SCATTERING OF LIGHT AND SURFACE PLASMON POLARITONS FROM ROUGH SURFACES

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Final Report

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### Abstract
Results of studies of several properties of surface plasmon polaritons on structured surfaces are described, together with results for the scattering of surface plasmon polaritons by various surface defects. In addition, results of rigorous calculations of the scattering of electromagnetic waves from clean or coated two-dimensional randomly rough metal surfaces, and from two-dimensional randomly rough perfectly conducting and dielectric surfaces are presented. Properties of partially coherent light are investigated, and several types of metallic microlenses are studied.

### Subject Terms
Surface plasmon polaritons, structured surface, surface defects, scattering, randomly rough surface, partially coherent fields, metallic microlenses

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1. Summary

The research carried out with support from AFRL contract FA9453-08-C-0230 during the period 01/23/2008 – 03/31/2013 falls mainly into two categories: the scattering of light from, and its transmission through, rough surfaces, and the propagation of surface electromagnetic waves on structured surfaces. We describe the highlights of the results obtained in each of these categories, and in two others as well.

2. Introduction

Several papers have been published that are devoted to surface plasmon polariton analogues of volume electromagnetic waves. The results of these investigations reveal new properties of these surface electromagnetic waves, which might be useful in device applications.

Thus, the negative refraction of a surface plasmon polariton was studied in two papers. In the first [1], all-angle negative refraction of a surface plasmon polariton was predicted for a system consisting of a vacuum in the region $x_3>d$, and a dielectric medium whose dielectric constant is $\varepsilon_d$ in the region $x_3<0$. The region $x_1<0$, $0<x_3<d$ is filled by a metal whose dielectric function is $\varepsilon_1(\omega)$, while the region $x_1>0$, $0<x_3<d$ is filled with a metal whose dielectric function is $\varepsilon_2(\omega)$. The dispersion curve for a surface plasmon polariton in each of the regions $x_1<0$ and $x_1>0$ consists of two branches. The lower frequency branch has an isotropic positive group velocity in the entire region of frequencies and two-dimensional wave vectors where the corresponding surface plasmon polariton exists. The higher frequency branch has an isotropic negative group velocity throughout the entire region of frequencies and wave vectors within which the corresponding surface plasmon polariton exists. A surface plasmon polariton of frequency $\omega$ from the lower branch of its dispersion curve is incident on the interface between the two metals from the region $x_1<0$. Its angle of incidence, measured counterclockwise from the negative $x_1$ axis, is $\theta$. The surface plasmon polariton of frequency $\omega$ transmitted through the interface $x_1=0$ into the region $x_1>0$ falls on the higher frequency branch of its dispersion curve. Its angle of transmission, measured counterclockwise from the positive $x_1$ axis is $\theta_t$. The relation between $\theta_t$ and $\theta$ was sought as $\theta$ varies from $-90^\circ$ to $+90^\circ$. This relation was solved by calculating the transmitted electromagnetic field with the use of effective boundary conditions at the plane $x_3=0$ between the electromagnetic fields in the vacuum and in the dielectric substrate. The singular integral equations that arise from the use of such boundary conditions were solved by means of the Wiener-Hopf method. It was found that this planar structure effects all angle negative refraction of a surface plasmon polariton incident on it, due to the isotropic negative group velocity of the transmitted surface plasmon polariton.

In the second paper [2] a system was studied that consists of a vacuum that is in contact with two metals that each fill a quarter space with a periodic boundary between them. The interface of each of the metals with the vacuum supports a surface plasmon polariton of frequency $\omega$.

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When a surface plasmon polariton passes through the periodic boundary between the metals it transforms partly into the Bragg beams of the refracted and reflected surface plasmon polaritons, and partly into volume waves in the vacuum. A suitable choice of the period ensures that the only propagating Bragg beams of the refracted surface plasmon polariton are the specular (0)-order beam and the (-1)-order beam, which is negatively refracted. With a suitable choice of the form of the periodic boundary and its amplitude the specular beam can be made to vanish, so that the incident surface plasmon polariton is negatively refracted. This simple structure does not produce all-angle negative refraction of the surface plasmon polariton like the structure proposed in [1]. Nevertheless it produces negative refraction over a sufficiently large range of angles of incidence to be useful in applications where this property is desired. It also has the attractive feature of being a planar structure, which minimizes the transformation of the incident surface plasmon polariton into volume electromagnetic waves due to the discontinuity of the dielectric functions across the periodic interface between the two metals.

When light is transmitted through a one-dimensional periodic structure, the image of that structure is found to repeat itself periodically with increasing distance of the image plane from the structure. This self-imaging of the periodic structure was discovered by H.F. Talbot in 1836\(^2\) and was explained by Lord Rayleigh in 1881\(^3\). We have studied the surface plasmon polariton analogue of this Talbot effect[3]. The system we studied consisted of vacuum in the region \(x_3>0\), and a metal whose dielectric function is \(\varepsilon_1(\omega)\) in the region \(x_3<0\). Deposited on this surface is a periodic row of dots of a material whose dielectric function is \(\varepsilon_2(\omega)\). The radius of each dot is \(R\), and they are centered at the points \((0,nb,0)\) where \(n=0, \pm 1, \pm 2, \ldots\), with \(R>b/2\). The interface between vacuum and the metal whose dielectric function is \(\varepsilon_2(\omega)\) supports a surface plasmon polariton of frequency \(\omega\). A surface plasmon polariton of this frequency is incident on the row of dots from the region \(x_1<0\). The transmission of the incident surface plasmon polariton by this periodic structure was studied on the basis of an impedance boundary condition on the surface \(x_3=0\). The field of the transmitted surface plasmon polaritons displayed periodic self images of the row of dots that are separated from it by multiples of a characteristic distance. This pattern is the analogue for surface plasmon polaritons of the Talbot effect for volume electromagnetic waves.

The transmission of a surface plasmon polariton through a slit of finite length on a planar metal surface was calculated as a function of the frequency of the incident surface plasmon polariton [1]. The structure studied in this work consisted of vacuum in the region \(x_3>0\), while the region \(x_3<0\) was filled with a metal whose dielectric function is \(\varepsilon_1(\omega)\), except in the regions \(|x_1|<L/2, |x_2|>d\), which are filled with a medium whose dielectric function is \(\varepsilon_2(\omega)\). Thus the region \(|x_1|<L/2, |x_2|<d\) is a slit of length \(L\) and width \(2d\) on the surface \(x_3=0\). The dielectric function \(\varepsilon_2(\omega)\) was chosen such that the slit acts as a waveguide that supports one or more guided surface plasmon polaritons, depending on the frequency \(\omega\). A surface plasmon polariton beam of

\(^2\) H.F. Talbot, “Facts relating to optical science, no. IV,” Phil Mag. 9, 401-407(1836).

