Chaotic Motions in the Real Fuzzy Electronic Circuits

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Abstract

In this paper, Fuzzy Electronic Circuit (FEC) is firstly introduced, which is implementing Takagi-Sugeno (T-S) fuzzy chaotic systems on electronic circuit. In the research field of secure communications, the original source should be blended with other complex signals. Chaotic signals are one of the good sources to be applied to encrypt high confidential signals, because of its high complexity, sensitiveness of initial conditions and unpredictability. Consequently, generating chaotic signals on electronic circuit to produce real electrical signals applied to secure communications are an exceeding important issue. However, nonlinear systems are always composed of many complex equations and are hard to realize on electronic circuits. Takagi-Sugeno (T-S) fuzzy model is a powerful tool, which is described by fuzzy IF-THEN rules to express the local dynamics of each fuzzy rule by a linear system model. Accordingly, in this paper, we produce the chaotic signals via electronic circuits through T-S fuzzy model and the numerical simulation results provided by MATLAB are also proposed for comparison. T-S fuzzy chaotic Lorenz and Chen-Lee systems are used for examples and are given to demonstrate the effectiveness of the proposed electronic circuit.

1. Introduction

Nonlinear dynamics, commonly called the chaos theory, changes the scientific way of looking at the dynamics of natural and social systems, which has been intensively studied over the past several decades [1-10]. The phenomenon of chaos has attracted widespread attention amongst mathematicians, physicists, engineers and has also been extensively studied in many fields, such as chemical reactions [11-12], biological systems [13-14], information processing [15-16], secure communications [17-20] and the rest.

The mathematical meteorologist E.N. Lorenz who discovered chaos in a simple system of three autonomous ordinary differential equations in order to describe the simplified Rayleigh – Benard problem [21] in 1963 is the most popular system for studying [22-26]. Chen and Lee reported a new chaotic system [27] in 2004, which is now called the Chen-Lee system [28]. The chaotic Chen-Lee system was developed based on Euler equations for the motion of rigid body. It was proved that this system is the governing set of equations for gyro motion with feedback control. Recently, studies were conducted on this system to explore the dynamic behavior of this system, including fractional order behavior, the generation of hyperchaos and perturbation analysis, control and anti-control of chaos, synchronization, etc. [29-30].

Since the fuzzy set theory [31] and fuzzy logic [32] were initiated by Zadeh in 1965 and 1973, fuzzy logic has received much attention as a powerful tool for the nonlinear filed. Among various kinds of fuzzy methods, Takagi-Sugeno fuzzy system is widely accepted as a tool for design and analysis of fuzzy control system [33]. The T–S fuzzy model, proposes a successful method to deal with certain complex nonlinear systems via some local linear subsystems. There are plenty of researches using the Takagi-Sugeno (T-S) fuzzy model to represent typical chaotic models and then apply some effective fuzzy techniques [34-42]. However, there are still no real experimental models in electronic circuit for Takagi-Sugeno (T-S) fuzzy-based chaotic systems. In this article, we carry out the powerful tool, Takagi-Sugeno (T-S) fuzzy model, in electronic circuit and show good agreement between computer simulations in MATLAB and experimental results in our circuits.

The layout of the rest of the paper is as follows. In Section 2, Takagi-Sugeno fuzzy model is introduced. In Section 3, experimental results and configurations in electronic circuits for T-S fuzzy chaotic Lorenz and Chen-Lee systems are presented. In Section 4, conclusions are given.

2. Takagi-Sugeno fuzzy model

In system analysis and design, it is important to select an appropriate model representing a real system. As an expression model of a real plant, we use the fuzzy implications and the fuzzy reasoning method suggested by Takagi and Sugeno. The Takagi-Sugeno (T-S) fuzzy model is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of the T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model. The overall fuzzy model of the system is achieved by fuzzy blending of the linear system models. Consider a continuous-time nonlinear dynamic system as follows:

Rule i:

IF $x_1(t)$ is M_{i1}... and $x_n(t)$ is M_{in}

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$, $\qquad\qquad\qquad$ (2-1)

where

$$x(t) = [x_1(t), x_2(t),...,x_n(t)]^T,$$

$$u(t) = [u_1(t), u_2(t),...u_n(t)]^T,$$

$i = 1,2,...,r$ (r is the number of IF-THEN rules), M_{ij} are fuzzy sets, and $x(t) = A_i x(t) + B_i u(t)$ is the output from the ith IF-THEN rule. Given a pair of ($x(t)$, $u(t)$), the final output of the fuzzy system is inferred as follows:

$$\dot{x} = \frac{\sum_{i=1}^{r} \omega_i(x(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} \omega_i(x(t))}, \qquad\qquad (2\text{-}2)$$

where

$$\omega_i(x(t)) = \prod_{j=1}^{n} M_{ij}(x(t)) \qquad\qquad\qquad (2\text{-}3)$$

For all t, and $M_{ij}(x(t))$ is the grade of membership $x_j(t)$ of in M_{ij}.

The open-loop system of (2-2) is

$$\dot{x} = \frac{\sum_{i=1}^{r} \omega_i(x(t)) A_i x(t)}{\sum_{i=1}^{r} \omega_i(x(t))} \tag{2-4}$$

where it is assumed that

$$\sum_{i=1}^{r} \omega_i(x(t)) > 0$$

$$\omega_i(x(t)) \geq 0, \, i = 1, 2, \ldots, r.$$

By introducing $h_i(x(t)) = \omega_i(x(t)) / \sum_{i=1}^{r} \omega_i(x(t))$ instead of $\omega_i(x(t))$, (2-2) and (2-4)

can be rewritten as:

$$\dot{x} = \sum_{i=1}^{r} h_i(x(t))\{A_i x(t) + B_i u(t)\}, \tag{2-5}$$

$$\dot{x} = \sum_{i=1}^{r} h_i(x(t)) A_i x(t), \tag{2-6}$$

Note that

$$\sum_{i=1}^{r} h_i(x(t)) = 1,$$

$$h_i(x(t)) \geq 0, \, i = 1, 2, \ldots, r.$$

for all t. $h_i(x(t))$ can be regarded as the normalized weight of the IF-THEN rules.

