STIR: Multistability and Chaos in a Driven Nanowire System

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14. ABSTRACT During the nine-month period of this STIR project, three things were done: (1) we discovered anti-phase synchronization in microelectromechanical (MEM) systems, (2) we uncovered a number of complex dynamical phenomena in nanoelectromechanical (NEM) systems, and (3) we developed an efficient, completely data-based method to
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Ying-Cheng Lai
ABSTRACT

During the nine-month period of this STIR project, three things were done: (1) we discovered anti-phase synchronization in microelectromechanical (MEM) systems, (2) we uncovered a number of complex dynamical phenomena in nanoelectromechanical (NEM) systems, and (3) we developed an efficient, completely data-based method to detect unstable periodic orbits (UPOs) in high-dimensional chaotic systems.

For (1), we showed that anti-phase synchronization can emerge in a pair of electrically coupled micro-mechanical beams. Under impulsive perturbation, desynchronization occurs, distorting the output of each beam. We derived a formula for the relaxation rate and verified it numerically. We also found that the difference between the displacements of the two beams, or the differential signal, is robustly immune to impulsive perturbation, implying that the system can effectively counter external disturbances. This can have significant applications in developing various micro-scale devices, which we elaborated using MEM resonators. For (2), we addressed the fundamental question of whether multistability can arise in high-dimensional physical systems. Motivated by the ever increasing widespread use of nanoscale systems, we investigated a prototypical class of NEM systems: electrostatically driven Si-nanowires, mathematically described by a set of driven, nonlinear partial differential equations. We developed a computationally efficient algorithm to solve the equations, and found that multistability and complicated structures of basin of attraction are common types of dynamics, and the latter can be attributed to extensive transient chaos. We also explored implications of these phenomena to device operations. For (3), we developed a framework, integrating the approximation theory of neural networks and adaptive synchronization, to address the problem of time-series based detection of UPOs in high-dimensional chaotic systems, and demonstrated the methodology using time series from the classic Mackey-Glass equation. The significance lies in the fact that detecting UPOs in chaotic systems based solely on time series has been a fundamental but extremely challenging problem in nonlinear dynamics, and previous approaches were applicable but mostly or low-dimensional chaotic systems.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

05/12/2013  2.00 Huanfei Ma, Wei Lin, Ying-Cheng Lai. Detecting unstable periodic orbits in high-dimensional chaotic systems from time series: Reconstruction meeting with adaptation, Physical Review E, (05 2013): 50901. doi: 10.1103/PhysRevE.87.050901


TOTAL: 3

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

None

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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**TOTAL:**

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### Patents Awarded

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Student Metrics
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Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense:...... 0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

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<td>Xuan Ni</td>
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### Sub Contractors (DD882)

### Inventions (DD882)

### Scientific Progress
1. Complex dynamics in nanosystems

Multistability and transient chaos are common in nonlinear dynamical systems. Phenomena associated with multistability such as fractal basin boundaries, riddled and intermingled basins, and noise-induced hopping have been extensively studied in the past three decades. However, most previous studies on multistability were focused on relatively low-dimensional dynamical systems that, mathematically, are often described by ordinary differential equations (ODEs). While multistability in micro-scale systems had been previously uncovered and studied, such as a mixed behavior in nonlinear micromechanical resonators and multistable micro actuator with serially connected bistable elements, there were no research on multistability in nanosystems.

During the STIR project period, we explored multistability with respect to complex dynamics and implications in a class of high-dimensional, physically significant, nanoelectromechanical (NEM) systems at the frontier of interdisciplinary research: electrostatically driven nanowire systems. Such systems are characterized by their small size, extremely low power consumption, and ultra fast speed. Applications range from Zeptogram scale mass sensing and single electron spin detection to RF communication, semiconductor superlattice and many others.

A fundamental goal of science is to have an experimentally validated, predictive theory based on a set of physical laws. With such a theory, a question of concern is whether the final state can be predicted from an initial state chosen in the vicinity of a basin boundary, due to the inevitable error in the specification of the initial state. Here, the basin of attraction of an attractor is the set of initial conditions in the phase space that approach asymptotically the attractor, and the basin boundary separates the initial states leading to different final asymptotic states or attractors. In nonlinear dynamical systems, it is common for basin boundaries to be fractal. In this case, the ability to predict the final attractor of the system may be compromised dramatically. Whether multistability can arise in nanosystems and its dynamical consequences on device performance are thus fundamental issues that need to be investigated for the design and development of nanoscale devices.

There were previous works on nonlinear dynamics in nanosystems, such as synchronized oscillations in coupled nanomechanical oscillators, signal amplification and stochastic resonance in silicon-based nanomechanical resonators, and extensive chaos in driven nanowire systems. However, to explore multistability and complex basin structures in driven nanowire systems is extremely challenging, because a physically realistic model of such systems is mathematically described by a set of nonlinear partial differential equations (PDEs), and it is necessary to examine solutions from a very large number of initial states. In the traditional framework of finite-element method (FEM), the solution is obtained by solving a matrix equation, where the matrix elements need to be evaluated in an iterative manner, a task that can be computationally extremely demanding especially for physically detailed models. Taking advantage of the specific physics associated with the driven nanowire system, we found that, surprisingly, a large set of matrix elements arising from the finite-element paradigm can in fact be evaluated analytically, reducing tremendously the integration time.

Our main finding was that multistability can occur in wide parameter regime of the driven Si-nanowire system, and the origin of complex basin dynamics can be attributed to high-dimensional transient chaos permeating the phase space. A practical implication is that, because of the intrinsic difficulty to predict the final state of the system, and because of the tendency for the system to occasionally switch from one stable state to another under disturbances, parameter regimes in which multistability and complex basin dynamics arise should be avoided in the design and development of nanowire devices.

