere and stratosphere) is assumed to be given by an exponential function h as already mentioned.

It is assumed on excellent theoretical basis also that the drag depends upon the velocity of the projectile in such a way that $D/(mv^2)$, or E/v, is a function of u ~~alone~~, where u, the "adjusted speed", called also Mach's number, is given by v/a, where in turn a, previously mentioned, is the ratio of the velocity of sound aloft at the actual temperature to the standard velocity of sound at the ground, or at standard temperature (15° C. = 59° F.)

For two projectiles of the same physical finish and geometric form and with ~~the~~ similar distribution of mass, we

[redacted]

[redacted]

assume that, at least approximately the value of $D/(mv^2) = E(v)/v$ varies directly as the density of the air and as the area of cross section of the projectile, and inversely as the mass. We may represent by $b(v)$ the corresponding value of $D/(mv^2)$ for standard atmospheric conditions and for a theoretical "model" projectile similar to the given projectile but of one inch caliber and one pound weight. There remains to be considered the air density regarded as a function of the altitude alone. We then write for standard conditions, $D/(mv^2) = b(v)\,F(y)$ where $F(y)$, accounting for air density and characteristics of the projectile, is accepted by convention, to be (for "standard conditions") an exponential function of the altitude in feet. The constant, $1/F(0)$, is called the ballistic coefficient, C, and the ratio $F(u)/F(0)$ indicated by $h(y)$ is called the standard density ratio aloft. Here $h(y)$ is independent of projectile, angle of departure, and muzzle velocity. It is a conventionalized meteorological standard function for ballistic purposes. Under non-standard conditions of air density aloft, the ratio $h(y)$ is modified appropriately by an additive correction term, δh, itself a function of the altitude, y. A large value for C corresponds to a ballistically good projectile, other things being equal since then the retardation per unit distance traveled is small.

7. <u>The Regression.</u>

By the <u>regression</u>, r, will be meant the time integral of the drag coefficient, E, which of course is always positive.

$$r = \int_0 E\,dt,$$

$$= (1/C)\int_0 vbh\,dt = \int_0 b(h/C)v\,dt = \int_0 b(h/C)\,ds = \int_0 b\,dj$$

Here $r_0 = 0$, and r increases along the trajectory, reaching its maximum, r_*, at the level point. Relative to the ballistic coefficient, the terminal value of the regression, r_*, decreases as C is increased, reaching zero for a vacuum trajectory.

Since furthermore from the original relations

$$d^2x/dt^2 = -E\,dx/dt$$

or on integrating, $x' = x_0' - \int_0 Ex'\,dt$ or $x' = x_0' - \int_0 x'\,dr$.

We desire that r be dimensionless. It then follows from $r = \int_0 b(h/C)\,ds$, that b must be in reciprocal feet to compensate for s being in feet.

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8. Reduction of Observed Range Firings.

In an actual sequence of range firings, there will be many conditions differing in some measure from those assumed as standard for the firing tables which are to be produced. Some of the changed conditions, resulting in slight discrepancy in weight of projectile or in muzzle velocity, are inferred from carefully noted measurements and can be accounted for by adjustments based upon theory and justified by experiment. The most serious modifications are those due to non-standard weather conditions.

In using the Differential Analyzer for the reduction of firings, only the range effects (occurring in the vertical plane of departure) are dealt with directly on the machine. All lateral displacements from this plane are more easily handled separately, as has been suggested earlier.

In practice three slowly changing corrections, $\delta w'$, δa, δh, (to be explained) are handled by finite increments introduced at preassigned altitudes. The air aloft is regarded as subdivided by horizontal planes into a sequence of strata called altitude zones. The use of these zones is purposely staggered among the three corrections so that at any one shift from one zone to the next, one correction only (or at most two) need be made by hand cranks, the remaining cranks (or crank) being locked at this particular zone bound.

Direct observation is made of direction and magnitude of wind, temperature, and air pressure at altitudes up to (or nearly up to) that of the summit of the trajectory. These observations are then interpreted to give for each of the corresponding adopted zones, the average component of wind velocity in the direction of abscissas, $-\delta w'$, the average algebraic increase in the air density, namely δh, and the average algebraic increase in the speed of sound in air at the observed altitude to that at the ground at standard temperature, namely δa.

To distinguish among functions for non-standard conditions, hereafter let v represent the magnitude of the velocity with respect to the moving air, as distinguished from velocity with respect to the ground. Hence v^2 is no longer representable as $x'^2 + y'^2$, but rather as $w'^2 + y'^2$, where w' is the horizontal velocity component with respect to the air. Then $w_* = \int_0 w' \, dt$ would represent the horizontal component of travel with respect to the air. Hence $w_* - x_*$ is the total rearward displacement of the air during the time of flight. The

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algebraic difference, $w' - x' = \delta w'$, is the velocity of the head wind with appropriate sign, while $-\delta w'$ is the corresponding algebraic velocity of the range component of the wind.

Designating the standard functions heretofore called h and a by $h_s(y)$ and $a_s(y)$, to distinguish them from the actual functions, we have

$$w' = x' + \delta w'$$

$$a = a_s + \delta a$$

$$h = h_s + \delta h$$

where w', a, h, a_s, h_s, $\delta w'$, δa, δh are all eventually determinate functions of y alone.

Both the range attained and the time of flight are observed.

The method of using the values of $\delta w'$, δa, and δh for the computation of standard trajectories is as follows:

Several trajectories are computed for suitably restricted but yet arbitrarily chosen neighboring values of C. These trajectories make use of the corrected values of w', a, and h. For each of the chosen values of C the computed range and time of flight are observed. If by linear interpolation one can find a choice of C which yields acceptable estimates of the range and time of flight simultaneously, this value is adopted for use under standard conditions, and standard runs of the machine are carried out for this choice of ballistic coefficient. If, however, as often happens, no choice of C yields simultaneously acceptable estimates of both x_* and t_*, then further trial runs are carried out with the same corrected values for w', a, and h, but with b varied through the introduction of an arbitrary additive constant, γ. Eventually by interpolation one finds such a value for the algebraic constant γ added to b, together with such a value of C as to reproduce acceptably both the observed horizontal range and the total time of flight.

In current practice it is usually found to be unnecessary to make any additive correction in the application of the drag, that, γ is usually zero.

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9. The Primary Physical Variables.

The following physical variables are given explicit symbols. Also for convenience in later discussing the theory of the mechanical performance of the Differential Analyzer, most of them are assigned numbers. Temporarily we list these in a more psychological order while indicating the official number (if any) in parentheses at the right.

x, the abscissa (in feet), or horizontal range, of any desired point of the trajectory. (6)

y, the ordinate (in feet) or vertical range or altitude, of any desired point of the trajectory. (7)

h_s, the standard ratio of air density at altitude y (in absolute units) to the standard air density at sea level, assumed as expressible in the form $h_s(y) = e^{-h_1 y}$ (y in feet), h_1 given numerically above. (8)

δh, an algebraic additive correction to h_s (in absolute units) obtained by observations aloft. (3')

$h(y)$, the corrected ratio of air density at altitude y to the standard air density at sea level, expressed by $h(y) = h_s(y) + \delta h$. (15)

s, the geometric arc length (in feet) of the trajectory with respect to the air. (1)

j, the ballistic arc length (in feet) of the trajectory, with respect to the moving air, given by $j = (1/C)\int_0^s h(y)\,ds$. (2

t, the elapsed time (in seconds) from the instant of fire. (0)

y', the vertical velocity (in feet per second) corrected for the earth's rotation. (16)

x', the horizontal velocity (in feet per second) with respect to the ground, corrected for the earth's rotation. (17)

y_s', the vertical velocity (in feet per second) uncorrected for the earth's rotation. (12)

[redacted]

x_s', the horizontal velocity (in feet per second) with respect to the ground, uncorrected for the earth's rotation. (5)

w', horizontal velocity of the projectile with respect to the air (in feet per second). (13)

$\delta w'$, the magnitude of the head wind (in feet per second) at altitude y (feet). (1')

v, magnitude of the velocity of the projectile with respect to the air, $v^2 = w'^2 + y'^2$. (9)

a(y), the corrected ratio of the velocity of sound in air at altitude y in absolute units to the standard velocity of sound in air, at sea level, $a(y) = a_s(y) + \delta a$ where $a_s(y) = 1 - a_1 y$. (14)

δa, an algebraic additive correction to a_s (in absolute units) obtained by observation aloft. (2')

u, the adjusted [redacted] velocity with respect to the air (in feet per second), $u = v/a$. (10)

b(u), the corrected retardation coefficient (in reciprocal feet, 1/f), $b(u) = b_s(u) + \gamma$, where $b_s(u)$, the standard retardation coefficient is a tabular function of u alone, and where (11)

γ, an algebraic additive constant determined empirically in such a way as to bring into harmony the ballistic coefficient reproducing the observed total horizontal range with that reproducing the observed total time of flight. (iii)

r, the regression, (in absolute units), $r = \int_0 b \, dj$. (3)

k, the cumulative vertical velocity correction due to regression (in feet per second), $k = \int_0 y' \, dr$. (4)

λ, the earth rotation coefficient, used in the relations $y' = y_s' + \lambda x$, $x' = x_s' - \lambda y$, (in reciprocal seconds). (9)

We have also the following:

v_0, the magnitude of the nominal muzzle velocity, with respect to the air (in feet per second).

[redacted]

y_o', the vertical component of the nominal muzzle velocity (in feet per second).

x_o', the horizontal component of the nominal muzzle velocity with respect to the ground (in feet per second).

w_o', the horizontal component of the nominal muzzle velocity with respect to the air (in feet per second).

t_*, the total time of flight (to the level point), (in seconds).

x_*, the total (horizontal) range (to the level point) (in feet).

v_*, the magnitude of the terminal velocity, (at the level point) with respect to the air (in feet per second).

$y_{max.}$, the maximum ordinate, or summital ordinate, (in feet).

Finally corresponding respectively to the six canonical variables T, X, Y, V, Y', X' are the six recorder variables $\underline{T}$, $\underline{X}$, $\underline{Y}$, $\underline{V}$, $\underline{Y}'$, $\underline{X}'$, numbered respectively I, II, III, IV, V, VI.

In review we may summarize the following facts with respect to physical dimensions.

Dimensionless, C, a, a_s, δa, h, h_s, δh, r.

In units of feet, x, y, s, x_*, $y_{max.}$, $j(s)$.

In units of reciprocal feet, a_1, h_1, $b(u)$, $b_s(u)$, γ.

In units of seconds, t, t_*.

In units of reciprocal seconds, λ.

In units of feet per second, v, y', x', y_s', x_s', w' u, k, v_o, y_o', x_o', w_o', v_*.

In units of feet per square seconds, g.

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B. The Canonical System.

1. Various systems of variables.

The algebraic and differential equations that might naturally first occur to an engineer in setting up his problem in physical variables here designated by small letters (expressed in units such as feet, seconds, feet per second, degrees of arc, etc.) should usually be modified to adapt the problem for more effective handling by mechanical computing devices. By a change of units the equations may be phrased in terms of absolute dimensionless variables, each of which might be regarded as having merely the number, unity, as its unit. However, it is often convenient to further modify this plausible choice of dimensionless variables by introducing some dimensionless canonical units not necessarily equal to unity. The equations become thus reduced to canonical form and expressed in terms of canonical variables here designated by capital letters. Of these variables one will be the single independent variable, the others being all dependent thereon. The canonical variables appearing explicitly and given each its own identifying symbol, may be called the primary canonical variables. As contrasted to both physical and canonical variables are the recorded variables here designated by underscored capital letters. Each recorded variable is proportional to a corresponding canonical variable, with a recorder ratio. The purpose of introducing these recorder ratios will be discussed later.

For a given physical variable say z_i, let Z_i be the associated canonical variable and $\underline{Z}_i$ the corresponding recorded variable. Let p_i be the physical proportionality factor, for which one physical unit is equivalent to p_i absolute dimensionless units. Let k_i be the canonical coefficient, for which one absolute unit is equivalent to k_i canonical units. Hence one physical unit is equivalent to $p_i k_i$ canonical units, or $Z_i = p_i k_i z_i$. Let r_i be the recorder ratio, for which one canonical unit is equivalent to r_i recorded units. Let us designate by m_i the total machine multiplier, where $m_i = p_i k_i r_i$. Then each physical unit is equivalent to m_i units of the corresponding recorded variable, or $\underline{Z}_i = m_i z_i$.

2. The Canonical Variables and Parameters.

As mentioned above, canonical variables and parameters will be indicated by capital letters, an exception being C for the ballistic coefficient, not a canonical parameter, and which requires no analogous canonical symbol.

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Those canonical variables and parameters which are obtained directly from dimensionless physical quantities exclusively are listed as follows:

H_o, $= 1/C$, the reciprocal of the ballistic coefficient. (ii)

ΔH, $= \delta h/C$, the canonical algebraic additive correction to H_s. (3')

$H(Y)$, $= h(y)/C$, the canonical air density ratio, in comparison to the unit projectile. (15)

$H_s(Y)$, $= h_s(y)/C$, the canonical standard air density ratio in comparison to the unit projectile. # ~~$h_s(y)/C$.~~ (8)

R, $= r$, the canonical regression. (3)

The physical constant h_1, used in designating standard air density aloft is in units of reciprocal feet, but so also is a_1, a second physical constant, used in designating standard sound velocity in air aloft. Hence the ratio, h_1/a_1 is a dimensionless multiplier. This multiplier is used to convert the dimensionless sound speed ratio $a(y)$ and related quantities into canonical quantities as follows:

$A(Y)$, $= a(y)\ (h_1/a_1)$, the canonical corrected sound velocity aloft given by the formula, $A(Y) = A_s(Y) + \Delta A$, where $A_s(Y)$, $= a_s(y)\ (h_1/a_1)$, is the canonical standard sound velocity aloft given by the formula $A_s(Y) = A_o - Y$. Y is discussed below. (14)

A_o, $= h_1/a_1$, the canonical standard sound velocity at sea level. (i)

ΔA, $= \delta a\ (h_1/a_1)$, the canonical algebraic additive correction to the canonical standard sound velocity aloft. (2')

As remarked above, the physical constant, h_1, used in designating standard air density aloft, is in units of reciprocal feet. Hence any physical measure in feet, if multiplied by h_1, gives a dimensionless quantity. We obtain the following canonical quantities in this manner:

X, $= xh_1$, the canonical abscissa, or horizontal range, of any desired point of the trajectory. (6)

[redacted]

Y, = yh_1, the canonical ordinate, or vertical range, or altitude of any desired point of the trajectory. (7)

S, = sh_1, the canonical geometric arc-length of the trajectory with reference to the air. (1)

X_*, = $x_* h_1$, the canonical total horizontal range.

$Y_{max.}$ = $y_{max.} h_1$, the canonical maximum ordinate.

$J(S)$ = jh_1, the canonical ballistic arc-length of the trajectory with reference to the air and in comparison to a unit projectile. (2)

Similarly by using the reciprocal multiplier, $1/h_1$, one secures the following canonical quantities:

$B(U)$, = $b(u)/h_1$, the canonical corrected retardation coefficient. (11)

Here $B(U) = B_s(U) + \Gamma$, where $B_s(U)$, = $b_s(u)/h_1$, the canonical standard retardation coefficient is a tabular function of U alone, (β)

and Γ, = γ/h_1, is an algebraic additive constant determined in such a way as to bring into harmony the observed total (horizontal) range and the observed total time of flight. (iii)

The physical constant g is in units of f/s^2, and hence $\sqrt{gh_1}$ is in units of 1/sec. Hence any physical measure in seconds, if multiplied by $\sqrt{gh_1}$, gives a dimensionless quantity. We obtain the following canonical quantities in this manner:

T, = $t\sqrt{gh_1}$, the canonical elapsed time, from the instant of fire. (o)

T_*, = $t_*\sqrt{gh_1}$, the total canonical time of flight (to the level point).

Similarly by using the reciprocal multiplier, we have as canonical parameter, Λ, = $\lambda/\sqrt{gh_1}$, the canonical earth rotation coefficient, used in the relations $Y' = Y_s' + \Lambda X$, $X' = X_s' - \Lambda Y$. (κ)

From the two physical constants g and h_1, we obtain also the factor, $\sqrt{h_1/g}$ which is in units s/f. Hence any physical measure in feet per second, if multiplied by $\sqrt{h_1/g}$ gives a

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dimensionless quantity. We obtain the following canonical quantities in this manner.

$V, = v\sqrt{h_1/g}$, magnitude of the canonical velocity of the projectile with respect to the air. $V^2 = W'^2 + Y'^2$. (9)

$Y', = y'\sqrt{h_1/g}$, the canonical vertical velocity, corrected for the earth's rotation. (16)

$X', = x'\sqrt{h_1/g}$, the canonical horizontal velocity with respect to the ground, corrected for the earth's rotation. (17)

$Y'_s, = y'_s\sqrt{h_1/g}$, the canonical vertical velocity, uncorrected for the earth's rotation. (12)

$X'_s, = x'_s\sqrt{h_1/g}$, the canonical horizontal velocity with respect to the ground, uncorrected for the earth's rotation. (5)

$W', = w'\sqrt{h_1/g}$, the canonical horizontal velocity of the projectile with respect to the air. (13)

$\Delta W', = \delta w'\sqrt{h_1/g}$, the canonical ballistic head wind. (I')

$K, = k\sqrt{h_1/g}$, the canonical cumulative vertical velocity correction due to regression, $K = \int_0 Y'\,dR$. (4)

$V_o, = v_o\sqrt{h_1/g}$, the magnitude of the canonical nominal muzzle velocity with respect to the air. (_)

$Y'_o, = y'_o\sqrt{h_1/g}$, the vertical component of the canonical nominal muzzle velocity. (iv)

$X'_o, = x'_o\sqrt{h_1/g}$, the horizontal component of the canonical nominal muzzle velocity with respect to the ground. (v)

$\Delta W'_o, = \delta w'_o\sqrt{h_1/g}$, the magnitude of the canonical surface head wind.

$V_*, = v_*\sqrt{h_1/g}$, the magnitude of the canonical terminal velocity (at the level point) with respect to the air.

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A little less direct than in the cases just mentioned is the relation between U and u, since here the dimensionless multiplier A_o, $= h_1/a_1$, is also involved. U, $= u\ a_1/\sqrt{gh_1} = u\sqrt{h_1/g}\ /A_o$, the canonical adjusted horizontal velocity with respect to the air, $U = V/A$. (10)

3. <u>The serial list of canonical equations and supplementary variables and parameters.</u>

We list serially as follows the canonical equations, and supplementary variables and parameters. The order chosen is suggested by the classification according to types of operations performed, and in the case of integrations, by the desirability of listing close together any two integrations where the integral resulting from one supplies the differential of the other.

For preliminary data, we have

(α) $\Lambda = .0035342 \sin\angle$

(β) $B_s(\bar{U})$, (tabulated)

(i) A_o, $= 10.5274$

(ii) H_o, $= 1/C$

(iii) Γ, obtained empirically, by inverse interpolation

(iv) $Y_o' = (\text{M.V.}) \sin\angle(\text{Dep.}) \times .000991098$

(v) $X_o' = (\text{M.V.}) \cos\angle(\text{Dep.}) \times .000991098$

(1') $\Delta W'$, obtained from observations

(2') ΔA, obtained from observations

(3') ΔH, obtained from observations

For the simultaneous system of algebraic and differential equations, we have, as the reader may check for himself

(0) T

(1) $S = \int_0 V\, dT$

[redacted]

(2) $J = \int_0 H \, dS$

(3) $K = \int_0 Y' \, dR$

(4) $R = \int_0 B(U) \, dJ$

(5) $X'_s = X'_0 - \int_0 W' \, dR$

(6) $X = \int_0 X' \, dT$

(7) $Y = \int_0 Y' \, dT$

(8) $H_s = H_0 - \int_0 H_s(Y) \, dY, \; (H_0 = 1/C)$

(9) $V = \sqrt{W'^2 + Y'^2}$

(10) $U = V/A$

(11) $B(U) = B_s(U) + \Gamma$

(12) $Y'_s = Y'_0 - (T + K)$

(13) $W' = X' + \Delta W'$

(14) $A(Y) = A_s(Y) + \Delta A = (A_0 - Y) + \Delta A, \; (A_0 = 10.5274)$

(15) $H(Y) = H_s(Y) + \Delta H$

(16) $Y' = Y'_s + \Lambda X \quad (\Lambda = .0035342 \sin \alpha)$

(17) $X' = X'_s - \Lambda Y \quad (\Lambda = .0035342 \sin \alpha)$

While the correction terms, $\Delta W'$, ΔA, ΔH may be positive, zero, or negative, and may be increasing, stationary, or negative, the eighteen canonical variables (save for Y' and Y'_s) are all nonnegative throughout the trajectory (as far as the level point). Their behavior along the trajectory, as functions of T may be described as follows:

(0) T Increasing from 0 to T_*.

(1) S Increasing from 0 to S_*.

(2) J Increasing from 0 to J_*.

(3) R Increasing from 0 to R_*.

[redacted]

(4) K Increasing from 0 to K_m (at summit) then decreasing to K_*.

(5) X'_s Decreasing from X'_o to X'_{s*} $(X'_o \leq X'_m)$

(6) X Increasing from 0 to X_*.

(7) Y Increasing from 0 to Y_m, then decreasing to $Y_* = 0$.

(8) H Decreasing from H_o to H_{min}, then increasing to H_o.

(9) V Decreasing from V_o to V_{min}, then usually increasing to V_*.

(10) U Decreasing from U_o to U_{min}, then usually increasing to U_*.

(11) B Lies between B_{min} and B_m.

(12) Y'_s Decreasing from Y'_o through zero to the negative value Y'_{s*}.

(13) W' Decreasing from W'_o to W'_*.

(14) A Decreasing from $A_o + \Delta A_o$ to A_{min}, then increasing to $A_{*}, = A_o \pm \Delta A_o$.

(15) H Decreasing from $H_o + \Delta H_o$ to H_{min}, then increasing to $H_*, = H_o \pm \Delta H_o$.

(16) Y' Decreasing from Y'_o through zero to the negative value Y'_*.

(17) X' Decreasing from X'_o to X'_*.

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PART V

CONVERSION COEFFICIENTS

A. Equations Among Conversion Coefficients.

1. Conversion Coefficients Introduced.

Let us recall briefly four types of multipliers or proportionality factors already introduced: (i) The physical proportionality factor, p_i, which converts a physical unit into a dimensionless unit. (ii) The canonical coefficient, k_i, which converts an elementary dimensionless unit into a canonical unit. (iii) The recorder ratio, r_i, which converts a canonical unit into a recorder unit. (iv) The machine multiplier, m_i, which converts a physical unit directly into a recorder unit.

The multipliers mentioned above have no special reference to the mechanical structure of the Differential Analyzer. We now turn to the question of <u>conversion coefficients</u> which while analogous to the preceding multipliers are not (in most cases) immediately expressible in terms of them.

In the Differential Analyzer each primary canonical variable enters into the physical operation of the machine only as proportional to the total turn of some one or more rotating bus shafts. If a given bus shaft represents by its total turn the canonical variable Z_i, then the coefficient c_i, which is such that one complete rotation of the shaft yields c_i units increase in the variable, is called the <u>conversion coefficient</u> for the combination of given shaft and given canonical variable. A given variable has a separate conversion coefficient with respect to each shaft representing it, while also a given shaft may represent several variables, (necessarily mutually proportional) but with a separate conversion coefficient for each. The symbols c_1', c_2', c_3', c_0, c_1, ..., c_{17}, c_I, ..., c_{VI} will be used for the conversion coefficients for the corresponding canonical variables (1', 2', 3', 0, 1, 2, ..., 17, I, ... VI) as represented on the correspondingly numbered "primary" bus shaft which serves as output for the correspondingly numbered submachine and machine element. For instance, the regression R is the variable with official number 3. It is the output of the integrator unit #3, which is also the Submachine #3. This primary output shaft is also Machine Element #3. The conversion coefficient from primary shaft to variable is designated by c_3, a signed quantity.