3. Implementation of T-S fuzzy systems on electronic circuit

This section shows the electronic circuit Implementations of the T-S fuzzy model of classical Lorenz system and Chen-Lee system. The experimental results are going to be compared with the simulation results given by MATLAB.

3.1 Fuzzy modeling of Lorenz system

For Lorenz system [21]:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_1 x_3 - x_2 \\ \dot{x}_3 = x_1 x_2 - bx_3 \end{cases} \tag{3-1}$$

where a, b, c are the parameters. When $a=10$, $b=8/3$, $c=28$ and initial states is (-0.1, 0.2, 0.3), the dynamic behavior is chaotic. Assume that $x_1 \in [-d, d]$ and $d > 0$, then

Lorenz system can be exactly represented by T-S fuzzy model as following [43]:

Rule 1: IF x is M_1, THEN $\dot{X}(t) = A_1 X(t)$, (3-2)

Rule 2: IF x is M_2, THEN $\dot{X}(t) = A_2 X(t)$. (3-3)

where

$$X = [x_1, x_2, x_3]^T,$$

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix}, \quad A_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix},$$

$$M_1(x) = \frac{1}{2}(1 + \frac{x_1}{d}), \quad M_2(x) = \frac{1}{2}(1 - \frac{x_1}{d}),$$

Choosing $d=30$. M_1 and M_2 are fuzzy sets of Lorenz system. Here, we call (3-2) the first liner subsystem under the fuzzy rule and (3-3) the second liner subsystem under the fuzzy rule. The final output of the fuzzy Lorenz system is inferred as follows:

$$\dot{X}(t) = \sum_{i=1}^{2} h_i A_i X(t) \tag{3-4}$$

$$= \begin{bmatrix} h_1 \\ h_1 \\ h_1 \end{bmatrix}^T \begin{bmatrix} a(x_2 - x_1) \\ cx_1 - dx_3 - x_2 \\ dx_2 - bx_3 \end{bmatrix} + \begin{bmatrix} h_2 \\ h_2 \\ h_2 \end{bmatrix}^T \begin{bmatrix} a(x_2 - x_1) \\ cx_1 + dx_3 - x_2 \\ -dx_2 - bx_3 \end{bmatrix}$$

where

$$h_1 = \frac{M_1}{M_1 + M_2}, \quad h_2 = \frac{M_2}{M_1 + M_2}$$

The configuration of electronic circuit in T-S fuzzy chaotic Lorenz system is shown in Fig.1 and the chaotic behavior in circuit and MATLAB are shown in Fig.2 and 3. It can be find out that the experimental result in our circuit is actually effective. It means that the nonlinear chaotic systems can be represented by real operations in electronic circuits, not just existing in simulation results.

3.2 Fuzzy modeling of Chen-Lee system

For Chen-Lee system:

$$\begin{cases} \dot{y}_1 = -y_2 y_3 + a_1 y_1 \\ \dot{y}_2 = y_1 y_3 + b_1 y_2 \\ \dot{y}_3 = y_1 y_2 / 3 + c_1 y_3 \end{cases} \tag{3-5}$$

where a_1, b_1, c are the parameters. When $a_1=5$, $b_1=-10$, $c_1=-38$ and initial states is (0.2, 0.2, 0.2), the dynamic behavior is chaotic. Assume that $y_1 \in [-d_1, d_1]$, $y_2 \in [-e_1, e_1]$ and $d_1 > 0$, $e_1 > 0$, then Chen-Lee system can be exactly represented by T-S fuzzy model as following:

Rule 1: IF y_1 is P_1 and IF y_2 is Q_1, THEN:

$$\dot{Y}(t) = B_1 Y(t),$$ (3-6)

Rule 2: IF y_1 is P_1 and IF y_2 is Q_2, THEN

$$\dot{Y}(t) = B_2 Y(t).$$ (3-7)

Rule 3: IF y_1 is P_2 and IF y_2 is Q_1, THEN

$$\dot{Y}(t) = B_3 Y(t).$$ (3-8)

Rule 4: IF y_1 is P_2 and IF y_2 is Q_2, THEN

$$\dot{Y}(t) = B_4 Y(t).$$ (3-9)

Where

$$Y = [y_1, y_2, y_3]^T,$$

$$B_1 = \begin{bmatrix} a & 0 & -e \\ 0 & b & d \\ 0 & \frac{1}{3}d & c \end{bmatrix}, \quad B_2 = \begin{bmatrix} a & 0 & e \\ 0 & b & d \\ 0 & \frac{1}{3}d & c \end{bmatrix},$$

$$B_1 = \begin{bmatrix} a & 0 & -e \\ 0 & b & -d \\ 0 & -\frac{1}{3}d & c \end{bmatrix}, \quad B_2 = \begin{bmatrix} a & 0 & e \\ 0 & b & -d \\ 0 & -\frac{1}{3}d & c \end{bmatrix},$$

$$P_1(y) = \frac{1}{2}(1 + \frac{y_1}{d_1}), \quad P_2(y) = \frac{1}{2}(1 - \frac{y_1}{d_1}),$$

$$Q_1(y) = \frac{1}{2}(1 + \frac{y_2}{e_1}), \quad Q_2(y) = \frac{1}{2}(1 - \frac{y_2}{e_1}),$$

