AAPSilver System Performance Validation

Distribution Statement A: Approved for public release; distribution is unlimited.

December 2012
NOTICE

This document is disseminated under the sponsorship of the Department of Homeland Security in the interest of information exchange. The United States Government assumes no liability for its contents or use thereof.

The United States Government does not endorse products or manufacturers. Trade or manufacturers’ names appear herein solely because they are considered essential to the object of this report.

This report does not constitute a standard, specification, or regulation.

James E. Fletcher
Environment & Waterways Branch Chief
United States Coast Guard
Research & Development Center
1 Chelsea Street
New London, CT 06320
AAPSilver System Performance Validation

The Visual Navigation Division (CG-NAV-1) requested that the Coast Guard Research and Development Center (RDC) examine the accuracy of the new AAPSilver software in calculating the geographic positions (latitude and longitude) of bearing and range offsets from reference locations. The USCG R&D Center tested the software by comparing the AAPSilver output to independently calculated results that were calculated by an algorithm obtained from the National Geodetic Survey. Combinations of test points and offsets were input to AAPSilver, which provided latitude-longitude pairs as outputs. These coordinates were then compared to calculations from the National Geodetic Survey’s FORWARD program, which served as the reference for the assessment of AAPSilver.

Offset positions agreed to within 8 cm in range and agreed exactly in bearing to the calculated reference measurements. These differences may be compared to uncertainties associated with the use of Differential Global Positioning System receivers used to position Coast Guard ATON vessels when the adequacy of the AAPSilver software is considered for distribution throughout the Coast Guard ATON fleet.

AAPSilver, ATON, FORWARD

Distribution Statement A: Approved for public release; distribution is unlimited.

UNCLAS/Public
EXECUTIVE SUMMARY

The Coast Guard uses a software package called the Automated Aid Positioning System (AAPS) to maintain and manage its Aids to Navigation (ATON) system. The Coast Guard will be updating AAPS to a new version called AAPSilver. The software update includes changes that could potentially affect its calculations of the geographic coordinates of ATON assets. The Visual Navigation Division (CG-NAV-1) has requested that the Coast Guard Research and Development Center (RDC) examine the performance of the new AAPSilver software in calculating the geographic positions (latitude and longitude) of bearing and range offsets from reference locations.

The USCG R&D Center tested the software by comparing the AAPSilver output to independently calculated results that were calculated by an algorithm obtained from the National Geodetic Survey. Combinations of test points and offsets were generated for test points in each of the four quadrants of the globe to test the accuracy of the system in the Coast Guard’s Areas of responsibility. We created eight random offset range and bearing combinations for each test point. These range-bearing pairs were input to AAPSilver which provided latitude-longitude pairs as outputs. These coordinates were then compared to calculations from the National Geodetic Survey’s FORWARD program, which served as the reference for the assessment of AAPSilver.

Offset positions agreed to within 8 cm in range and agreed exactly in bearing to the calculated reference measurements. These differences may be compared to uncertainties associated with the use of Differential Global Positioning System receivers used to position Coast Guard ATON vessels when the adequacy of the AAPSilver software is considered for distribution throughout the Coast Guard ATON fleet.
# TABLE OF CONTENTS

1  BACKGROUND .................................................................................................................................1
2  PROCEDURE ......................................................................................................................................2
3  RESULTS ...........................................................................................................................................4
4  CONCLUSIONS ................................................................................................................................10

APPENDIX A. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP ALPHA ... A-1
APPENDIX B. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP BRAVO ... B-1
APPENDIX C. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP CHARLIE .................................................. C-1
APPENDIX D. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP DELTA ... D-1
APPENDIX E. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP ECHO ...... E-1
LIST OF FIGURES

Figure 1. Assigned point and buoy port configuration. .................................................................2