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The conversion coefficients (as distinguished from recorder coupling coefficients) c_I to c_{VI} are for the recorder variables.

The gear couplings have been described earlier. Each is either a single pair of meshed spur gears (then constituting a <u>simple</u> gear coupling) or else is <u>compound</u> composed of a train of such gear couplings. When two bus shafts are geared together through a gear coupling, both shafts serve to represent the same canonical variable but with conversion factors in general different. The change in conversion factor introduced by passing the variable through the gear coupling is multiplication by a <u>coupling coefficient</u> (always a rational number) entirely determined by the mechanical relations within the gear coupling.

The <u>coupling coefficient</u> is defined as the (signed) number of revolutions required in the bus shaft entering the gear coupling to produce one revolution (in the same sense) of the bus shaft emerging from the gear coupling. Since the bus shafts are parallel, there is no ambiguity of sign involved. A coupling coefficient has determinate sign; it is, for instance, negative for each simple gear coupling.

The gear couplings are given serial numbers* from 18 through 56, corresponding to their position in the complete list of submachines for output. We may designate by c_J the coupling coefficient for the gear coupling #J. This produces no confusion with the conversion coefficient, since the two sets of subscript numberings concerned do not overlap.

To illustrate the relations among the variables, consider x, the physical variable for horizontal range, $\underline{X}$, the corresponding canonical variable, and $\underline{\underline{X}}$ the corresponding recorder variable. One has the equation, $\underline{X} = h_1 x$, connecting physical and canonical variables. It will be seen that c_6 is the conversion coefficient for X and c_{52} is the coupling coefficient from the primary bus shaft for X to the bus shaft of the recorder variable. For the variables here considered,

* There would be exceptions in the actual numbering of mechanical gear couplings, exceptions which do not affect the theory but which simplify the problem of setting up the machine.

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there is a common zero origin. Now one turn of the primary bus shaft gives by definition c_6 units increase in the canonical variable X. Hence Θ_x, the total turn (in revolutions) in the primary bus shaft for X, is equal to $X/c_6 = h_1x/c_6$. But it takes c_{52} turns of this primary bus shaft to effect one turn in the recorder bus shaft, and hence one unit of increase in the recorder variable, $\underline{X}$. Hence the variables, $\underline{X}$ and X, when carried together on this recorder bus shaft are so related that $\underline{X} = \Theta_x/c_{52} = h_1x/(c_6c_{52})$. Hence finally

$$x = c_6c_{52}\underline{X}/h_1.$$

2. Equational Principles Relating Conversion Coefficients.

We shall list herewith some principles adopted concerning equations among the conversion coefficients, using the lettering in most cases analogous to that for the variables and submachines.

(a) For a given canonical variable, Z, with conversion coefficient q on a given bus shaft, and for any given non-zero constant, n, the conversion coefficient for the variable nZ on this same bus shaft is nq. [This follows from the definition of conversion coefficient.]

(b) For the calibrating drums the conversion coefficients for the separate canonical parameters, A_o, H_o, Γ, Y_o', X_o' (respectively numbered (i), (ii), (iii), (iv), (v)) on their respective primary bus shafts are taken without loss of generality to be the same respectively as those for the primary canonical variables to which these parameters are respectively (algebraically) added, on these same shafts. [This means merely that no further couplings are actually or nominally interposed between the mechanism of the calibrating drum unit, and the shaft whose primary canonical variable is being calibrated.]

(c) See (k).

(d) On the initial bus shaft (chain-driven by the prime motor) the conversion factor for T is taken without loss of generality to be arbitrary save for inferior limitations to be discussed later. It is usual to take this conversion as small as is conveniently permitted by these inferior limitations.

(e) to (h), covered by (i) to (k).

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(i) (1) For any integrator, the conversion coefficient for the output variable on the output shaft is 32 times the product of the conversion coefficients of the two input variables of the integrator on their respective input shafts. [This follows from the nature of the integration process and from the calibrating constants of the integrator unit as a sub-machine.]

(2) When a given canonical variable serves as differential input for more than one integrator unit, the conversion coefficients are to be taken as equal for this variable on these several input shafts; similarly, when one canonical variable serves as integrand input for more than one integrator unit.

(j) (1) For the Vector Table (Table #1) the conversion coefficients of the three variables concerned, W', Y', V, are equal on their respective bus shafts of this submachine.

(2) For the Division Table (Table #2) the conversion coefficients of the three variables concerned, V, A, U, on their respective bus shafts of this machine are so related that the coefficient for the quotient, U, is 1/360 times the quotient of the respective conversion coefficients for the numerator, V, and for the denominator, A. [The factor, 1/360, arises for the following reason, when V and A are each at their limiting position of 90 revolutions of their respective shafts and associated lead screws, then U is also at 90 revolutions of its own lead screw. But this lead screw in turn is given, in the mechanical set-up of the table itself, a gear ratio producing four turns of the eventual output shaft to each turn of the output lead screw. Hence 360 turns of the eventual output shaft is required to yield one unit increase in U.]

(3) Note. For the Template Table (Table #3) no special rule is invoked.

(k) For any adder, the three canonical variables, say Z_i, Z_j, Z_k, whose relation $Z_i + Z_j + Z_k = 0$ is maintained by the adder upon the respective shafts entering the adder, have on their respective shafts the same conversion coefficient. [This is obvious from inspection of the construction of the adder.]

(l) For any printer, the coupling coefficient for yielding the recorder variable, say $\underline{Z}$, on its own primary bus shaft from the corresponding canonical variable, say Z, on its own primary bus shaft, is the corresponding recorder ratio.

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[This is seen from the following two facts: (i) The two variables are related by an equation of the form $\underline{Z} = rZ$, where r is by definition the corresponding recorder ratio; (ii) since a coupling connects directly the primary shafts for Z and $\underline{Z}$, one has a relation of the form $c_{(\underline{Z})} = c_{(Z)}c$, where c is the coupling coefficient concerned, and $c_{(\underline{Z})}$ and $c_{(Z)}$ are the conversion coefficients for Z and $\underline{Z}$ on their respective primary bus shafts.]

3. <u>The Equations in Serial Order.</u>

The submachines #1 to 17 inclusive with their respective equations and variables lead to the following equations among conversion coefficients, involving as multipliers the coupling coefficients concerned. The letters (i) to (l) are carried over from the general discussion.*

To illustrate the method of obtaining the equations among conversion coefficients, it will probably suffice to show how the first relation, namely $c_1 = 32\, c_0 c_9 c_{18} c_{19}$, is obtained. This is found by reference to the relation $S = \int_0 V\, dt$ provided by the first integrator unit. The output of this integrator unit is S carried on the primary shaft for S, namely shaft #1, and with conversion coefficient c_1. The shafts carrying T and V respectively into this integrator unit are not however the primary shafts for these variables. The variable T has been passed through the gear coupling #19, which has the coupling coefficient c_{19}. On the primary shaft for T the conversion coefficient is c_0, hence on the differential input shaft for integrator unit #1, T has the conversion coefficient $c_0 c_{19}$. In an entirely similar way V has on its input shaft into this integrator unit, the conversion coefficient $c_9 c_{18}$, the c_9 being the conversion coefficient of V on its own primary shaft and the c_{18} being the coupling coefficient for the gear coupling which carries V from its own primary shaft to the input shaft for integrator #1. By Principle (i), the integration gives 32 times the product of $c_0 c_{19}$ by $c_9 c_{18}$, or $32\, c_0 c_9 c_{18} c_{19}$ which in turn is to be equated to c_1 as desired.

By (i) (l)

1. From $S = \int_0 V\, dT$, $\qquad c_1 = 32\, c_0 c_9 c_{18} c_{19}$

2. From $J = \int_0 H\, dS$, $\qquad c_2 = 32\, c_1 c_{15} c_{20} c_{21}$

3. From $R = \int_0 B\, dJ$, $\qquad c_3 = 32\, c_2 c_{11} c_{22} c_{23}$

*Consult Supplements A and B.

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4. From $K = \int_0 Y'\, dR$, $\quad c_4 = 32\, c_3 c_{16} c_{24} c_{25}$

5. From $X'_s = X'_o - \int_0 W'\, dR$, $\quad c_5 = -32\, c_3 c_{13} c_{26} c_{27}$

6. From $X = \int_0 X'\, dT$, $\quad c_6 = 32\, c_0 c_{17} c_{28} c_{29}$

7. From $Y = \int_0 Y'\, dT$, $\quad c_7 = 32\, c_0 c_{16} c_{30} c_{31}$

8. From $H_s = H_o - \int_0 H_s\, dY$, $\quad c_8 = -32\, c_7 c_8 c_{32} c_{33}$

By (i) (2)

(i) From 1,6, and 7, $\quad c_{18} = c_{28} = c_{30}$

(ii) From 4 and 5, $\quad c_{24} = c_{26}$

(iii) From 4 and 7, $\quad c_{25} = c_{31}$

By (j) (1)

9. From $V = \sqrt{W'^2 + Y'^2}$, $\quad c_9 = c_{13} c_{34} = c_{16} c_{35}$

By (j) (2)

10. From $U = V/A$, $\quad c_{10} = c_9 c_{37}/(360\, c_{14} c_{36})$

11. From $B = B_s(U) + \Gamma$, no special equation is obtained.

By (k) (See also (a))

12. From $Y'_s = Y'_o - (T + K)$, $\quad c_{12} = c_0 c_{39} = c_4 c_{40}$

13. From $W' = X' + \Delta W'$, $\quad c_{13} = -c_{17} c_{41} = -c'_1 c_{42}$

14. From $A = (A_o - Y) + \Delta A$, $\quad c_{14} = c_7 c_{43} = -c'_2 c_{44}$

15. From $H = H_s + \Delta H$, $\quad c_{15} = -c_8 c_{45} = -c'_3 c_{46}$

16. From $Y' = Y'_s + \Lambda X$, $\quad c_{16} = -c_{12} c_{47} = -c_6 \Lambda c_{48}$ (using (a) also)

17. From $X' = X'_s - \Lambda Y$, $\quad c_{17} = -c_5 c_{49} = c_7 \Lambda c_{50}$ (using (a) also)

By (l)

I. From $\underline{T} = r_I T$, $\quad c_{51} = r_I$

II. From $\underline{X} = r_{II} X$, $\quad c_{52} = r_{II}$

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[redacted]

III. From $\underline{Y} = r_{III}Y$, $\qquad c_{53} = r_{III}$

IV. From $\underline{V} = r_{IV}V$, $\qquad c_{54} = r_{IV}$

V. From $\underline{Y}' = r_{V}Y'$, $\qquad c_{55} = r_{V}$

VI. From $\underline{X}' = r_{VI}X'$, $\qquad c_{56} = r_{VI}$

4. Solution of the Equations for Conversion Coefficients.

The system of equations in the preceding section involves many more unknowns than equations. It is desirable to separate relations involving coupling coefficients alone from those which involve primary conversion coefficients. The coefficients c_1', c_2', c_3', and c_{51} to c_{56} may be considered separately. It will be recalled that Λ is in some cases zero, and is in any case based upon the observation of the azimuth of fire. This number need not be easily representable exactly by gear train factors. However the use of the machine requires a gear train factor to represent Λ. Since the effect is in any case small, we adjust c_{48} and c_{50} to give acceptable values for $-c_6\Lambda c_{48}$ and $c_7\Lambda c_{50}$, as will be discussed presently.

It is found that c_8 may be retained at present as an independent parameter, to be adjusted later subject to inferior limitations. Leaving this parameter indeterminate, each other primary conversion coefficient in the list, c_0 to c_{17}, may be determined in terms of coupling coefficients. The determination is rational (involving c_8) save for a single quadratic irrationality, which may be taken to be c_0.

We proceed to the solution, using the numbering of equations given in Section 3.

From (8), since $c_8 \neq 0$,

$$c_7 = -(1/32)c_{32}^{-1}c_{33}^{-1} \qquad (1)'$$

From (14) and (1)',

$$c_{14} = -(1/32)c_{32}^{-1}c_{33}^{-1}c_{43} \qquad (2)'$$

From (12),

$$c_{12} = c_0c_{39} \qquad (3)'$$

$$c_4 = c_0c_{39}c_{40}^{-1} \qquad (4)'$$

[redacted]

From (16) and (3)',

$$c_{16} = -c_0 c_{39} c_{47} \tag{5'}$$

From (9) and (5)',

$$c_9 = -c_0 c_{35} c_{39} c_{47} \tag{6'}$$

$$c_{13} = -c_0 c_{34}^{-1} c_{35} c_{39} c_{47} \tag{7'}$$

From (13) and (7)',

$$c_{17} = c_0 c_{34}^{-1} c_{35} c_{39} c_{41}^{-1} c_{47} \tag{8'}$$

From (17) and (8)',

$$c_5 = -c_0 c_{34}^{-1} c_{35} c_{39} c_{41}^{-1} c_{47} c_{49}^{-1} \tag{9'}$$

From (10), (2)', and (6)',

$$c_{10} = (4/45) c_0 c_{32} c_{33} c_{35} c_{36}^{-1} c_{37} c_{39} c_{43}^{-1} c_{47} \tag{10'}$$

From (5), (7)', and (9)',

$$c_3 = -(1/32) c_{26}^{-1} c_{27}^{-1} c_{41}^{-1} c_{49}^{-1} \tag{11'}$$

From (4), (4)', (11)', and (5)', we obtain the following relation in coupling coefficients only,

$$c_{24} c_{25} c_{40} c_{47} = c_{26} c_{27} c_{41} c_{49}$$

which by (8 ii) reduces to

$$c_{25} c_{40} c_{47} = c_{27} c_{41} c_{49} \tag{12'}$$

This relation may be used to eliminate c_{49} in relations which would otherwise contain it. In particular from (9)' and (11)' we obtain respectively using also (8 ii),

$$c_5 = -c_0 c_{25}^{-1} c_{27} c_{34}^{-1} c_{35} c_{39} c_{40}^{-1} \tag{13'}$$

$$c_3 = -(1/32) c_{24}^{-1} c_{25}^{-1} c_{40}^{-1} c_{47}^{-1} \tag{14'}$$

From (7), (1)', (5)', and (8 iii), we obtain the quadratic relation for c_0 which latter may be chosen as positive, giving

$$1/c_0 = 32\sqrt{c_{25} c_{30} c_{32} c_{33} c_{39} c_{47}} \tag{15'}$$

From (6), (8)', (15)', and (8 1),

$$c_6 = (1/32)c_{25}^{-1}c_{29}c_{32}^{-1}c_{33}^{-1}c_{34}^{-1}c_{35}c_{41}^{-1} \qquad (16)'$$

From (1), (6)', (8 1), and (15)',

$$c_1 = -(1/32)c_{19}c_{25}^{-1}c_{32}^{-1}c_{33}^{-1}c_{35} \qquad (17)'$$

We shall retain c_8 as temporarily undetermined. Then (15) reads

$$c_{15} = -c_8c_{45} \qquad (18)'$$

From (2), (17)', and (18)', we have

$$c_2 = c_8c_{19}c_{20}c_{21}c_{25}^{-1}c_{32}^{-1}c_{33}^{-1}c_{35}c_{45} \qquad (19)'$$

From (3), (14)', and (19)', we have for $1/c_{11}$ in terms of c_8, the following,

$$1/c_{11} = -(32)^2c_8c_{19}c_{20}c_{21}c_{22}c_{23}c_{24}c_{32}^{-1}c_{33}^{-1}c_{35}c_{40}c_{45}c_{47} \qquad (20)'$$

Solving for Λ, we obtain using (16), (3)', and (16)',

$$\Lambda c_{48} = 32c_0c_{25}c_{29}^{-1}c_{32}c_{33}c_{34}c_{35}^{-1}c_{39}c_{41}c_{47} \qquad (21)'$$

Also from (17), (8)', and (1)',

$$\Lambda c_{50} = -32c_0c_{32}c_{33}c_{34}^{-1}c_{35}c_{39}c_{41}^{-1}c_{47} \qquad (22)'$$

Eliminating Λ between (21)' and (22)', we obtain

$$c_{25}c_{34}^2c_{41}^2c_{50} = -c_{29}c_{35}^2c_{48} \qquad (23)'$$

Turning finally to the gear coupling coefficients leading from the handcranks, we have from (13), (14), (15) respectively,

$$c'_1c_{42} = c_0c_{34}^{-1}c_{35}c_{39}c_{47} \qquad (24)'$$

$$c'_2c_{44} = (1/32)c_{32}^{-1}c_{33}^{-1}c_{43} \qquad (25)'$$

$$c'_3c_{46} = c_8c_{45} \qquad (26)'$$

For convenience in future reference, the preceding results are given as follows in sequence:

From

(15)' $1/c_0 = 32\sqrt{c_{25}c_{30}c_{32}c_{33}c_{39}c_{47}}$ (0)

(17)' $1/c_1 = -32\, c_{19}^{-1} c_{25}c_{32}c_{33}c_{35}^{-1}$ (1)

(19)' $c_2 = (c_8)\cdot c_{19}c_{20}c_{21}c_{25}^{-1}c_{32}^{-1}c_{33}^{-1}c_{35}c_{45}$ (2)

(14)' $1/c_3 = -32\, c_{24}c_{25}c_{40}c_{47}$ (3)

(4)' $c_4 = (c_0)\cdot c_{39}c_{40}^{-1}$ (4)

(13)' $c_5 = -(c_0)\cdot c_{25}^{-1}c_{27}c_{34}^{-1}c_{35}c_{39}c_{40}^{-1}$ (5)

(16)' $1/c_6 = 32\, c_{25}c_{29}^{-1}c_{32}c_{33}c_{34}c_{35}^{-1}c_{41}$ (6)

(1)' $1/c_7 = -32\, c_{32}c_{33}$ (7)

c_8 arbitrary $(\neq 0)$ (8)

(6)' $c_9 = -(c_0)\, c_{35}c_{39}c_{47}$ (9)

(10)' $c_{10} = (4/45)(c_0)\cdot c_{32}c_{33}c_{35}c_{36}^{-1}c_{37}c_{39}c_{43}^{-1}c_{47}$ (10)

(20)' $1/c_{11} = -(32)^3(c_8)\cdot c_{19}c_{20}c_{21}c_{22}c_{23}c_{24}c_{32}^{-1}c_{33}^{-1}c_{35}c_{40}c_{45}c_{47}$ (11)

(3)' $c_{12} = (c_0)\, c_{39}$ (12)

(7)' $c_{13} = -(c_0)\cdot c_{34}^{-1}c_{35}c_{39}c_{47}$ (13)

(2)' $1/c_{14} = -32\, c_{32}c_{33}c_{43}^{-1}$ (14)

(18)' $c_{15} = -(c_8)\cdot c_{45}$ (15)

(5)' $c_{16} = -(c_0)\cdot c_{39}c_{47}$ (16)

(8)' $c_{17} = (c_0)\cdot c_{34}^{-1}c_{35}c_{39}c_{41}^{-1}c_{47}$ (17)

(21)' $\Lambda c_{48} = 32\, c_0c_{25}c_{29}^{-1}c_{32}c_{33}c_{34}c_{35}^{-1}c_{39}c_{41}c_{47}$ (18)

Also

(8 i) $c_{18} = c_{28} = c_{30}$ (19)

(8 ii) $c_{24} = c_{26}$ (20)

(8 iii) $c_{25} = c_{31}$ (21)

(12)' $c_{25}c_{40}c_{47} = c_{27}c_{41}c_{49}$ (22)

[redacted]

[redacted]

(23)' $c_{25}\, c_{34}^{2}\, c_{41}^{2}\, c_{50} = -c_{29} c_{35}^{2}\, c_{48}$ (23)

To this list we may append for the recorder ratios

$$r_{I} = c_{51}$$
$$r_{II} = c_{52}$$
$$r_{III} = c_{53}$$
$$r_{IV} = c_{54}$$
$$r_{V} = c_{55}$$
$$r_{VI} = c_{56}$$

For the crank gear coupling coefficients, we have

(24)' $c_1' c_{42} = c_0 c_{34}^{-1} c_{35} c_{39} c_{47}$

(25)' $c_2' c_{44} = (1/32) c_{32}^{-1} c_{33}^{-1} c_{43}$

(26)' $c_3' c_{46} = c_8 c_{45}$

B. Inequalities for Conversion and Coupling Coefficients.

I. Travel Limits.

The travel limits for the separate submachines were mentioned in the description of the Differential Analyzer. Some additional comments are called for in this connection due to the form and use of the canonical equations.

The following ballistic functions have ordinarily for their maximum values on any one trajectory the initial values, and in any case (allowing a little margin) the initial values may be used as satisfactory approximations to the maximum values. This holds even in the case of $U = V/A$, despite the decrease of A with altitude.

$$X_s', H_s, V, U, Y_s', W', A, H, Y', X'$$

The respective initial values may be designated as follows, since in particular X_s' and X' have the same initial value, and also Y_s' and Y' have the same initial value.

$$X_o', H_{s_o}, V_o, U_o, Y_o', W_o', A_o, H_o, Y_o', X_o'.$$

[redacted]

We may use H_m as the greater of H_{s_0} and H_0, and V_0/A_0 for U_0.

For lower bounds on the values of the variables given above, we may use with a margin of safety the value zero in each case save for Y'_s and Y' for both of which the negative value $-Y'_0$ may be used.

In contrast with the foregoing it is convenient to use a special notation B_Δ for the positive interval between minimum and maximum values of B(U) for the template considered. This interval, B_Δ, will depend upon the type of projectile used but not upon the ballistic coefficient, muzzle velocity, or angle of departure of the particular trajectory.

A given template has for its maximum abscissa a determinate value of U. If larger values of U are needed on some trajectory, a new template is called for. This maximum value of U for which the given template, giving B(U), can be used may be designated by U_B. It may of course happen that a given trajectory does not use the full range of the template, so that U_B may exceed U_0. If however a group of trajectories have throughout a common muzzle velocity, V_0, such that V_0/A_0 is very much less than the U_B of the given template, a new template may be used displaying the same functional relationship B(U), but with greatly reduced numerical range for U and hence with greatly increased scale and correspondingly improved accuracy.

In somewhat similar manner, one may remark that ordinarily in the computation of trajectories, the work is not confined to a single trajectory, but rather to a set of trajectories alike save for such differences as follow from changing the angle of departure from one trajectory to another or lowering the muzzle velocity for nearer zones. Since the angles of departure are usually distributed from nearly 0° to nearly 90°, the maximum value, say X'_m, used for X' and W' on the set of trajectories is likely to be approximately V_0, and similarly the maximum value, say Y'_m, of Y' is also likely to be approximately V_0, even though X'_m and Y'_m cannot both be used on any single trajectory.

2. The Inequalities Based on Travel, Derived.

On each of the integrating units the integrand as expressed in revolutions of the lead screw is free to range from approximately -40 to +40. The discussion for the first integrator unit, giving $S = \int_0 V\,dT$ may serve to illustrate the method for the others. The maximum value of V is V_0. Now, by

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definition of conversion coefficient, one complete turn of the primary shaft of V yields an increment of c_9 units in V, and also one complete turn of the secondary shaft carrying V, (which shaft is gear coupled to its primary shaft by a gear coupling with coupling coefficient c_{19}) it yields c_9c_{19} units algebraic increment in V. Hence 40 turns yields $40|c_9c_{19}|$ units increase in V, so that one has

$$40\ |c_9c_{19}| \geq V_m \tag{1'}$$

In a similar manner for the other integrator units one obtains,

$$40\ |c_{15}c_{21}| \geq H_m \tag{2'}$$

$$40\ |c_{11}c_{23}| \geq B_m \tag{3'}$$

$$40\ |c_{16}c_{25}| \geq Y'_m \tag{4'}$$

$$40\ |c_{13}c_{27}| \geq X'_m \tag{5'}$$

$$40\ |c_{17}c_{29}| \geq X'_m \tag{6'}$$

$$40\ |c_{16}c_{31}| \geq Y'_m \tag{7'}$$

$$40\ |c_8c_{33}| \geq H_m \tag{8'}$$

There are similar inequalities derivable from the limitations on travel for the tables, as follows.

