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Technical Report ARDSM-TR-12001

SMALL STRAIN COMPATIBILITY CONDITIONS OF AN ELASTIC SOLID IN CYLINDRICAL COORDINATES

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April 2013



U.S. ARMY ARMAMENT RESEARCH, DEVELOPMENT AND
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REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-01-0188		
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1. REPORT DATE (DD-MM-YYYY) April 2013		2. REPORT TYPE		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE SMALL STRAIN COMPATIBILITY CONDITIONS OF AN ELASTIC SOLID IN CYLINDRICAL COORDINATES			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHORS D. Carlucci, N. Payne and I. Mehmedagic			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army ARDEC, DSM/METC Computational Structural Modeling and Fuze & Precision Armaments Technology Directorate (RDAR-DSM/MEF-E) Picatinny Arsenal, NJ 07806-5000			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army ARDEC, ESIC Knowledge & Process Management (RDAR-EIK) Picatinny Arsenal, NJ 07806-5000			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) Technical Report ARDSM-TR-12001		
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The design and analysis of projectiles and gun tubes is often most conveniently accomplished using a cylindrical coordinate system where the coordinates of r , θ , and z represent the radial, circumferential, and longitudinal coordinates, respectively. The compatibility conditions under small strains are not conveniently found in the literature, although it is certain that they have been developed. In this technical report, the six equations of compatibility are documented for convenient use and reference.					
15. SUBJECT TERMS Strain Elastic solid Compatibility Cylindrical coordinates					
16. SECURITY CLASSIFICATION OF:		17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON	
a. REPORT	b. ABSTRACT			c. THIS PAGE	Don Carlucci
U	U	U	16	19b. TELEPHONE NUMBER (Include area code) (973) 724-2486	
SAR					

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INTRODUCTION

The design and analysis of projectiles and gun tubes is often most conveniently accomplished using a cylindrical coordinate system where the coordinates of r , θ , and z represent the radial, circumferential, and longitudinal coordinates, respectively. This geometry is depicted in figure 1, which shows the coordinate system superimposed on a cylinder of length, L , spinning with angular velocity, ω . This is typical of a projectile problem.

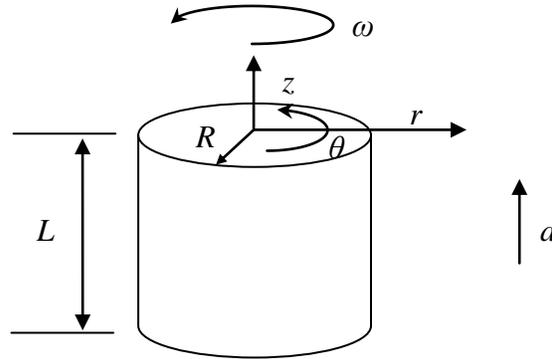


Figure 1
Cylindrical coordinate geometry

The compatibility conditions under small strains are not conveniently found in the literature, although it is certain that they have been developed. References 1 through 8 all contain developments of the compatibility conditions in rectangular Cartesian coordinates and also develop a large number of solutions that require cylindrical coordinates, but only reference 8 explicitly states one of the compatibility conditions (for a planar, two-dimensional geometry). References 9 and 10 develop the equations in both rectangular Cartesian and cylindrical coordinate, but do not have them expressed in a long form likely due to space limitations.

The purpose of this document is simply to list the equations in one place as a reference for future work.

STRAIN-DISPLACEMENT RELATIONS

In considering a solid subjected to small strains, reference 1 develops the strain-displacement relations in cylindrical coordinates as

$$\varepsilon_{rr} = \frac{\partial u}{\partial r} \quad (1)$$

$$\varepsilon_{\theta\theta} = \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} \quad (2)$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \quad (3)$$

$$\gamma_{r\theta} = 2\varepsilon_{r\theta} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \quad (4)$$

$$\gamma_{rz} = 2\varepsilon_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \quad (5)$$

$$\gamma_{\theta z} = 2\varepsilon_{\theta z} = \frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial w}{\partial \theta} \quad (6)$$