Choose $d_1=40$ and $e_1=30$. N_1 and N_2 are fuzzy sets of Chen-Lee system. Here, we call (3-6) the first liner subsystem under the fuzzy rule and (3-7) the second liner subsystem under the fuzzy rule. The final output of the fuzzy Chen-Lee system is inferred as follows:

$$\dot{Z}(t) = \sum_{i=1}^{2} I_i B_i Z(t)$$ (3-8)

$$
= \begin{bmatrix} I_1 \\ I_1 \\ I_1 \end{bmatrix}^T \begin{bmatrix} -e_1 y_3 + a_1 y_1 \\ d_1 y_3 + b_1 y_2 \\ d_1 y_2/3 + c_1 y_3 \end{bmatrix} + \begin{bmatrix} I_2 \\ I_2 \\ I_2 \end{bmatrix}^T \begin{bmatrix} e_1 y_3 + a_1 y_1 \\ d_1 y_3 + b_1 y_2 \\ d_1 y_2/3 + c_1 y_3 \end{bmatrix} + + \begin{bmatrix} I_3 \\ I_3 \\ I_3 \end{bmatrix}^T \begin{bmatrix} -e_1 y_3 + a_1 y_1 \\ -d_1 y_3 + b_1 y_2 \\ -d_1 y_2/3 + c_1 y_3 \end{bmatrix}
$$

$$
+ \begin{bmatrix} I_4 \\ I_4 \\ I_4 \end{bmatrix}^T \begin{bmatrix} e_1 y_3 + a_1 y_1 \\ -d_1 y_3 + b_1 y_2 \\ -d_1 y_2/3 + c_1 y_3 \end{bmatrix}
$$

where

$$
I_1 = \frac{P_1 \times Q_1}{P_1 + P_2 + Q_1 + Q_2} , \quad I_2 = \frac{P_1 \times Q_2}{P_1 + P_2 + Q_1 + Q_2}
$$

$$
I_3 = \frac{P_2 \times Q_1}{P_1 + P_2 + Q_1 + Q_2} , \quad I_4 = \frac{P_2 \times Q_2}{P_1 + P_2 + Q_1 + Q_2}
$$

The configuration of electronic circuit in T-S fuzzy chaotic Chen-Lee system is shown in Fig.4 and the chaotic behavior in circuit and MATLAB are shown in Figs.5 and 6. This experimental result in T-S fuzzy chaotic Chen-Lee system is exactly effective as well.

Two illustrations given in sections 3.1 and 3.2 all show the agreement between our experimental and MATLAB simulation results. It means the T-S fuzzy model would no longer to be just a mathematical tools, it can be applied to electronic circuits for various kinds of applications in practice.

4. Conclusions

The implementations of Takagi-Sugeno (T-S) fuzzy chaotic systems on electronic circuits are proposed in this paper. We construct the powerful tool, Takagi-Sugeno (T-S) fuzzy model, on electronic circuit and show good agreement between computer simulations in MATLAB and experimental results in our circuits. Through our effort, the powerful Takagi-Sugeno (T-S) method is no more than just a numerical strategies, it can be applied to electronic circuits for various kinds of applications in practice. Implementations of electronic circuits for Takagi-Sugeno

(T-S) fuzzy chaotic systems are only the beginning for secure communication and other kinds of applications, this paper also create both opportunities and challenges. Implementations of novel synchronization or control approaches on electronic circuits in nonlinear research filed would be definitely our future directions to achieve.