LIST OF TABLES

Table 1. Vessel buoy port (BPP) assigned positions used for this study. .........................................2
Table 2. Target position offsets used for each assigned position. ......................................................3
Table 3. Comparison between FORWARD and AAPSilver calculations for BPP Alpha at 41° N 72° W. ....5
Table 4. Comparison between FORWARD and AAPSilver calculations for BPP Bravo at 13° N 144° E.....6
Table 5. Comparison between FORWARD and AAPSilver calculations for BPP Charlie at 60° S 87° W....7
Table 6. Comparison between FORWARD and AAPSilver calculations for BPP Delta at 10° S 170° E.....8
Table 7. Comparison between FORWARD and AAPSilver calculations for BPP Echo at 60° S 87° W. ....9
# LIST OF ACRONYMS, ABBREVIATIONS, AND SYMBOLS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPS</td>
<td>Automated Aid Positioning System</td>
</tr>
<tr>
<td>AP</td>
<td>Assigned Position</td>
</tr>
<tr>
<td>ATON</td>
<td>Aids to Navigation</td>
</tr>
<tr>
<td>BPP</td>
<td>Buoy Port Position</td>
</tr>
<tr>
<td>CGDN</td>
<td>Coast Guard Data Network</td>
</tr>
<tr>
<td>CG-NAV-1</td>
<td>Coast Guard Headquarters Visual Navigation Division</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>I-ATONIS</td>
<td>Integrated Aids to Navigation Information System</td>
</tr>
<tr>
<td>NMEA</td>
<td>National Marine Electronics Association</td>
</tr>
<tr>
<td>OSC</td>
<td>Coast Guard Operations Systems Center, Martinsburg, WV</td>
</tr>
<tr>
<td>RDC</td>
<td>Research and Development Center</td>
</tr>
</tbody>
</table>
(This page intentionally left blank.)
1 BACKGROUND

The Coast Guard is responsible for maintaining a network of fixed and floating aids to navigation to support the nation’s marine transportation system. These aids must be maintained and physically positioned, as necessary, in order to mark the boundaries of our navigable waters. The Coast Guard manages information on more than 100,000 federal and private ATONs using the Integrated Aids to Navigation Information System (I-ATONIS). I-ATONIS is a distributed and centralized database system that is maintained by the Operations Systems Center (OSC), located in Martinsburg, WV, which holds the I-ATONIS Database for the entire Coast Guard. Coast Guard units and District offices access the central I-ATONIS database using the Coast Guard Data Network (CGDN+) to track and schedule maintenance, in order to keep the nationwide ATON system operating properly.

Field users use the Automated Aid Positioning System (AAPS) software interface to document the status of the system and to manage and maintain the ATON system. The information exchanged through the AAPS interface includes:

- Physical configuration and status of the aid
- Location and location accuracy
- Light beacon and electrical power system characteristics
- Mooring system details
- The history of ATON maintenance

Coast Guard units or contractors conduct periodic visits to each ATON in the system. The position of each floating buoy is checked each time it is visited, and a determination is made as to whether the buoy is on or off station, in accordance with standards contained in the Aids to Navigation Manual – Positioning, COMDTINST M16500.1 (series). The USCG publishes the positions of floating and fixed aids to navigation in the 8 volumes of the Light List.

The Visual Navigation Division (CG-NAV-1) has updated the AAPS software to a new version called AAPSilver. The software update includes changes that could potentially affect its calculations of the geographic coordinates of ATON assets. CG-NAV-1 has asked the Coast Guard Research and Development Center (RDC) to examine the accuracy of the new AAPSilver software in calculating geographic positions (latitude and longitude) of bearing and range offsets from reference vessel locations.

This report documents a test of a function, in which a Coast Guard ATON vessel would be placing an ATON in an Assigned Position (AP). APs are conveyed to mariners through the Light Lists and other automated processes that produce charts and related hydrographic products. It is therefore of great importance to ensure that buoy and beacon positions meet Coast Guard positioning standards. The Coast Guard vessel handles and positions the ATON from a buoy port on the vessel whose position, the buoy port position (BPP), is calculated from a GPS fix, the ship’s heading, and the position offset of the buoy port relative to the GPS antenna. AAPSilver provides ranges and bearings from the BPP to the AP, to assist the vessel in converging on the AP to within Coast Guard positioning tolerances.
2 PROCEDURE

The AAPSilver test focused on comparing calculations made by AAPSilver with those made by a reference software application. Eight APs were placed around each BPP, with two APs in each quadrant relative to the BPP (Figure 1).

