Coastal Engineering

Technical Note

DIRECTIONAL WAVE SPECTRA USING
NORMAL SPREADING FUNCTION

PURPOSE: To present a parameterized model of a directional spectrum of the sea surface using an energy spectrum and a value for the mean wave direction. The model used in this Technical Note is the Normal Spreading Function.

BACKGROUND: With the growing interest in directional spectral estimation and modeling of so-called "Spreading Functions," there is a need for methods by which the directional spectral density can be easily computed using various types of data. If one has data from directional type measuring devices, it is then possible to estimate a directional spectrum directly. In many cases, the raw data are not available but have been reduced to estimates of the energy spectrum and a mean direction estimate. This is the situation that will be addressed in this note.

There are many functional models in the literature with which to represent the directional spectra of the sea surface. This CETN will present the normal spreading function which is one of the most simple and easily used models. This model has been shown to fit prototype data (Sand 1984). For a more detailed explanation of directional wave spectra, see Panicker (1974).

DIRECTIONAL SPECTRA: The directional spectrum of the sea surface is modeled as the energy spectrum times a function of angular spreading, usually called a spreading function. The directional spectrum is:

$$E(f,\theta) = E(f) \, D(f,\theta)$$

where

$$E(f) \quad = \quad \text{energy spectral density function}$$

1. REPORT DATE **MAR 1985**	2. REPORT TYPE	3. DATES COVERED **00-00-1985 to 00-00-1985**
4. TITLE AND SUBTITLE **Directional Wave Spectra Using Normal Spreading Function**		5a. CONTRACT NUMBER
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **U.S. Army Engineer Waterways Experiment Station,Coastal Engineering Research Center,Vicksburg,MS,39180**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

With the growing interest in directional spectral estimation and modeling of so-called "Spreading Functions," there is a need for methods by which the directional spectral density can be easily computed using various types of data. If one has data from directional then possible to estimate a directional spectrum raw data are not available but have been reduced spectrum and a mean direction estimate. This is addressed in this note.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**5**	

$D(f,\theta)$ = spreading function

$E(f,\theta)$ = directional spectral density function

f = frequency in cycles per second

θ = direction in radians

The function $D(f,\theta)$ can be represented by a wrapped normal function of the following form:

$$D(f,\theta) = \sum_{k=-1}^{1} \frac{\exp\left[-\frac{1}{2}\left(\frac{\theta - \theta_o - 2\pi k}{\sigma}\right)^2\right]}{\sqrt{2\pi}\,\sigma}$$

where

σ = the directional spreading parameter in radians

θ_o = the mean wave direction in radians

for $\sigma < \pi/3$ radians.

If θ_o is not close to zero (or 2π), then the spreading function can be approximated by a normal bump centered at θ_o, and the terms for $k = -1$ and $k = 1$ may be dropped. Thus

$$D(f,\theta) = \frac{\exp\left[-\frac{1}{2}\left(\frac{\theta - \theta_o}{\sigma}\right)^2\right]}{\sqrt{2\pi}\,\sigma}$$

This is the well known normal probability function where θ_o and θ are functions of frequency. The most commonly used spreading function is the COSINE 2s model (Long 1980). The normal model is used here as an approximation due to its mathematical simplicity.

EXAMPLE: Suppose a given location is partially sheltered by an obstacle such that only the wave energy associated with the angular band θ_1, θ_2 is of interest (Mathiesen 1984) (see Figure 1). For this example, assume that no diffraction or refraction occurs at the inlet entrance. If we are given the energy spectrum of the sea surface and a mean direction θ_o offshore of the
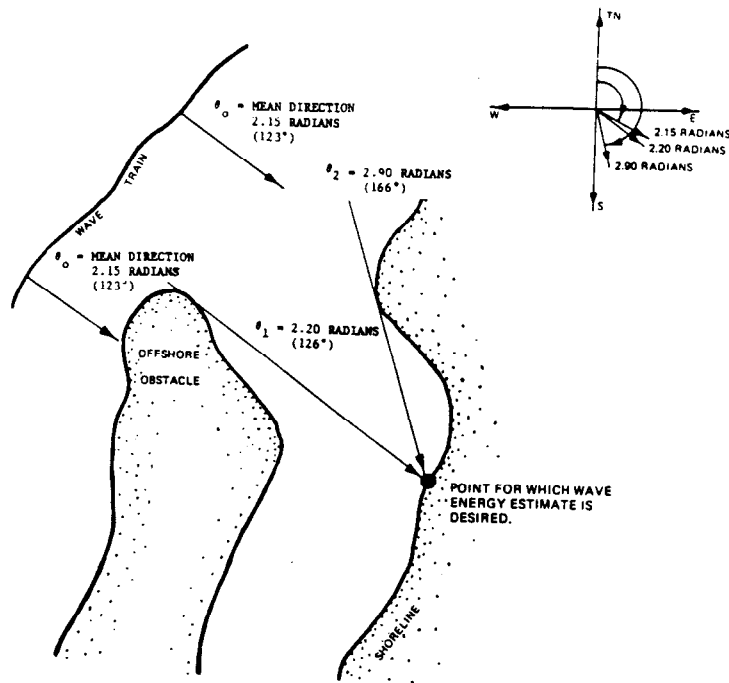
Figure 1. Partially sheltered location