\(^3\) Lord Rayleigh, “On copying diffraction-grating and on some phenomena connected therewith,” Phil. Mag. 11, 196-205(1881).
frequency $\omega$ is incident on the slit from the region $x_1<-L/2$. The field of the surface plasmon polariton transmitted through the slit into the region $x_1>L/2$ was calculated by the use of a surface impedance boundary condition on the surface $x_3=0$, and the Wiener-Hopf method. The results obtained display a step-wise increase in the frequency dependence of the transmissivity at each frequency at which an infinitely long slit can support an additional guided wave surface plasmon polariton.

The interference fringes observed by Thomas Young\(^{(4)}\) in the intensity distribution of light transmitted through a pair of nearby pinholes in an opaque screen was the first experimental evidence for the wave nature of light. The same kind of interference pattern is also obtained when the pinholes of Young’s experiment are replaced by narrow slits\(^{(5)}\). The surface plasmon polariton analogue of Young’s double slit experiment has been studied for a structure that consisted of vacuum in the region $x_3>0$, while the region $x_3<0$ is filled with a metal whose dielectric function is $\varepsilon_1(\omega)$, except in the regions $|x_1|<L/2$, $|x_2|>d$, and $|x_1|<L/2$, $|x_2|<d-\Delta$, which are filled with a dielectric whose (real, positive) dielectric constant is $\varepsilon_2(\omega)$. These portions of the surface $x_3=0$ do not support surface plasmon polaritons. Thus the regions $|x_1|<L/2$, $d-\Delta<|x_2|<d$ are slits of length $L$ and width $\Delta$ on the surface $x_3=0$. A surface plasmon polariton of frequency $\omega$ in the form of a plane wave is incident on the pair of slits from the region $x_1<-L/2$ on the surface $x_3=0$. The field of the surface plasmon polaritons transmitted into the region $x_1>L/2$ of this surface was calculated by means of an impedance boundary condition and a numerical solution of the integral equations to which the use of this boundary condition gives rise. The surface plasmon polariton beams emerging from the two slits spread with increasing distance from the exits of the slits due to diffraction. When the diffracted surface waves overlap an interference pattern is produced, whose signature is a central maximum. Thus, with increasing distance from the exits of the slits the two-peaked intensity distribution close to the slits evolves into a three-peaked distribution with a dominant central peak.

When an electromagnetic beam of finite cross section is incident from an optically more dense medium on its planar interface with an optically less dense medium, and the polar angle of incidence is greater than the critical angle for total internal reflection, the reflected beam undergoes a lateral displacement along the interface, as if it has been reflected from a plane in the optically less dense medium parallel to the physical interface. This lateral displacement of the reflected beam is called the Goos-Hänchen effect\(^{(6)}\). A surface plasmon polariton can display a Goos-Hänchen effect [5]. This was shown in calculations for a structure that consists of vacuum in the region $x_3>0$, while the region $x_1<0$, $x_3<0$ is filled with a metal whose dielectric

\(^{4}\) T. Young, A Course of Lectures on Natural Philosophy and the Mechanical Arts (J. Johnson, London, 1807).


function is \( \varepsilon_1(\omega) \) (gold) and the region \( x_1>0, x_3<0 \) is filled with a metal whose dielectric constant is \( \varepsilon_2(\omega) \) (aluminum), so that \( |\varepsilon_2(\omega)| > |\varepsilon_1(\omega)| \). The planar interface \( x_3=0 \) between vacuum and each of the metals supports a surface plasmon polariton of frequency \( \omega \). A surface plasmon polariton beam of this frequency is incident from the region \( x_1<0 \) on the interface \( x_3=0 \) between the two metals. The electromagnetic field of the surface plasmon polariton beam reflected from this interface back into the region \( x_1<0 \) is calculated by the use of an impedance boundary condition on the plane \( x_3=0 \), and a numerical solution of the integral equations that result from the use of this boundary condition. The results show that the surface plasmon polariton beam reflected from the boundary \( x_3=0 \) is shifted along the boundary when the angle of incidence of the incident surface plasmon polariton is greater than the critical angle for total internal reflection from that boundary. The calculated lateral shift of the reflected surface plasmon polariton beam appears to be large enough to be experimentally observable.

A great deal of interest at the present time attaches to theoretical approaches to the realization of an electromagnetic cloak that renders a macroscopic object surrounded by it invisible. A structure has been devised that significantly reduces the scattering of surface plasmon polaritons from an object within it [6]. It consists of two concentric rings of equally spaced point scatterers on a planar metal surface at \( x_3=0 \). Each point scattered is represented by an effective polarizability that defines its scattering properties. This circular structure is illuminated by a surface plasmon polariton incident from the region \( x_1<0 \) along the \( x_1 \) axis. The polarizability of each scatterer is determined iteratively and self-consistently by requiring that the total electric field inside the array vanish, while the total electric field outside the array coincide with the incident field there. When a surface defect is introduced into the interior of the array, the total electric field on the surface is unchanged from what it is in the case of an empty cloak, demonstrating the effectiveness of the concentric cloaking arrangement. An attractive feature of this cloaking structure is that it is independent of the optical and geometrical properties of the object being cloaked.

In 1989 de Raedt et al. [7] studied theoretically the propagation of a beam of light along the axis of a random array of parallel dielectric rods. They found that on propagation the beam expanded only to a cross-sectional size determined by the localization length in the random medium. This effect, named “transverse localization”, has been demonstrated in experiments carried out on a two-dimensional photonic crystal that is disordered in the two transverse directions but is invariant in the propagation direction [8]. Surface plasmon polaritons can also display transverse localization [7]. This has been demonstrated in theoretical calculations carried out for a system that consists of a parallel array of semi-infinite strips of width \( a \) in the region \( x_1>0 \) of a metal whose dielectric function is \( \varepsilon_2(\omega) \) that are deposited on the planar surface \( x_3=0 \) of a metal whose dielectric function is \( \varepsilon_1(\omega) \). The region \( x_3>0 \) is vacuum. The midlines of these strips are


displaced in the $x_2$ direction from periodic positions given by $x_2=(2n+1)a$, $n=0,\pm 1,\pm 2,\ldots$, by random distances $\{d_{2n+1}\}$ drawn from a uniform distribution in the interval $[-b/2, b/2]$, where $b<a/2$. The interface between the vacuum and each metal at $x_3=0$ supports a surface plasmon polariton of frequency $\omega$. The region between consecutive strips of the metal whose dielectric function is $\varepsilon_2(\omega)$ form slits through which a surface plasmon polariton Gaussian beam incident from the region $x_1<0$ along the $x_1$ axis on the surface $x_3=0$ propagates. The electric field distribution of the surface plasmon polariton beam in the region $x_1>0$ is calculated by the use of an impedance boundary condition on the surface $x_3=0$, and a numerical solution of the coupled integral equations to which this boundary condition gives rise. It shows evidence of a saturation of diffractive spreading of the beam with increasing distance into the array of slits. The intensity distribution of the field in this region in the $x_2$ direction evolves from a Gaussian form at the entrance to the array of slits into a negative exponential dependence of the form $\exp(-|x_2|/R)$ with increasing $x_1$.