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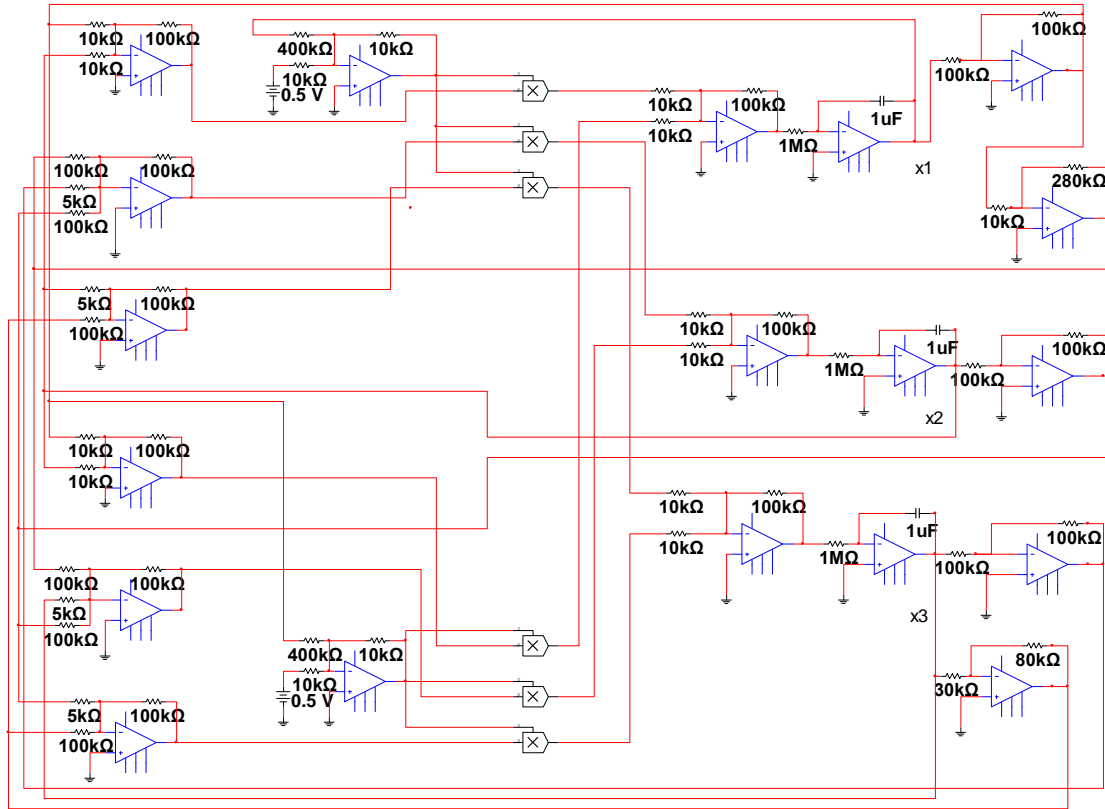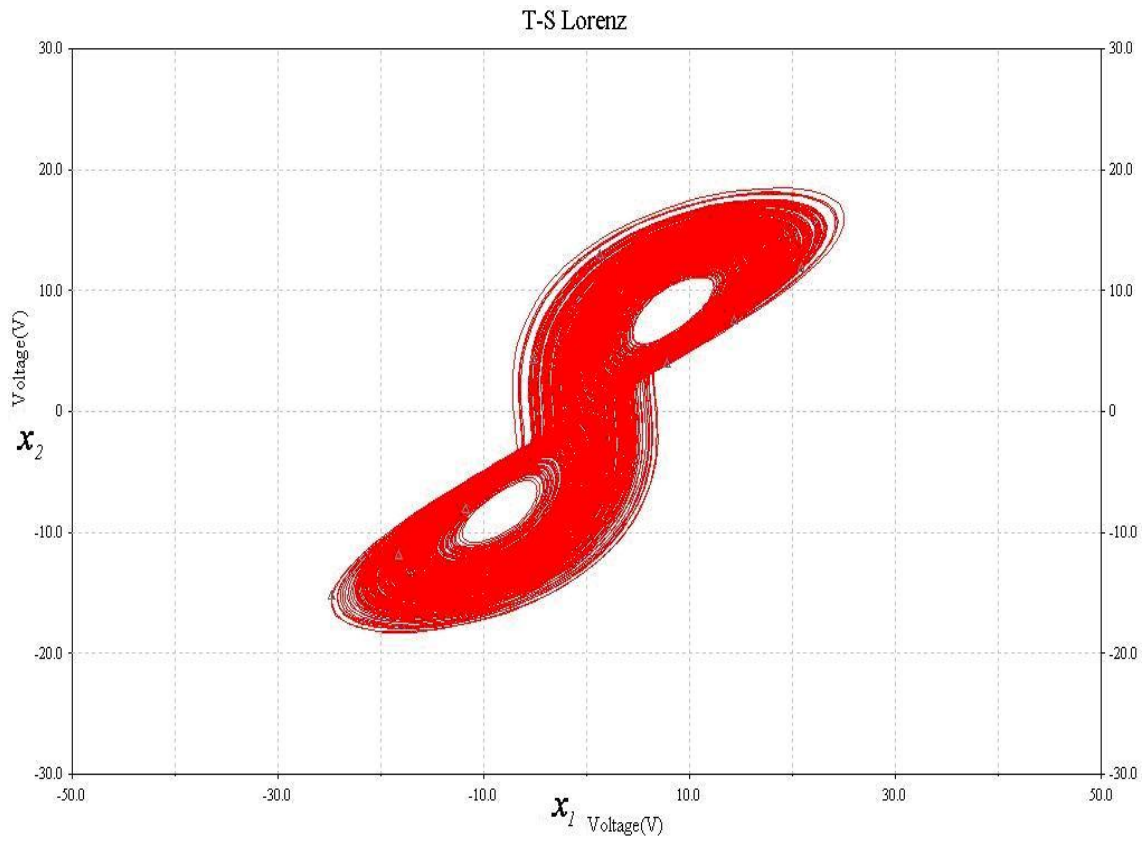
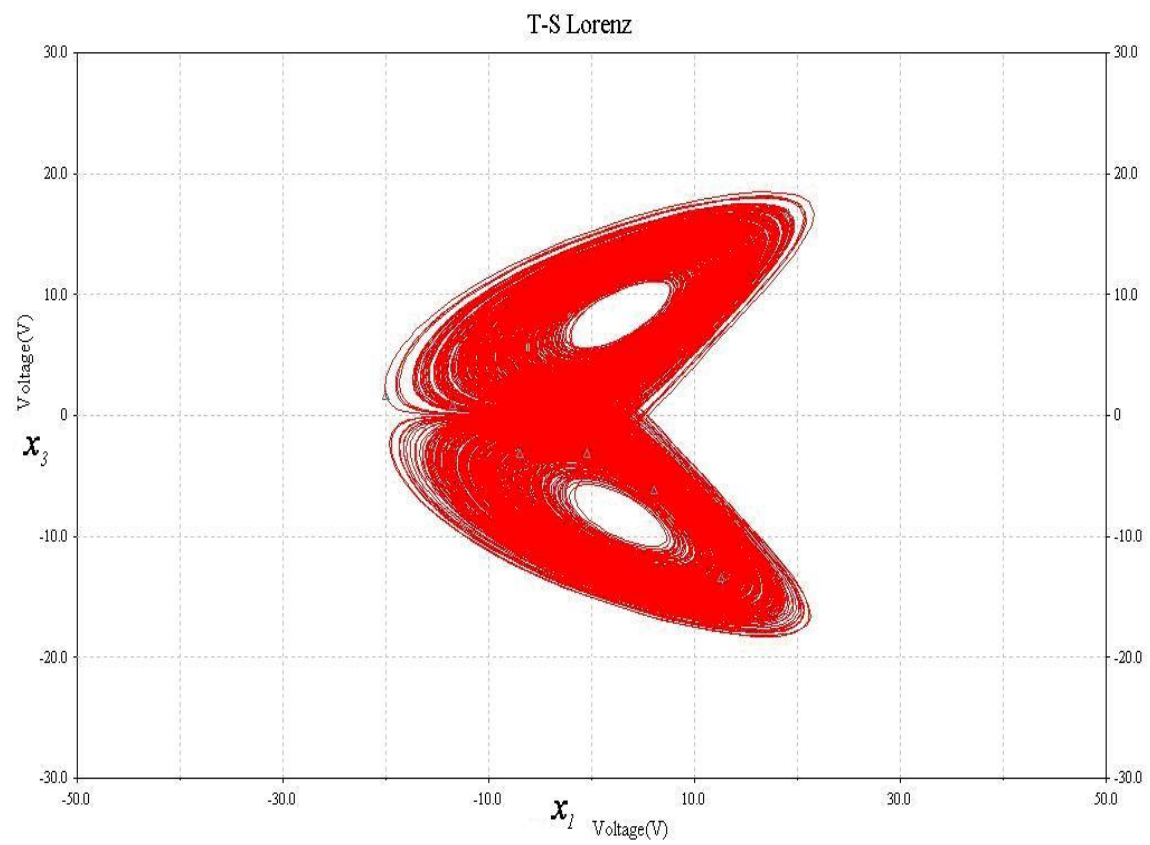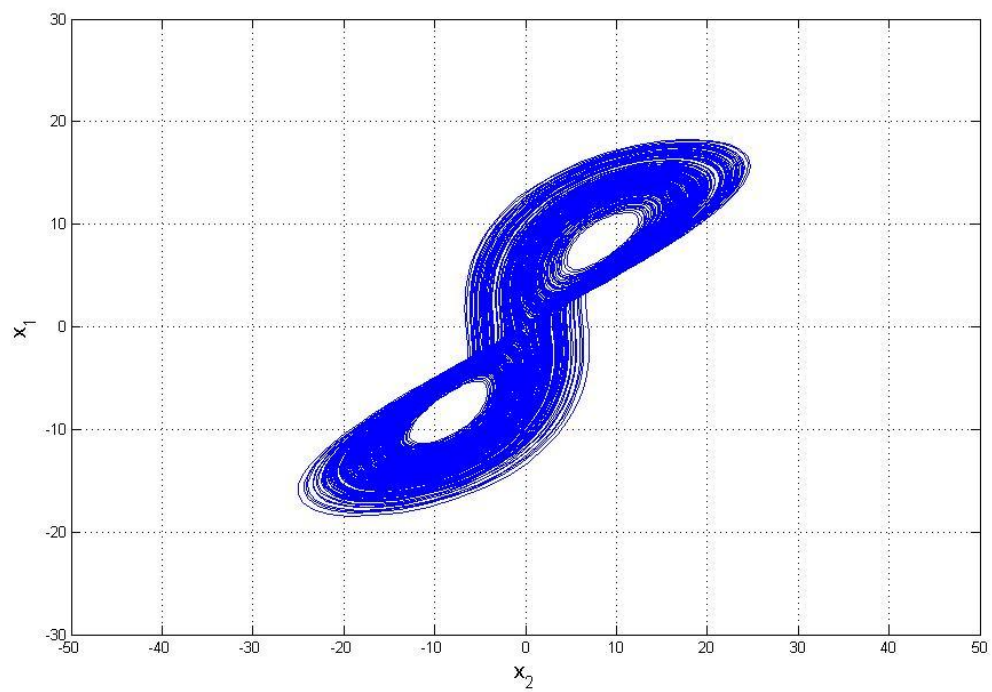Fig.1 The fuzzy electronic circuit for chaotic Lorenz system.