![Diagram of assigned point and buoy port configuration]

Figure 1. Assigned point and buoy port configuration.

Comparisons were made at four BPPs that were placed around the globe. The positions chosen for the four BPPs (Table 1) mirrored those of an earlier RDC study, to provide calculations that were comparable with earlier versions of AAPS.

Table 1. Vessel buoy port position (BPP) assigned positions used for this study.

<table>
<thead>
<tr>
<th>Vessel Assigned Position (AP)</th>
<th>Latitude (dd mm ss H)</th>
<th>Longitude (ddd mm ss H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>41 00 00 N</td>
<td>072 00 00 W</td>
</tr>
<tr>
<td>Bravo</td>
<td>13 00 00 N</td>
<td>144 00 00 W</td>
</tr>
<tr>
<td>Charlie</td>
<td>60 00 00 S</td>
<td>087 00 00 E</td>
</tr>
<tr>
<td>Delta</td>
<td>10 00 00 S</td>
<td>170 00 00 E</td>
</tr>
</tbody>
</table>

The AP offsets (Table 2) were also those used in the 1999 evaluation, to permit a comparison of results, if desired. The National Geodetic Survey’s FORWARD application was used to calculate the geodetic position (latitude and longitude in degrees, minutes, and seconds) of each AP from each BPP based on the AP’s azimuth and range. FORWARD, which is run on-line, uses a reference position (i.e. BPP), a range, and a bearing as inputs, and calculates the resulting latitude and longitude (AP) as outputs. FORWARD was used to generate eight latitude/longitude APs at various ranges and bearings (“offsets”) around each BPP. The same set of offsets was used at each of the four BPPs.

During normal use, AAPSilver accepts an output string from the ship’s GPA receiver to determine the ship’s BPP. The receiver’s position is contained as a NMEA $GPGGA message. For this test, AAPSilver
Table 2. Target position offsets used for each assigned position.

<table>
<thead>
<tr>
<th>Test Point ID</th>
<th>Range (yds)</th>
<th>Azimuth ° True</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP1</td>
<td>100</td>
<td>30</td>
</tr>
<tr>
<td>AP2</td>
<td>75</td>
<td>59</td>
</tr>
<tr>
<td>AP3</td>
<td>60</td>
<td>110</td>
</tr>
<tr>
<td>AP4</td>
<td>20</td>
<td>157</td>
</tr>
<tr>
<td>AP5</td>
<td>87</td>
<td>182</td>
</tr>
<tr>
<td>AP6</td>
<td>300</td>
<td>241</td>
</tr>
<tr>
<td>AP7</td>
<td>640</td>
<td>294</td>
</tr>
<tr>
<td>AP8</td>
<td>31</td>
<td>345</td>
</tr>
</tbody>
</table>

was modified by the technical support contractor at the Coast Guard Operations Systems Center (OSC), Martinsburg, WV to permit the manual insertion of each BPP in a synthetic $GPGGA string. The AP positions were input using the “New Federal” button in the AAPSilver ATON page. The Range and Bearing calculated by AAPS were recorded and compared to the “True” Range and Bearing value used for the FORWARD calculation for each BPP.

The procedure for the test was as follows:

1) A spreadsheet was assembled for the input and output data that contained a separate worksheet for BPPs ALPHA through DELTA.

2) The FORWARD program was run to create the AP coordinates for each BPP. Eight APs were entered on each worksheet, and formatted in the spreadsheet for input to AAPSilver. The FORWARD inputs are the BPP and the Assigned Point offset (Range and Azimuth). The output of FORWARD is the resulting latitude and longitude, which are then formatted in the spreadsheet for input to AAPSilver.