In these equations- u , v , and w represent the displacements in the r , θ , and z directions, respectively; while ε_{rr} , $\varepsilon_{\theta\theta}$, and ε_{zz} represent the normal strains in the r , θ , and z directions, respectively, and $\varepsilon_{r\theta}$, $\varepsilon_{\theta z}$, and ε_{rz} represent the shearing strains in each direction with associated engineering shear strains of $\gamma_{r\theta}$, $\gamma_{\theta z}$, and γ_{rz} . Only six independent equations exist because it will assumed the material behaves in an isotropic manner such that

$$\gamma_{r\theta} = \gamma_{\theta r} \quad (7)$$

$$\gamma_{rz} = \gamma_{zr} \quad (8)$$

$$\gamma_{\theta z} = \gamma_{z\theta} \quad (9)$$

The appendix contains a listing of the first and second partial derivatives and mixed partial derivatives of the strains in terms of the displacements.

COMPATIBILITY CONDITIONS

Using the appropriate combinations of partial and mixed-partial derivatives found in the appendix, the compatibility conditions in cylindrical coordinates can be expressed as

$$\frac{1}{r} \frac{\partial^2 \varepsilon_{rr}}{\partial \theta^2} + \frac{\partial}{\partial r} \left\{ r \frac{\partial \varepsilon_{\theta\theta}}{\partial r} - (\varepsilon_{rr} - \varepsilon_{\theta\theta}) \right\} = \frac{\partial}{\partial \theta} \left\{ \frac{\partial \gamma_{r\theta}}{\partial r} + \frac{\gamma_{r\theta}}{r} \right\} \quad (10)$$

$$\frac{1}{r^2} \frac{\partial^2 \varepsilon_{zz}}{\partial \theta^2} + \frac{\partial^2 \varepsilon_{\theta\theta}}{\partial z^2} + \frac{1}{r} \frac{\partial \varepsilon_{zz}}{\partial r} = \frac{1}{r} \frac{\partial}{\partial z} \left\{ \frac{\partial \gamma_{\theta z}}{\partial \theta} + \gamma_{rz} \right\} \quad (11)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial r^2} + \frac{\partial^2 \varepsilon_{rr}}{\partial z^2} = \frac{\partial^2 \gamma_{rz}}{\partial r \partial z} \quad (12)$$

$$\frac{2}{r} \frac{\partial^2 \varepsilon_{rr}}{\partial \theta \partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left\{ r \frac{\partial \gamma_{r\theta}}{\partial z} - \gamma_{\theta z} \right\} + \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial \gamma_{rz}}{\partial \theta} - \frac{\partial \gamma_{\theta z}}{\partial r} \right\} + \frac{1}{r} \frac{\partial \gamma_{r\theta}}{\partial z} + \frac{\gamma_{\theta z}}{r^2} \quad (13)$$

$$2 \frac{\partial}{\partial z} \left\{ \frac{\partial \varepsilon_{\theta\theta}}{\partial r} - \left(\frac{\varepsilon_{rr} - \varepsilon_{\theta\theta}}{r} \right) \right\} = \frac{1}{r} \frac{\partial}{\partial \theta} \left\{ \frac{\partial \gamma_{\theta z}}{\partial r} - \frac{1}{r} \frac{\partial \gamma_{rz}}{\partial \theta} + \frac{\partial \gamma_{r\theta}}{\partial z} \right\} + \frac{1}{r^2} \frac{\partial \gamma_{\theta z}}{\partial \theta} \quad (14)$$

$$\frac{2}{r} \frac{\partial}{\partial \theta} \left\{ \frac{\partial \varepsilon_{zz}}{\partial r} - \frac{\varepsilon_{zz}}{r} \right\} = \frac{\partial}{\partial z} \left\{ \frac{\partial \gamma_{\theta z}}{\partial r} + \frac{1}{r} \frac{\partial \gamma_{rz}}{\partial \theta} - \frac{\partial \gamma_{r\theta}}{\partial z} - \frac{\gamma_{\theta z}}{r} \right\} \quad (15)$$

With the conditions of equations 10 through 15 it is possible to formulate the general differential equations for stress and strain in a cylindrical geometry.

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APPENDIX
LISTING OF FIRST, SECOND, AND MIXED PARTIAL DERIVATIVES

This appendix is simply a listing of the first and second partial derivatives and mixed partial derivatives of the strains in terms of the displacements utilizing a cylindrical coordinate system. These equations are developed directly from equation (1) through (6). The equation numbering sequence is continued from the main text.