For the Vector Table, giving $V = \sqrt{W'^2 + Y'^2}$, W' is allowed to vary over positive values up to about X'_m. The Table seemingly offers for positive values measured from its center, 240 turns of the abscissa lead screw. However, the radial bar carrying V on the face of the table is adapted to only a slight margin over a travel in V of 180 turns of its lead screw. Since both W' and Y' remain in absolute value essentially not greater than V, we have as practical limits, $W' < V_m$, $|Y'| < V_m$, $V \leq V_m$. These lead respectively to the inequalities

$$180\ |c_{13}c_{34}| \geq V_m \tag{9'}$$

$$180\ |c_{16}c_{35}| \geq V_m \tag{10'}$$

$$180\ |c_9| \geq V_m \tag{11'}$$

For the Division Table giving $U = V/A$, the extreme bound is imposed by the initial conditions. At the start A is evaluated for the group and has the known constant value A_o.

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Here V is bounded by V_m. To make the most use of the range provided by the Table, the initial setting should be for the radial bar which carries the follower marking U on the face of the Table to be in approximately a 45° position. Then A_0 is out at 180 turns of its lead screw, and V_m at 180 turns of its lead screw. In this angular position the follower for the quotient, U, has its displacement fixed. The output of the machine is provided by a heavy lead screw placed parallel to the abscissa lead screw and with a ten turn per inch pitch. The nut carries a sleeve in which slides a bar set rigidly at right angles to the radial bar in the face of the table. In extreme position, it is also a little more than nine inches from the center on this lead screw, and due to a 4 to 1 gear reduction built into the table, records the value of 360 turns for the output shaft. Hence for the Division Table we have (save for a little margin) the following,

$$180\,|c_{14}c_{36}| \geq A_o,$$

$$180\,|c_9 c_{37}| \geq V_m,$$

$$360\,|c_{10}| \geq V_m/A_o.$$

The exact value of A_o to six significant places is 10.5274. However due to the available margin, we may use 21/2 in place of A_o for the purpose of these inequalities. In place of the first inequality stated above, we write an equation

$$180\,|c_{14}c_{36}| = 21/2$$

and in place of the third, we write the slightly stronger inequality

$$360\,|c_{10}| \geq 2\,V_m/21$$

Hence we write

$$|c_{14}c_{36}| = 7/120 \qquad (12)'$$

$$|c_9 c_{37}| \geq V_m/180 \qquad (13)'$$

$$|c_{10}| \geq V_m/3780 \qquad (14)'$$

The Template Table giving directly $B_s(U)$ as a function of U, and through calibration giving $B(U) = B_s(U) + \Gamma$ where Γ is an empirical constant on any given trajectory, involves (in contrast to the other tables) only two lead screws, the abscissa as input, the ordinate as output. The abscissa lead screw on this table has a total travel of 480 turns corresponding to values of U from 0 to U_m. The ordinate lead screw has a total travel of 360 turns corresponding to the total interval of B_Δ. Now $U = V/A$ is actually a maximum at the start. Hence for the Template Table instead of using the direct relation $480|c_{10}c_{38}| \geq V_m/A_o$, and recalling that $V_m/A_o < 2\,V_m/21$, we adopt the nominally stronger inequality (15)' below. We write

$$|c_{10}c_{38}| \geq V_m/5040 \qquad (15)'$$

$$|c_{11}| \geq B_\Delta/360 \qquad (16)'$$

Substituting the previously obtained values for the conversion coefficients c_1 to c_{17}, obtained in Section 1, we obtain the following from (1)' to (16)' respectively. Save for the equation (12), these are all inequalities.

From

(1)' $|c_0| \cdot |c_{19}c_{35}c_{39}c_{47}| \geq V_m/40$ (1)

(2)' $|c_8| \cdot |c_{21}c_{45}| \geq H_m/40$ (2)

(3)' $|c_8|^{-1} \cdot |c_{19}^{-1} c_{20}^{-1} c_{21}^{-1} c_{22}^{-1} c_{24}^{-1} c_{32}c_{33}c_{35}^{-1} c_{40}^{-1} c_{45}^{-1} c_{47}^{-1}| \geq 128B_m/5$ (3)

(4)' $|c_0| \cdot |c_{25}c_{39}c_{47}| \geq Y'_m/40$ (4)

(5)' $|c_0| \cdot |c_{27}c_{34}^{-1} c_{35}c_{39}c_{47}| \geq X'_m/40$ (5)

(6)' $|c_0| \cdot |c_{29}c_{34}^{-1} c_{35}c_{39}c_{41}^{-1} c_{47}| \geq X'_m/40$ (6)

(7)' Duplicate of (4), since $c_{25} = c_{31}$ (7)

(8)' $|c_8| \cdot |c_{33}| \geq H_m/40$ (8)

(9)' $|c_0| \cdot |c_{35}c_{39}c_{47}| \geq V_m/180$ (9)

(10)' Duplicates (9) (10)

(11)' Duplicates (9) (11)

(12)' $|c_{32}^{-1} c_{33}^{-1} c_{36}c_{43}| = 28/15$ (12)

(13)' $|c_0| \cdot |c_{35}c_{37}c_{39}c_{47}| \geq V_m/180$ (13)

(14)' $|c_0| \cdot |c_{32}c_{33}c_{35}c_{36}^{-1} c_{37}c_{39}c_{43}^{-1} c_{47}| \geq V_m/336$ (14)

(15)' $|c_0| \cdot |c_{32}c_{33}c_{35}c_{36}^{-1} c_{37}c_{38}c_{39}c_{43}^{-1} c_{47}| \geq V_m/448$ (15)

(16)' $|c_8|^{-1} \cdot |c_{19}^{-1} c_{20}^{-1} c_{21}^{-1} c_{22}^{-1} c_{23}^{-1} c_{24}^{-1} c_{32}c_{33}c_{35}^{-1} c_{40}^{-1} c_{45}^{-1} c_{47}^{-1}| \geq 128B_{\Delta}/45$ (16)

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3. Nature of Load Limits.

Most of the connections within the Differential Analyzer are either firmly locked internally or operate through an automatic follower with fresh power input. Questions of back lash, friction and load arise, but roughly speaking, if a submachine (other than an integrator unit) operates at all under a given load, it operates satisfactorily. In the case of an integrator unit, however, where furthermore the output passes through a torque amplifier, there is always the possibility of slippage between wheel and disc of the integrator but even more between band and drum of the torque amplifier. This slippage is often spasmodic and sometimes probably undetected.

The torque amplifier, with its own motor drive provides force, also torque, energy, and power. In order to keep the discussion clear it is well for the reader to refresh his memory upon the distinctions among these terms. Assuming that the word "force" is adequately understood, the reader should recall that torque is a moment. The torque of a shaft is the product of the tangential force by the radial distance. The introduction of a gear coupling changes the torque transmitted. The output bus shaft of a gear coupling on the Differential Analyzer has the same radius as the input bus shaft but may provide a greater (tangential) force by being so geared as to turn less rapidly than the input shaft. The energy, however, save for loss by friction, remains constant on passing through a gear coupling. The energy of a steadily rotating bus shaft (ignoring inertia) is proportional to the product of the (constant) torque by the angular distance through which this (constant) torque acts. When a gear coupling increases the torque, it is at the cost of reducing the relative speed of rotation proportionately thus keeping constant the energy (save for loss through friction). The power of a steadily rotating shaft is its rate of suppling energy, or more familiarly, of doing work. One could, if desired, think of power rather than energy being transmitted through the bus shafts and one would carry out all computations in terms of power. The power supplied would depend upon the speed of the prime motor at given torque. Since the functioning of the Differential Analyzer involves essentially only relative rather than absolute speeds, it seems better to omit reference to comparisons of power as such. If such comparisons were made, the speed of the prime motor would cancel out leaving only relations among energies and torques.

In computing loads we ignore the speeds of rotation

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and consider essentially only tangential forces, or more conveniently since bus shafts are concerned, only torques. This is appropriate for frictional loads under steady rotation. A submachine element being driven by an integrator output shaft consumes to maintain its operation a certain torque rather than a certain quantity of energy. By sufficient reduction of relative speeds, any given set of submachines can be operated by a single integrator unit through its torque amplifier.

It seems safe to assume that in ordinary practice the loads placed upon motor driven elements other than the torque amplifier operating integrator output never exceeds the torque provided, since when no further precautions on load are adopted, there is no stoppage of bus shafts.

Now the load borne by a machine element ordinarily is carried no further. The machine element either is a natural terminus as in the case of a printer unit, or else the submachine of which it is an input element supplies fresh power to its output. The adders, however, constitute a notable exception to this general remark. Any load carried by the output element of an adder is also carried by one or the other or both of the input elements of this adder. As a simple, although crude method of obtaining an estimate of the total load borne by a given integrator output shaft, we adopt the procedure of computing any loads carried by an adder which is gear-connected to a given integrator output shaft, as though the second input of this adder remains stationary-- save for one exceptional type of case which we now note. In the case only of the adders yielding earth rotation effects, $Y' = Y'_s + \Lambda X$, and $X' = X'_s - \Lambda Y$, due to the smallness of the correction terms ΛX, and $-\Lambda Y$ respectively, the load on the shafts bearing these correction variables is reckoned as not including any further loads carried by the outputs, Y' and X' respectively.

4. The List of Loaded Gear Couplings for the Separate Integrator Output Shafts.

There are eight integrator units which have been assigned the numbers 1 to 8. These same numbers are also assigned to the respective output shafts. Each of these shafts lead to at least one input machine element through a gear coupling. The input machine element and the coupling through which it is driven have been assigned a common number. We shall presently list these numbers.

To simplify the problem of tracing the load

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carried by a given integrator output, we shall underscore in each case the number of any adder whose output has further loads contributing to the total load of the integrator output. This further load will be indicated by placing in parentheses the numbers of such input machine elements as driven through the adder concerned. If among these occurs another adder with further transmitted load, the number of this second adder will be underscored in the parentheses and will contribute a subparenthetical sequence of numbers.

For convenience in computing the loads we shall classify by type the separate loads concerned in each case and indicate their total number. The types will be as follows: (i) Differential input to an integrator unit, (ii) Integrand input to an integrator unit, (iii) Input to a table, (iv) Input to an adder, (v) Input to a printer.

It may prove enlightening to trace through one of the more complicated cases, leaving easier cases to be carried through in like manner by the reader. We select for illustration, the Integrator Unit #5. This yields (with proper calibration) $X'_s = X'_o - \int_o W'\, dR$. Only a single gear coupling is attached directly to the output shaft for X'_s, namely the adder #49, which gives $X' = X'_s - \Lambda Y$. The entire load for X' is to be counted as part of the load for X'_s. The list of gear couplings for this integrator unit reads (at the start) as follows, 49 ( ). The underscoring of 49 is for the reason explained above. The contents of the parentheses are exactly as for X' itself. The primary shaft for X' is not an integrator output shaft but is the output of the adder (#49). The following are the numbers for the gear couplings and thence of the submachine elements driven by the primary bus shaft for X': 29, 41, 56. Here #29 gives the integrand input for Integrator Unit #6, yielding $X = \int_o X'\, dt$. Similarly #56 gives the Printer VI for X'. Both of these are terminal for load. In contrast #41 gives an input to Adder #1, yielding $W' = X' + \Delta W'$. Hence the load of W' is also to be counted as part of the load for X'_s. The notation thus far discussed for the load elements for X'_s yields 49 (29, 41, 56 ( )), where the uncompleted secondary set of parentheses marks the load for #41, or W'. The primary bus shaft for W' has two attached gear connections, namely #27 and #34. Of these #27 leads to the integrand of Integrator Unit #5, yielding $X'_s = X'_o - \int_o W'\, dR$, incidentally, the equation with which we started. Similarly, #34 leads to an input of Table #1, the Vector Table, giving $V = \sqrt{W'^2 + Y'^2}$. Both of these are terminal as to load. Our complete list of loads for the output of Integrator Unit #5 reads as follows: 49 (29, 41, 56 (27, 34)).

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This list consists of 6 loads, comprising 2 integrand inputs, 1 table input, 2 adder inputs, and 1 printer.

The complete list is as follows:

Integrator #	Equation	Gear Coupling #s also Machine Element #s	Differential Inputs	Integrand Inputs	Table Inputs	Adder Inputs	Printers	Total
1	$S = \int_0 V \, dT$	20	1					1
2	$J = \int_0 H \, dS$	22	1					1
3	$R = \int_0 B \, dJ$	24,26	2					2
4	$K = \int_0 Y' \, dR$	40(47(25,31,35,55))		2	1	2	1	6
5	$X'_s = X'_o - \int_0 W' \, dR$	49(29,41,56(27,34))		2	1	2	1	6
6	$X = \int_0 X' \, dT$	48,52				1	1	2
7	$Y = \int_0 Y' \, dT$	32,43,50,53,(36)	1		1	2	1	5
8	$H_s = H_o - \int_0 H_s \, dY$	33,45,(21)		2		1		3

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5. Quantitative Principles for Load Limits.

Let τ represent the (approximately constant) torque of a single torque amplifier output in operation. If a gear coupling with coupling coefficient c be introduced, then on passing through the gear coupling, the angular speed is divided by c. Some torque is consumed in friction in the gear coupling, but the torque delivered is approximately c times the initial torque. The energy remains unchanged save for loss by friction in the gear coupling.

Experiments with the safe loads upon the torque amplified output of an integrator unit have led to the formulation of the following principles:

(1) The torque required to operate the differential input of an integrator unit (and its interposed gear couplings) is approximately equal to $\tau/3$.

(2) The torque required to operate the integrand input of an integrator unit (and its interposed gear couplings) is approximately equal to τ.

(3) The torque required to operate one input to a table (and its interposed gear couplings) is approximately equal to $\tau/2$.

(4) The torque required to operate one input to an adder (and its interposed gear couplings) is approximately equal to $\tau/5$.

(5) In order to avoid unnecessary feeding back of vibrations experience seems to suggest that the gear coupling from an output of an integrator unit to the differential input of an integrator unit should yield a reduction of relative speed to at most 4/5 of the output speed.

(6) In order to avoid unnecessary vibration in an adder, it is regarded as desirable that no shaft leading into an adder rotates at more than half the speed of an integrator output shaft driving it by gear-coupling.

(7) As a practical procedure, if an integrator output shaft drives several loads, n in number, then one part in n of the total energy output is allotted to each load in computing load limits. The load due to a printer is counted in determining this n, although the restrictions on energy arising from a printer alone are found to be more than covered by the other principles. Furthermore the torque transmitted

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through an adder to a more distant load is computed as though it were delivered directly from the integrator output shaft.

(8) For the reduction of internal friction in an adder no gear coupling coefficient less than unity should be used leading immediately into an adder if the output of this adder is to be geared immediately with a gear coupling coefficient correspondingly greater than unity.

6. Inequalities due to Load Limits.

Let us illustrate the method of obtaining inequalities due to load limits by resuming the treatment of the loads upon the output of Integrator Unit #5, which gives $X'_m = X'_o - \int_o W' \, dR$. As we have seen the load numbers for this variable, X'_s, are <u>49</u>(29,<u>41</u>,56(27,34)) and consist of six loads, namely 2 integrand inputs, 1 table input, 2 adder inputs, 1 printer input. One sixth of the original output energy from Integrator Unit #5, is to be allotted directly to each of these loads, in accordance with Principle #7. The respective torques supplied are then as follows, assuming that in the case of each adder considered, the other input may be treated as though stationary.

To	Type	Torque
49	adder	$\lvert c_{49} \rvert \, (T/6)$
29	integrand	$\lvert c_{29} c_{49} \rvert \, (T/6)$
56	printer	$\lvert c_{49} c_{56} \rvert \, (T/6)$
41	adder	$\lvert c_{41} c_{49} \rvert \, (T/6)$
27	integrand	$\lvert c_{27} c_{41} c_{49} \rvert \, (T/6)$
34	table	$\lvert c_{34} c_{41} c_{49} \rvert \, (T/6)$

By Principles (1) to (4), we conclude (omitting the printer):

$$\lvert c_{49} \rvert \, (T/6) \geq T/5$$

$$\lvert c_{29} c_{49} \rvert \, (T/6) \geq T$$

$$\lvert c_{41} c_{49} \rvert \, (T/6) \geq T/5$$

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$$|c_{27} c_{41} c_{49}| \ (T/6) \geq T$$

$$|c_{34} c_{41} c_{49}| \ (T/6) \geq T/2$$

Eliminating T, we have the following five inequalities:

$$|c_{49}| \geq 6/5$$

$$|c_{29} c_{49}| \geq 6$$

$$|c_{41} c_{49}| \geq 6/5$$

$$|c_{27} c_{41} c_{49}| \geq 6$$

$$|c_{34} c_{41} c_{49}| \geq 3$$

Following the pattern of this example, the reader may verify each of the following complete list obtained by use of Principles (1), (2), (3), (4), (7) alone.

For Integrator #1

$$|c_{20}| \geq 1/3 \qquad (1)'$$

For Integrator #2

$$|c_{22}| \geq 1/3 \qquad (2)'$$

For Integrator #3

$$|c_{24}| \geq 2/3 \qquad (3)' \ (i)$$

$$|c_{26}| \geq 2/3 \qquad (3)' \ (ii)$$

For Integrator #4

$$|c_{40}| \geq 6/5 \qquad (4)' \ (i)$$

$$|c_{40} c_{47}| \geq 6/5 \qquad (4)' \ (ii)$$

$$|c_{25} c_{40} c_{47}| \geq 6 \qquad (4)' \ (iii)$$

$$|c_{31} c_{40} c_{47}| \geq 6 \qquad (4)' \ (iv)$$

$$|c_{35} c_{40} c_{47}| \geq 3 \qquad (4)' \ (v)$$

For Integrator #5, as already obtained

$$|c_{49}| \geq 6/5 \qquad (5)' \ (i)$$

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$|c_{29}c_{49}| \geq 6$ (5)' (ii)

$|c_{41}c_{49}| \geq 6/5$ (5)' (iii)

$|c_{27}c_{41}c_{49}| \geq 6$ (5)' (iv)

$|c_{34}c_{41}c_{49}| \geq 3$ (5)' (v)

For Integrator #6

$|c_{48}| \geq 2/5$ (6)'

For Integrator #7

$|c_{32}| \geq 5/3$ (7)' (i)

$|c_{50}| \geq 1$ (7)' (ii)

$|c_{43}| \geq 1$ (7)' (iii)

$|c_{36}c_{43}| \geq 5/2$ (7)' (iv)

For Integrator #8

$|c_{33}| \geq 3$ (8)' (i)

$|c_{45}| \geq 3/5$ (8)' (ii)

$|c_{21}c_{45}| \geq 3$ (8)' (iii)

Applying Principle (5), we obtain the following:

$|c_{20}| \geq 5/4$ (9)' (i)

$|c_{22}| \geq 5/4$ (9)' (ii)

$|c_{24}| \geq 5/4$ (9)' (iii)

$|c_{26}| \geq 5/4$ (9)' (iv)

$|c_{32}| \geq 5/4$ (9)' (v)

Applying Principle (6), we obtain

$|c_{40}| \geq 2$ (10)' (i)

$|c_{43}| \geq 2$ (10)' (ii)

$|c_{45}| \geq 2$ (10)' (iii)

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$|c_{48}| \geq 2$ (10)' (iv)

$|c_{49}| \geq 2$ (10)' (v)

$|c_{50}| \geq 2$ (10)' (vi)

It becomes obvious even upon cursory inspection that these inequalities are not all independent. For example, (1)' states that $|c_{20}| \geq 1/3$, while (9)' (i) states that $|c_{20}| \geq 5/4$.

A revised list making use also of the following relations, previously stated, $c_{18}=c_{28}=c_{30}$, $c_{24}=c_{26}$, and $c_{25}=c_{31}$, is as follows arranged in order of increasing subscript in any given sublist.

From (9)' (i) and (1)'	$\lvert c_{20}\rvert \geq 5/4$	(1)
From (9)' (ii) and (2)'	$\lvert c_{22}\rvert \geq 5/4$	(2)
From (9)' (iii) and (3)' (i)	$\lvert c_{24}\rvert \geq 5/4$	(3)

Using $c_{24}=c_{26}$, (9)'(iv) and (3)' (ii) yield nothing new.

From (9)' (v) and (7)' (i)	$\lvert c_{32}\rvert \geq 5/3$	(4)
From (8)' (i)	$\lvert c_{33}\rvert \geq 3$	(5)
From (10)' (i) and (4)' (i)	$\lvert c_{40}\rvert \geq 2$	(6)
By Principle 8	$\lvert c_{41}\rvert \geq 1$	(7)
From (10)' (ii) and (7)' (iii)	$\lvert c_{43}\rvert \geq 2$	(8)
From (10)' (iii) and (8)' (ii)	$\lvert c_{45}\rvert \geq 2$	(9)
By Principle 8	$\lvert c_{47}\rvert \geq 1$	(10)
From (10)' (iv) and (6)'	$\lvert c_{48}\rvert \geq 2$	(11)
From (10)' (v) and (5)' (i)	$\lvert c_{49}\rvert \geq 2$	(12)
From (10)' (vi) and (7)' (ii)	$\lvert c_{50}\rvert \geq 2$	(13)

Next for products of two coupling coefficients, arranged as to lower subscript,

From (8)' (iii)	$\lvert c_{21}c_{45}\rvert \geq 3$	(14)
From (5)'(ii)	$\lvert c_{29}c_{49}\rvert \geq 6$	(15)

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From (7)' (iv) $\qquad |c_{36}c_{43}| \geq 5/2 \qquad (16)$

The condition (4)' (ii) follows from (6) and (10). The condition (5)' (iii) similarly follows from (7) and (12).

Next for products of three coupling coefficients,

From (4)' (iii) $\qquad |c_{25}c_{40}c_{47}| \geq 6 \qquad (17)$

From (5)' (iv) $\qquad |c_{27}c_{41}c_{49}| \geq 6 \qquad (18)$

The condition (4)' (iv) follows from (17) by use of the equation, $c_{25}=c_{31}$.

From (5)' (v) $\qquad |c_{34}c_{41}c_{49}| \geq 3 \qquad (19)$

From (4)' (v) $\qquad |c_{35}c_{40}c_{47}| \geq 3 \qquad (20)$

7. Speed Limits of Primary Bus Shafts.

As stated previously, the actual speed at which the machine is run does not affect the recorded results, if one observes a few safeguards. In practice the prime motor though rated at 1725 r.p.m. actually attains nearly 2400 r.p.m. whenever it is driven at full speed but will not run at greater speed on the present circuit. Several conditions serve usually to keep the speed more nearly at or under the rated value.

Let us list explicitly some principles concerning speed limits.

(1) The primary shaft for T should never rotate faster than 800 r.p.m. (automatically assured at present).

(2) At the start of a trajectory the primary shaft for T should not rotate faster than about 480 r.p.m., due to the number of integrands then at their extreme value.

(3) For the proper working of the torque amplifier in connection with an integrator unit output, experience indicates an upper limit of 600 r.p.m. for the output shaft of any integrator unit.

In application of the third of these principles an upper bound of 800 r.p.m. for the main time shaft might be made the basis of computation of gear couplings. In practice, however, the operator watches the tachometers

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attached to the T and X shafts, which shafts alone show critical speeds and verify that not more than 480 r.p.m. is registered for T at the start of the trajectory and 600 r.p.m. for X at any time. In case of either bound being surpassed, the speed of the entire machine would be slowed down by the hand control until values below the allowable limits are reached.