3) AAPSilver was started. In the settings menu, right-click on the screen to enable manual $GPGGA entry, and paste the $GPGGA string for the first BPP. This tells the software that the vessel is at the BPP. In the “Vessel” screen, “Jump Kit” was selected as the GPS device. This sets the GPS antenna and BPP positions to the same location (i.e. zero offset).

4) Enter the ATON screen, select the search icon, and the Search Aids screen will come up. Select the “New Federal” icon to identify a new federal aid. Paste in the AP name and position from the worksheet file. Select “Add”, and then highlight the icon in the upper left corner. This step simulates the position of the aid at the AP.

5) Go to the “Positioning” screen. The vessel position will be displayed in the lower left corner. The range and bearing to the AP will be displayed in the upper right corner: that range and bearing are compared to the values input to the FORWARD Program in step 2.
3 RESULTS

An initial comparison for the eight APs at the four BPPs: Alpha, Bravo, Charlie, and Delta, is presented in the top eight rows of Tables 3 through 6. The range and bearing offsets calculated by AAPSilver were typically identical to those input into FORWARD. The observed differences ranged from 1-4 cm. The differences were examined, and in most cases could be attributed to roundoff in the seconds field of the latitude or longitude. In other words, the difference was typically due to a difference in the fourth digit in seconds of latitude or longitude.

The largest differences between the AAPSilver and FORWARD calculations were observed at BPP Charlie, and appeared to scale with range. In addition, we decided also to conduct a brief exercise of the performance of the AAPSilver software at cardinal points of the compass. A fifth BPP, Echo, was created to exercise AAPSilver at longer ranges and at the 90° and 180° bearings, and at ranges out to 2000 yards. The initial test produced range error disparity increased at an approximately linear rate, to a maximum of 0.08 yards at 2000 yards. A more significant result was that the bearing from BPP to Assigned Points at the 180° bearing was in error by 180°, so that the bearing reported by AAPSilver was 0° instead of 180° (not shown in Table 7).

This problem was reported to the sponsor and the contractor at OSC, Martinsburg. The contractor traced the problem to a bug in the software that would only occur in South latitudes and West longitudes. The contractor corrected the problem, and the BPP Echo cases were re-run. The results, shown in Table 7, were now consistent with those for the other BPPs: Range errors were on the order of a few centimeters, and increased to a maximum difference of 8 cm at 2000 yards. The bearing error problem was corrected.

Another set of runs was also conducted for BPPs Alpha, Bravo, and Delta. The results are in Tables 3, 4, and 6 as Assigned Points 9 and greater (> AP9). These runs confirmed to our satisfaction that the bearing error had been rectified. It also confirmed that the error that had once existed in the South and West quadrant of the globe did not exist in the other quadrants.
Table 3. Comparison between FORWARD and AAPSilver calculations for BPP Alpha at 41° N 72° W.

<table>
<thead>
<tr>
<th>Reference Position (BPP)</th>
<th>Assigned Point Offset</th>
<th>Assigned Point ID</th>
<th>AAPSilver INPUT:</th>
<th>AAPSilver Calculated Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>AP Latitude</td>
<td>AP Longitude</td>
</tr>
<tr>
<td>Latitude</td>
<td>Longitude</td>
<td>Range</td>
<td>Azimuth</td>
<td>dd-mm-ss.ssssH</td>
</tr>
<tr>
<td>Hdd</td>
<td>Hdd</td>
<td>yds</td>
<td>m</td>
<td>° True</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>100</td>
<td>91.44</td>
<td>30</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>75</td>
<td>68.58</td>
<td>59</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>60</td>
<td>54.864</td>
<td>110</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>87</td>
<td>79.5528</td>
<td>182</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>300</td>
<td>274.32</td>
<td>241</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>640</td>
<td>585.216</td>
<td>294</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>31</td>
<td>28.3464</td>
<td>345</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>1000</td>
<td>914.4</td>
<td>0</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>1000</td>
<td>914.4</td>
<td>90</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>1000</td>
<td>914.4</td>
<td>180</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>1000</td>
<td>914.4</td>
<td>270</td>
</tr>
<tr>
<td>N41</td>
<td>W72</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* Changing this value to 072°-00'-10.2657"W drops the range to 300.00 yards
‡ Changing this value to 072°-00'-22.8763" drops the range to 640.00 yards
Table 4. Comparison between FORWARD and AAPSilver calculations for BPP Bravo at 13° N 144° E.