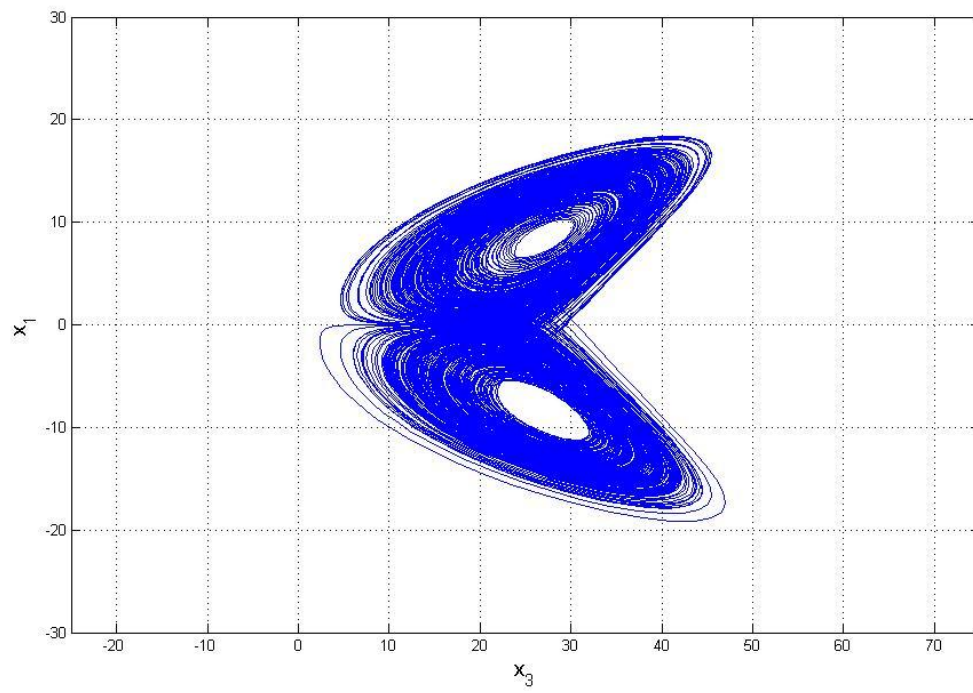Fig.2 Projection of phase portraits outputs in fuzzy electronic circuit for Lorenz
system.