In making the computations various approximate estimates are adopted. One may note that for an integrator unit the maximum value of the integrand is approximately for forty turns of the integrand lead screw. For the integrand disc at this position the output shaft rotates 40/32 times, 5/4 times as rapidly as the differential input shaft, since, as noted earlier, the output shaft and differential input shaft rotate at the same rate when the integrand has the value 32 (turns), and for given rate of differential input the output is proportional to the integrand.

Let us make some estimate for selected primary variables under initial conditions computed separately for zero angle of departure, for 45° angle of departure, and as a limiting case, for vertical fire.

(a) At zero angle of departure.

The variable S on the output shaft of the first integrator unit will be turning at not more than 600 r.p.m. This limit, given by Principle 3, may be actually obtained at the start of the trajectory. At that moment the input shaft for T will be rotating at about the rate $480/c_{18}$, by Principle (2), and therefore the output shaft at about $(5/4)(480/c_{18}) = 600/c_{18}$ r.p.m. This output is slowed down to $600/(c_{18}c_{20})$ r.p.m. before entering as differential input into an integrator unit. With H at the edge of the second integrator disc, the primary shaft for J is restricted to $(5/4)600/(c_{18}c_{20})$ or $750/(c_{18}c_{20})$ r.p.m. In entirely similar manner (although B is not necessarily at a maximum initially) the primary shaft for R is restricted to $1875/(2c_{18}c_{20}c_{22})$ r.p.m. Since we are now taking zero as the value of Y', the primary shaft for K is approximately stationary; however, as above, and with a change of sign, the primary shaft for X'_s is restricted absolutely to $9375/(8c_{18}c_{20}c_{22}c_{26})$ r.p.m.

Starting afresh from the T shaft, we obtain in similar manner $600/c_{28}$ r.p.m. as maximum for the primary shaft for X. For the assumed angle of departure, the speeds for Y', Y, H_s, etc. are negligible.

For use in the first table, we must obtain an

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estimate of the maximum for W' under conditions for which Y' is zero or nearly zero. This involves reference to Adders #5 and #1. In Adder #5, one can ignore for the present purpose the zero term, $-\Lambda Y$, so that, from what went before, the primary shaft for X' will have on its own primary shaft as upper bound $9375/(8c_{18}c_{20}c_{22}c_{26}c_{49})$ r.p.m., and therefore the primary shaft for W', ignoring the term $\Delta W'$, will have as maximum $9375/(8c_{18}c_{20}c_{22}c_{26}c_{41}c_{49})$ r.p.m. On entering Table #1, the shaft for W' has then as an upper bound approximately $9375/(8c_{18}c_{20}c_{22}c_{26}c_{34}c_{41}c_{49})$, which value therefore is, under assumed angle of departure, also an upper bound for the primary shaft for V.

(b) At 45° angle of departure.

At the angle of departure of 45°, the previous upper bounds for S, J, and R continue to hold. However, Y' is no longer negligible. But the maximum value to be used for Y' on this trajectory cannot be regarded as located on the edge of the disc at 40 complete turns. Indeed the maximum value of Y' when used for the limiting case of vertical fire is V_0, and only $V_0/\sqrt{2}$ is available here, or about $(7/10)V_0$, which will be at about 28 complete turns of the integrand lead screw. Since 28/32 is 7/8, we have for an estimate for K, $[1875/(2c_{18}c_{20}c_{22}c_{24})](7/8)$ r.p.m. Thus the primary bus shaft for K, under the conditions stated, is restricted to an angular speed of not more than about $13125/(16c_{18}c_{20}c_{22}c_{24})$ r.p.m. Similarly, for the primary bus shaft for X_s' we now obtain as upper bound approximately $13125/(16c_{18}c_{20}c_{22}c_{26})$ r.p.m.

For integrator units #6 and #7, placing X' and Y' at 28 turns of the integrand lead screw, we obtain, under conditions assumed, $(7/8)(480/c_{28}) = 420/c_{28}$ r.p.m. for X, and similarly $420/c_{30}$ r.p.m. for Y, on their respective primary shafts. We shall not need an estimate for an upper bound for H_s.

To obtain an upper bound for V, we use formula #9, for Table #1, and follow the variables through adders until we reach integrator outputs. In Adder #0, we have initially for T the upper bound $480/c_{39}$ r.p.m. and for K the upper bound, $13125/(16c_{18}c_{20}c_{22}c_{24}c_{40})$ r.p.m. Hence on the primary shaft for Y_s' we have as upper bound on the speed the value $[480/c_{39}] + [13125/(16c_{18}c_{20}c_{22}c_{24}c_{40})]$ r.p.m. Carrying this to Adder #4, and ignoring the initially zero term ΛX, we have as upper bound for Y' the value

$$[480/(c_{39}c_{47})] + [13125/(16c_{18}c_{20}c_{22}c_{24}c_{40}c_{47})] \text{ r.p.m.}$$

In Adder #5, neglecting the initially zero term, $-\Delta Y$, an upper bound for X_s' is $13125/(16c_{18}c_{20}c_{22}c_{26}c_{49})$ r.p.m., which value is also an upper bound for X' on its own primary shaft. Neglecting the correction term, $\Delta W'$, one then has from Adder #1, the value of $13125/(16c_{18}c_{20}c_{22}c_{26}c_{41}c_{49})$ r.p.m. as an upper bound for W' on its own primary shaft.

From the relation $V^2 = W'^2 + Y'^2$, we have

$$V\,dV = W'\,dW' + Y'\,dY'$$

where furthermore, under the conditions here assumed, initially $W' = Y' = V/\sqrt{2}$. Using this estimate we have

$$dV = 1/\sqrt{2}\,(dW' + dY').$$

Substituting the upper bounds for speeds of W' and Y' found above with fresh gear coupling coefficients for use in Table #1, one obtains as an upper bound for the speed of V the following

$$\left[480/(c_{35}c_{39}c_{47})\right] + \left[13125/(16c_{18}c_{20}c_{22}c_{24}c_{35}c_{40}c_{47})\right]$$
$$+ \left[13125/(16c_{18}c_{20}c_{22}c_{26}c_{34}c_{41}c_{49})\right] \text{ r.p.m.}$$

(c) For vertical fire (used as a limiting case).

For vertical fire the previous upper bounds for S, J, R continue to apply. For K, on Integrator Unit #4, Y' will now be V_0 and will be set at the edge of the integrator disc. Hence for K on its own primary shaft an upper limit for the speed under the conditions here assumed, will be $9375/(8c_{18}c_{20}c_{22}c_{24})$ r.p.m. Turning next to Adder #0, we obtain for an upper limit for the angular speed of the primary shaft for Y_s', the sum,

$$\left[480/c_{39}\right] + \left[9375/(8c_{18}c_{20}c_{22}c_{24}c_{40})\right] \text{ r.p.m.}$$

On passing through Adder #4, and dropping the initially zero term ΔX, one has as an upper bound for the angular speed of Y' on its own primary shaft,

$$\left[480/(c_{39}c_{47})\right] + \left[9375/(8c_{18}c_{20}c_{22}c_{24}c_{40}c_{47})\right] \text{ r.p.m.}$$

Since at this angle of departure W' is negligible, we obtain as an upper bound for the angular speed of V on its own primary shaft,

$$\left[480/(c_{35}c_{39}c_{47})\right] + \left[9375/(8c_{18}c_{20}c_{22}c_{24}c_{35}c_{40}c_{47})\right] \text{ r.p.m.}$$

For Y we have ready, on its own primary shaft, $600/c_{30}$ r.p.m.

Applying the conditions of the third principle stated above, we have inequalities originally phrased in terms of speed, but by use of the first principle, these are expressible in terms of gear coupling coefficients (as far as operation after the initial slow speed stage for the prime motor is past). Assuming that the primary T shaft rotates at approximately 800 r.p.m., the condition that no integrator unit output shaft shall rotate faster than 600 r.p.m., could be made to yield inequalities. However, after the initial slow speed stage is past, the integrand variables will not in general be at their extreme values and no easily usable form of inequality results. Instead we use initial conditions only. Principle (2), rather than (1), will regulate the maximum to be used for T. We shall list the inequalities of chief interest so obtained, in some cases distinguishing among the three special cases (a), (b), (c) discussed above.

(1) For S

From $600/|c_{18}| \leq 600$, we have

$$|c_{18}| \geq 1 \tag{1}$$

(2) For J

From $(5/4)\ 600/|c_{18}c_{20}| \leq 600$, we have

$$|c_{20}| \geq (5/4)\ 1/|c_{18}| \tag{2}$$

(3) For R

From $(5/4)^2 600/|c_{18}c_{20}c_{22}| \leq 600$, we have

$$|c_{22}| \geq (5/4)^2 1/|c_{18}c_{20}| \tag{3}$$

(4) For K

In case (c), the only case of significance for a limitation of value on K, we have

$$(5/4)^3 600/|c_{18}c_{20}c_{22}c_{24}| \leq 600, \text{ giving}$$

$$|c_{24}| \geq (5/4)^3\ 1/|c_{18}c_{20}c_{22}| \tag{4}$$

(5) For X_s'

In case (a), the only case of significance for a limitation of value on X_s', we have

$$(5/4)^3 600/|c_{18}c_{20}c_{22}c_{26}| \leq 600, \text{ giving}$$

$$|c_{26}| \geq (5/4)^3 1/|c_{18}c_{20}c_{22}| \qquad (5)$$

(6) For X

In case (a), the only case of significance for a limitation of value on X, we have the same situation as for S. From $600/|c_{28}| \leq 600$, we find

$$|c_{28}| \geq 1 \qquad (6)$$

(7) For Y

In case (c), the only case of significance for a limitation of value of Y, we have similarly, from $600/|c_{30}| \leq 600$,

$$|c_{30}| \geq 1 \qquad (7)$$

(8) For H_s

In case (c), from $(5/4)600/|c_{30}c_{32}| \leq 600$, we have

$$|c_{32}| \geq (5/4)1/|c_{30}| \qquad (8)$$

It will be sufficient, but not wholly necessary, to accept in place of these, the following

$$|c_{18}| = |c_{28}| = |c_{30}| \geq 1 \qquad (1)'$$

$$|c_{20}| \geq 5/4 \qquad (2)'$$

$$|c_{22}| \geq 5/4 \qquad (3)'$$

$$|c_{24}| \geq 5/4 \qquad (4)'$$

$$|c_{26}| \geq 5/4 \qquad (5)'$$

$$|c_{32}| \geq 5/4 \qquad (6)'$$

8. Speed Limits for the Printers.

The six printer units which make up the operating part of the Recorder, have the respective speeds determined by the corresponding speeds of the respective bus shafts for the canonical variables T, X, Y, V, Y', X', save for the interposition of gear couplings, themselves subject to optimal determination. On the basis of experiment the following principle has been enunciated.

Principle. For the accurate hitting n integer values of some recorder variable by a printer, there is a best speed for the printer which usually lies between 120 r.p.m. and 150 r.p.m. No printer should be operated at faster than 200 r.p.m.

We shall obtain separately inequalities for each printer corresponding to the conditions (a), (b), (c) of the preceding section.

(I) Recorder Variable T.

Since the primary shaft for T connects with the printer shaft for T through the gear coupling #51, with coupling coefficient $c_{51} = r_I$, it follows that to maintain a printer speed of not more than 200 r.p.m. over the entire trajectory, a reduction to one fourth of the maximum speed of 800 r.p.m., for T, is required. Hence

$$\text{(I)} \quad c_{51}, = r_I \geq 4$$

This holds for all cases, (a), (b), (c).

(II) Recorder Variable X.

For X, we distinguish as to cases (a), (b), (c), but (c) is trivial, and (b) is covered by (a). We obtain for (a) (the only case of significance)

$$600/(c_{28}c_{52}) \leq 200 \quad \text{or}$$

$$c_{52}, = r_{II}, \geq 3/c_{28}.$$

(III) Recorder Variable Y.

For Y as for X, we have only one case of significance, this time (c), giving

$$600/(c_{30}c_{53}) \leq 200 \quad \text{or}$$

$$c_{53}, = r_{III}, \geq 3/c_{30}$$

(IV) Recorder Variable $\underline{V}$.

The three cases for $\underline{V}$ must be handled separately. For (a), from $9375/(8c_{18}c_{20}c_{22}c_{26}c_{34}c_{41}c_{49})$ for V on its own primary shaft, we have

$$9375/(8c_{18}c_{20}c_{22}c_{26}c_{34}c_{41}c_{49}c_{54}) \leq 200, \text{ or}$$

$$c_{54'} = r_{IV'} \geq \left|375/(64c_{18}c_{20}c_{22}c_{26}c_{34}c_{41}c_{49})\right| \quad \text{(a)}$$

For (b) and (c), similarly

$$\begin{aligned} c_{54'} = r_{IV'} \geq\ & \left|12/(5c_{35}c_{39}c_{47})\right| + \\ & \left|525/(128c_{18}c_{20}c_{22}c_{24}c_{35}c_{40}c_{47})\right| + \\ & \left|525/(128c_{18}c_{20}c_{22}c_{26}c_{34}c_{41}c_{49})\right| \quad \text{(b)} \end{aligned}$$

$$\begin{aligned} c_{54'} = r_{IV'} \geq\ & \left|12/(5c_{35}c_{39}c_{47})\right| + \\ & \left|375/(64c_{18}c_{20}c_{22}c_{24}c_{35}c_{40}c_{47})\right| \quad \text{(c)} \end{aligned}$$

(V) Recorder Variable $\underline{Y}'$.

For the recorder variable $\underline{Y}'$, only case (c) is of significance. We obtain in this case

$$\begin{aligned} c_{55'} = r_{V'} \geq\ & \left|12/(5c_{39}c_{47})\right| + \\ & \left|375/(64c_{18}c_{20}c_{22}c_{24}c_{40}c_{47})\right| \end{aligned}$$

(VI) Recorder Variable $\underline{X}'$.

For the recorder variable, $\underline{X}'$, only case (a) is of significance. We obtain

$$c_{56'} = r_{VI'} \geq \left|375/(64c_{18}c_{20}c_{22}c_{26}c_{49})\right| .$$

C. Determination of Conversion and Coupling Coefficients.

1. The System of Equations and Inequalities for Gear Coupling Coefficients.

The equations and inequalities of the previous sections involve coupling coefficients in many ways. The original relations are not all independent, as was seen in a partial reduction and elimination of dependent inequalities discussed earlier. There is indeed a freedom of choice in obtaining mathematically a solution of the formal system of equations and inequalities. One is not interested, however, in all such possible theoretical solutions. There are practical considerations which would serve to reject as trivial many formally correct solutions. It is desirable to avail oneself of substantially all the permissible travel of movable parts in order that the scale be throughout as large as conveniently possible. For example, if a given template were such that for an extended program of trajectory computation only a small horizontal interval of the template were used, one would ordinarily prepare a new template reproducing the same functional relationship in the part used, but spreading this used interval of the variable, U, over the whole available horizontal range for this table.

In seeking a solution it is convenient to distinguish between those data which depend upon the particular program of trajectory computation, (or even more immediately upon the particular trajectory) and those which remain essentially constant for the machine as a whole when serving for the computation of trajectories. It is found that the freedom of choice is ample to permit assigning at once certain values from among the limiting values provided by the inequalities already given for travel, load, and speed. We might proceed arbitrarily to assign certain values and relations and justify this choice by furnishing later a method for satisfying all the equations and inequalities already stated.

It will be convenient, however, prior to such arbitrary assigning, to list for reference all those equations and inequalities among gear coupling coefficients, other than those for cranks and printers. Comparing the lists previously obtained, we have the following, involving (save for Λ in 7 and c_o in 6) coupling coefficients only:

Equations. From the final list in Part V, Section A 4, we copy the following

From

(19) $$c_{18} = c_{28} = c_{30} \qquad (1)$$

(20) $$c_{24} = c_{26} \qquad (2)$$

(21) $$c_{25} = c_{31} \qquad (3)$$

(22) $$c_{25}c_{40}c_{47} = c_{27}c_{41}c_{49} \qquad (4)$$

[redacted]

(23) $c_{25}c_{34}^{2}c_{41}^{2}c_{50} = -c_{29}c_{35}^{2}c_{48}$ (5)

(0) $1/c_{0} = 32\sqrt{c_{25}c_{30}c_{32}c_{33}c_{39}c_{47}}$ (6)

(18) $\Lambda c_{48} = 32c_{0}c_{25}c_{29}^{-1}c_{32}c_{33}c_{34}c_{35}^{-1}c_{39}c_{41}c_{47}$ (7)

Also from Part V, Section B 2, Equation 12

$28\,|c_{32}c_{33}| = 15\,|c_{36}c_{43}|$ (8)

<u>Inequalities.</u>

From

B 7 (1)' $|c_{18}| \geq 1$ (9)

B 6 (1) and B 7 (2)' $|c_{20}| \geq 5/4$ (10)

B 6 (2) and B 7 (3)' $|c_{22}| \geq 5/4$ (11)

B 6 (3) and B 7 (4)' $|c_{24}| \geq 5/4$ (12)

B 7 (5)' $|c_{26}| \geq 5/4$ (13)

(1) above $|c_{28}| \geq 1$ and $|c_{30}| \geq 1$ (14)

B 6 (4) and B 7 (6)' $|c_{32}| \geq 5/3$ (15)

B 6 (5) $|c_{33}| \geq 3$ (16)

B 6 (6) $|c_{40}| \geq 2$ (17)

B 6 (7) $|c_{41}| \geq 1$ (18)

B 6 (8) $|c_{43}| \geq 2$ (19)

B 6 (9) $|c_{45}| \geq 2$ (20)

B 6 (10) $|c_{47}| \geq 1$ (21)

B 6 (11) $|c_{48}| \geq 2$ (22)

B 6 (12) $|c_{49}| \geq 2$ (23)

B 6 (13) $|c_{50}| \geq 2$ (24)

Also the following

B 6 (14) $|c_{21}c_{45}| \geq 3$ (25)

[redacted]

B 6 (17) $|c_{25}c_{40}c_{47}| \geqq 6$ (26)

B 6 (18) $|c_{27}c_{41}c_{49}| \geqq 6$ (27)

B 6 (15) $|c_{29}c_{49}| \geqq 6$ (28)

B 6 (19) $|c_{34}c_{41}c_{49}| \geqq 3$ (29)

B 6 (20) $|c_{35}c_{40}c_{47}| \geqq 3$ (30)

From (8), (15), (16) above, $|c_{36}c_{43}| \geqq 28/3$ (31)

[redacted]

We have also from Part V, Section B 2 the following involving parameters,

From

B 2 (8)' $|c_8| \cdot |c_{33}| \geq H_m/40,$ (Integ. #8) (32)

B 2 (2)' $|c_8| \cdot |c_{21} c_{45}| \geq H_m/40,$ (Integ. #2) (33)

B 2 (9)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \geq V_m/180,$ (Vector Table) (34)

B 2 (1)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \cdot |c_{19}| \geq V_m/40,$ (Integ. #1) (35)

B 2 (13)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \cdot |c_{37}| \geq V_m/180,$ (Div. Table, V) (36)

B 2 (14)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \cdot |c_{37}| \cdot |c_{32} c_{33} c_{36}^{-1} c_{43}^{-1}| \geq V_m/336,$ (Div. Table, U) (37)

B 2 (15)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \cdot |c_{37}| \cdot |c_{32} c_{33} c_{36}^{-1} c_{43}^{-1}| \cdot |c_{38}| \geq V_m/448,$ (Template Table) (38)

B 2 (4)' $|c_0| \cdot |c_{25} c_{39} c_{47}| \geq Y'_m/40,$ (Integ. #4) (39)

B 2 (5)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \cdot |c_{27}| \geq X'_m \; |c_{34}|/40,$ (Integ. #5) (40)

B 2 (6)' $|c_0| \cdot |c_{35} c_{39} c_{47}| \cdot |c_{29}| \cdot |c_{41}^{-1}| \geq X'_m \; |c_{34}|/40,$ (Integ. #6) (41)

B 2 (3)' $|c_{32} c_{33}| \geq 128 \; B_m \; |c_8| \cdot |c_{19} c_{20} c_{21} c_{22} c_{24} c_{35} c_{40} c_{45} c_{47}|/5,$ (Integ. #3) (42)

B 2 (16)' $|c_{32} c_{33}| \geq 128 \; B_\Delta \; |c_8| \cdot |c_{23}| \cdot |c_{19} c_{20} c_{21} c_{22} c_{24} c_{35} c_{40} c_{45} c_{47}|/45,$

(Template Table) (43)

The inequalities here may be somewhat simplified by splitting so as to obtain a set of sufficient conditions, no one of which by itself can be strengthened by a numerical factor, but which are not as a system completely necessary.

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Recalling that by (16), $|c_{33}| \geq 3$, and that by (25), $|c_{21}c_{45}| \geq 3$ from the information given above we replace the first two inequalities, (32) and (33), by the single inequality

$$|c_8| \geq H_m/120 \tag{44}$$

To reduce the five inequalities, (34) to (38), respectively, to a common form, (34), we accept in addition to (8) above the following,

$$|c_{19}| = 9/2 \tag{45}$$

$$|c_{37}| = 1 \tag{46}$$

$$|c_{38}| = 3/4 \tag{47}$$

While it is true that on a given trajectory X_m' may differ greatly from Y_m', yet as remarked earlier, under ordinary procedure there will be a set of trajectories for which a single arrangement of gear couplings is to be used throughout irrespective of the angle of departure. As these angles range through the first quadrant, the value of X_m' which covers all the trajectories of the set will be practically equal to V_m, and similarly the value of Y_m' will be practically V_m. For such a set of trajectories, there is no essential loss of generality in taking X_m' as equal to Y_m' and both as approximately equal to V_m. With this consideration in mind, we seek to reduce to a similar form not only inequalities (40) and (41), involving X_m', but also (39) involving Y_m', and all these three in comparison with (34). We therefore write

$$2\,|c_{25}| = 9\,|c_{35}| \tag{48}$$

$$2\,|c_{27}| = 9\,|c_{34}| \tag{49}$$

$$|c_{29}| = |c_{27}c_{41}| \tag{50}$$

2. <u>Signs of Gear Coupling Coefficients.</u>

As was remarked earlier, the bus shafts lie mutually parallel. A bus shaft is regarded as rotating positively if, and only if, it is rotating in the same angular direction as the primary time shaft, or what amounts to the same thing as the output shaft of the prime motor when this rotates in its normal direction. A gear coupling which preserves sense has a positive gear coupling coefficient. One which reverses sense has a negative coefficient. It is true that one might reverse the sense throughout some subnetwork of the analyzer by changing the signs of all connections connecting this subnetwork with the rest of the machine. We shall assume however positive gear connections wherever possible. Increasing functions of T will then be carried on positively turning bus shafts. If on passing through a gear coupling no algebraic change of sign for the variable occurs, this gear coupling has a positive coefficient. With these principles in mind signs for conversion coefficients and gear coupling coefficients are readily chosen as follows:

The conversion coefficients c_1' to c_3', c_0 to c_{17} are all positive. The gear coupling coefficients c_{18} to c_{56} are positive with the following exceptions, c_{27}, c_{33}, c_{41}, c_{42}, c_{44}, c_{45}, c_{46}, c_{47}, c_{49}, which are definitely negative, while the signs of c_{48}, c_{50}, (when Λ is not zero) are such that Λc_{48} is negative and Λc_{50} is positive.