In the 1950s Wannier (9) studied the motion of an electron in a periodic potential while being accelerated by a constant, homogeneous, external electric field. He showed that the energy spectrum of the electron in this case consists of equidistant energy levels, which have come to be known as a Wannier-Stark ladder. The possibility of the existence of a Wannier-Stark ladder for surface plasmon polaritons was studied theoretically in Ref.[7]. The system assumed in this work consisted of vacuum in the region $x_3>0$, and a metal whose dielectric function is $\varepsilon(\omega)$ in the region $x_3<0$. In the region $x_1>0$ of the metal surface $x_3=0$ are deposited $N$ parallel thin strips of a dielectric material. The role of each strip is to change the effective dielectric function of the portion of the surface that it covers, and hence the contribution to the surface impedence of that portion of the surface. The width of each dielectric strip is the same, and the width of each metal strip between consecutive dielectric strips is the same, although the two widths need not be the same. The periodicity in the positions of the strips is the analogue of the periodic potential in which an electron moves in Wannier’s theory. The thickness of the dielectric strips increases from strip to strip in such a way that the effective dielectric function of the surface increases linearly with increasing distance along the surface normal to the strips. This dependence mimics the effect of the external electric field in the electronic case. The transmission coefficient of a surface plasmon polariton incident on this structure from the region $x_1<0$, plotted as a function of its frequency for frequencies in the vicinity of the gap in the band structure between the first and second bands in the absence of the linear increase of the dielectric function of the surface, displays a succession of equally spaced peaks, whose separation is proportional to the rate of increase of the dielectric function. These peaks are the signature of the existence of a Wannier-Stark ladder for a surface plasmon polariton.

A rigorous approach to the scattering of a surface plasmon polariton incident normally on nanoscale one-dimensional surface defects on an otherwise planar lossy metal surface, that is free from the assumptions of a lossless metal and the use of an impedance boundary condition, has been developed [8]. It is based on the assumption of a source that excites both forward- and

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backward-propagating surface plasmon polaritons, the use of Green’s second integral identity in the plane, and a numerical solution of the integral equations to which this identity gives rise.

One of the most commonly used methods in the theoretical studies of the scattering of volume electromagnetic waves from one- and two-dimensional rough surfaces is the Kirchhoff approximation\textsuperscript{(10)}. In this approximation scattering and transmission are treated as reflection from or transmission through the plane tangent to the surface at each point of it. It is a single-scattering approximation that is valid when the wavelength of the incident field is small compared to the radii of curvature of the surface at each point of it. When the conditions for its validity are satisfied it yields qualitatively and quantitatively accurate results for the scattered field. To simplify calculations of the scattering of surface plasmon polaritons by certain types of surface structures a Kirchhoff approximation for surface plasmon polaritons was developed in Ref.[9]. Specifically, in this methodological work the use of an impedance boundary condition, the Weiner-Hopf method, and Green’s second integral identity in the plane are used to obtain a Kirchhoff approximation for the reflection amplitude of a surface plasmon polariton incident from one metal surface onto its rough boundary with a co-planar surface of a second metal. The system studied consists of vacuum in the region \(x_3>0\), a metal whose dielectric function is \(\varepsilon_1(ω)\) in the region \(x_1<\zeta(x_2)\times_3<0\) and \(\varepsilon_2(ω)\) in the region \(x_1>\zeta(x_2), x_3<0\). The boundary profile function \(\zeta(x_2)\) is assumed to be a single-valued function of \(x_2\) that is differentiable. The reflection amplitude is obtained in the form of an integral, and its use is illustrated for the case of a boundary profile function give by \(\zeta(x_2)=x_2^3/x_0^3\), where \(x_0\) is a characteristic length. The intensity distribution of the electric field of the reflected surface plasmon polariton shows that this surface wave propagates along a curved path with an intensity that decreases with increasing distance from the boundary, and resembles a volume Airy beam that has been the object of considerable theoretical and experimental interest in recent years\textsuperscript{(11)}.

The propagation of a wave packet that is a superposition of three s-polarized guided waves with different frequencies in a planar waveguide consisting of a dielectric medium with a linearly graded index of refraction, sandwiched between perfectly conducting walls, has been studied theoretically\textsuperscript{[10]}. The electric field at each point within the waveguide was calculated, and it was shown that each of the constituent modes ceases to propagate at a specific distance along the waveguide that depends on its frequency and on the geometrical and material parameters defining the waveguide. This simple model displays the phenomenon of wave trapping of guided waves in an explicit fashion, without the use of a negative index metamaterial.

An effective boundary condition was used to derive a two-dimensional integral equation for the amplitude in the Rayleigh representation of the electric field of a surface plasmon polariton scattered from a localized two-dimensional dielectric surface defect\textsuperscript{[11]}. This integral equation


was solved by transforming it into a set of coupled one-dimensional integral equations that were solved numerically, by representing the dependence of the amplitude on the azimuthal angle in the form of a rotational expansion. Numerical results were obtained for dielectric defects that had the shape of an anisotropic Gaussian protuberance or a hemiellipsoid with a finite footprint on an otherwise planar metal surface. The surface plasmon polariton incident on each of these defects from an arbitrary direction was either a plane wave or a Gaussian beam in the plane of the metal surface. An inspection of the near-field distribution of the radiation caused by the scattering revealed a localized increase of surface plasmon polariton intensity in the vicinity of the dielectric defect (Fig. 1), which indicates focusing of either form of the incident surface plasmon polariton.

![Figure 1. Total Electric Field Intensity](image)

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It has been known for a long time\textsuperscript{12} that an s-polarized surface plasmon polariton cannot exist at a planar dielectric-metal interface. The situation is quite different if the dielectric metal interface is not planar but curved. In a recent paper [12] the propagation of an s-polarized electromagnetic wave circumferentially around a portion of a circular interface between vacuum and a metal has been studied. The electromagnetic fields in this case are not required to be single valued. It was found that when the metal is concave toward the vacuum the curvature of the interface can localize the wave to its vicinity, producing thereby an s-polarized surface plasmon polariton. The dispersion curve of this surface wave possesses many branches (Fig. 2). As the radius of the cylindrical interface increases, the number of branches increases, and their separation in frequency decreases. In addition, the number of nodes in the radial dependence of the electric field of each mode equals the branch number, if the lowest frequency branch is labeled the zero branch.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{dispersion_curves.png}
\caption{Dispersion Curves of S-Polarized Surface Plasmon Polaritons on a Circular Boundary}
\end{figure}