A possible experimental scheme to test our findings is as follows. Due to the extensive nature of transient chaos leading to basin boundaries permeating the phase space, random perturbations can cause the system to "hop" from one attractor to another in an intermittent manner. Experimentally one can add a stochastic voltage signal to the sinusoidal driving and monitor the motion of the nanowire. The occurrence of intermittency, i.e., the system's exhibiting one type of periodic motion for a finite duration of time and then switching to another, is strong indication of multistability. Persistence of the intermittent behavior, regardless of how the amplitude of the stochastic voltage signal changes, implies complex basin structure and extensive transient chaos.

2. Anti-phase synchronization in microelectromechanical systems and effect of impulsive perturbation

Synchronization is a universal phenomenon in a variety of natural and engineering systems. In recent years, interest in synchronization in micro- or nano-scale systems has emerged, motivated by the potential that synchronization can be exploited for significant applications in nanoscience and nanotechnology. In this regard, phase locking in a pair of mechanically coupled nano-beams was demonstrated. Quite recently, the idea of using optical coupling to synchronize micro-mechanical oscillators was exploited for potential application in realizing massive optomechanical oscillator arrays.

During the project period, we articulated a class of electrically coupled, micro-mechanical oscillator systems and showed that
anti-phase synchronization can arise in such a system. Our system consists of a pair of nearly identical micro-mechanical beams driven by a differential electrical signal. Anti-phase synchronization means that the two beams oscillate in opposite directions at any time. We developed a realistic model, incorporating multiple physical effects such as beam bending, fluid-pressure forces, and electrostatic force. One particularly interesting issue concerned about the consequence of impulsive perturbation which, in reality, can be mechanical shocks, sudden drop of the device, or disturbances from extreme operational environment. When the system is perturbed in such a fashion, anti-phase synchronization will be destroyed temporally but the system can relax to the anti-phase synchronous state after a certain time. We obtained a theoretical formula for the relaxation rate and verified it numerically. An interesting finding is that the differential displacement between the two beams depends on the electrical driving but is extremely insensitive to impulsive perturbation, regardless of whether anti-phase synchronization is achieved. This implies that our electrically coupled, double-beam or differential configuration represents a novel class of microelectromechanical (MEM) systems with superior capability to counter impulsive perturbation. Developing effective strategies to mitigate such perturbation in MEM systems is a problem of tremendous engineering and technological interest. Thus our work and finding not only contribute to the basic nonlinear dynamics of micro-scale systems, but also have the potential to lead to effective shock-immune MEM systems for a variety of significant, state-of-the-art technological applications.

3. Detecting unstable periodic orbits in high-dimensional chaotic systems from time series

The most fundamental building blocks of any chaotic set, attracting or non-attracting, are unstable periodic orbits (UPOs). Consider, for example, a chaotic attractor. The motion of a typical trajectory can be regarded as consisting of intermittent “epochs” of visits to the neighborhoods of various UPOs and, as a result, the natural measure of the attractor is determined by the unstable eigenvalues of the UPOs. A similar picture arises for non-attracting chaotic sets leading to transient chaos, in that the natural measure of such a set can be characterized by UPOs in a way similar to that for chaotic attractors. UPOs are also pivotal for many other areas of research, such as controlling chaos where a central task is to stabilize the system about some UPO that gives rise to desirable performance. In the field of quantum chaos, the celebrated Gutzwiller formula expresses the quantum density of states in terms of classical periodic orbits. It is no surprise then that investigations of UPOs played an extremely important role in the development of nonlinear dynamics and chaos.

In the experimental study of nonlinear systems, a common situation is that the system equations are not known but one is interested in detecting UPOs. Consequently, one must rely on measured time series to accomplish this task, and there has been a significant amount of previous work on this topic, where some pioneering approaches were based on the recurrence of chaotic trajectories in the reconstructed phase space including the approach articulated by Kostelich and Lathrop (LK). In particular, given a time series, one first reconstructs a phase-space trajectory by using Takens’ delay-coordinate embedding method. One next follows the phase-space evolution and record the recurrence time, the time that it takes for the trajectory to return to a small neighborhood of some point, a recurrent point. Statistical-significance test can then be conducted to determine whether the recurrent point belongs to some UPO. This approach is not only applicable to chaotic attractors, but also to detecting UPOs from transiently chaotic systems where only short segments of informative time series are available. In spite of its wide usage, a basic limitation of the LK method lies in the difficulty with high-dimensional chaotic systems. This is especially the case when detection of UPOs of long periods is attempted, due to the difficulty to identify long recurrences. In fact, due to the basic characteristics of the dynamical recurrences in chaotic systems, the LK method is best suited for detecting UPOs from low-dimensional chaotic systems. The problem of detecting UPOs in high-dimensional chaotic systems remains to be an outstanding problem in applied nonlinear dynamics.

During the project period, we articulated a general method to detect UPOs in high-dimensional chaotic systems by integrating the approximation theory of neural networks and adaptive delayed feedback control. In particular, our method consists of three steps: (1) reconstructing from time series the phase space of the underlying system using the standard delay-coordinate embedding technique, (2) adaptively training proper neural networks in a bounded region in the phase space to obtain an estimate of the vector field of the underlying system, and (3) using adaptive control or synchronization to detect UPOs. We demonstrated that the method is capable of detecting UPOs from high-dimensional chaotic systems modeled, for example, by delay-differential equations. We expect our method to find applications in experimental study of high-dimensional nonlinear dynamical systems.

Technology Transfer