While considering conversion coefficients the reader may start to wonder how it happens that a variable can change sign when all the gear trains are left unaltered, and hence when no gear coupling coefficient changes sign. The variable may change sign mechanically as a consequence for example of any one of the following three events: (i) The integrand disc carriage in an integrator unit may move continuously so that the roller of the differential input slides past the zero position. When this occurs, the output of this integrator unit changes sign. (ii) The radial arm in the vector table may in turning pass continuously through the horizontal position, and so correspond to a change of sign of Y'. (iii) Two variables of opposite sign being fed into an adder may so change in relative magnitude that the sum changes in sign. One should recall that the direction of rotation of a shaft determines whether the variable is increasing or decreasing, (when once the associated constant conversion coefficient is known) but does not determine whether the variable is positive or negative. The sign of the variable may indeed be altered (in many cases) by the simple expedient of changing the additive constant carried by a calibrating drum.

3. Partial Determination of Gear Coupling Coefficients.

In order to secure a practical solution for the extensive system of equations and inequalities already laid down, it will be convenient to specialize at once most of the gear coupling coefficients, by assigning to them the minimum absolute values (not less than unity) permitted by the foregoing simultaneous system, where the signs of the coefficients are chosen as described above. We therefore choose the following,

From

Section C 1	(9)	$c_{18} = 1$
	(45)	$c_{19} = 9/2$
	(10)	$c_{20} = 5/4$
By definition,		$c_{21} = 1$
Section C 1	(11)	$c_{22} = 5/4$
	(12)	$c_{24} = 5/4$
	(13)	$c_{26} = 5/4$
	(14)	$c_{28} = 1$
	(14)	$c_{30} = 1$
	(16)	$c_{33} = -3$
By definition,		$c_{36} = 1$
Section C 1	(46)	$c_{37} = 1$
	(47)	$c_{38} = 3/4$
	(17) and (30)	$c_{40} = 3$
	(18)	$c_{41} = -1$
	(25) and (4) above	$c_{45} = -3$
	(21)	$c_{47} = -1$

We shall accept also the following equalities,

$$c_{25} = -c_{27} = c_{29} = c_{31} \qquad (1)$$

Hence by (1), (48), and (49),

$$c_{34} = c_{35} \qquad (2)$$

From (5), (18), (1), and (2)

$$c_{48} = -c_{50} \tag{3}$$

$$c_{49} = -3 \tag{4}$$

It follows from 1,(29) that $|c_{34}| \geq 1$. In view of 1,(48) and 1,(49), and to simplify the gear trains, we adopt the following,

$$c_{34} = 1,$$

$$c_{35} = 1$$

and hence by 1,(48) and 1,(49), and by 3,(1),

$$c_{25} = 9/2$$

$$c_{27} = -9/2$$

$$c_{29} = 9/2$$

$$c_{31} = 9/2$$

By 1,(8) it follows that

$$c_{43}/c_{32} = 28/5, \tag{5}$$

From 1 (6), $\quad 1/c_0 = 96\sqrt{3c_{32}c_{39}/2} \qquad (6)$

1 (7), $\quad \Lambda c_{50} = \sqrt{2/(3\, c_{32}c_{39})} \qquad (7)$

1 (32), $\quad c_8 \geq H_m/120 \qquad (8)$

(6) and 1 (34), $\quad c_{39} \geq 32\, c_{32} V_m^2/75 \qquad (9)$

1 (42), $\quad c_{32} \geq 675\, B_m c_8 \qquad (10)$

1 (43), $\quad c_{32} \geq 75\, c_8 c_{23} B_\Delta \qquad (11)$

4. Determination of Remaining Gear Coupling Coefficients.

We solve the conditions of Section 3 in the following order

From (8) choose c_8 (a conversion coefficient) so that

$$c_8 = H_m/120$$

From (11) then choose c_{32} to approximate from above, $675\, B_m c_8$,

$$c_{32} \searrow 675\, B_m c_8$$

This condition will be found in practice to satisfy Part C 1(15).

From 3 (5), c_{43} is then determined.

$$c_{43} = 28 c_{32}/5$$

(It is permissible to use an equality sign here since new gears are being made so that a train of gears equalling 28/5 may be used.)

This choice of c_{43} suggests the following single practical modification of the theoretical arrangement of bus shafts and connection gear couplings. In place of leading Y directly from its own primary shaft to Adder #2,-- exhibiting the equation $A = (A_o - Y) + \Delta A$,-- through gear coupling #43, we adopt the following. A permanent gear coupling $c_{43/32}$, satisfying

$$c_{43/32} = 28/5$$

is introduced leading from the differential input of Integrator #8 to Adder #2. This yields an ultimate gear coupling coefficient, c_{43}, of the magnitude desired but avoids introducing the complicated gear train corresponding to c_{32}, more than once.

Next choose c_{23} as an approximation from below to satisfy 3 (11)

$$c_{23} \nearrow c_{32}/(75 c_8 B_\Delta)$$

Next choose c_{39} to satisfy 3 (9)

$$c_{39} \searrow 32\, c_{32} V_m^2/75$$

Save for the gear couplings which connect the hand cranks to the rest of the machine, there remains to be considered only c_{50}. By 3 (7)

$$c_{50} = \sqrt{2/(3\, c_{32} c_{39})}/\Lambda$$

Here Λ is numerically a small quantity determined with reasonable accuracy from observation and adequately approximated by an obtainable gear coupling coefficient. It may happen that Λ is near zero. This would cause c_{50} to be numerically extremely large. Theoretically c_{50} could be infinite without causing any difficulty. This situation would involve the shafts carrying ΛX and ΛY respectively to be stationary. For a standard trajectory where rotation effects are ignored, we replace Λ by zero.

5. Coupling Coefficients for Gears Connecting Cranks with Adders.

There are three hand cranks which feed in the corrections, (based on meteorological observations), to W', A, H respectively. These cranks turn (abbreviated) bus shafts, numbered 1', 2', 3', respectively, and pass their motions through gear couplings numbered 42, 44, 46 respectively, to further bus shafts similarly numbered 42, 44, 46 respectively, leading into adders mentioned below. The conversion coefficients for $\Delta W'$, ΔA, ΔH, respectively on these latter bus shafts must be equal to the conversion coefficients for W', A, H respectively on the bus shafts which also enter the Adders numbered respectively 1, 2, 3, or as submachines numbered 13, 14, 15 respectively. These considerations have already led to the equations

From

Section A 4 (24)' $\quad c_1' c_{42} = (c_0)\, c_{34}^{-1} c_{35} c_{39} c_{47}$

Section A 4 (25)' $\quad c_2' c_{44} = (1/32)\, c_{32}^{-1} c_{33}^{-1} c_{43}$

Section A 4 (26)' $\quad c_3' c_{46} = (c_8)\, c_{45}$

which may now be simplified if desired.

There are no inherent relations which would serve to isolate the values of c_{42}, c_{44}, and c_{46} from out of the values of the corresponding products, $c_1' c_{42}$, $c_2' c_{44}$, $c_3' c_{46}$ respectively. The only practical considerations seem to be that it shall not be necessary to turn these cranks at an uncomfortably high speed or so slowly as to make careful following of tabulated data impracticable. The following values have been found convenient and are not changed between runs.

$$c_{42} = -4, \quad c_{44} = -1/4, \quad c_{46} = -1.$$

6. Recorder Ratios.

The six recorder ratios r_I, r_{II}, r_{III}, r_{IV}, r_V, r_{VI}, are respectively equal to the gear coupling coefficients c_{51}, c_{52}, c_{53}, c_{54}, c_{55}, c_{56}, as has been noted already. They carry the canonical variables T, X, Y, V, Y', X', respectively into recorder variables $\underline{T}$, $\underline{X}$, $\underline{Y}$, $\underline{V}$, $\underline{Y}'$, $\underline{X}'$. Upper limits on the speeds of the printer discs have led to certain limits on gear coupling coefficients already noted. To make accurate interpolation convenient, it is desirable to maintain each printer train rotating at approximately the maximum speed permitted under the restrictions laid down. This is particularly true in case of the variable $\underline{T}$.

Substituting previously obtained values we find, choosing lower bounds, from Section 8 of Part B for $c_{54} = c_{55}$,

$$c_{51} = 4$$

$$c_{52} = 3$$

$$c_{53} = 3$$

$$c_{54} = \left[12/(5c_{39})\right] + \left[7/5\right]$$

$$c_{55} = \left[12/(5c_{39})\right] + \left[7/5\right]$$

$$c_{56} = 1$$

7. The Conversion Coefficients for the Canonical Variables.

Making use of known values of the gear coupling coefficients, while leaving in literal form, c_8, c_{23}, c_{32}, c_{39}, we have the following from Section 4 of Part A and Section 3 of Part C.

$$c_0 = \sqrt{2}/(96\sqrt{3\ c_{32}c_{39}})$$

$$c_1 = 1/(96\ c_{32})$$

$$c_2 = 5(c_8)/(4(c_{32}))$$

$$c_3 = 1/540$$

$$c_4 = \sqrt{2\ c_{39}}/(288\sqrt{3\ c_{32}})$$

$c_5 = \sqrt{2\ c_{39}}/(288\ \sqrt{3\ c_{32}})$

$c_6 = 1/(96\ c_{32})$

$c_7 = 1/(96\ c_{32})$

c_8, determined as in Part B, Section 4.

$c_9 = \sqrt{2\ c_{39}}/(96\ \sqrt{3\ c_{32}})$

$c_{10} = \sqrt{2\ c_{39}}/(2016\ \sqrt{3\ c_{32}})$

$c_{11} = c_{32}/(27000\ c_8 c_{23})$

$c_{12} = \sqrt{2\ c_{39}}/(96\ \sqrt{3\ c_{32}})$

$c_{13} = \sqrt{2\ c_{39}}/(96\ \sqrt{3\ c_{32}})$

$c_{14} = 7/120$

$c_{15} = 3\ c_8$

$c_{16} = \sqrt{2\ c_{39}}/(96\ \sqrt{3\ c_{32}})$

$c_{17} = \sqrt{2\ c_{39}}/(96\ \sqrt{3\ c_{32}})$

[redacted]

The practical details of arranging the gear connections, of setting the integrand discs of integrator units, of using the meteorological data, and so forth, as well as the whole prior problem of arranging the actual set-up of bus shafts on the analyzer frame is left for the Supplements. There one finds, as the result of a special study, a set-up diagram proposed in which for practical convenience, various modifications from the notation hitherto used have been incorporated. All gear connections which can be maintained unchanged from one program to the next have been deprived of their special serial numbering and indicated merely by the gear ratio involved. The adders and tables have been given special names, the integrator units have been numbered according to their location, and the adjustable gears have been specially designated.

For the method of using this set-up diagram and of proceeding directly from data to recorded runs, Computing Note No. 2, "Directions for Filling Out 'Record for Analyzer'", is appended. This note refers to the blank form, a copy of which is also appended, entitled "Record for Analyzer", and calls for the use of Computing Note No. 1, also appended, which is a list of gear ratios obtainable with so few as three pairs of available gears.

Albert A. Bennett

Albert A. Bennett,
Major, Ordnance Dept.

[redacted]

I. OUTLET ELEMENTS AND SHAFTS

SUB-MACHINES (FOR OUTPUT)	TYPE	NO.		MACHINE ELEMENTS (OUTPUT ELEM.) TYPE	NO.		PRIMARY (OUTPUT) SHAFTS JOINING: MACH. ELEM. NO.	AND GEAR COUPLINGS NOS.	EQUATIONS: VAR. = FORMULA
α									$\Lambda = .0035342 \ \mathrm{SIN}\ \alpha$
β									$B_s(U) = b_s(u)/h_1 = b_s(u) \times 31663.3$
(A) (i)	CALIBR	1	OUTPUT ONLY	CALIBR.	1				$A_0 = 10.5274$
(B) (ii)	"	2		"	2				$H_0 = 1/C$
(iii)	"	3		"	3				Γ —
(iv)	"	4		"	4				$Y'_0 = (\mathrm{M.V.})\ \mathrm{SIN}\angle(\mathrm{DEP}) \times .000991098$
(v)	"	5		"	5				$X'_0 = (\mathrm{M.V.})\ \mathrm{COS}\angle(\mathrm{DEP}) \times .000991098$
(C) 1'	CRANK	1		CRANK	1	FOR REDUCTION RUNS ONLY	1'	42, (27, 34)	$\Delta W'$ —
2'	"	2		"	2		2'	44, (36)	ΔA —
3'	"	3		"	3		3'	46, (21)	ΔH —
(D) 0	PRIME MOTOR			PRIME MOTOR			0	18, 28, 30, 39, 51, (47, (25, 31, 35, 56))	T —
(E) 1	INTEGRATOR	1	POWER DRIVEN OUTPUT	INTEGRATOR	1		1	20	$S = \int_0 V\,dT$
2	"	2		"	2		2	22	$J = \int_0 H\,dS$
3	"	3		"	3		3	24, 26	$R = \int_0 B\,dJ$
4	"	4		"	4		4	40, (47, (25, 31, 35, 55))	$K = \int_0 Y'\,dR$
5	"	5		"	5		5	49 (29, 41, 56, (27, 34))	$X'_s = X'_0 - \int_0 W'\,dR$
6	"	6		"	6		6	48, 52	$X = \int_0 X'\,dT$
7	"	7		"	7		7	32, 43, 50, 53, (36)	$Y = \int_0 Y'\,dT$
8	"	8		"	8		8	33, 45, (21)	$H_s = H_0 - \int_0 H_B\,dY$
(F) 9	TABLE	1		TABLE	1		9	19, 37, 54	$V = \sqrt{W'^2 + Y'^2}$
10	"	2		"	2		10	38	$U = V/A$
11	"	3		"	3		11	23	$B = B_s(U) + \Gamma$
(G) 12	ADDER	0		ADDER	0		12	47, (25, 31, 35, 55)	$Y'_s = Y'_0 - (T + K)$
13	"	1		"	1		13	27, 34	$W' = X' + \Delta W'$ (FOR REDUCTION ONLY)
14	"	2		"	2		14	36	$A = (A_0 - Y) + \Delta A$ (FOR REDUCTION ONLY)
15	"	3		"	3		15	21	$H = H_s + \Delta H$ (FOR REDUCTION ONLY)
16	"	4		"	4		16	25, 31, 35, 55	$Y' = Y'_s + \Lambda X$ (FOR REDUCTION ONLY)
17	"	5		"	5		17	29, 41, 56 (27, 34)	$X' = X'_s - \Lambda Y$ (FOR REDUCTION ONLY)

FUNDAMENTAL EQUATIONS: S through X' above.

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2. GEAR COUPLINGS, INPUT ELEMENTS AND SHAFTS (TO BE CONTINUED)

	SUB-MACHINES (FOR OUTPUT) TYPE	NO.		MACHINE ELEMENTS (INPUT ELEM.) TYPE	NO.	DIR.		(INPUT) SHAFTS JOINING GEAR COUPLING NO.	AND MACH. ELEM. NO.
18	COUPLING	18		INTEGRATOR	1	↔		18	18
19	"	19		"	"	↕		19	19
20	"	20		"	2	↔		20	20
21	"	21		"	"	↕		21	21
22	"	22		"	3	↔		22	22
23	"	23		"	"	↕		23	23
24	"	24		"	4	↔		24	24
25	"	25		"	"	↕		25	25
26	"	26		"	5	↔	(I)	26	26
27	"	27		"	"	↕		27	27
28	"	28	(H)	"	6	↔		28	28
29	"	29		"	"	↕		29	29
30	"	30		"	7	↔		30	30
31	"	31		"	"	↕		31	31
32	"	32		"	8	↔		32	32
33	"	33		"	"	↕		33	33
34	"	34		TABLE	1	↔		34	34
35	"	35		"	"	↕		35	35
36	"	36		"	2	↔	(J)	36	36
37	"	37		"	"	↕		37	37
38	"	38		"	3	↔		38	38

EQUATIONS

EQ. NO.	VAR. NO.	VAR. SYM.	FORMULA
1	0 9	T V	$\int_0 V\,dT = S$
2	1 15	S H	$\int_0 H\,dS = J$
3	2 11	J B	$\int_0 B\,dJ = R$
4	3 16	R Y′	$\int_0 Y'\,dR = K$
5	3 13	R W′	$X'_0 - \int_0 W'\,dR = X'_s$
6	0 17	T X′	$\int_0 X'\,dT = X$
7	0 16	T Y′	$\int_0 Y'\,dT = Y$
8	7 8	Y H_s	$H_0 - \int_0 H_s\,dY = H_s$
9	13 16	W′ Y′	$\sqrt{W'^2 + Y'^2} = V$
10	14 9	A V	$V/A = U$
11	10	U	$B_s(U) + \Gamma = B$

3. GEAR COUPLINGS, INPUT ELEMENTS AND SHAFTS (CONTINUED)

	SUB-MACHINES (FOR OUTPUT) TYPE	NO.	LOAD	MACHINE ELEMENTS (INPUT ELEM.) TYPE	NO.	(INPUT) SHAFTS JOINING GEAR COUPLING NO.	AND MACH. ELEM. NO.	EQUATIONS EQ.	VAR. NO.	VAR. SYM.	FORMULA
39	COUPLING	39	→	ADDER	0	39	39	12	0	T	$Y'_o - (T+K) = Y'_s$
40	"	40	→	"	"	40	40		4	K	
41	"	41	→	"	1	41	41	13	17	X'	$X' + \Delta W' = W'$
42	"	42	→	"	"	42	42		1′	$\Delta W'$	
43	"	43	→	"	2	43	43	14	7 (32)	Y	$(A_o - Y) + \Delta A = A$
44	"	44	→	"	"	44	44		2′	ΔA	
45	"	45	→	"	3 (K)	45	45	15	8	H_s	$H_s + \Delta H = H$
46	"	46	→	"	"	46	46		3′	ΔH	
47	"	47	→	"	4	47	47	16	12	Y'_s	$Y'_s + \Lambda X = Y'$
48	"	48	(H)	"	"	48	48		6	X	
49	"	49	→	"	5	49	49	17	5	X'_s	$X'_s - \Lambda Y = X'$
50	"	50		"	"	50	50		7	Y	
51	"	51		PRINTER (INPUT ONLY)	I	51	51	I	0	T	$r_{I}\, T = \underline{T}$
52	"	52		"	II	52	52	II	6	X	$r_{II}\, X = \underline{X}$
53	"	53		"	III	53	53	III	7	Y	$r_{III}\, Y = \underline{Y}$
54	"	54		"	IV (L)	54	54	IV	9	V	$r_{IV}\, V = \underline{V}$
55	"	55		"	V	55	55	V	16	Y'	$r_{V}\, Y' = \underline{Y'}$
56	"	56		"	VI	56	56	VI	17	X'	$r_{VI}\, X' = \underline{X'}$

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EXPLANATION OF LETTERS USED

QUANTITY	SYMBOL	FORMULA
EARTH ROTATION COEFFICIENT	Λ	.0035342 SIN AZIMUTH
SOUND SPEED CORRECTION	ΔA	(FROM OBSERVATION)
DENSITY RATIO CORRECTION	ΔH	(FROM OBSERVATION)
HEAD WIND	$\Delta W'$	(FROM OBSERVATION)
TIME	T	(INDEPENDENT)
RANGE (HORIZONTAL) (CORRECTED FOR ROTATION)	X	$\int_0 X' dT$
ALTITUDE (CORRECTED FOR ROTATION)	Y	$\int_0 Y' dT$
GROUND SPEED (CORRECTED FOR ROTATION)	X'	$X'_s - \Lambda Y$
VERTICAL VELOCITY (CORRECTED FOR ROTATION)	Y'	$Y'_s + \Lambda X$
VELOCITY, WITH RESPECT TO AIR	V	$\sqrt{W'^2 + Y'^2}$
ARC LENGTH, WITH RESPECT TO AIR	S	$\int_0 V dT$
SOUND SPEED RATIO (ACTUAL)	A	$(A_0 - Y) + \Delta A$ $(A_0 = 10.5274)$
AIR DENSITY RATIO (STANDARD)	H_s	$H_0 - \int_0 H_s dY$ $(H_0 = 1/C)$
AIR DENSITY RATIO (ACTUAL)	H	$H_s + \Delta H$
AIR SPEED (HORIZONTAL)	W'	$X' + \Delta W'$
GROUND SPEED (UNCORRECTED FOR ROTATION)	X'_s	$X'_0 - \int_0 W' dR$ (X'_0 = HORIZONTAL COMPONENT OF MUZZLE VELOCITY)
VERTICAL VELOCITY (UNCORRECTED FOR ROTATION)	Y'_s	$Y'_0 - (T + K)$ (Y'_0 = VERTICAL COMPONENT OF MUZZLE VELOCITY)
ADJUSTED VELOCITY (WITH RESPECT TO AIR)	U	V/A
DRAG COEFFICIENT (ADJUSTED FOR TIME OF FLIGHT)	B	$B_s(U) + \Gamma$ ($B_s(U)$ FROM TEMPLATE, Γ ADJUSTMENT PARAMETER)
REGRESSION	R	$\int_0 B dJ$
"J"-INTEGRAL	J	$\int_0 H dS$
"K"-INTEGRAL	K	$\int_0 Y' dR$

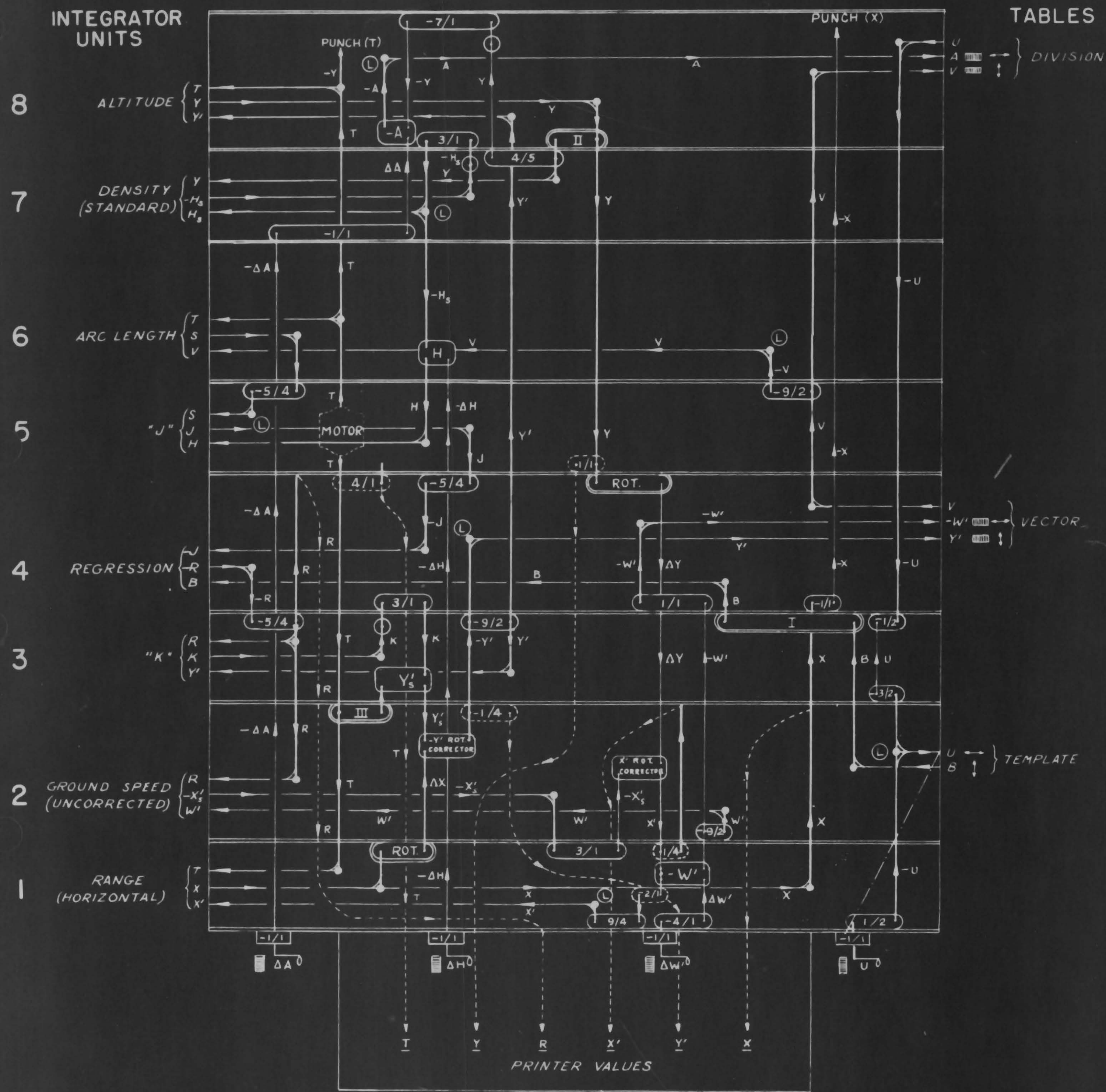
BRL 319

AP-727

6a

DIAGRAM OF SET-UP OF DIFFERENTIAL ANALYZER

JANUARY 1943



SYMBOLS

LOWER SHAFT

UPPER SHAFT

SPIRAL GEARS

(L) SIGNIFIES LEFT-HAND

GEAR TRAIN (FIXED) (COUPLING COEFFICIENT INDICATED)

ADDER

CRANK

GEAR TRAIN (ADJUSTABLE)

FRONT-LASH UNIT

COUNTER

PRINTER GEAR TRAIN

NOTE: A COUPLING COEFFICIENT GREATER THAN UNITY INDICATES AN INCREASE IN TORQUE AND REDUCTION IN SPEED

→ ARROWS INDICATE DIRECTION OF POWER FLOW

REVISION – MARCH 3, 1943

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RECORD FOR ANALYZER (Plan of January 1943)

Schedule No.	Runs: No. to No.
FT	Template No.
Gun	Maximum u = f/s
Projectile	Maximum b =
Fuze	Minimum b =
Drag Function	Range of b = =n(turns)
Estim. Max. v =	Computed by_ _ _ _ _ _ _ _ _ _ _ _ _ _ _
Estim. Min. C =	Checked by _ _ _ _ _ _ _ _ _ _ _ _ _ _ _
Az. Fire: α =	Date:
I = =	Punch ( )=
II = =	= —
III = =	[$]^{-1}$+[$]^{-1}$
Prop. factor, p=	Rot.= =

		INTEGRANDS				TABLES
8.	y'	1 f/s= t.l.s.	Div.	Top	a	1 unit= 180.00 t.l.s.
7.	h_s	1 unit= t.l.s.		Bot.	v	1 f/s
6.	v	(same as 8)	Vect.	Top	w'	1 f/s = t.l.s.
5.	h	(same as 7)		Bot.	y'	1 f/s
4.	b	1 unit= t.l.s.	Templ. Top		u	1 f/s= t.l.s.
3.	y'	(same as 8)	METRO		δa	1 unit= 45.00 t.l.s.
2	w'	(same as 8)	CRANKS		δh	(same as 7)
1.	x'	(same as 8)			$\delta w'$	1 mi/h= t.l.s.