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In recent work a procedure is described for designing metal-metal boundaries for the strong attenuation of surface plasmon polaritons without the introduction of reflections or scattering effects [13]. The structure studied is depicted in Fig. 3. It is assumed that the medium in the region $x_3 > 0$ is vacuum, so that $\varepsilon_d = \mu_d = 1$. A surface plasmon polariton propagating from left to right on medium 1 ($\varepsilon_1, \mu_1$) impinges on the vertical boundary between media 1 and 2. The dielectric permittivity and magnetic permeability of medium 2 ($\varepsilon_2, \mu_2$) are chosen such that the surface plasmon polariton on this medium is strongly attenuated without the introduction of significant reflection or coupling into radiative modes. This is achieved by requiring that the transverse profile of the surface plasmon polariton wave must be the same on the two sides of the vertical interface. This condition minimizes the scattering losses due to the interface. To minimize the reflection of the surface plasmon polariton from the interface, the real part of the wave number of the surface plasmon polariton should be the same in both media 1 and 2. For given values of $\varepsilon_1$ and $\mu_1$, namely $\varepsilon_1 = -40.44 + i2.97$ and $\mu_1 = 1$, corresponding to gold at the vacuum wavelength $\lambda = 980 \text{nm}$, and values of $\text{Im} \varepsilon_2 = 3.0$ and $\text{Im} \mu_2 = 0$, the values of $\text{Re} \varepsilon_2 = -10.0$ and $\text{Re} \mu_2 = 0.19$ were found to make medium 2 an effective absorber of the incident surface plasmon polariton, without the interface perturbing the incident wave significantly (Fig. 4). In the numerical calculations the incident surface plasmon polariton, propagating to the right, was excited by illuminating an array of 5 rectangular grooves, positioned to the left of the vertical interface, by a Gaussian beam at an angle of incidence

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$\theta_0 = -6.5^\circ$. The technique developed in this paper can be used effectively in plasmonic and nanophotonic calculations.

A surface structure that transmits a surface plasmon polariton incident on it from one side, but nearly does not transmit one incident on it from the opposite direction is described in a recent work [14]. This nonreciprocal transmission of a surface plasmon polariton does not require either electrical nonlinearity or the presence of a magnetic field, but is a consequence solely of the geometry of the structure. This structure consists of a square array of scatterers, whose edges are parallel to the $x_1$ and $x_2$ axes, arranged in a triangular mesh with a lattice constant $a_0$ on a planar metal surface (Fig. 5).

Figure 4. The Magnetic Field of a Surface Plasmon Polariton After It Strikes a Perfectly Absorbing Medium

Figure 5. A Surface Structure for Asymmetric Transmission
A diffraction structure with a period $4\ a_0$ in the $x_2$ direction is attached to the left side of the periodic array. A surface plasmon polariton of frequency $\omega$ propagating along the $x_1$ axis impinges on the structure from the region $x_1 < 0$ to the left of it, or from the region $x_1 > 0$ to the right of it. The surface plasmon polariton is described by its electric field component parallel to the $x_3$ axis, evaluated on the surface $x_3 = 0$. Each scatterer is represented by an effective polarizability $\alpha$. The surface plasmon polariton incident on this structure is represented by a Gaussian beam, whose width is of the order of, or smaller than, the lateral dimension of the array of point scatterers. Its frequency lies in a range that includes the gap in the band structure of an infinitely extended triangular surface plasmon polaritonic crystal. The transmissivities for incidence from the right ($T_R$) and from the left ($T_L$) of the structure were calculated as functions of the wavelength of the surface plasmon polariton, and a contrast transmissivity ratio $R_c(\lambda) = \left| \frac{T_R - T_L}{T_R + T_L} \right|$ was constructed from these results (Fig. 6).

![Figure 6. The Asymmetric Transmissivity of a Surface Plasmon Polariton](image)

It is seen to be nonzero in the range $700\ \text{nm} < \lambda < 720\ \text{nm}$, which contains the gap in the surface plasmon polaritonic band structure. While $R_c$ does not reach unity in these calculations, it is expected that it will come much closer to this value with some modifications of the structure.

It has been shown that an s-polarized surface electromagnetic wave exists at the planar interface between a semi-infinite homogeneous dielectric medium, e.g. vacuum ($z<0$), and a dielectric medium ($z>0$) whose electric permittivity decreases continuously with increasing distance into
the medium from the interface until it saturates at a bulk value greater than unity [15]. This permittivity is independent of frequency, and is given by

\[ \varepsilon(z) = n_0^2[1 - \frac{1}{g} \frac{1}{g(1 + \frac{z}{L})^2}], \]

where \( n_0 \) is the real index of refraction at \( z=0 \). For frequencies below a certain critical value \( \Omega_c \) the dispersion curve of this surface wave consists of a single branch that exists in a narrow spectral range (Fig. 7).

![Figure 7. Dispersion Curve of a Surface Electromagnetic Wave on a Graded-Index Dielectric Medium](image)

Its electric field decays exponentially with increasing distance into each dielectric medium (Fig. 8). For frequencies above this critical value the dispersion curves possesses several branches. The corresponding electric field decays exponentially with increasing distance into the homogeneous dielectric medium, and decays in an oscillatory fashion with increasing distance into the graded-index medium. The number of nodes in the latter field equals the branch number, starting with zero for the lowest frequency branch. Thus, the modes in this frequency range have the nature of guided waves supported by a graded-index waveguide.
The scattering of surface plasmon polaritons by surface defects continues to attract interest. A surface plasmon polariton incident on a surface defect is partly scattered into other surface plasmon polaritons, and is partly converted into volume electromagnetic waves in the vacuum above the surface. The scattering out of the beam caused by the presence of surface defects decreases the propagation length of the surface plasmon polariton, and it is important for applications of these surface electromagnetic waves to be able to calculate the cross section for such scattering. At the same time surface defects of particular forms and sizes can scatter surface plasmon polaritons in desirable ways. To exploit the possibilities this offers it is also necessary to be able to calculate the scattering of surface plasmon polaritons by such surface defects. Much of the emphasis in recent work on this problem has focused on searches for new methods of calculating this scattering that are accurate and computationally tractable. In recent work [16] a reduced Rayleigh equation for the scattering of a surface plasmon polariton incident normally on a one-dimensional ridge or groove on an otherwise planar metal surface is solved by a purely numerical approach. The solution is used to calculate the reflectivity and transmissivity of the surface plasmon polariton, and its conversion into volume electromagnetic waves in the vacuum above the metal surface. The surface plasmon polariton is assumed to be propagating in the positive $x_1$ direction on the plane $x_3 = 0$. The shape of the defect is given by $x_3 = \zeta(x_1) = A \exp(-x_1^2 / R^2)$, so that it describes a ridge when $A>0$, and a groove when $A<0$, and by $x_3 = \zeta(x_1) = -h + (h / a)|x_1|$ for $0 \leq |x_1| \leq a$, and $x_3 = 0$ for $|x_1| > a$. The frequency dependence of the transmittance, reflectance, emittance, and their sum when the surface plasmon polariton is incident on these on a silver surface is depicted in Fig. 9 for the Gaussian defect the case that $R = 250\text{nm}$ and $|A| = 80\text{nm}$.
Figure 9. Transmittance, Reflectance, and Emittance of a Surface Plasmon Polariton Scattered by a Surface Defect

The closeness of the sum to unity indicates how well unitarity is satisfied in these calculations. The angular dependence of the emittance when a surface plasmon polariton is scattered by the Gaussian defect is presented in Fig. 10 for a fixed value of R and several values of |A|. For each value of |A| the radiation pattern has a maximum at a scattering angle $\theta_s = 30^\circ$. Moreover a groove is seen to convert more of the energy in the incident surface plasmon polariton into volume electromagnetic waves in the vacuum than does a ridge, for each value of |A|.