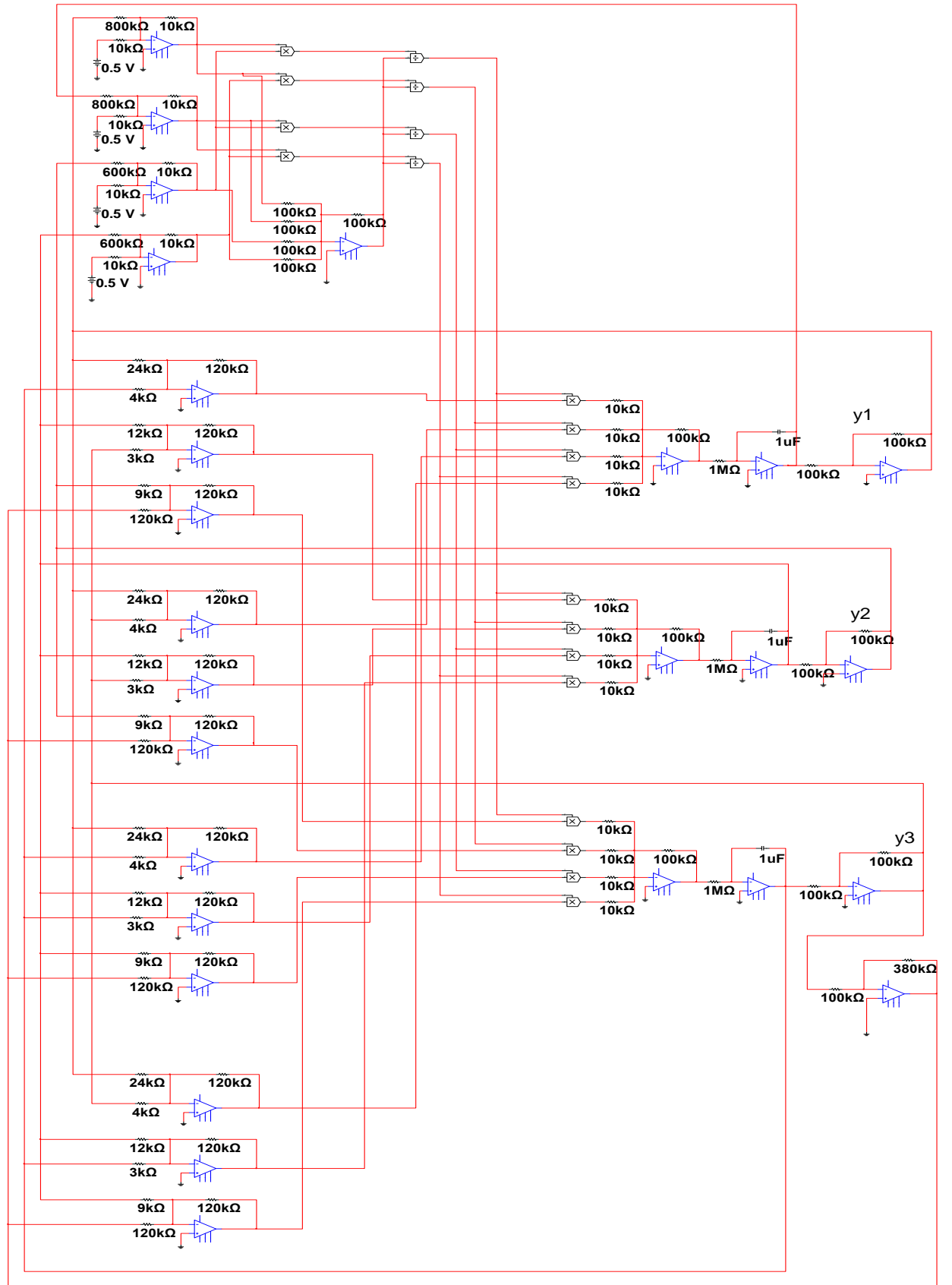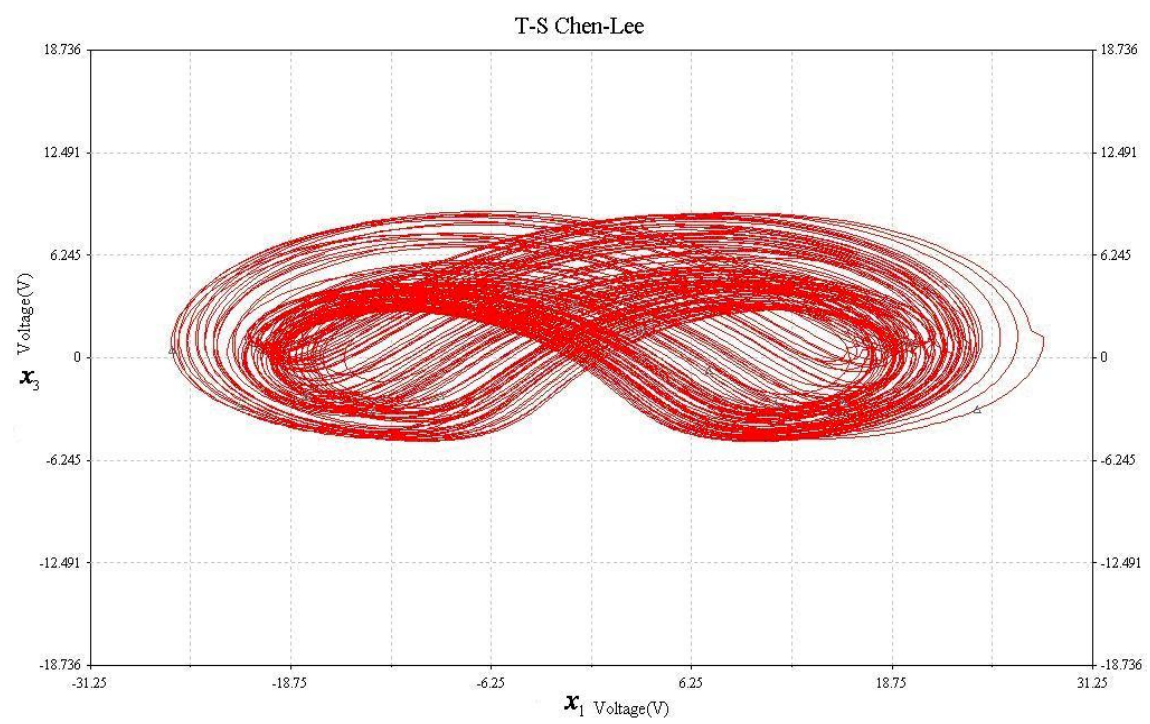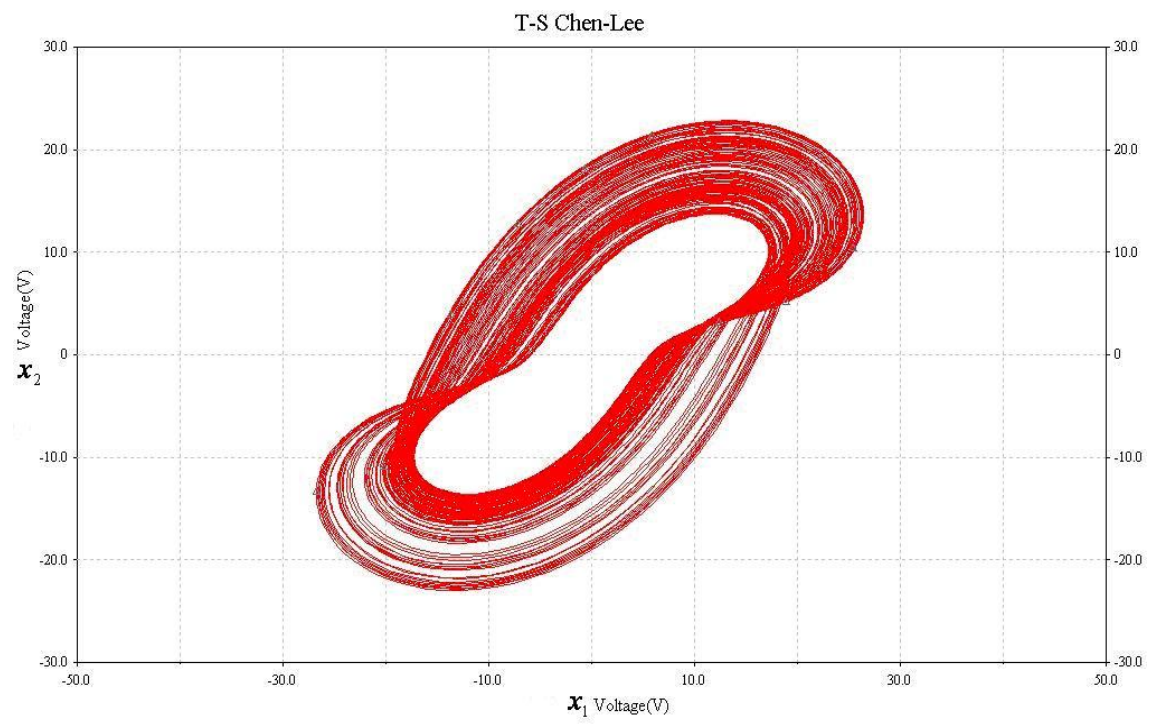Fig. 3 Projection of phase portraits in MATLAB for fuzzy chaotic Lorenz system.