<table>
<thead>
<tr>
<th>Reference Position (BPP)</th>
<th>Assigned Point Offset</th>
<th>Assigned Point ID</th>
<th>AAPSilver INPUT:</th>
<th>AAPSilver Calculated Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Latitude</td>
<td>Longitude</td>
<td>Range</td>
<td>Azimuth</td>
</tr>
<tr>
<td></td>
<td>Hdd</td>
<td>Hdd</td>
<td>yds</td>
<td>m</td>
</tr>
<tr>
<td>N13  E144</td>
<td>100</td>
<td>91.44</td>
<td>30</td>
<td>BRAVO AP1</td>
</tr>
<tr>
<td>N13  E144</td>
<td>75</td>
<td>68.58</td>
<td>59</td>
<td>BRAVO AP2</td>
</tr>
<tr>
<td>N13  E144</td>
<td>60</td>
<td>54.864</td>
<td>110</td>
<td>BRAVO AP3</td>
</tr>
<tr>
<td>N13  E144</td>
<td>20</td>
<td>18.288</td>
<td>157</td>
<td>BRAVO AP4</td>
</tr>
<tr>
<td>N13  E144</td>
<td>87</td>
<td>79.552</td>
<td>182</td>
<td>BRAVO AP5</td>
</tr>
<tr>
<td>N13  E144</td>
<td>300</td>
<td>274.32</td>
<td>241</td>
<td>BRAVO AP6</td>
</tr>
<tr>
<td>N13  E144</td>
<td>640</td>
<td>585.216</td>
<td>294</td>
<td>BRAVO AP7</td>
</tr>
<tr>
<td>N13  E144</td>
<td>31</td>
<td>28.346</td>
<td>345</td>
<td>BRAVO AP8</td>
</tr>
<tr>
<td>N13  E144</td>
<td>1000</td>
<td>914.4</td>
<td>0</td>
<td>BRAVO AP9</td>
</tr>
<tr>
<td>N13  E144</td>
<td>1000</td>
<td>914.4</td>
<td>90</td>
<td>BRAVO AP10</td>
</tr>
<tr>
<td>N13  E144</td>
<td>1000</td>
<td>914.4</td>
<td>180</td>
<td>BRAVO AP11</td>
</tr>
<tr>
<td>N13  E144</td>
<td>1000</td>
<td>914.4</td>
<td>270</td>
<td>BRAVO AP12</td>
</tr>
</tbody>
</table>

* - With latitude rounded down to 12°-59'-57.4128"N, the range calculated by AAPSilver is 87.0 yards.
# AAPSilver System Performance Validation

Table 5. Comparison between FORWARD and AAPSilver calculations for BPP Charlie at 60° S 87° W.

<table>
<thead>
<tr>
<th>Reference Position (BPP)</th>
<th>Assigned Point Offset</th>
<th>Assigned Point ID</th>
<th>AAPSilver INPUT:</th>
<th>AAPSilver Calculated Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Latitude</td>
<td>Longitude</td>
<td>Range</td>
<td>Azimuth</td>
</tr>
<tr>
<td></td>
<td>Hdd</td>
<td>Hdd</td>
<td>yds</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S60 W87</td>
<td>100</td>
<td>91.44</td>
<td>30</td>
<td>CHARLIE AP1</td>
</tr>
<tr>
<td>S60 W87</td>
<td>75</td>
<td>68.58</td>
<td>59</td>
<td>CHARLIE AP2</td>
</tr>
<tr>
<td>S60 W87</td>
<td>60</td>
<td>54.864</td>
<td>110</td>
<td>CHARLIE AP3</td>
</tr>
<tr>
<td>S60 W87</td>
<td>20</td>
<td>18.288</td>
<td>157</td>
<td>CHARLIE AP4</td>
</tr>
<tr>
<td>S60 W87</td>
<td>87</td>
<td>79.5528</td>
<td>182</td>
<td>CHARLIE AP5</td>
</tr>
<tr>
<td>S60 W87</td>
<td>300</td>
<td>274.32</td>
<td>241</td>
<td>CHARLIE AP6</td>
</tr>
<tr>
<td>S60 W87</td>
<td>640</td>
<td>585.216</td>
<td>294</td>
<td>CHARLIE AP7</td>
</tr>
<tr>
<td>S60 W87</td>
<td>31</td>
<td>28.3464</td>
<td>345</td>
<td>CHARLIE AP8</td>
</tr>
</tbody>
</table>