The dispersion and damping of surface plasmon polaritons on a randomly rough metal surface have been studied theoretically by several authors. It is interesting that in these studies the randomly rough surface has been a two-dimensional randomly rough surface. The dispersion of surface plasmon polaritons on a one-dimensional randomly rough metal surface has hardly been considered. These properties were mentioned briefly and qualitatively in $^{14, 15}$, but no results were presented in these papers.

Such surfaces can now be fabricated with specified statistical properties. Thus, it seemed worthwhile to fill this gap in the literature. This was done in Ref. [17] where by the use of the reduced Rayleigh equation for the amplitude of a surface plasmon polariton on a one-dimensional randomly rough metal surface the dispersion and damping of the surface electromagnetic wave was calculated to the lowest nonzero order in the rms height of the surface. It was found that the frequency of the surface plasmon polariton is depressed by the surface

roughness. The attenuation of the surface plasmon polariton in the long wavelength limit is due primarily to its scattering into other surface plasmon polaritons, while at shorter wavelengths it is due primarily to its roughness-induced conversion into volume electromagnetic waves in the vacuum above the surface. The energy mean free path of the surface plasmon polariton is shorter on a randomly rough lossless metal surface than it is on a lossy planar metal surface (Fig. 11), and the surface plasmon polariton is more tightly bound to a rough surface than to a planar one.

![Figure 11. Mean Free Path of a Surface Plasmon Polariton on Randomly Rough Surface](image)

Until now the great majority of the calculations of the scattering of a surface plasmon polariton by a surface defect have been carried out for scattering by one-dimensional defects, i.e. by grooves and ridges. In the case of two-dimensional defects only scattering from indentations (dimples) or protuberances formed from the same metal as the substrate have been studied\(^{18}\) by rigorous methods. Scattering from a two-dimensional dielectric defect on a planar metal surface has been little studied. A rigorous calculation by means of a Green’s function approach for the scattering of a surface plasmon polariton by a dielectric rectangular parallelepiped on the surface of a planar metal film in the Kretschmann attenuated total reflection geometry has been carried


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out\textsuperscript{19} and a calculation of the scattering from an anisotropic Gaussian and an anisotropic hemiellipsoidal dielectric defect on a planar surface of a semi-infinite metal has been studied by the use of an effective boundary condition \cite{11}. The interaction of a surface plasmon polariton with a localized dielectric surface defect is of interest\textsuperscript{20}, and deserves additional study. In recent work \cite{18} a pair of coupled inhomogeneous two-dimensional integral equations is derived from which the amplitudes of the \textit{p} and \textit{s}-polarized components of the electric field scattered into the vacuum region above the surface can be determined, as well as the amplitude of the (\textit{p}-polarized) electric field of the scattered surface plasmon polariton. These equations are valid for any single-valued shape of the two-dimensional surface defect. However, when the defect is circularly symmetric the pair of coupled two-dimensional integral equations is transformed into a set of coupled one-dimensional integral equations. This is the case studied in Ref. \cite{18}. These equations are solved nonperturbatively by a purely numerical approach. The incident surface plasmon polariton is assumed to be propagating in the positive \textit{x}_{1} direction and to have the form of a plane wave in the plane of the surface \textit{x}_{3}=0. The dielectric defect is assumed to have the shape of a Gaussian protuberance defined by \( x_{3} = \zeta(x_{||}) = A \exp(-x_{||}^{2}/R^{2}) \), where \( x_{||} = (x_{1},x_{2},0) \) and \( A>0 \). The solutions are used to calculate the angular dependence of the differential cross section for scattering into the vacuum (Fig. 12).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure12.png}
\caption{Cross Section for Scattering a Surface Plasmon Polariton into Vacuum}
\end{figure}


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The maximum of the scattered intensity occurs at $\theta_x = 70^\circ, \phi_x = 0^\circ$, where $\theta_x$ and $\phi_x$ are the polar and azimuthal angles of the vector $x$ from the position of the defect to the point of observation. The angular dependence of the differential cross section for scattering into other surface plasmon polaritons (Fig. 13) shows that there is no scattering into the backward directions, and that the scattering is predominantly in the forward direction, with no shadow behind the defect. Thus, there are significant differences between the results of Ref.[18] and those of Shchegrov et al., where scattering from a dimple in a planar metal surface produced a maximum of the scattered intensity at $\theta_x = 28^\circ, \phi_x = 0^\circ$, while the cross section for scattering into other surface plasmon polaritons displayed a shadow behind the defect.

![Figure 13. Cross Section for Scattering a Surface Plasmon Polariton Into Other Surface Plasmon Polaritons](image)

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3. Methods, Assumptions, and Procedures

The straightforward use of the Rayleigh hypothesis in calculations of the scattering of light from, and its transmission through, a film with two one-dimensional rough surfaces leads to the necessity of solving a system of four coupled one-dimensional inhomogeneous integral equations to obtain the scattering and transmission amplitudes, due to the two boundary conditions that have to be satisfied at each of the two surfaces. In [19] manipulation of these four integral equations yields a single integral equation for the scattering amplitude and for the transmission amplitude for light of s polarization incident on a free-standing or supported film, both of whose surfaces are one-dimensional rough surfaces. The price paid for this significant reduction in the number of equations that have to be solved is that each of the single integral equations is a two-dimensional integral equation. However, these equations provide a convenient starting point for calculations of these amplitudes by small-amplitude perturbation theory through terms of third order in the surface profile functions. They are illustrated by applying them to the calculation of the contributions to the mean differential reflection and transmission coefficients from the light scattered or transmitted incoherently, in which enhanced backscattering and enhanced transmission peaks, as well as satellite peaks, are clearly visible.

Calculations of the angular dependence of the intensity of light scattered from two-dimensional randomly rough surfaces present severe computational challenges. As a result, most of the advanced calculations of such scattering have been perturbative in nature\textsuperscript{21,22,23}.