T sec.	X'	f/s	X'	p.u.=1 f/s
Y ft.	Y'		Y'	
R 0.00231481 un.	X	yd.	Prints intervals in ( ) of_ _ _ _ _ _	yd. sec.

Equivalent to one printer unit.(p.u.)

REMARKS

Ballistic Research Laboratory - Computing Branch

Computing Note No. 1 November 1942.

List of decimals from 0.5 to 2.0, each expressible with three or fewer available Absolute Gear Ratios of the Analyzer.

Note:- For values outside this range introduce appropriate positive powers of 4/1 or 1/4.

1 $.5000 = \frac{1}{2} = \frac{1}{2}$

3 $.5062 = \frac{245}{484} = \frac{5}{4} \left(\frac{7}{11}\right)^2$

3 $.5079 = \frac{32}{63} = \frac{8}{7} \left(\frac{2}{3}\right)^2$

2 $.5091 = \frac{28}{55} = \frac{4}{5} \cdot \frac{7}{11}$

3 $.5101 = \frac{176}{345} = \frac{22}{23} \cdot \frac{4}{5} \cdot \frac{2}{3}$

3 $.5104 = \frac{49}{96} = \left(\frac{7}{8}\right)^2 \frac{2}{3}$

3 $.5120 = \frac{64}{125} = \left(\frac{4}{5}\right)^3$

2 $.5227 = \frac{23}{44} = \frac{23}{22} \cdot \frac{1}{2}$

3 $.5238 = \frac{11}{21} = \frac{11}{7} \cdot \frac{2}{3} \cdot \frac{1}{2}$

3 $.5303 = \frac{35}{66} = \frac{5}{4} \cdot \frac{2}{3} \cdot \frac{7}{11}$

3 $.5322 = \frac{322}{605} = \frac{23}{22} \cdot \frac{4}{5} \cdot \frac{7}{11}$

3 $.5326 = \frac{539}{1012} = \frac{22}{23} \cdot \frac{7}{8} \cdot \frac{7}{11}$

2 $.5333 = \frac{8}{15} = \frac{4}{5} \cdot \frac{2}{3}$

3 $.5465 = \frac{529}{968} = \left(\frac{23}{22}\right)^2 \frac{1}{2}$

3 $.5466 = \frac{88}{161} = \frac{8}{7} \cdot \frac{22}{23} \cdot \frac{1}{2}$

3 $.5469 = \frac{35}{64} = \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{1}{2}$

3 $.5556 = \frac{5}{9} = \left(\frac{2}{3}\right)^2 \left(\frac{5}{4}\right)$

2 $.5568 = \frac{49}{88} = \frac{7}{8} \cdot \frac{7}{11}$

3 $.5576 = \frac{92}{165} = \frac{23}{22} \cdot \frac{4}{5} \cdot \frac{2}{3}$

3 $.5580 = \frac{138}{77} = \frac{22}{23} \cdot \frac{7}{8} \cdot \frac{2}{3}$

3 $.5600 = \frac{14}{25} = \left(\frac{4}{5}\right)^2 \left(\frac{7}{8}\right)$

3 $.5625 = \frac{9}{16} = \left(\frac{3}{2}\right)^2 \left(\frac{1}{4}\right)$

2 $.5714 = \frac{4}{7} = \frac{8}{7} \cdot \frac{1}{2}$

3 $.5818 = \frac{32}{55} = \frac{8}{7} \cdot \frac{4}{5} \cdot \frac{7}{11}$

3 $.5821 = \frac{1127}{1936} = \frac{23}{22} \cdot \frac{7}{8} \cdot \frac{7}{11}$

3 $.5822 = \frac{3388}{5819} = \left(\frac{22}{23}\right)^2 \left(\frac{7}{11}\right)$

2 $.5833 = \frac{7}{12} = \frac{7}{8} \cdot \frac{2}{3}$

3 $.5893 = \frac{33}{56} = \frac{11}{7} \cdot \frac{3}{2} \cdot \frac{1}{4}$

3 $.5974 = \frac{46}{77} = \frac{8}{7} \cdot \frac{23}{22} \cdot \frac{1}{2}$

3 $.5978 = \frac{55}{92} = \frac{5}{4} \cdot \frac{22}{23} \cdot \frac{1}{2}$

3 $.6000 = \frac{3}{5} = \frac{3}{2} \cdot \frac{4}{5} \cdot \frac{1}{2}$

3 $.6074 = \frac{147}{242} = \frac{3}{2} \cdot \left(\frac{7}{11}\right)^2$

2 $.6087 = \frac{154}{253} = \frac{22}{23} \cdot \frac{7}{11}$

3 $.6095 = \frac{64}{105} = \frac{8}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$

3 $.6098 = \frac{161}{264} = \frac{23}{22} \cdot \frac{7}{8} \cdot \frac{2}{3}$

3 $.6100 = \frac{968}{1587} = \left(\frac{22}{23}\right)^2 \left(\frac{2}{3}\right)$

3 $.6122 = \frac{352}{575} = \left(\frac{4}{5}\right)^2 \frac{22}{23}$

3 $.6125 = \frac{49}{80} = \left(\frac{7}{8}\right)^2 \frac{4}{5}$

3 $.6173 = \frac{121}{196} = \left(\frac{11}{7}\right)^2 \frac{1}{4}$

2 $.6250 = \frac{5}{8} = \frac{5}{4} \cdot \frac{1}{2}$

3 $.6286 = \frac{22}{35} = \frac{11}{7} \cdot \frac{4}{5} \cdot \frac{1}{2}$

1 $.6364 = \frac{7}{11} = \frac{7}{11}$

2 $.6377 = \frac{44}{69} = \frac{22}{23} \cdot \frac{2}{3}$

2 $.6400 = \left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^2$

3 $.6531 = \frac{32}{49} = \left(\frac{8}{7}\right)^2 \frac{1}{2}$

3 $.6534 = \frac{115}{176} = \frac{5}{4} \cdot \frac{23}{22} \cdot \frac{1}{2}$

3 $.6563 = \frac{21}{32} = \frac{3}{2} \cdot \frac{7}{8} \cdot \frac{1}{2}$

2 $.6653 = \frac{161}{242} = \frac{23}{22} \cdot \frac{7}{11}$

1 $.6667 = \frac{2}{3} = \frac{2}{3}$

3 $.6691 = \frac{184}{275} = \frac{23}{22} \left(\frac{4}{5}\right)^2$

3 $.6696 = \frac{77}{115} = \frac{22}{23} \cdot \frac{7}{8} \cdot \frac{4}{5}$

3 $.6699 = \frac{343}{512} = \left(\frac{7}{8}\right)^3$

3 $.6875 = \frac{11}{16} = \frac{11}{7} \cdot \frac{7}{8} \cdot \frac{1}{2}$

3 $.6955 = \frac{3703}{5324} = \left(\frac{23}{22}\right)^2 \left(\frac{7}{11}\right)$

3 $.6957 = \frac{16}{23} = \frac{8}{7} \cdot \frac{22}{23} \cdot \frac{7}{11}$

3 $.6960 = \frac{245}{352} = \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{7}{11}$

2 $.6970 = \frac{23}{33} = \frac{23}{22} \cdot \frac{2}{3}$

3 $.6984 = \frac{44}{63} = \frac{11}{7} \left(\frac{2}{3}\right)^2$

2 $.7000 = \frac{7}{10} = \frac{7}{8} \cdot \frac{4}{5}$

3 $.7143 = \frac{5}{7} = \frac{5}{4} \cdot \frac{8}{7} \cdot \frac{1}{2}$

3 $.7174 = \frac{33}{46} = \frac{3}{2} \cdot \frac{22}{23} \cdot \frac{1}{2}$

2 $.7273 = \frac{8}{11} = \frac{8}{7} \cdot \frac{7}{11}$

3 $.7287 = \frac{529}{726} = \left(\frac{23}{22}\right)^2 \frac{2}{3}$

3 $.7288 = \frac{352}{483} = \frac{8}{7} \cdot \frac{22}{23} \cdot \frac{2}{3}$

3 $.7292 = \frac{35}{48} = \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{2}{3}$

3 $.7314 = \frac{128}{175} = \frac{8}{7} \left(\frac{4}{5}\right)^2$

3 $.7318 = \frac{161}{220} = \frac{23}{22} \cdot \frac{7}{8} \cdot \frac{4}{5}$

3 $.7319 = \frac{1936}{2645} = \left(\frac{22}{23}\right)^2 \left(\frac{4}{5}\right)$

3 $.7323 = \frac{539}{736} = \left(\frac{7}{8}\right)^2 \left(\frac{22}{23}\right)$

2 $.7500 = \frac{3}{4} = \frac{3}{2} \cdot \frac{1}{2}$

3 $.7516 = \frac{121}{161} = \frac{11}{7} \cdot \frac{22}{23} \cdot \frac{1}{2}$

3 $.7603 = \frac{92}{121} = \frac{8}{7} \cdot \frac{23}{22} \cdot \frac{7}{11}$

3 $.7609 = \frac{35}{46} = \frac{5}{4} \cdot \frac{22}{23} \cdot \frac{7}{11}$

2 $.7619 = \frac{16}{21} = \frac{8}{7} \cdot \frac{2}{3}$

3 $.7636 = \frac{42}{55} = \frac{3}{2} \cdot \frac{4}{5} \cdot \frac{7}{11}$

2 $.7652 = \frac{88}{115} = \frac{22}{23} \cdot \frac{4}{5}$

2 $.7656 = \frac{49}{64} = \left(\frac{7}{8}\right)^2$

3 $.7813 = \frac{25}{32} = \left(\frac{5}{4}\right)^2 \frac{1}{2}$

3 $.7841 = \frac{69}{88} = \frac{3}{2} \cdot \frac{23}{22} \cdot \frac{1}{2}$

2 $.7857 = \frac{11}{14} = \frac{11}{7} \cdot \frac{1}{2}$

2 $.7955 = \frac{35}{44} = \frac{5}{4} \cdot \frac{7}{11}$

3 $.7965 = \frac{184}{231} = \frac{8}{7} \cdot \frac{23}{22} \cdot \frac{2}{3}$

3 $.7971 = \frac{55}{69} = \frac{5}{4} \cdot \frac{22}{23} \cdot \frac{2}{3}$

1 $.8000 = \frac{4}{5} = \frac{4}{5}$

3 $.8004 = \frac{1127}{1408} = \frac{23}{22} \left(\frac{7}{8}\right)^2$

3 $.8006 = \frac{847}{1058} = \left(\frac{22}{23}\right)^2 \left(\frac{7}{8}\right)$

3 $.8099 = \frac{98}{121} = 2 \left(\frac{7}{11}\right)^2$

3 $.8214 = \frac{23}{14} = \frac{11}{7} \cdot \frac{23}{22} \cdot \frac{1}{2}$

3 $.8312 = \frac{64}{77} = \left(\frac{8}{7}\right)^2 \left(\frac{7}{11}\right)$

3 $.8316 = \frac{805}{968} = \frac{5}{4} \cdot \frac{23}{22} \cdot \frac{7}{11}$

2 $.8333 = \frac{5}{6} = \frac{2}{3} \cdot \frac{5}{4}$

3 $.8352 = \frac{147}{176} = \frac{3}{2} \cdot \frac{7}{8} \cdot \frac{7}{11}$

2 $.8364 = \frac{46}{55} = \frac{23}{22} \cdot \frac{4}{5}$

2 $.8370 = \frac{77}{92} = \frac{22}{23} \cdot \frac{7}{8}$

3 $.8381 = \frac{88}{105} = \frac{11}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$

3 $.8485 = \frac{28}{33} = 2 \cdot \frac{2}{3} \cdot \frac{7}{11}$

3 $.8571 = \frac{6}{7} = \frac{3}{2} \cdot \frac{8}{7} \cdot \frac{1}{2}$

3 $.8707 = \frac{128}{147} = \left(\frac{8}{7}\right)^2 \left(\frac{2}{3}\right)$

3 $.8712 = \frac{115}{132} = \frac{5}{4} \cdot \frac{23}{22} \cdot \frac{2}{3}$

3	.8744	$= \frac{529}{605} = \left(\frac{23}{22}\right)^2 \frac{4}{5}$
3	.8745	$= \frac{704}{805} = \frac{8}{7} \cdot \frac{22}{23} \cdot \frac{4}{5}$
1	.8750	$= \frac{7}{8} = \frac{7}{8}$
3	.8752	$= \frac{10648}{12167} = \left(\frac{22}{23}\right)^3$
3	.8889	$= \frac{8}{9} = 2 \left(\frac{2}{3}\right)^2$
3	.8980	$= \frac{44}{49} = \frac{11}{7} \cdot \frac{8}{7} \cdot \frac{1}{2}$
3	.9091	$= \frac{10}{11} = \frac{5}{4} \cdot \frac{8}{7} \cdot \frac{7}{11}$
3	.9130	$= \frac{21}{23} = \frac{3}{2} \cdot \frac{22}{23} \cdot \frac{7}{11}$
2	.9143	$= \frac{32}{35} = \frac{8}{7} \cdot \frac{4}{5}$
2	.9148	$= \frac{161}{176} = \frac{23}{22} \cdot \frac{7}{8}$
2	.9149	$= \frac{484}{529} = \left(\frac{22}{23}\right)^2$
3	.9167	$= \frac{11}{12} = \frac{11}{7} \cdot \frac{7}{8} \cdot \frac{2}{3}$
3	.9375	$= \frac{15}{16} = \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{1}{2}$
3	.9524	$= \frac{20}{21} = \frac{5}{4} \cdot \frac{8}{7} \cdot \frac{2}{3}$
2	.9545	$= \frac{21}{22} = \frac{3}{2} \cdot \frac{7}{11}$
3	.9558	$= \frac{368}{385} = \frac{8}{7} \cdot \frac{23}{22} \cdot \frac{4}{5}$
3	.9564	$= \frac{3703}{3872} = \left(\frac{23}{22}\right)^2 \frac{7}{8}$
1	.9565	$= \frac{22}{23} = \frac{22}{23}$
3	.9570	$= \frac{245}{256} = \left(\frac{5}{4}\right) \left(\frac{7}{8}\right)^2$
3	.9600	$= \frac{24}{25} = \frac{3}{2} \left(\frac{4}{5}\right)^2$
3	.9821	$= \frac{55}{56} = \frac{11}{7} \cdot \frac{5}{4} \cdot \frac{1}{2}$
3	.9943	$= \frac{175}{176} = \left(\frac{5}{4}\right)^2 \left(\frac{7}{11}\right)$
3	.9979	$= \frac{483}{484} = \frac{3}{2} \cdot \frac{23}{22} \cdot \frac{7}{11}$
1	1	$= 1 = \frac{1}{1}$
3	1.0021	$= \frac{484}{483} = \frac{11}{7} \cdot \frac{22}{23} \cdot \frac{2}{3}$
3	1.0057	$= \frac{176}{175} = \frac{11}{7} \left(\frac{4}{5}\right)^2$
3	1.0182	$= \frac{56}{55} = 2 \cdot \frac{4}{5} \cdot \frac{7}{11}$
3	1.0417	$= \frac{25}{24} = \left(\frac{5}{4}\right)^2 \left(\frac{2}{3}\right)$
3	1.0449	$= \frac{256}{245} = \left(\frac{8}{7}\right)^2 \left(\frac{4}{5}\right)$
1	1.0455	$= \frac{23}{22} = \frac{23}{22}$
3	1.0456	$= \frac{3872}{3703} = \left(\frac{8}{7}\right) \left(\frac{22}{23}\right)^2$
3	1.0462	$= \frac{385}{368} = \frac{5}{4} \cdot \frac{22}{23} \cdot \frac{7}{8}$
2	1.0476	$= \frac{22}{21} = \frac{11}{7} \cdot \frac{2}{3}$
3	1.0500	$= \frac{21}{20} = \frac{3}{2} \cdot \frac{7}{8} \cdot \frac{4}{5}$
3	1.0667	$= \frac{16}{15} = 2 \cdot \frac{4}{5} \cdot \frac{2}{3}$
3	1.0909	$= \frac{12}{11} = \frac{3}{2} \cdot \frac{8}{7} \cdot \frac{7}{11}$
2	1.0930	$= \frac{529}{484} = \left(\frac{23}{22}\right)^2$
2	1.0932	$= \frac{176}{161} = \frac{8}{7} \cdot \frac{22}{23}$
2	1.0938	$= \frac{35}{32} = \frac{5}{4} \cdot \frac{7}{8}$
3	1.0952	$= \frac{23}{21} = \frac{11}{7} \cdot \frac{23}{22} \cdot \frac{2}{3}$
3	1.1000	$= \frac{11}{10} = \frac{11}{7} \cdot \frac{7}{8} \cdot \frac{4}{5}$
3	1.1136	$= \frac{49}{44} = 2 \cdot \frac{7}{8} \cdot \frac{7}{11}$
3	1.1250	$= \frac{9}{8} = \left(\frac{3}{2}\right)^2 \frac{1}{2}$
3	1.1427	$= \frac{12167}{10648} = \left(\frac{23}{22}\right)^3$

1 $1.1429 = \frac{8}{7} = \frac{8}{7}$

3 $1.1435 = \frac{805}{704} = \frac{5}{4}\cdot\frac{23}{22}\cdot\frac{7}{8}$

3 $1.1437 = \frac{605}{529} = \left(\frac{22}{23}\right)^2\left(\frac{5}{4}\right)$

3 $1.1478 = \frac{132}{115} = \frac{3}{2}\cdot\frac{22}{23}\cdot\frac{4}{5}$

3 $1.1484 = \frac{147}{128} = \left(\frac{3}{2}\right)\left(\frac{7}{8}\right)^2$

3 $1.1667 = \frac{7}{6} = 2.\frac{7}{8}\cdot\frac{2}{3}$

3 $1.1786 = \frac{33}{28} = \frac{11}{7}\cdot\frac{3}{2}\cdot\frac{1}{2}$

3 $1.1932 = \frac{105}{88} = \frac{3}{2}\cdot\frac{5}{4}\cdot\frac{7}{11}$

2 $1.1948 = \frac{92}{77} = \frac{8}{7}\cdot\frac{23}{22}$

2 $1.1957 = \frac{55}{46} = \frac{5}{4}\cdot\frac{22}{23}$

3 $1.1973 = \frac{176}{147} = \frac{11}{7}\cdot\frac{8}{7}\cdot\frac{2}{3}$

2 $1.2000 = \frac{6}{5} = \frac{3}{2}\cdot\frac{4}{5}$

3 $1.2025 = \frac{968}{805} = \frac{11}{7}\cdot\frac{22}{23}\cdot\frac{4}{5}$

3 $1.2031 = \frac{77}{64} = \frac{11}{7}\left(\frac{7}{8}\right)^2$

3 $1.2174 = \frac{308}{253} = 2.\ \frac{22}{23}\cdot\frac{7}{11}$

3 $1.2347 = \frac{121}{98} = \left(\frac{11}{7}\right)^2\left(\frac{1}{2}\right)$

3 $1.2491 = \frac{1058}{847} = \left(\frac{23}{22}\right)^2\left(\frac{8}{7}\right)$

3 $1.2493 = \frac{1408}{1127} = \left(\frac{8}{7}\right)^2\left(\frac{22}{23}\right)$

1 $1.2500 = \frac{5}{4} = \frac{5}{4}$

3 $1.2545 = \frac{69}{55} = \frac{3}{2}\cdot\frac{23}{22}\cdot\frac{4}{5}$

3 $1.2554 = \frac{231}{184} = \frac{3}{2}\cdot\frac{22}{23}\cdot\frac{7}{8}$

2 $1.2571 = \frac{44}{35} = \frac{11}{7}\cdot\frac{4}{5}$

2 $1.2727 = \frac{14}{11} = 2.\ \frac{7}{11}$

3 $1.2754 = \frac{88}{69} = 2.\ \frac{22}{23}\cdot\frac{2}{3}$

3 $1.2800 = \frac{32}{25} = 2\ \left(\frac{4}{5}\right)^2$

2 $1.3061 = \frac{64}{49} = \left(\frac{8}{7}\right)^2$

2 $1.3068 = \frac{115}{88} = \frac{5}{4}\cdot\frac{23}{22}$

3 $1.3095 = \frac{55}{42} = \frac{11}{7}\cdot\frac{5}{4}\cdot\frac{2}{3}$

2 $1.3125 = \frac{21}{16} = \frac{3}{2}\cdot\frac{7}{8}$

3 $1.3143 = \frac{506}{385} = \frac{11}{7}\cdot\frac{23}{22}\cdot\frac{4}{5}$

3 $1.3152 = \frac{121}{92} = \frac{11}{7}\cdot\frac{22}{23}\cdot\frac{7}{8}$

3 $1.3306 = \frac{161}{121} = 2.\ \frac{23}{22}\cdot\frac{7}{11}$

2 $1.3333 = \frac{4}{3} = 2.\ \frac{2}{3}$

3 $1.3655 = \frac{736}{539} = \left(\frac{8}{7}\right)^2\left(\frac{23}{22}\right)$