* - When this position was rounded down to 60°-00'-02.5690"S, the range dropped to 87.00 yards.
Table 6. Comparison between FORWARD and AAPSilver calculations for BPP Delta at 10° S 170° E.

<table>
<thead>
<tr>
<th>Reference Position (BPP)</th>
<th>Assigned Point Offset</th>
<th>Assigned Point ID</th>
<th>AAPSilver INPUT:</th>
<th>AAPSilver Calculated Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Latitude</td>
<td>Longitude</td>
<td>Range</td>
<td>Azimuth</td>
</tr>
<tr>
<td></td>
<td>Hdd</td>
<td>Hdd</td>
<td>yds</td>
<td>m</td>
</tr>
<tr>
<td>S10 E170</td>
<td>100</td>
<td>91.44</td>
<td>30</td>
<td>DELTA AP1</td>
</tr>
<tr>
<td>S10 E170</td>
<td>75</td>
<td>68.58</td>
<td>59</td>
<td>DELTA AP2</td>
</tr>
<tr>
<td>S10 E170</td>
<td>60</td>
<td>54.864</td>
<td>110</td>
<td>DELTA AP3</td>
</tr>
<tr>
<td>S10 E170</td>
<td>20</td>
<td>18.288</td>
<td>157</td>
<td>DELTA AP4</td>
</tr>
<tr>
<td>S10 E170</td>
<td>87</td>
<td>79.5528</td>
<td>182</td>
<td>DELTA AP5</td>
</tr>
<tr>
<td>S10 E170</td>
<td>300</td>
<td>274.32</td>
<td>241</td>
<td>DELTA AP6</td>
</tr>
<tr>
<td>S10 E170</td>
<td>640</td>
<td>585.216</td>
<td>294</td>
<td>DELTA AP7</td>
</tr>
<tr>
<td>S10 E170</td>
<td>31</td>
<td>28.3464</td>
<td>345</td>
<td>DELTA AP8</td>
</tr>
<tr>
<td>S10 E170</td>
<td>1000</td>
<td>914.4</td>
<td>0</td>
<td>DELTA AP9</td>
</tr>
<tr>
<td>S10 E170</td>
<td>1000</td>
<td>914.4</td>
<td>90</td>
<td>DELTA AP10</td>
</tr>
<tr>
<td>S10 E170</td>
<td>1000</td>
<td>914.4</td>
<td>180</td>
<td>DELTA AP11</td>
</tr>
<tr>
<td>S10 E170</td>
<td>1000</td>
<td>914.4</td>
<td>270</td>
<td>DELTA AP12</td>
</tr>
<tr>
<td>S10 E170</td>
<td>1000</td>
<td>914.4</td>
<td>360</td>
<td>DELTA AP13</td>
</tr>
</tbody>
</table>
Table 7. Comparison between FORWARD and AAPSilver calculations for BPP Echo at 60° S 87° W.