A major thrust in our theoretical studies of the scattering of light from rough surfaces has been the rigorous calculation of the scattering of p- and s- polarized light from two-dimensional randomly rough perfectly conducting [20-22] and metallic [23] surfaces. In the case of the scattering of light from perfectly conducting surfaces, the pair of coupled inhomogeneous integral equations for two independent tangential components of the magnetic field on the surface are converted into matrix equations by the method of moments, which are then solved by the biconjugate gradient stabilized method. The solutions are used to calculate the mean differential reflection coefficient for given angles of incidence and specified polarizations of the incident and scattered fields [20, 21]. In [21] the full angular distribution of the intensity of the scattered light is also obtained. In [22] the same approach is used to calculate the scattering of a p-polarized electromagnetic beam from a two-dimensional randomly rough perfectly conducting surface defined by an anisotropic power spectrum of the surface roughness. The surface anisotropy gives rise to interesting and pronounced signatures in the angular distribution of the intensity of the scattered light. In [23] the scattering of p- and s- polarized light from a two-dimensional randomly rough penetrable surface (e.g. a metal surface) is studied. The use of the


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Müller equations\textsuperscript{24} and an impedance boundary condition for a two-dimensional rough surface\textsuperscript{25} yields a pair of coupled inhomogeneous integral equations for the source functions in terms of which the scattered field is expressed through the Franz formulas\textsuperscript{26}. These equations are solved numerically as was done in [20,21]. The full angular distribution of the intensity of the scattered light is calculated from the solutions, and unitarity is very well satisfied by the results in [20,22] and for the nonabsorbing case of [23]. The work presented in [21] and [23] is the first in which the full angular distribution of the intensity of the scattered light has been calculated.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{intensity.png}
\caption{Intensity of Light Scattered from a Two-Dimensional Randomly Rough Silver Surface}
\end{figure}

In Ref. [24] a purely numerical solution of the reduced Rayleigh equation\textsuperscript{27} for the scattering of p- and s- polarized light from two-dimensional rough penetrable surfaces was used to calculate

\begin{thebibliography}{9}
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the mean differential reflection coefficient and the full angular distribution of the intensity of s-
polarized light scattered from two-dimensional randomly rough metallic and dielectric surfaces
(Fig. 14). It was shown that the results for the metallic surface obtained by this approach were in
good agreement with results for the same metallic surface obtained by the rigorous
computational approach.

All of the information about the polarization transformations light undergoes when it is scattered
from rough surfaces is contained in the Mueller matrix\textsuperscript{28, 29, 30}. All sixteen elements of this
matrix have been calculated for the scattering of light from a two-dimensional, randomly rough,
lossy metal surface [25]. The calculations were carried out by means of a rigorous,
nonperturbative, purely numerical solution of the reduced Rayleigh equations for the scattering
of p- and s- polarized light from a two-dimensional rough penetrable surface\textsuperscript{27}. The calculations
can be carried out for arbitrary polar and azimuthal angles of incidence \((\theta_0, \phi_0)\), and in fact were
carried out for angles of incidence \((\theta_0, \phi_0) = (2^\circ, 45^\circ)\) and \((25^\circ, 45^\circ)\). At normal incidence the
isotropy of the surface statistics require the elements of the Mueller matrix to possess certain
symmetry properties\textsuperscript{31}. These symmetry properties are present in the results obtained in Ref.[25]
when the angles of incidence are \((\theta_0, \phi_0) = (2^\circ, 45^\circ)\). The ability to calculate the elements of the
Mueller matrix will lead to better understanding of the polarimetric properties of randomly rough
surfaces. Such knowledge may be critical for improved photovoltaic and remote sensing
applications. It also has the potential to be used in engineering surfaces that produce well-
defined polarization properties in scattered light. This work was published as a Rapid
Communication in Physical Review A.

It should also be noted that although the computational approach described in Ref.[25] was
applied to the determination of the elements of the Mueller matrix in the case of the scattering of
light from two-dimensional randomly rough surfaces, it can also be applied to the calculation of
these elements in the case of the scattering of light from a deterministic rough surface, or even
from isolated surface defects.

Sometimes a metal surface has to be coated by a dielectric film. This can be to prevent oxidation
of the surface, or to reduce the reflectivity of the surface. One or both surfaces of the dielectric
film can be a two-dimensional randomly rough surface. It can be important to know how the
roughness of the surface(s) affects the optical properties of the coated metal surface. As a step in
that direction in recent work [26] a rigorous, nonperturbative, purely numerical, solution of the
reduced Rayleigh equation\textsuperscript{23, 32} for the scattering of p- and s- polarized light from a dielectric
film with a two-dimensional randomly rough surface deposited on the planar surface of a
metallic substrate, has been carried out. Both the mean differential reflection coefficient (Fig.

\textsuperscript{29} W.S. Bickel and W.M. Bailey, “Stokes vectors, Mueller matrices, and polarized scattered
\textsuperscript{31} N.C. Bruce, “Calculations of the Mueller matrix for scattering of light from two-dimensional
\textsuperscript{32} T.A. Leskova (unpublished work).
and the full angular distribution of the intensity of the scattered field were calculated for two polar angles of incidence. An interesting result of these calculations is that satellite peaks are present in the angular dependence of the elements of the mean differential reflection coefficient, in addition to an enhanced backscattering peak. This result resolves a conflict between the results of earlier, perturbative, studies of scattering from this system\textsuperscript{33}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.png}
\caption{Mueller Matrix Elements for Light Scattered from a Two-Dimensional Randomly Rough Silver Surface}
\end{figure}

The calculations described in the preceding paragraphs have all dealt with the direct scattering problem. In [27] an inverse scattering problem was considered. In this work an optimization algorithm was developed that uses the intensity of light transmitted into the far field to reconstruct the profile of the one-dimensional rough interface between the medium of incidence and the medium of transmission. The algorithm was used to determine the profile function of the


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rough interface between photoresist and air from experimental data for the intensity of light transmitted through it. The reconstructed profile agreed well with the profile measured by an atomic force microscope.

A different type of inverse scattering problem was studied in Ref.[28], where on the basis of the geometrical optics limit of the Kirchhoff approximation, a one-dimensional random interface between two dielectric media was designed that refracts p- or s- polarized light incident on it at an arbitrary angle of incidence $\theta_0$ from one of them into the other at an arbitrary but specified angle of transmission $\theta$ that is not defined in terms of $\theta_0$ by Snell’s law. This kind of transmission was called nonstandard refraction.

In related work as a method for designing a two-dimensional random phase screen that transforms a beam with a specified intensity distribution incident normally on it into a different specified intensity distribution (e.g. transforms a Gaussian beam into a flat-top beam) was devised [29]. In addition, it has been shown how to design a two-dimensional, circularly symmetric, radially random phase screen that produces a pseudo-nondiffracting beam in transmission when illuminated at normal incidence by a plane wave [29].