obstacle, then the energy E associated with the angle interval $\theta_1 \to \theta_2$ is approximately

$$E = m_o \int_{\theta_1}^{\theta_2} D(\theta) \, d\theta \qquad (1)$$

where m_o is the variance of the sea surface

$$m_o = \int_{-\infty}^{\infty} E(f) \, df$$

The dependency on frequency has been dropped from $D(f,\theta)$ leaving $D(\theta)$. This implies that all frequencies have the same mean direction and speed. The validity of this assumption depends on the narrow bandedness of the energy spectral density function. For fairly narrow spectra (e.g., a swell train), the assumption is good. For situations where the spectral density is not

3

narrow banded (e.g., a locally generated sea train) this method will usually yield incorrect results.

The energy reduction factor due to sheltering A can be shown as the integral in the following manner:

$$A = \int_{\theta_1}^{\theta_2} D(\theta) \, d\theta \qquad (2)$$

This integral is evaluated in the following way:

$$A = \int_{\frac{\theta_1 - \theta_o}{\sigma}}^{\frac{\theta_2 - \theta_o}{\sigma}} z(x) \, dx, \qquad (3)$$

where Z(x) is the standard normal probability function. The integral in equation 3 is further reduced to

$$A = \int_{-\infty}^{\frac{\theta_2 - \theta_o}{\sigma}} Z(x) \, dx, \quad - \quad \int_{-\infty}^{\frac{\theta_1 - \theta_o}{\sigma}} Z(x) \, dx,$$

$$= P\left(\frac{\theta_2 - \theta_o}{\sigma}\right) - P\left(\frac{\theta_1 - \theta_o}{\sigma}\right) \qquad (4)$$

The values for P (Z) are available in standard normal tables such as the CRC Standard Mathematical Tables (Selby 1981).

Suppose that θ_2 = 2.90 radians and θ_1 = 2.20 radians where θ_o = 2.15 radians. A value of σ = 0.524 radians (based on $30°$ spread) is assumed. If an approximate value for σ is not known and one is unwilling to assume a value, then a general range of 15 to 60 deg may be assumed and the analysis computed at the end points of this range to obtain upper and lower bounds on

the resulting wave energy estimates. This range corresponds to the range for the COSINE 2s model that has been shown to hold for data obtained under various conditions (Sand 1981).

Then equation 4 becomes:

$$A = P(1.43) - P(0.10)$$
$$A = 0.92 - 0.54 = 0.38$$

Finally, $E = AM_o = 0.38 M_o$ represents the approximate energy associated with the angles of interest. This represents a 62 percent reduction in wave energy due to sheltering.

CONCLUSION: The directional spectrum is useful in applications where the angular distribution of wave energy is of interest. One such example has been presented. The wrapped normal spreading function is a good model for obtaining an approximation to the directional spectrum of the sea surface. This is due to its relative simplicity, and it can be evaluated using existing tables.

ADDITIONAL INFORMATION: Contact Mr. J. Michael Hemsley, CERC Prototype Measurement and Analysis Branch, (601) 634-2075.

REFERENCES:

Long, R. B. 1980. "The Statistical Evaluation of Directional Spectrum Estimates Derived from Pitch/Roll Buoy Data," Journal of Physical Oceanography, Vol 10, p 944.

Mathiesen, M. 1984 (Feb). "Personal Communication."

Panicker, N. N. 1974. "Review of Techniques for Directional Wave Spectra," Ocean Wave Measurement and Analysis Proceedings, ASCE, New Orleans, LA, Vol 1, pp 669-688.

Sand, S. E. 1981 (Sep). "Short and Long Wave Directional Spectra," Proceedings of the Conference on Directional Wave Spectra Applications, ASCE, New York, N.Y.

_____. 1984. "Deterministic Decomposition of Pitch-and-Roll Buoy Measurements," Coastal Engineering, Vol 8, pp 243-263.

Selby, S. M. 1981. CRC Standard Mathematical Tables, 25th ed., Boca Raton, Florida.