First Partial Derivatives

$$\frac{\partial \varepsilon_{rr}}{\partial r} = \frac{\partial^2 u}{\partial r^2} \quad (1)$$

$$\frac{\partial \varepsilon_{rr}}{\partial \theta} = \frac{\partial^2 u}{\partial r \partial \theta} \quad (2)$$

$$\frac{\partial \varepsilon_{rr}}{\partial z} = \frac{\partial^2 u}{\partial r \partial z} \quad (3)$$

$$\frac{\partial \varepsilon_{\theta\theta}}{\partial r} = \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{r} \frac{\partial^2 v}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial v}{\partial \theta} \quad (4)$$

$$\frac{\partial \varepsilon_{\theta\theta}}{\partial \theta} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{1}{r} \frac{\partial^2 v}{\partial \theta^2} \quad (5)$$

$$\frac{\partial \varepsilon_{\theta\theta}}{\partial z} = \frac{1}{r} \frac{\partial u}{\partial z} + \frac{1}{r} \frac{\partial^2 v}{\partial \theta \partial z} \quad (6)$$

$$\frac{\partial \varepsilon_{zz}}{\partial r} = \frac{\partial^2 w}{\partial r \partial z} \quad (7)$$

$$\frac{\partial \varepsilon_{zz}}{\partial \theta} = \frac{\partial^2 w}{\partial \theta \partial z} \quad (8)$$

$$\frac{\partial \varepsilon_{zz}}{\partial z} = \frac{\partial^2 w}{\partial z^2} \quad (9)$$

$$\frac{\partial \gamma_{r\theta}}{\partial r} = \frac{1}{r} \frac{\partial^2 u}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial u}{\partial \theta} + \frac{\partial^2 v}{\partial r^2} - \frac{1}{r} \frac{\partial v}{\partial r} + \frac{v}{r^2} \quad (10)$$

$$\frac{\partial \gamma_{r\theta}}{\partial \theta} = \frac{1}{r} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 v}{\partial r \partial \theta} - \frac{1}{r} \frac{\partial v}{\partial \theta} \quad (11)$$

$$\frac{\partial \gamma_{r\theta}}{\partial z} = \frac{1}{r} \frac{\partial^2 u}{\partial \theta \partial z} + \frac{\partial^2 v}{\partial r \partial z} - \frac{1}{r} \frac{\partial v}{\partial z} \quad (12)$$

$$\frac{\partial \gamma_{rz}}{\partial r} = \frac{\partial^2 u}{\partial r \partial z} + \frac{\partial^2 w}{\partial r^2} \quad (13)$$

$$\frac{\partial \gamma_{rz}}{\partial \theta} = \frac{\partial^2 u}{\partial \theta \partial z} + \frac{\partial^2 w}{\partial r \partial \theta} \quad (14)$$

$$\frac{\partial \gamma_{rz}}{\partial z} = \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial r \partial z} \quad (15)$$

$$\frac{\partial \gamma_{\theta z}}{\partial r} = \frac{\partial^2 v}{\partial r \partial z} + \frac{1}{r} \frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial w}{\partial \theta} \quad (16)$$

$$\frac{\partial \gamma_{\theta z}}{\partial \theta} = \frac{\partial^2 v}{\partial \theta \partial z} + \frac{1}{r} \frac{\partial^2 w}{\partial \theta^2} \quad (17)$$

$$\frac{\partial \gamma_{\theta z}}{\partial z} = \frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \frac{\partial^2 w}{\partial \theta \partial z} \quad (18)$$

Second and Mixed Partial Derivatives

$$\frac{\partial^2 \varepsilon_{rr}}{\partial r^2} = \frac{\partial^3 u}{\partial r^3} \quad (19)$$

$$\frac{\partial^2 \varepsilon_{rr}}{\partial r \partial \theta} = \frac{\partial^3 u}{\partial r^2 \partial \theta} \quad (20)$$

$$\frac{\partial^2 \varepsilon_{rr}}{\partial r \partial z} = \frac{\partial^3 u}{\partial r^2 \partial z} \quad (21)$$

$$\frac{\partial^2 \varepsilon_{rr}}{\partial \theta \partial z} = \frac{\partial^3 u}{\partial r \partial \theta \partial z} \quad (22)$$