A combined experimental and theoretical investigation of the coherent (specular) and incoherent (diffuse) scattering of p- and s- polarized light from two-dimensional randomly rough metal (gold) surfaces was carried out [30]. The wavelength of the incident light was 10.6 $\mu$m. The experimental data for the reflectivity were compared with predictions obtained by small-amplitude perturbation theory, phase perturbation theory, self-energy perturbation theory, and the Kirchhoff approximation. The experimental data for the incoherent scattering were compared with predictions obtained by small-amplitude perturbation theory, phase perturbation theory, and the Kirchhoff approximation. The theoretical results obtained by means of phase perturbation theory were closest to all experimental results for the reflectivity in both p and s polarization. Phase perturbation theory also produced the best overall agreement with the experimental results for the contribution to the mean differential reflection coefficient from the light scattered incoherently for in-plane, co-polarized scattering.

4. Results and Discussion

Properties of partially coherent light have also been studied during the period of the contract. In [31] the interference of light produced by a pair of mutually correlated Schell-model sources was studied theoretically and experimentally. The spatial distributions of the electromagnetic fields produced by the two sources were inverted with respect to each other through their common center in the source plane. When the beams were in phase $\phi=0$, a bright spot appeared in the center of the spatial distribution of the beam intensity. When the beams have a phase shift $\phi=\pi$, a dark spot appears in the center of the spatial distribution of the beam intensity. Experimental results were obtained that showed the existence of these bright and dark spots. Both the bright and dark spots diverge more slowly with increasing distance from the sources than the beams themselves. These results gave rise to the suggestion that the interference of the beams produced by two mutually correlated Schell-model sources can be used to create a pseudo-nondiffracting beam. It was also shown that when the spectral density (intensity) of each source has a Gaussian form, while the spectral degree of coherence of each source has a Lorentzian form, and the two

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sources are in phase, $\phi=0$, the initial beams evolve on propagation in such a way that in the far field the angular divergence of the initial beams coincides with the angular divergence of the interference peak.

In [32] the evolution of a beam radiated into the region $x_3=0$ by a circularly symmetric Schell-model source in the plane $x_3=0$ was investigated. The spectral density of the electromagnetic field in the source plane was assumed to have a Gaussian form. The spectral degree of coherence in the source plane was assumed to have a Gaussian form, a Lorentzian form, and the square-root Lorentzian form. The angular divergence of each radiated beam was determined. The beam produced by the square-root Lorentzian spectral degree of coherence had the smallest angular divergence, the beam produced by the Lorentzian spectral degree of coherence had a larger angular divergence, and the beam produced by the Gaussian spectral degree of coherence had the largest angular divergence. These results show that the more slowly the spectral degree of coherence decreases monotonically in the radial direction the smaller is the angular divergence of the corresponding radiated beam. This result suggests the possibility that if propagation distances of at most a few tens of centimeters are required, Schell-model sources could be produced that radiate beams that diverge so little over such distances that they could be used in applications where pseudo-nondiffracting beams are used today. The small angular divergence of beams produced by a source whose spectral degree of coherence has the square-root Lorentzian form suggests also the possibility of using such beams in long distance optical communications. It seems worthwhile to investigate the propagation of such beams through a turbulent atmosphere.

An approach to generating a Collett-Wolf beam by means of an optical feedback technique was devised as part of an effort to reduce speckle in applications such as imaging, illumination, and pointing. This approach was validated by experiments in which it was implemented [33].

5. Conclusions

The reflection of $p$-polarized light from, and its transmission through, a metal film sandwiched between air and a dielectric substrate whose surfaces are modeled by two aligned and reversed arrays of nanogrooves of finite depth, have been studied in two papers. In the first [34] the scattering of $p$-polarized light from, and its transmission through, a supported thin silver film with aligned and reversed double-groove gratings on both its surfaces, i.e. gratings with two grooves in each period, was studied by means of a Green’s function formalism. Peaks in the transmissivity spectra associated with the excitation of surface plasmon polaritons on both surfaces by the incident light are observed, as in the case of transmission through such a film with single-groove gratings. However, some of the modes in the latter structure are split into symmetric and antisymmetric modes in the case of the film with double-groove gratings due to electromagnetic coupling between the two grooves in a single period. Peaks in the transmission spectra corresponding to these split modes are observed.

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In the second paper [35] the spatial dependence of the intensity of p-polarized light transmitted through a thin metal (gold) film sandwiched between air and a dielectric substrate whose surfaces are modeled by two aligned and reversed arrays of a finite number of nanogrooves of finite depth is determined. The interest in such structures arises from the possibility of using them as metallic nanoscale lenses. Metallic nanoscale lenses have been fabricated earlier in the form of a metal film pierced by a one-dimensional array of slits with subwavelength widths.  

There are several reasons for interest in such lenses. Nanoscale slit lenses retain their focusing ability as their dimensions shrink to the wavelength level, in contrast with dielectric refractive lenses whose focusing ability degrades in this limit. Due to their planar geometries they are easier to fabricate at these dimensions than dielectric lenses, and the use of metal rather than a dielectric provides a higher index contrast that leads to a stronger focusing capability. Finally, they focus light in the far field rather than in the near field. One of the reasons for choosing a structure with nanogrooves of a finite depth instead of a slits in [35] was the opportunity it provides to examine the role of surface plasmon polaritons through the slits of a metallic lens in its focusing capability, since these surface electromagnetic waves do not exist in the structure studied in [35]. A second reason for assuming this structure is that it provides additional degrees of freedom in comparison with arrays of slits, namely the ability to change the shape of each nanogroove as a way of improving the focusing capability of the resulting lens. Thus, the surfaces of the film were defined by \( x_3 = \zeta(x_1) \) and \( x_3 = -L - \zeta(x_1) \), where the surface profile function had the form \( \zeta(x_1) = -L \sum_{j=-M}^{M} \exp\left[-\frac{x - jd}{b_j}\right] \). Here \( L \) was the thickness of the unpatterned metal film, \( 2M+1 \) is the total number of grooves on each surface, \( t < L/2 \) is the groove depth, and \( d \) is the distance between the centers of consecutive grooves. The exponent \( s \) controls the shape of each groove, and was given the values \( s=2 \) and \( 4 \), while \( b_j = \alpha + \beta j + j^2 \) defines the width of each groove. The film was illuminated from either the air or the dielectric side, by light of p polarization, whose plane of incidence was the \( x_1x_3 \) plane, and the spatial distribution of the squared modulus of the transmitted magnetic field, \( |H_2(x_1,x_3)|^2 \), was calculated. If the groove pattern is only a few wavelengths long and the distance between adjacent grooves is considerably smaller than the wavelength of the incident light, both p- and s- polarized light exhibit a focus, but due to the enhanced transmission mediated by surface plasmon polaritons propagating along the vacuum/metal film and metal film/dielectric interfaces the intensity of the transmitted p-polarized field is several orders of magnitude greater than that of the s-polarized field. The position of the maximum of the field intensity (the focus distance) is determined by the diffraction on the aperture of the groove pattern, as long as the wavelength of light in the substrate is small enough to ensure that the focal point lies in the far field. Thus there is no need to have slits that pierce the film and surface plasmon polaritons that propagate through them across the film to achieve focusing of the transmitted field. A study was carried out that indicated the best way to improve the quality of the focus by changing the groove profile and the groove width variation. It was also found that when light is incident on the film from the air and is transmitted into the dielectric substrate, the focal distance is inversely proportional to the wavelength of the light in the substrate in accordance with Fresnel diffraction by the aperture. When light is incident from the dielectric substrate and is transmitted into the air, the intensity of

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the transmitted field is weaker, and practically independent of the wavelength. The longer wavelength of the light in the air shifts the maximum of the field intensity closer to the film surface, into the region where it becomes independent of the wavelength.