3 $1.3662 = \frac{2645}{1936} = \left(\frac{23}{22}\right)^2\left(\frac{5}{4}\right)$

3 $1.3665 = \frac{220}{161} = \frac{5}{4}\cdot\frac{8}{7}\cdot\frac{22}{23}$

3 $1.3672 = \frac{175}{128} = \left(\frac{5}{4}\right)^2\ \frac{7}{8}$

3 $1.3714 = \frac{48}{35} = \frac{3}{2}\cdot\frac{8}{7}\cdot\frac{4}{5}$

3 $1.3722 = \frac{483}{352} = \frac{3}{2}\cdot\frac{23}{22}\cdot\frac{7}{8}$

3 $1.3724 = \frac{726}{529} = \frac{3}{2}\cdot\left(\frac{22}{23}\right)^2$

2 $1.3750 = \frac{11}{8} = \frac{11}{7}\cdot\frac{7}{8}$

3 $1.3939 = \frac{46}{33} = 2.\frac{23}{22}\cdot\frac{2}{3}$

3 $1.4000 = \frac{7}{5} = 2.\ \frac{7}{8}\cdot\frac{4}{5}$

2 $1.4286 = \frac{10}{7} = \frac{5}{4}\cdot\frac{8}{7}$

3 $1.4318 = \frac{63}{44} = (\frac{3}{2})^2(\frac{7}{11})$

2 $1.4348 = \frac{33}{23} = \frac{3}{2} \cdot \frac{22}{23}$

3 $1.4367 = \frac{352}{245} = \frac{11}{7} \cdot \frac{8}{7} \cdot \frac{4}{5}$

3 $1.4375 = \frac{23}{16} = \frac{11}{7} \cdot \frac{23}{22} \cdot \frac{7}{8}$

3 $1.4378 = \frac{5324}{3703} = (\frac{11}{7})(\frac{22}{23})^2$

3 $1.4545 = \frac{16}{11} = 2 \cdot \frac{8}{7} \cdot \frac{7}{11}$

3 $1.4927 = \frac{512}{343} = (\frac{8}{7})^3$

3 $1.4935 = \frac{115}{77} = \frac{5}{4} \cdot \frac{8}{7} \cdot \frac{23}{22}$

3 $1.4946 = \frac{275}{184} = (\frac{5}{4})^2 (\frac{22}{23})$

1 $1.5000 = \frac{3}{2} = \frac{3}{2}$

2 $1.5031 = \frac{242}{161} = \frac{11}{7} \cdot \frac{22}{23}$

3 $1.5238 = \frac{32}{21} = 2 \cdot \frac{8}{7} \cdot \frac{2}{3}$

3 $1.5304 = \frac{176}{115} = 2 \cdot \frac{22}{23} \cdot \frac{4}{5}$

3 $1.5313 = \frac{49}{32} = 2(\frac{7}{8})^2$

2 $1.5625 = \frac{25}{16} = (\frac{5}{4})^2$

2 $1.5682 = \frac{69}{44} = \frac{3}{2} \cdot \frac{23}{22}$

1 $1.5714 = \frac{11}{7} = \frac{11}{7}$

3 $1.5909 = \frac{35}{22} = 2 \cdot \frac{5}{4} \cdot \frac{7}{11}$

2 $1.6000 = \frac{8}{5} = 2 \cdot \frac{4}{5}$

3 $1.6198 = \frac{196}{121} = 4 \cdot (\frac{7}{11})^2$

3 $1.6327 = \frac{80}{49} = (\frac{8}{7})^2(\frac{5}{4})$

3 $1.6335 = \frac{575}{352} = (\frac{5}{4})^2(\frac{23}{22})$

3 $1.6395 = \frac{1587}{968} (\frac{23}{22})^2 \frac{3}{2}$

3 $1.6398 = \frac{264}{161} = \frac{3}{2} \cdot \frac{8}{7} \cdot \frac{22}{23}$

3 $1.6406 = \frac{105}{64} = \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{7}{8}$

2 $1.6429 = \frac{23}{14} = \frac{11}{7} \cdot \frac{23}{22}$

3 $1.6463 = \frac{242}{147} = (\frac{11}{7})^2(\frac{2}{3})$

3 $1.6666 = \frac{5}{3} = 2 \cdot \frac{5}{4} \cdot \frac{2}{3}$

3 $1.6727 = \frac{92}{55} = 2 \cdot \frac{23}{22} \cdot \frac{4}{5}$

3 $1.6739 = \frac{77}{46} = 2 \cdot \frac{22}{23} \cdot \frac{7}{8}$

3 $1.6970 = \frac{56}{33} = 4 \cdot \frac{2}{3} \cdot \frac{7}{11}$

2 $1.7143 = \frac{12}{7} = \frac{3}{2} \cdot \frac{8}{7}$

3 $1.7175 = \frac{529}{308} = (\frac{23}{22})^2 \frac{11}{7}$

3 $1.7178 = \frac{1936}{1127} = \frac{11}{7} \cdot \frac{8}{7} \cdot \frac{22}{23}$

3 $1.7188 = \frac{55}{32} = \frac{11}{7} \cdot \frac{5}{4} \cdot \frac{7}{8}$

2 $1.7500 = \frac{7}{4} = 2 \cdot \frac{7}{8}$

3 $1.7778 = \frac{16}{9} = 4 (\frac{2}{3})^2$

3 $1.7857 = \frac{25}{14} = (\frac{5}{4})^2 \frac{8}{7}$

3 $1.7922 = \frac{138}{77} = \frac{3}{2} \cdot \frac{8}{7} \cdot \frac{23}{22}$

3 $1.7935 = \frac{165}{92} = \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{22}{23}$

2 $1.7959 = \frac{88}{49} = \frac{11}{7} \cdot \frac{8}{7}$

3 $1.8000 = \frac{9}{5} = (\frac{3}{2})^2 \frac{4}{5}$

3 $1.8286 = \frac{64}{35} = 2 \cdot \frac{8}{7} \cdot \frac{4}{5}$

3 $1.8295 = \frac{161}{88} = 2 \cdot \frac{23}{22} \cdot \frac{7}{8}$

3 $1.8299 = \frac{968}{529} = 2(\frac{22}{23})^2$

2 $1.8750 = \frac{15}{8} = \frac{3}{2} \cdot \frac{5}{4}$

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3 $\quad 1.8776 = \frac{92}{49} = \frac{11}{7} \cdot \frac{8}{7} \cdot \frac{23}{22}$

3 $\quad 1.8789 = \frac{605}{322} = \frac{11}{7} \cdot \frac{5}{4} \cdot \frac{22}{23}$

3 $\quad 1.8857 = \frac{66}{35} = \frac{11}{7} \cdot \frac{3}{2} \cdot \frac{4}{5}$

3 $\quad 1.9091 = \frac{21}{11} = 2 \cdot \frac{3}{2} \cdot \frac{7}{11}$

2 $\quad 1.9130 = \frac{44}{23} = 2 \cdot \frac{22}{23}$

3 $\quad 1.9531 = \frac{125}{64} = \left(\frac{5}{4}\right)^3$

3 $\quad 1.9592 = \frac{96}{49} = \left(\frac{8}{7}\right)^2 \left(\frac{3}{2}\right)$

3 $\quad 1.9602 = \frac{345}{176} = \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{23}{22}$

2 $\quad 1.9643 = \frac{55}{28} = \frac{11}{7} \cdot \frac{5}{4}$

3 $\quad 1.9688 = \frac{63}{32} = \left(\frac{3}{2}\right)^2 \frac{7}{8}$

3 $\quad 1.9755 = \frac{484}{245} = \left(\frac{11}{7}\right)^2 \frac{4}{5}$

1 $\quad 2 = 2 = \frac{2}{1}$

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Ballistic Research Laboratory - Computing Branch

Computing Note No. 2 Revision, February, 1943

Directions for Filling Out "Record for Analyzer"

Introduction. This Note is for the purpose of enabling a computer, not already familiar with the procedure, to fill out properly the form entitled "Record for Analyzer (Set-Up Plan of January, 1943)". This Note presupposes the use of Computing Note No. 1 which latter exhibits a sequence of decimals representing gear ratios obtainable by gear trains each using not more than three of the gear-pairs now available for the Differential Analyzer. The present Note also relates to the notions and concepts in the BRL "Report on the Differential Analyzer", December, 1942. In particular, of course, the reader of this Note is assumed to have at hand a copy of the blank form, "Record for Analyzer", mentioned above. Since for practical convenience the notation on this form departs in some respects from that used in most of the Report, reference to the final diagram of analyzer set-up appearing as a Supplement of that Report is appropriate.

Save for the heading and the final space reserved for "Remarks" the blank form of the "Record" may be viewed as divided into four parts respectively (1) "Data", (2) "Gears", (3) "Number of Turns of Lead Screw per Physical Unit", (4) "Number of Physical Units per Printer Unit". Each of these parts will be discussed in turn. For brevity the names of these separate parts are not printed on the "Record". A fifth section of this Note displays conversion factors for the Integrator Units.

1. Data

At the top left is a place for the "Schedule No." of the sheet. Each sheet formulates a separate "Schedule"; these schedules are numbered serially. A "Schedule" applies to a group of trajectories designed for the same firing table but having for example different angles of departure. Sometimes the muzzle velocity is common to the whole group of trajectories covered by a given schedule, but often there will be different "zones" each with its own distinct muzzle velocity. Furthermore the ballistic coefficient may vary slightly with the muzzle velocity or elevation. A given "run" presents numerical values on a single trajectory (for given muzzle velocity, angle of departure, ballistic coefficient, azimuth of fire and meteorological conditions). The runs are numbered serially in a single list. The serial numbers of the many runs for the same schedule will form a consecutive set to be indicated as usual by writing the first and last numbers of the set, separated by the word "to" at the upper right.

In carrying out the separate runs based upon a common schedule,

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further data beyond those that fill the "Record" are needed. Usually two additional sheets of data are prepared for each run. These sheets, not further discussed in this Note, incorporate the exact muzzle velocity (rather than merely an upper bound), and the exact ballistic coefficient (rather than merely a lower bound), the angle of departure, and so forth, and show the settings to be used at the various tables, integrands, and hand cranks, for the particular run concerned.

The "FT" at the left refers to the official Firing Table designation, such as "FT 75-B-4" which is for a certain 75 MM gun and ammunition. The "Gun", "Projectile", and "Fuze" spaces at the left call for no further explanation. The "Drag Function" makes reference to the appropriate one of the six functions $G_1, \ldots\, G_6$, (or sometimes also designated as $J_1, \ldots\, J_6$) of which, however, G_3 and G_4 are practically obsolete, or else to the corresponding function divided by v, called respectively $B_1, \ldots\, B_6$. The particular drag function used depends upon the shape of the projectile. Other characteristics of the projectile are for practical purposes essentially to be lumped in with the ballistic coefficient, C. By "Estim. Max. v" is meant an (approximate) maximum velocity for all runs to be carried out on the given schedule. This Max. v is either a muzzle velocity, or else an "adjusted velocity", also called u, obtained by comparison with the velocity of sound; it is to be carefully distinguished from "Maximum u" listed in the right hand column which is a number fixed for any given template independent of the individual runs of a schedule. For a many-zoned program this "Estim. Max. v" would be the velocity used for the farthest zone. The ballistic coefficient, often regarded as a constant determined by the projectile alone, is a parameter adjusted to fit observed results, and in practice may vary with angle of elevation and muzzle velocity even when accepted as constant during any one run. In order to secure the efficient use of the permitted travel, on the tables and integrator units, there is need not only for an upper bound on muzzle velocity but also for a suitable lower bound for the various values of the ballistic coefficient (C) corresponding to different runs on the given schedule. An estimate of the minimum value of C is to be recorded in the space marked "Estim. Min. C". The actual muzzle velocity and ballistic coefficient are eventually used on each run, but those are listed on other sheets and enter the computations in a different connection. By "Az. Fire, α," is meant the azimuth of the vertical plane of fire measured from north through east. Unfortunately the practice at the Proving Ground is to report azimuths clockwise but from the south. The azimuth is used for eliminating the effect of earth's rotation in reduction of observed firings or for estimating its predicted effect. Sometimes more than one azimuth is involved, but since the azimuth is used, if at all, in determining one of the gear ratios adopted, any one schedule is substantially either for all azimuths or else for a single azimuth only. A rather crude approximation to the azimuth will be adequate except in cases of very long range.

Let us now turn to the data entries on the right hand side of the top section of the sheet. A given drag function, say G_1 or B_1 may be represented by more than onetemplate corresponding to the range of velocities embraced. It is preferable to use a template with maximum listed velocity about ten percent in excess of the maximum to be encountered in the runs for the given schedule. The templates for a given drag function are themselves numbered for example, "Template #14(B)". After deciding upon both the "Drag Function" and "Estim. Max. v", one can pick an appropriate template among those already on hand, or if occasion demands, can arrange to have a new template cut.

A given template for a "B" function has as independent argument, not velocity (with respect to ground or to air), but the quotient, $u = v/a$, where v is the velocity of the projectile with respect to the air, and a (the sound speed ratio) is the velocity of sound at the given altitude (for standard temperature structure) divided by the corresponding velocity of sound for standard sea-level conditions. A given template carries the values of u in feet per second, from u = 0 to a certain "Maximum u", (maximum for the particular template). The "Maximum u" for one template differs from that for another. In the process of computing the effect of (i) increase of muzzle velocity, or (ii) decrease of temperature (as affecting elasticity) or (iii) head wind, there may well result an increase of some 10% in u over that for the standard trajectory, since $u = v/a$ where v is the velocity with respect to the air, and a is the ratio of the velocity of sound at given temperature to the velocity of sound at sea level under standard conditions. It is necessary therefore to be sure that the estimated maximum muzzle velocity be not more than about nine tenths of the maximum u permitted by the template. With this precaution u will not fall beyond the template for the study of such special effects.

The values of the "B" function in the selected domain of u, lie between a minimum, ("Minimum b") taken on for u shortly below the velocity of sound and a maximum ("Maximum b") taken for u shortly above the velocity of sound. The template is preferably so planned that the maximum and minimum are respectively at the top and at the bottom of the table, separated by 18 inches, or 360 turns of the ordinate lead screw. Let n be the actual number of turns of the lead screw separating the the maximum from the minimum. The range of dependent variable exhibited by the template spans the difference, $b_\delta = b_{max} - b_{min}$. The "Record" exhibits the values of "Maximum u", (for the template), "Maximum b", "Minimum b", and (vertical) "range of b", in turns of vertical lead screw, taking this information from the legend cut on the template.

The spaces "Computed by" and "Checked by" call for no special remark. The "Date" refers to the date of computation of the schedule sheet, rather than to any date of completion for analyzer runs, firings, reductions, or other computations.

2. Gears.

There are four to six gear trains to be determined afresh for each schedule (but left unaltered from run to run in the schedule), the other gear trains in the machine being laid down in the set-up plan, and remaining unaltered from one schedule to the next. Each gear train is either (a) "Numbered", being one of three marked respectively I, II, III, (b) "Punch", being the gear train concerned wholly with operating the punch, or (c) "Rot", being one of two gear trains identical save for location, and concerned only with effects due to the earth's rotation, and hence labelled briefly "Rot". Associated with the gear trains is a numerical proportionality factor called "p".

Each of these gear trains yields a coupling coefficient expressible as a product of "simple" negative ratios where each of these "simple" negative ratios is given by a single gear-pair. Each is one of the following: -4/1, -2/1, -11/7, -3/2, -5/4, -8/7, -23/22, -1/1, -22/23, -7/8, -4/5, -2/3, -7/11, -1/2, -1/4. Here the reciprocal of each ratio on the list is also on the list.

By use of the accompanying list of ratios (Computing Note #1), one finds a suitable product ratio for each of the distinct gear trains in the list. We discuss these individually.

(a) The Numbered Gear Trains, and the Proportionality Factor.

(I) Gear train I, carrying b(u), reduces the speed of rotation from the output shaft of the Template Table to the integrand of Integrator Unit 4, whose output in turn is the regression, r (save for sign). Now on the Template Table, b_δ occupies a given number of turns of the ordinate lead screw, usually 360 turns, but more generally some specified number, say n, not exceeding 360. The gear train I is chosen so as to reduce "Maximum b" on the integrand of Integrator Unit 4, to about 36 turns. This permits an approximate increase in b of 10% over recorded maximum to fall within the 40 turns which constitute approximately the maximal radial distance available on the integrand disk. By this means an additive correction may be imposed upon the "B" function to adjust for the observed times of flight of the projectile.

The first problem is to find how many turns of the vertical lead screw on the Template Table would correspond to the interval from $b = 0$ to $b = b_{max}$. It may be remarked that only ratios are involved, so that the units in which b is measured are immaterial. Suppose to make the problem general, one reads on the template that (in customary units) $b_{max} = 0.00008430$, and that $b_{min} = 0.00003407$ and one finds on measuring upon the template a spread of 16.5 inches (instead of full 18) or $n = 330$ turns from b_{min} to b_{max}. Here $b_\delta = b_{max} - b_{min} = 0.00005023$, and this spread in b is covered in 330 turns. The number of turns that would be required on the same scale to reach from $b = 0$ to b_{max} is then

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(8430/5023)330 or about 554 turns. The gear coupling coefficient of I which is to reduce this to some 36 turns or less, is about 15.4 A larger coefficient is allowable, but not one much smaller, although perhaps the use of 15.00 would be acceptable. Taking out a factor of 4^2, we might turn to the table of available gear coupling coefficients to find some number about equal to 15.4/16, = 0.96 approx. or somewhat larger. For simplicity, thus keeping the gear train short, we may use 1.00 instead, or, for the whole coupling coefficient, 16 itself. This is a positive number. It corresponds to two gear pairs of -4/1 each. Had the coupling coefficient to be approximated been about 13, we would have removed 4^2 and looked up in Computing Note No. 1, 13/16, = 0.81, approx. We would have found $0.8099 = (2)(7/11)^2$. Hence (to keep the number of gear pairs even) we could in this case have used the positive gear train represented by (-4/1)(-2/1)(-2/1)(-2/1)(-7/11)(-7/11). This, however, involves six pairs of gears, which is unnecessarily heavy. Instead, we would have used $(-4/1)^2(-2/3)(-5/4)$, a four gear-pair train, with coupling coefficient of 40/3 = 13.3. For n = 360, the normal number of turns to be expected to be available for the ordinate lead screw on the Template Table, the gear coupling coefficient I, must satisfy approximately

$$(8430/5023)360 \leq I(36)$$

or

$$I \geq 16.7838$$

Removing the factor 16, we have I/16 approximating from above the number, 1.0490 approximately. We have two possible convenient choices to approximate 1.0490 from above, namely (-11/7)(-2/3), = 1.0476, and $(-23/22)^2$ = 1.0930. While the former might well suffice, we prefer to use the latter, and so we write

$$I = (-4/1)^2(-23/22)^2 = 17.4876.$$

A simple check on the correctness of the computation of I (this gear train always involves an even number of gear pairs, usually four) is to note that with b_{max} at about 36 turns or a little less, on the integrand of Integrator Unit 4, b_{min} on this same integrand must have such a number of turns as to reproduce for the integrand the proper ratio, b_{max}/b_{min}: Hence by subtraction, one finds the number of turns of the integrand lead screw corresponding to b_δ. Then I must carry n (usually 360) into this number. The general formula is

$$I \geq (b_{max}\ n)/(b_\delta\ 36) \text{ approx.} \qquad (1)$$

which for the usual case of n = 360, reduces to

$$I \geq 10\ b_{max} / b_\delta \quad \text{approx.} \qquad (1')$$

(II) Gear train II, carrying y and a, regulates the travel of h on Integrand No. 5, and of h_s on Integrand No. 7. The conversion coefficient giving the number of turns per unit of h or h_s is in each case expressible in the following general formula $(9000\ I\ b_\delta)/(C\ h_1\ II\ n)$. Here h_1 is a physical constant, = 0.0000315823, I, b_δ, n, are as above, C is the ballistic coefficient, and II is the gear coupling coefficient under discussion. In order to allow for as much as some 10% margin, since for reduction runs, C as first chosen is only an estimate, this travel should not exceed about 36 turns. Using, therefore, C_{min}, available for the entire schedule, we have the general formula,

$$II \geq (250\ I\ b_\delta)/(C_{min}\ h_1\ n) \text{ approx.}$$

or

$$II \geq (I\ b_\delta/n)(7.9158 \times 10^6/C_{min}) \text{ approx.} \quad (2)$$

Since $I\ b_\delta/n \geq b_{max}/36$, approx., and ordinarily closely approximates the latter, it often suffices to take the simpler formula

$$II \geq 2.1988 \times 10^5\ b_{max}/C_{min} \quad (2')$$

For a convenient typical schedule, with b_δ = 0.00005023, n = 360, and b_{max} = 0.00008430, as used above, and I = 17.4876 as above, we have as suitable value of C_{min}, the number, 2.15. Hence II ≥ 8.98 approx. from the more exact formula, or II ≥ 8.62 approx. from the less exact formula. The number, 8.98, we approximate from above by the four factor ~~gear~~ gear coupling coefficient

$$II = 9.0000 = (-3/2)^2(-4/1)(-1/1)$$

A similar computation would apply in any case.

(III, p) Gear train III, carrying t into the adder for y_s, enters the computation only in the ratio III/p^2, where p is the proportionality factor mentioned earlier. It would be possible to keep III constant and absorb all the variation in this variable gear train determination in the number p which enters only as a conversion coefficient. The factor p (entered in the left-hand column immediately under I, II, III) permits ~~enables~~ as much use of the tables as possible (the maximum value being established by template) while there is a safety margin on the integrands. However, it is desirable to keep p very close to but not less than unity, and at the same time to make III/p^2 a very close approximation to an expression involving I, II, and physical constants. Thus p may be regarded as taking up the slack when using for III, a convenient available approximation. The formula obtained by comparing the number of turns per second of time along the trajectory, on two mutually geared bus shafts, is as follows:

$$III/p^2 = (32\ II\ h_1\ u_m^2)/(75\ g)$$

where g is taken to be 32.1522 f/s^2. Hence in general

$$III/p^2 = 0.000000419103 \text{ II } u_m^2$$

In the example considered above II = 9, and u_m = 2124.4 f/s, as marked upon the template. Hence in this example

$$III/p^2 = 17.0230$$

We wish therefore to choose for III, a convenient gear train, with gear coupling coefficient near but not less than 17.0230, = 16 x 1.06394. The most convenient approximation to yield a light gear train, is

$$III = (-4/1)^2(-23/22)^2, = 17.4876$$

which, quite by coincidence, is in this example, identical with I. For p, we have, in this example, the relation

$$p^2 = 17.4876/17.0230 = 1.02729,$$

hence for this example,-

$$p = 1.01355+.$$

In any schedule the value of p should not exceed about 1.10.

(b) "Punch".

For the "Punch" gear train it is necessary to use a very close approximation to the "true" value. The punch operates for convenient equal intervals (usually 500 yards) of the horizontal range x or else (one or five seconds) of the time t, according to the use to which the trajectory is put. On the schedule blank, marks of parenthesis follow the word "punch", for insertion of the letter "x" or "t" as the case may be.

We wish "Punch" when printing x, to approximate closely the ratio which would give exactly the number of turns of the primary bus shaft for X for five hundred yards (or fifteen hundred feet) increase in x, (the horizontal range to any point of the trajectory). This is fifteen hundred times the number 96 II h_1, which represents the number of turns of this shaft for one foot increase in x. Hence "Punch" approximates 96 x 0.0000315823 x 1500 x II, or

$$\text{"Punch (x)"} = 4.54785 \times \text{II} \qquad (4_x)$$

To effect a close approximation, more close than can be easily accomplished by means of a gear train composed of a few available gear pairs alone, one may have recourse to the device of removing first a factor thus representable, and then expressing the quotient which will be near unity, by use of an adder in the form $\pm(1 \pm \delta)$, where δ is

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once more approximated by a gear train.