Fig. 4 The fuzzy electronic circuit for chaotic Chen-Lee system.
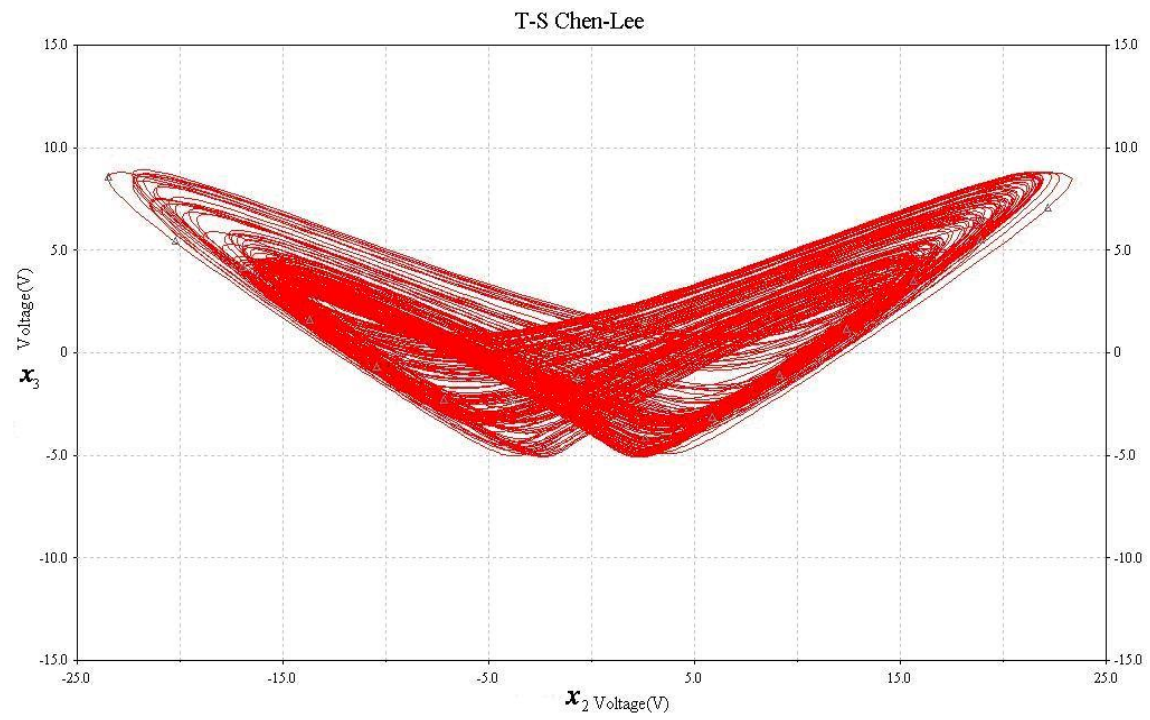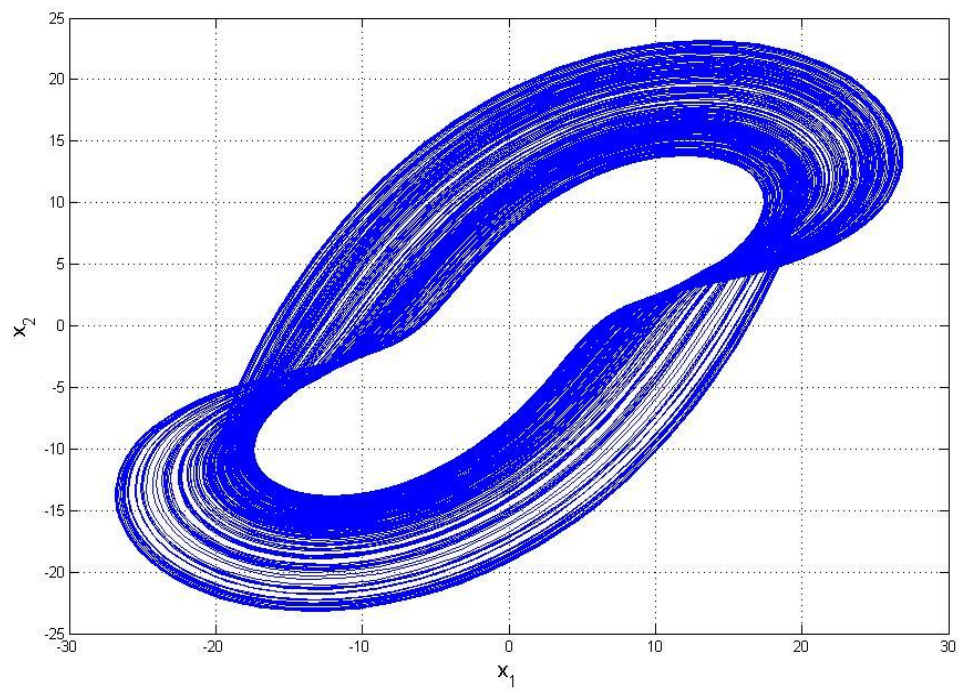
T-S Chen-Lee