A summary of this work was published in the SPIE Newsroom [36].
References


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Appendix A - Presentations

The research results described above have been presented at several national and international conferences during the review period.

1. Professor A.A. Maradudin presented a contributed talk “Scattering of electromagnetic waves from two-dimensional randomly rough perfectly conducting surfaces”, at the 25th Annual Review of Progress in Applied Computational Electromagnetics (ACES), held in Monterey, California, March 9-11, 2009. Coauthors were T.A. Leskova and I. Simonsen.

2. Professor Maradudin gave a invited talk, “Transformations of surface plasmon polaritons by surface structures”, at the 8th International Meeting on the Electrical, Transport, and Optical Properties of Inhomogeneous Media (ETOPIM8), held in Rethymnon, Crete, Greece, June 7-12, 2009. Coauthor was T.A. Leskova.

3. Three talks based on research supported by the Air Force Research Laboratory contract AF9453-08-C-0230 was presented at the Workshop on Waves in Complex Media, held in Yountville, California, June 28 – July 1, 2009. They are as follows:

   3a. A.A. Maradudin, “Surface plasmon polariton analogues of volume electromagnetic wave effects.” Coauthor was T.A. Leskova.

   3b. T.A. Leskova, “A Kirchhoff approximation for surface plasmon polaritons.” Coauthor was A.A. Maradudin.

   3c. I. Simonsen, “The scattering of electromagnetic waves from two-dimensional randomly rough surfaces.” Coauthors were T.A. Leskova and A.A. Maradudin.

4. Professor Maradudin gave a plenary talk, “Surface plasmon polariton analogues of volume electromagnetic wave effects,” at the SPIE meeting held in San Diego, California, August 2-6, 2009. Coauthors were F. Huerkamp and T.A. Leskova.

5. Four papers were presented by Professor Maradudin at the Progress of Electromagnetics Research Symposium (PIERS 2009), held in Moscow, Russia, August 18-21, 2009. They are as follows.

   5a. “Surface plasmon polariton analogues of volume electromagnetic wave effects.” Coauthor was T.A. Leskova.

   5b. “The scattering of electromagnetic waves from two-dimensional randomly rough surfaces.” Coauthors were T.A. Leskova and I. Simonsen.

   5c. “The scattering of a surface plasmon polariton by a one-dimensional defect on an otherwise planar surface of a lossy metal.” Coauthors were T.A. Leskova, E.E. García-Guerrero, and E.R. Méndez.

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5d. “Nonstandard refraction of light from ID quasi periodic surfaces.” Authors were Zu-Han Gu and A. Wang.

5e. Dr. Danhong Huang (AFRL) presented a paper “Optical spectrum and electromagnetic field distribution at double-groove metallic surface gratings.” Coauthors were L.D. Wellems, T.A. Leskova, and A.A. Maradudin.

6. Professor Maradudin presented two contributed talks at the SPIE meeting in San Diego, CA, August 11-12, 2008. These were as follows.

   6a. “Reduced Rayleigh equations in the scattering of s-polarized from, and its transmission through, a film with two one-dimensional rough surfaces.” Coauthor was Dr. T.A. Leskova.

   6b. The design of random surfaces that produce nonstandard refraction of light.” Coauthors were Dr. T.A. Leskova and Dr. I. Simonsen.

7. Dr. T.A. Leskova presented two contributed talks at the annual Radio Science Meeting in Boulder, Colorado, January 5-8, 2009. They were:

   7a. “The Talbot effect for a surface plasmon polariton,” Coauthor was Professor A.A. Maradudin.

   7b. “Quantized transmission of a surface plasmon polariton through a slit.” Coauthor was Professor A.A. Maradudin.


9. Several talks describing work supported by AFRL contract FA 9453-08-C-0230 were presented at the annual SPIE meeting in San Diego, California, August 3-5 (2010).

   9a. Professor A.A. Maradudin presented a contributed talk, “A Kirchhoff approximation for surface plasmon polaritons.” Coauthor was Dr. T.A. Leskova.

   9b. Dr. T.A. Leskova presented a contributed talk, “A partially coherent slowly diffracting beam.” Coauthors were Dr. E.R. Méndez, Dr. E.E. García-Guerrero, Dr. Zu-Han Gu, and Professor A.A. Maradudin.

   9c. Dr. Zu-Han Gu presented a contributed talk, “Nonstandard refraction of light from one- and two-dimensional dielectric surfaces.” Coauthor was Dr. A. Wang.

   9d. Dr. Zu-Han Gu presented a contributed talk, “Coherence effect: From spectral change to non-diffraction.”

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9e. Dr. Danhong Huang presented a contributed talk, “Wavelength-tunable focal length of a nanopatterned metallic planar lens.” Coauthor was Dr. L.D. Wellems.


12. A talk, “Scattering of surface plasmon polaritons by a localized dielectric surface defect,” by R.E. Arias and A.A. Maradudin, will be presented by Professor Arias at the 6th International Conference on Surface Plasmon Photonics, to be held May 26-31 (2013) in Ottawa, Canada.

13. Professor Maradudin will be giving an invited talk “Surface Plasmon Polariton on Structured Surfaces,” at the Fourth International Symposium on Plasma Nanoscience, which will be held at the Asilomar Beach, California on August 25-29, 2013.
Appendix B-Outreach Activities

A two and a half day workshop on Waves in Complex Media was held at the Napa Valley Lodge June 28 – July 1, 2009. It was organized by Professor Alexei A. Maradudin, with financial support from the Air Force Research Laboratory and the University of California, Irvine’s Institute for Surface and Interface Science. Extended abstracts of the presentations at the workshop are available on its website: http://www.physics.uci.edu/Yountville-2009.

Changes in research objectives: None

Change in AFOSR program manager: None

Extension on granted or milestones slipped: Two no-cost extensions requested and approved

New Discoveries, inventions, or patent disclosures during this review period: None

Persons supported by this contract:

Prof. Ingve Simonsen

Dr. Tamara A. Leskova

Dr. Wolfgan Zierau

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