$$\frac{\partial^2 \varepsilon_{rr}}{\partial \theta^2} = \frac{\partial^3 u}{\partial r \partial \theta^2} \quad (23)$$

$$\frac{\partial^2 \varepsilon_{rr}}{\partial z^2} = \frac{\partial^3 u}{\partial r \partial z^2} \quad (24)$$

$$\frac{\partial^2 \varepsilon_{\theta\theta}}{\partial r^2} = \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{2}{r^2} \frac{\partial u}{\partial r} + \frac{2u}{r^3} + \frac{1}{r} \frac{\partial^3 v}{\partial r^2 \partial \theta} - \frac{2}{r^2} \frac{\partial^2 v}{\partial r \partial \theta} + \frac{2}{r^3} \frac{\partial v}{\partial \theta} \quad (25)$$

$$\frac{\partial^2 \varepsilon_{\theta\theta}}{\partial r \partial \theta} = \frac{1}{r} \frac{\partial^2 u}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial u}{\partial \theta} + \frac{1}{r} \frac{\partial^3 v}{\partial r \partial \theta^2} - \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} \quad (26)$$

$$\frac{\partial^2 \varepsilon_{\theta\theta}}{\partial r \partial z} = \frac{1}{r} \frac{\partial^2 u}{\partial r \partial z} - \frac{1}{r^2} \frac{\partial u}{\partial z} + \frac{1}{r} \frac{\partial^3 v}{\partial r \partial \theta \partial z} - \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta \partial z} \quad (27)$$

$$\frac{\partial^2 \varepsilon_{\theta\theta}}{\partial \theta \partial z} = \frac{1}{r} \frac{\partial^2 u}{\partial \theta \partial z} + \frac{1}{r} \frac{\partial^3 v}{\partial \theta^2 \partial z} \quad (28)$$

$$\frac{\partial^2 \varepsilon_{\theta\theta}}{\partial \theta^2} = \frac{1}{r} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r} \frac{\partial^3 v}{\partial \theta^3} \quad (29)$$

$$\frac{\partial^2 \varepsilon_{\theta\theta}}{\partial z^2} = \frac{1}{r} \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial^3 v}{\partial \theta \partial z^2} \quad (30)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial r^2} = \frac{\partial^3 w}{\partial r^2 \partial z} \quad (31)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial r \partial \theta} = \frac{\partial^3 w}{\partial r \partial \theta \partial z} \quad (32)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial r \partial z} = \frac{\partial^3 w}{\partial r \partial z^2} \quad (33)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial z \partial \theta} = \frac{\partial^3 w}{\partial \theta \partial z^2} \quad (34)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial \theta^2} = \frac{\partial^3 w}{\partial \theta^2 \partial z} \quad (35)$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial z^2} = \frac{\partial^3 w}{\partial z^3} \quad (36)$$

$$\frac{\partial^2 \gamma_{r\theta}}{\partial r^2} = \frac{1}{r} \frac{\partial^3 u}{\partial r^2 \partial \theta} - \frac{2}{r^2} \frac{\partial^2 u}{\partial r \partial \theta} + \frac{2}{r^3} \frac{\partial u}{\partial \theta} + \frac{\partial^3 v}{\partial r^3} - \frac{1}{r} \frac{\partial^2 v}{\partial r^2} + \frac{2}{r^2} \frac{\partial v}{\partial r} - \frac{2v}{r^3} \quad (37)$$

$$\frac{\partial^2 \gamma_{r\theta}}{\partial r \partial \theta} = \frac{1}{r} \frac{\partial^3 u}{\partial r \partial \theta^2} - \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^3 v}{\partial r^2 \partial \theta} - \frac{1}{r} \frac{\partial^2 v}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial v}{\partial \theta} \quad (38)$$

$$\frac{\partial^2 \gamma_{r\theta}}{\partial r \partial z} = \frac{1}{r} \frac{\partial^3 u}{\partial r \partial \theta \partial z} - \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta \partial z} + \frac{\partial^3 v}{\partial r^2 \partial z} - \frac{1}{r} \frac{\partial^2 v}{\partial r \partial z} + \frac{1}{r^2} \frac{\partial v}{\partial z} \quad (39)$$