Consideration of the character of a coefficient which provides the number of positive turns of a lead screw or bus shaft per unit increase in the associated physical variable, indicates that the sigh of addition must occur not between two such coefficients but between their reciprocals. Indeed "Punch" must be of the form $-a/\left\{(b)^{-1} + (c)^{-1}\right\}$, where a, b, c are coupling coefficients, each of which is numerically greater than unity when an increase of torque but decrease in speed occurs by virtue of the gear train concerned.

Let us illustrate this procedure for the schedule already mentioned, for which II = 9. Here "Punch" = 40.9306. As a start removing powers of 4, we have

$$\text{"Punch"} = 4^3 \times 0.639541 = 4^3/1.56362.$$

From the list of available gear ratios we find as an approximation to 1.56362, the fraction, $(5/4)^2 = 1.56250$. Hence by subtraction,

$$\text{"Punch"} = 4^3/\left\{\left[1/(4/5)^2\right] + 0.00112\right\}.$$

Multiplying 0.00112 by successive powers of 4 until we have a number in our range of available gears we have $0.00112 \cong (1/4)^5\ 1.15 = (1/4)^5(8/7)$ approx. Hence in this case

$$\text{"Punch"} = -(-4/1)^3/\left\{\left[(-4/5)^2\right]^{-1} + \left[(-4/1)^5(-7/8)\right]^{-1}\right\}$$

Interpreted in the actual set-up of gear trains, this complex fraction indicates that from the x-shaft two gear trains with gear coupling coefficients $(-4/5)^2$ and $(-4/1)^5(-7/8)$ respectively lead to shafts meeting in an adder. The output of the adder is then affected with a gear train of $(-4/1)^3$, after which reduction, a shaft carries x at the desired speed. One further complication arises, however. The shaft to the punch is carrying -x, so the final external gear train used is $(-1/1)(-4/1)^3$.

In the case of t as regularly printed variable, the procedure is essentially the same. Again we seek an expression of the form $-a/\left\{(b)^{-1} + (c)^{-1}\right\}$ for the punch gear train coefficient.

Taking the case of one second (of trajectory time) as uniform increment, we wish "Punch" to approximate closely the ratio which would give exactly the number of turns of the primary bus shaft for T for increase of one second in t. This latter is in general the number

$$384\ \text{II}\ h_1\ u_m\ p/5, = 0.00242552\ \text{II}\ u_m\ p$$

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or in general

$$\text{"Punch (t)", for seconds, } = 0.00242552 \ \mathrm{II} \ u_m \ p. \qquad (4_t)$$

In the schedule discussed hitherto, II = 9, u_m = 2124.4 f/s, p = 1.01355. Hence

$$\text{"Punch"} = 47.0034, = 4^3 \times 0.734428.$$

Next we approximate 0.734428 by $(7/8)^2(22/23)$, = 0.7323 approx. But we desire to use this information in reciprocal form. Whether at this stage or earlier we start to write reciprocals is immaterial.

$$1/0.734428 = 1.36160$$

$$(8/7)^2(23/22) = 1.36549$$

Hence in this schedule,

$$\text{"Punch (t)"} = 4^3/\left\{\left[(7/8)^2(22/23)\right]^{-1} - 0.00389\right\}$$

Since $(1/4)^4 = 1/256 = 0.003906$, this will serve as satisfactory approximation. Watching signs throughout, we have finally in this schedule,

$$\text{"Punch (t)"} = -(-1/1)(-4/1)^3/\left\{\left[(-7/8)^2(-22/23)\right]^{-1} + \left[(-4/1)^4\right]^{-1}\right\}$$

The external (-1/1) is introduced here to keep the proper sign for the shaft which carries t itself, and not as in the previous case because the shaft carried -x instead of x.

(c) The Rotation Gear Trains.

There are two gear trains, identical save for position in the machine, used. These are employed, when at all, incomputing effects of the earth's rotation upon x and y. Each is labeled "Rot". For trajectories of, at most, moderate length these gear connections into the rotation adders are locked out by the simple device of matching against each other on the same bus shafts, two gear pairs of markedly differing ratios while the bus shaft connections from the integrator units are broken. The reader will recall that a large coupling coefficient means a large increase in torque and a corresponding slowing down in rate of rotation. The limiting case of complete locking would be represented by an infinite coupling coefficient. In practice therefore one may start to compute the value of "Rot", and if this computed value turns out to be very large, one merely locks the gear connections, treating the computed value as not differing from infinity for any practical purpose.

At the Vector Table, one turn of the Y' shaft gives $u_m p/180$ f/s increase in y'. Hence also between the "-y' Rot Corrector" and "Rot" gear train, one turn of the Λ X shaft gives $u_m p/180$ f/s increase in λx or $u_m p/(180 \lambda)$ ft. increase in x. Between Integrand of Integrator Unit No.1, and the "Rot" gear train, one turn of the X shaft gives $u_m p/(\text{Rot } 180 \lambda)$ ft. increase in x. But from Integrator Unit No. 1, one turn of the X shaft gives $1/(96 \text{ II } h_1)$ ft. increase in x. Hence

$$u_m p/(\text{Rot } 180 \quad) = 1/(96 \text{ II } h_1),$$

or

$$\text{"Rot"} = u_m p \; 96 \text{ II } h_1/(180 \lambda)$$

or since $h_1 = 0.0000315823$ and $\lambda = 0.00011262 \sin\alpha$, we have in general,

$$\text{"Rot"} = 0.14956 \text{ II } u_m p/\sin\alpha \qquad (5)$$

In the schedule used in the previous illustrative examples, $\text{II} = 9$, $u_m = 2124.4$ f/s, $p = 1.01355$, while α, the azimuth of the plane of fire, depends upon the particular firing. For this case we have

$$\text{"Rot"} = 2898.3/\sin\alpha$$

On the Proving Ground azimuths are reported as clockwise from the south. For example, a reported azimuth of 38° means a direction of $180^\circ + 38^\circ = 218^\circ$ from the north through east. For such a case $\sin\alpha = -\sin 38^\circ = -0.61566$. In this case

$$\text{"Rot"} = -4707.7$$

Except for an unusually long trajectory, this would probably suggest that the rotation be locked out.

Choice of a gear train for "Rot" = -4707.7, if used, would be: (taking out powers of 4)

$$-4707.7 = 4^6 \times 1.14934 \qquad \text{or}$$

From Computing Note #1,

$$(-4/1)^6(-3/2)(-7/8)^2 = -4704.0, \text{ gear train used for "Rot".}$$

3. <u>Number of Turns of Lead Screw per Physical Unit.</u>

The part devoted to "Number of Turns of Lead Screw per Physical Unit", falls itself into three parts, concerned respectively with (a) <u>Integrands</u>, (given in the left-hand column), (b) <u>Tables</u>, (given in the upper part of the right-hand column), and (c) <u>Metro Cranks</u>, (given in the lower part of the right-hand column).

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The abbreviations "t.l.s.", stand for "turns of the lead screw". The Integrator Units, Tables, and Metro Cranks appear on the Record sheet arranged to suggest their arrangement on the Analyzer, as viewed from the front.

We shall state the needed formulas briefly. A more complete table giving the general conversion formulas for all three parts of all eight Integrator Units will be exhibited at the end of this Note.

For Integrand No. 8, $1\ f/s = 40 / (u_m\ p)$	t.l.s.
For Integrand No. 7*, $1\ \text{unit} = 2.84970 \times 10\ \ I\ b_\delta / (C\ \ II\ n)$	t.l.s.
For Integrand No. 4, $1\ \text{unit} = n/(I\ b_\delta)$	t.l.s.
For Vector Table, $1\ f/s = 180/(u_m\ p)$	t.l.s.
For Template Table (Top), $1\ f/s = 480/u_m$	t.l.s.
For Metro Crank*, δh, $1\ \text{unit} = 2.84970 \times 10\ \ I\ b_\delta / (C\ II\ n)$ (the same as for Integrand No. 7)	t.l.s.
For Metro Crank, $\delta w'$, $1\ mi/h = 1056/(u_m\ p)$	t.l.s.

* The quantity C in this formula, remains as a letter and is not constant throughout the schedule. It changes with the run. This is the "actual C", not the C_{min}.

4. <u>Number of Physical Units per Printer Unit</u>.

A flexible shaft leading to a printer train turns once for each unit change indicated by the printer train. This printer train prints also tenths of a printer unit (indicated by "p.u."). One has in succession,

One turn of $\underline{T}$ represents $1649.13/(II\ u_m\ p)$	sec.
One turn of $\underline{Y}$ represents $329.83/II$	ft.
One turn of $\underline{H}$ represents $1/432, = 2.3148 \times 10^{-3}$	units
One turn of $\underline{X}'$ represents $u_m\ p/720$	f/s
One turn of $\underline{Y}'$ represents $u_m\ p/720$	f/s
One turn of $\underline{X}$ represents $109.942/II$	yd.

Reciprocally, for $720/(u_m\ p)$ turns for $\underline{X}'$ or $\underline{Y}'$ represents one f/s.

There is a place at the bottom of the right hand column to record what equal intervals are being used for operating the punch, whether 500 yards of range, 1 second in time, or 5 seconds in time.

There is also at the end of the sheet a place for any pertinent "Remarks".

Following through the illustrative example for parts (3) and (4) when as used above $I = (-4/1)^2(-23/22)^2$; $II = (-3/2)^2(-4/1)(-1/1)$; $p = 1.01355+$; $b_\delta = 0.00005023$; $u_m = 2124.4$ f/s; $n = 360$, one has:

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		INTEGRANDS				TABLES
8.	y'	1 f/s = 0.0185771 t.l.s.	Div.	Top	a	1 unit = 180.00 t.l.s.
7.	h_s	1 unit = 77.25872/C t.l.s.		Bot.	v	1 f/s
6.	v	(same as 8)	Vect.	Top	w'	1 f/s } = 0.0835971 t.l.s.
5.	h	(same as 7)		Bot.	y'	1 f/s
4.	b	1 unit = 409835.0 t.l.s.	Templ.	Top	u	1 f/s = 0.225946 t.l.s.
3.	y'	(same as 8)	METRO CRANKS		δ a	1 unit = 45.00 t.l.s.
2.	w'	(same as 8)			δ h	(same as 7)
1.	x'	(same as 8)			δ w'	1 mi/h = 0.490436 t.l.s.

T	0.0851003	sec.	X', Y'	2.990535	f/s	X', Y' 0.334388 p.u. = 1 f/s
Y	36.64778	ft.				
R	0.0023148	un.	X	12.21578	yd.	Prints intervals in (t) of _ _ sec. _ _ _ sec.

Equivalent to one printer unit (p.u.).

5. Conversion Coefficients for Integrator Units.

Since it is customary to number the integrator units serially from front to rear, the listing, if it is to correspond to the arrangement in space as one looks down upon the machine from the operator's position in front, must run down from No. 8 to No. 1 in order. In each integrator unit the shafts from far to near are respectively (i) Differential, (ii) Integral, (iii) Integrand, the central one being the output from the unit, the other two being inputs into the unit. The list showing the number of turns per physical unit, runs as follows:

	For increase of	in	the... shaft	for the ...	turns...times
	1 sec.	t	T	differential	2.42852×10^{-3} II u_m p
No. 8	1 ft.	y	Y	integral	3.03190×10^{-3} II
	1 f/s	y'	Y'	integrand	$40/(u_m\ p)$

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	For increase of	in	the... shaft	for the ...	turns...times
	1 ft.	y	Y	differential	3.03190×10^{-3}
No. 7	1 unit	h_s	$-H_s$	integral	$6.54910 \times 10^{8}\ I\ b_\gamma/(C\ II\ n)$*
	1 unit	h_s	H_s	integrand	$2.84970 \times 10^{8}\ I\ b_\gamma/(CII\ n)$*
	1 sec.	t	T	differential	$2.42552 \times 10^{-3}\ II\ u_m\ p$
No. 6	1 ft.	s	S	integral	$3.03190 \times 10^{-3}\ II$
	1 f/s	v	V	integrand	$40/(u_m\ p)$
	1 ft.	s	S	differentdal	$2.42552 \times 10^{-3}\ II$
No. 5	1 ft.	j	J	integral	$21600\ I\ b_\gamma/n$
	1 unit	h	H	integrand	$2.84970 \times 10^{8}\ I\ b_\gamma/(C\ II\ n)$*
	1 ft.	j	$-J$	differential	$17280\ I\ b_\gamma/n$
No. 4	1 unit	r	$-R$	integral	540
	1/ft.	b	B	integrand	$n/(I\ b_\gamma)$
	1 unit	r	R	differential	432
No. 3	1 f/s	k	K	integral	$540/(u_m\ p)$
	1 f/s	y'	Y'	integrand	$40/(u_m\ p)$
	1 unit	r	R	differential	432
No. 2	1 f/s	x_s'	$-X_s'$	integral	$540/(u_m\ p)$
	1 f/s	w'	W'	integrand	$40/(u_m\ p)$

	For increase	in	the... shaft	for the ...	turns...times
	1 sec.	t	T	differential	2.42552×10^{-3} II u_m p
No. 1	1 ft.	x	X	integral	3.03190×10^{-3} II
	1 f/s	x'	X'	integrand	$40/(u_m\ p)$

*Note: the "C" remains as a letter in the formula, as discussed above.

6. Method of Computing Conversion Coefficients.

(a) For Tables

The template is cut so that the 480 turns of the abscissa lead screw correspond to a range of from 0 to $u_m = v_m / a_o$ units and so that to every value of u there corresponds exactly one value of b(u) where the values of B(u) range between b_{min} and b_{max}. Since there are 360 possible turns of the ordinate lead screw, it is desirable to have $b_{max} - b_{min} = b_\delta =$ 360 turns of the lead screw.

To get the setting for the template table we consider the equation,

u_m units = 480 turns of the lead screw,

or 1 unit = $480/u_m$ turns of the lead screw.

Now the initial setting, for a given trajectory where the initial value of u is u_o, would be $(480/u_m)u_o$ turns. The value, $480/u_m$, is called the conversion coefficient (i. e. the number of turns of the lead screw equivalent to one unit of the variable.)

Having determined earlier by means of speed limits, load limits, and travel limits, all but four gear ratios, we discover that all the conversion coefficients as well as the remaining gear ratios are fixed by mere consideration of the travel limits of the various parts of the machine.

We have seen that $480/u_m$ turns of the shaft entering the template table corresponds to 1 unit increase in u. This shaft is connected to the output shaft of the division table by a 3/4 gear. In terms of speed this means that the shaft leaving the division table is turning at a slower rate than the shaft entering the template table. In other words one unit of u corresponds to 3/4 as many turns on the shaft leaving the division table as on the shaft entering the template table.

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1 unit of $u = (3/4)(480/u_m), = (360/u_m)$ turns of the shaft leaving the division table.

As has been mentioned on page 25 of the Report, the output shaft of the division table is geared to its lead screw by a gear ratio of 1/4. As mentioned above this gearing causes the speed of the output shaft to be increased as it enters the analyser proper. Hence

1 unit of $u = (1/4)(360/u_m), = (90/u_m)$ turns of the division output lead screw

Now u_m is an upper bound for v; the u lead screw on the division table has 90 turns, the v lead screw has 180 turns. It might seem reasonable, therefore, so to select the conversion factor, that

1 f/s of $v = 2 \cdot 90/u_m = (180/u_m)$ turns of the vertical lead screw.

However at this point we introduce a "proportionality factor", p, (slightly in excess of unity) whose need will arise later, (for which reason its calculation will be explained later) and we write

1 f/s of $v = 180/(u_m\ p)$ turns of lead screw on division table.

This lead screw is connected to the output of the vector table by a 1/1 gear. Since there are 180 turns on each lead screw of the vector table in any one quadrant we see

1 f/s of $v = 180/(u_m\ p)$ turns of output shaft on the vector table.

When the angle of fire is 0°, $|w'| = |v|$ so that

180 turns on the lead screw for $w' = u_m \cdot p$ (f/s)

or 1 f/s of $w' = 180/(u_m\ p)$ turns of the horizontal lead screw.

Similarly, at 90°, $|y'| = |v|$; consequently

1 f/s of $y' = 180/(u_m\ p)$ turns of the vertical lead screw.

There are 180 turns of the horizontal lead screw on the division table, the variable "a" has a range of approximately 0 to 1. Hence (at the Division Table),

1 unit of a = 180 turns of the horizontal lead screw.

We list then the conversion coefficients for the tables; using "t.l.s.", to indicate "turns of the lead screw":

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I		480 turns = u_m (f/s)
Template	u	1 unit of u = $480/u_m$ t.l.s.
	B	b_δ, (= $b_{max} - b_{min}$) units of b = 360 t.l.s.
		1 unit of b = $360/b_\delta$ t.l.s.
II Vector	w'	u_m p units of w' = 180 t.l.s.
		1 unit of w' = $180/(u_m\ p)$ t.l.s.
	v	u_m p units of v = 180 t.l.s.
		1 unit of v = $180/(u_m\ p)$ t.l.s.
	y'	1 f/s in y' = $180/(u_m\ p)$ t.l.s.
III Division	u	u_m f/s of u = 90 t.l.s.
		1 f/s of u = $90/u_m$ t.l.s.
	a	1 unit of a = 180 t.l.s.
	v	u_m p f/s of v = 180 t.l.s.
		1 f/s in v = $180/(u_m\ p)$ t.l.s.

(b) __For Integrator Units.__

Let us find the conversion coefficients for the Integrators. We know that

1 unit of a = 180 turns of the output shaft of the adder for which the inputs are $1 - a_1 y$ and δa. Since we know that the conversion coefficients on the three shafts leading into an adder are identical we have:

1 unit increase in $a_1 y$ = 180 turns of the shaft labeled -Y

But 1 unit increase in a y means $(1/a_1)$ units increase in y. Hence

1 unit increase in y = 180 a_1 turns etc.

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Following the diagram we see that this shaft is connected to a shaft labeled Y by a gear train equal to -28/5. Now in passing in a direction opposite to the arrows a gear train with value greater than 1 tends to speed up the shaft. Hence

1 unit increase in $y = (28/5)\ 180\ a_1 = 1008\ a_1$, turns etc.

Similarly we see that the conversion coefficient on the output shaft of the 8th integrator is

1 unit increase in $y = \mathrm{II} \cdot 1008\ a$ turns etc.

To find the conversion coefficient for the integrand of No. 8, we note that this is connected to the y' shaft of the vector table by a (-9/2) gear train.

1 unit increase in $y' = (2/9)(180/(u_m\ p)) = (40/(u_m\ p))$ turns of the integrand shaft.

In Physical units we have the relation

$$y = \int y'\ dt \qquad (1)$$

In turns of the integrand, differential and output shafts, the relation is

$$Y = (1/32)\int Y'\ dT \qquad (2)$$

However from the definition of conversion coefficients we see at the integrand shaft of Integrator No. 8

$$Y' = (40/(u_m\ p))\ y'$$

At the output shaft of Integrator No. 8

$$Y = (\mathrm{II} \cdot 1008\ a_1)\ y$$

Replacing in the equation (2) given above,

$$(1/32)\int (40/(u_m\ p))\ y'\ dT = (\mathrm{II} \cdot 1008\ a_1)\ y$$

or

$$y = \int y'(1/32)(40/(u_m\ p))(1/\mathrm{II})(1/1008\ a_1)\ dT$$

Comparing this equation with (1) we see that

$$dt = (5/(4032\ u_m\ p\ \mathrm{II}\ a_1))\ dT$$

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or the conversion coefficient for T on the differential shaft of Integrator No. 8 is

$$4032\ II\ a_1\ u_m\ p\ /\ 5$$

or at the differential shaft of No. 8

$$T = (4032\ II\ a_1\ u_m\ p/5)t$$

Following the diagram we see that we have immediately the conversion coefficients for the differential and integrand shafts of Integrators No. 6 and No. 1.

For No. 6, therefore,

$$S = (1/32)\int V\ dT = (1/32)\int ((40/(u_m\ p))\ v\ (4032\ II\ a_1\ u_m\ p/5)\ dt$$

or $$S/(1008\ II\ a_1) = \int v\ dt = s$$

Hence $$S = (1008\ II\ a_1)\ s$$

For No. 1,

$$X = (1/32)\int X'\ dT = (1/32)\int (40/(u_m\ p))x'(4032\ II\ a_1 u_m\ p/5)dx$$

or $$X/(1008\ II\ a_1) = \int x'\ dt = x$$

Hence $$X = (1008\ II\ a_1)x$$

Let us next calculate the conversion coefficient for k. Following the arrows backward from the Y' shaft of the vector table to Integrator No. 3, we have:

1 unit increase in $y = 180/(u_m\ p)$ turns of Y' shaft

1 unit increase in $y_s' = 180/(u_m\ p)$ turns of Y_s' shaft

Finally, 1 unit increase in $k = (3)(180)/(u_m\ p)$ turns of output shaft of Integrator No. 3

The conversion coefficient for y' on the integrand shaft of No. 3 is the same as for No. 8 . We have then the relations

$$k = \int_0 y'\ dr, \quad K = (1/32)\int_0 Y'\ dR,$$

and at the integrand shaft of No. 3,

$$Y' = (40/(u_m\ p))\ y$$

and at the output shaft of No. 3,

$$K = 540 / (u_m\ p)\ k$$

$$(540/(u_m\ p))\ k = (1/32) \int (40/(u_m\ p))\ y'\ dR$$

or $$(40/((540)(32))\,dR = dr$$

at the differential shaft of No. 3,

$$R = 432\ r$$

At this point we see that we can progress from one integrator to the next where at each step we know two conversion coefficients and can calculate the third. In this way we may compute and check all the entries in the table given in Section 5.

We may determine in this manner the conversion coefficients for all the various shafts of the machine, including in particular, the coefficients for the cranks which feed in the meteorological data as corrections from standard values. It is of interest to verify that the same conversion coefficient is reached when it is computed in two independent ways.

We will do this for the T shaft, thereby calculating the value for III and also throwing light on the introduction of the proportionality factor, p.

Calculating the conversion coefficient from the 8th Integrator we see that

(3) $$T = (4 \cdot 28 \cdot 36 \cdot 2 \cdot h_1\ \text{II}\ u_m \cdot p / (5 \cdot 21))\ t = (96\ \text{II}\ h_1)\ t$$

Let us calculate this conversion coefficient starting with the y' shaft of the vector table.

1 unit increase in $y' = 180/(u_m\ p)$ t.l.s.

After passing through the $-y'$ Rot adder

1 unit increase in $y'_s = 180/(u_m\ p)$ t.l.s.

After passing through $-Y'_s$ adder and gear III (where $y'_s = y_o - (k + tg)$)

1 unit increase in $gt = 180/(u_m\ p)$ t.l.s.

1 unit increase in $t = 180\ g\ \text{III}/(u_m\ p)$ t.l.s.

From (3) then

$$\text{III}\ 180\ g/(u_m\ p) = 4 \cdot 28 \cdot 36 \cdot 2 \cdot h_1\ \ \text{II}\ u_m\ p/(5 \cdot 21)$$

(4) $\quad III/p^2 = 32\ II\ u_m^2 / (75\ g)$

We can now approximate III as follows

$$III \geqq 32\ II\ u_m^2 / (75\ g)$$

It is now possible to calculate p from the equality (4). Since III is almost equal to the right hand side of the inequality, p is very close to 1. The introduction of the proportionality factor, p, permits a rather crude and simple approximation to suffice for III.

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Revision February, 1943

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