T-S Chen-Lee

Fig. 5 Projection of phase portraits outputs in fuzzy electronic circuit for Chen-Lee
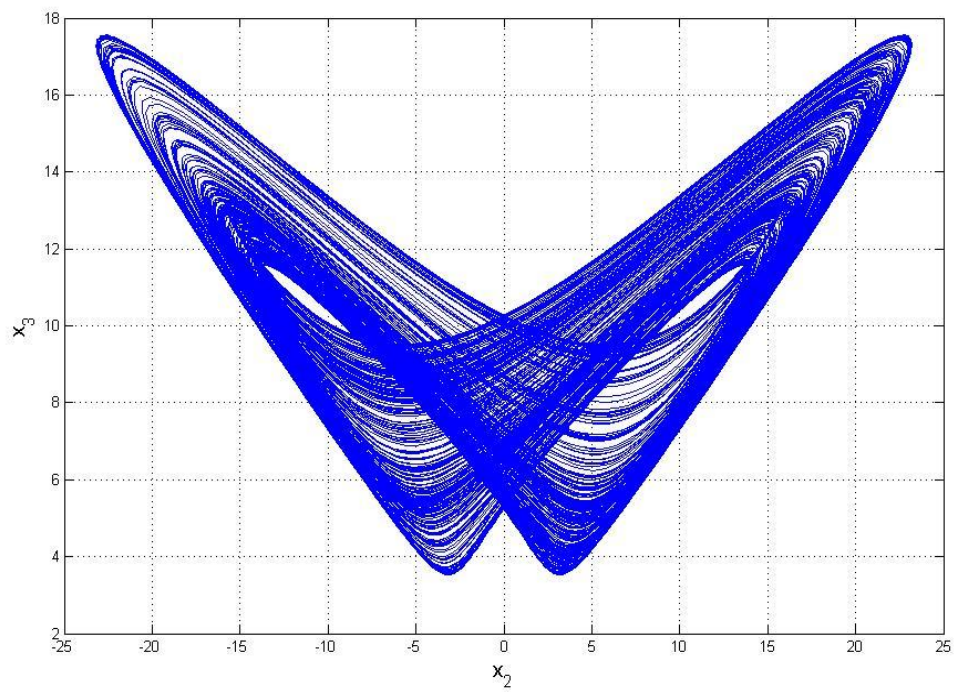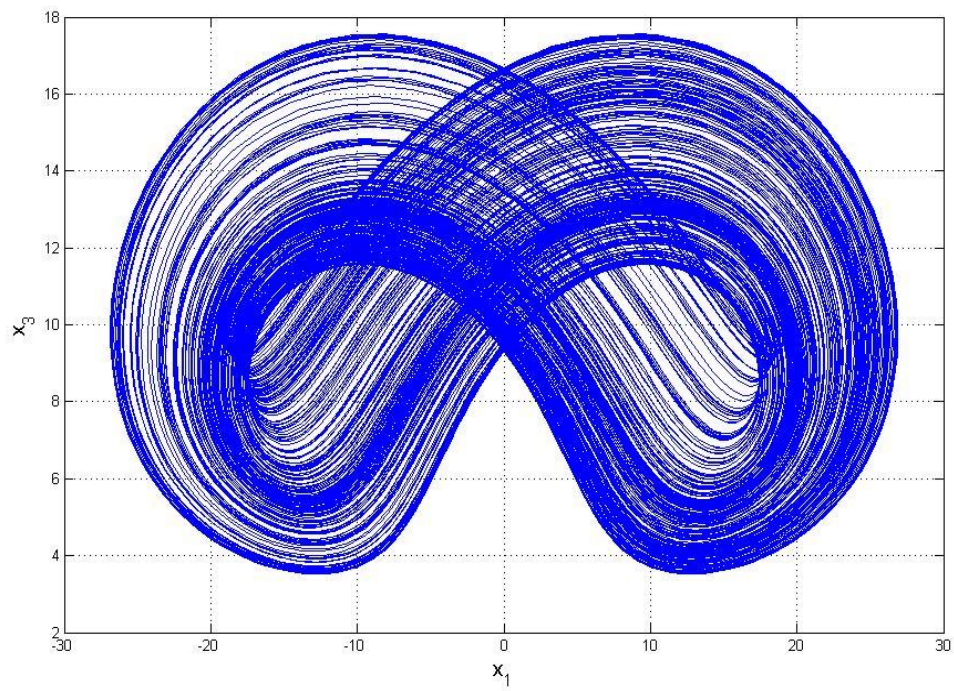
system.

Fig.6 Projection of phase portraits in MATLAB for fuzzy chaotic Chen-Lee system