$$\frac{\partial^2 \gamma_{r\theta}}{\partial \theta \partial z} = \frac{1}{r} \frac{\partial^3 u}{\partial \theta^2 \partial z} + \frac{\partial^3 v}{\partial r \partial \theta \partial z} - \frac{1}{r} \frac{\partial^2 v}{\partial \theta \partial z} \quad (40)$$

$$\frac{\partial^2 \gamma_{r\theta}}{\partial \theta^2} = \frac{1}{r} \frac{\partial^3 u}{\partial \theta^3} + \frac{\partial^3 v}{\partial r \partial \theta^2} - \frac{1}{r} \frac{\partial^2 v}{\partial \theta^2} \quad (41)$$

$$\frac{\partial^2 \gamma_{r\theta}}{\partial z^2} = \frac{1}{r} \frac{\partial^3 u}{\partial \theta \partial z^2} + \frac{\partial^3 v}{\partial r \partial z^2} - \frac{1}{r} \frac{\partial^2 v}{\partial z^2} \quad (42)$$

$$\frac{\partial^2 \gamma_{rz}}{\partial r^2} = \frac{\partial^3 u}{\partial r^2 \partial z} + \frac{\partial^3 w}{\partial r^3} \quad (43)$$

$$\frac{\partial^2 \gamma_{rz}}{\partial r \partial \theta} = \frac{\partial^3 u}{\partial r \partial \theta \partial z} + \frac{\partial^3 w}{\partial r^2 \partial \theta} \quad (44)$$

$$\frac{\partial^2 \gamma_{rz}}{\partial r \partial z} = \frac{\partial^3 u}{\partial r \partial z^2} + \frac{\partial^3 w}{\partial r^2 \partial z} \quad (45)$$

$$\frac{\partial^2 \gamma_{rz}}{\partial \theta \partial z} = \frac{\partial^3 u}{\partial \theta \partial z^2} + \frac{\partial^3 w}{\partial r \partial \theta \partial z} \quad (46)$$

$$\frac{\partial^2 \gamma_{rz}}{\partial \theta^2} = \frac{\partial^3 u}{\partial \theta^2 \partial z} + \frac{\partial^3 w}{\partial r \partial \theta^2} \quad (47)$$

$$\frac{\partial^2 \gamma_{rz}}{\partial z^2} = \frac{\partial^3 u}{\partial z^3} + \frac{\partial^3 w}{\partial r \partial z^2} \quad (48)$$

$$\frac{\partial^2 \gamma_{\theta z}}{\partial r^2} = \frac{\partial^3 v}{\partial r^2 \partial z} + \frac{1}{r} \frac{\partial^3 w}{\partial r^2 \partial \theta} - \frac{2}{r^2} \frac{\partial^2 w}{\partial r \partial \theta} + \frac{2}{r^3} \frac{\partial w}{\partial \theta} \quad (49)$$

$$\frac{\partial^2 \gamma_{\theta z}}{\partial r \partial \theta} = \frac{\partial^3 v}{\partial r \partial \theta \partial z} + \frac{1}{r} \frac{\partial^3 w}{\partial r \partial \theta^2} - \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \quad (50)$$

$$\frac{\partial^2 \gamma_{\theta z}}{\partial r \partial z} = \frac{\partial^3 v}{\partial r \partial z^2} + \frac{1}{r} \frac{\partial^3 w}{\partial r \partial \theta \partial z} - \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta \partial z} \quad (51)$$

$$\frac{\partial^2 \gamma_{\theta z}}{\partial \theta \partial z} = \frac{\partial^3 v}{\partial \theta \partial z^2} + \frac{1}{r} \frac{\partial^3 w}{\partial \theta^2 \partial z} \quad (52)$$

$$\frac{\partial^2 \gamma_{\theta z}}{\partial \theta^2} = \frac{\partial^3 v}{\partial \theta^2 \partial z} + \frac{1}{r} \frac{\partial^3 w}{\partial \theta^3} \quad (53)$$

$$\frac{\partial^2 \gamma_{\theta z}}{\partial z^2} = \frac{\partial^3 v}{\partial z^3} + \frac{1}{r} \frac{\partial^3 w}{\partial \theta \partial z^2} \quad (54)$$

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