<table>
<thead>
<tr>
<th>Reference Position (BPP)</th>
<th>Assigned Point Offset</th>
<th>Assigned Point ID</th>
<th>AAPSilver INPUT:</th>
<th>AAPSilver Calculated Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>AP Latitude</td>
<td>AP Longitude</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>dd-mm-ss.ssssH</td>
<td>ddd-mm-ss.ssssH</td>
</tr>
<tr>
<td>Latitude</td>
<td>Longitude</td>
<td>Range</td>
<td>Azimuth</td>
<td></td>
</tr>
<tr>
<td>Hdd</td>
<td>Hdd</td>
<td>yds</td>
<td>m</td>
<td>° True</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>100</td>
<td>91.44</td>
<td>90</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>500</td>
<td>457.2</td>
<td>90</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>1000</td>
<td>914.4</td>
<td>90</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>2000</td>
<td>1828.8</td>
<td>90</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>100</td>
<td>91.44</td>
<td>180</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>500</td>
<td>457.2</td>
<td>180</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>1000</td>
<td>914.4</td>
<td>180</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>2000</td>
<td>1828.8</td>
<td>180</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>100</td>
<td>91.44</td>
<td>270</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>500</td>
<td>457.2</td>
<td>270</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>1000</td>
<td>914.4</td>
<td>270</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>2000</td>
<td>1828.8</td>
<td>270</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>100</td>
<td>91.44</td>
<td>360</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>500</td>
<td>457.2</td>
<td>360</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>1000</td>
<td>914.4</td>
<td>360</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>2000</td>
<td>1828.8</td>
<td>360</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>100</td>
<td>91.44</td>
<td>359</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>500</td>
<td>457.2</td>
<td>359</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>1000</td>
<td>914.4</td>
<td>359</td>
</tr>
<tr>
<td>S60</td>
<td>W87</td>
<td>2000</td>
<td>1828.8</td>
<td>359</td>
</tr>
</tbody>
</table>

AAPSilver System Performance Validation
4 CONCLUSIONS

The Visual Navigation Division (CG-NAV-1) requested that the Coast Guard RDC examine the performance of the AAPSilver software in calculating the geographic positions (latitude and longitude) of bearing and range offsets from reference locations.

Our conclusion based on the testing described above is that the AAPSilver software performs as would be expected. The agreement between results produced by AAPSilver and the reference software was within 8 cm in range, and agreed exactly in bearing. These differences may be compared to uncertainties associated with the use of Differential Global Positioning System receivers used to position Coast Guard ATON vessels and aids when the AAPSilver software is considered for distribution throughout the Coast Guard ATON fleet.
APPENDIX A. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP ALPHA

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station : AP1
----------------
LAT = 41 0 0.0000 North
LON = 72 0 0.0000 West

Second Station : TP1
----------------
LAT = 41 0 2.56705 North
LON = 71 59 58.04370 West

Forward azimuth \( FAZ = 30 0 0.0000 \) From North
Back azimuth \( BAZ = 210 0 1.2835 \) From North
Ellipsoidal distance \( S = 91.4400 \) m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station : AP1
----------------
LAT = 41 0 0.0000 North
LON = 72 0 0.0000 West

Second Station : TP2
----------------
LAT = 41 0 1.14499 North
LON = 71 59 57.48470 West

Forward azimuth \( FAZ = 59 0 0.0000 \) From North
Back azimuth \( BAZ = 239 0 1.6502 \) From North
Ellipsoidal distance \( S = 68.5800 \) m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)
First Station : AP1
----------------
LAT =  41  0  0.00000 North  
LON =  72  0  0.00000 West

Second Station : TP3
-----------------
LAT =  40 59 59.39171 North
LON =  71 59 57.79404 West

Forward azimuth  FAZ = 110  0  0.0000 From North
Back azimuth    BAZ = 290  0  1.4472 From North
Ellipsoidal distance   S =        54.8640 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a     =    6378137.0000
Polar axis,         b     =    6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : AP1
-----------------
LAT =  41  0  0.00000 North
LON =  72  0  0.00000 West

Second Station : TP4
-----------------
LAT =  40 59 59.45429 North
LON =  71 59 59.69425 West

Forward azimuth  FAZ = 157  0  0.0000 From North
Back azimuth    BAZ = 337  0  0.2006 From North
Ellipsoidal distance   S =        18.2880 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a     =    6378137.0000
Polar axis,         b     =    6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : AP1
-----------------
LAT =  41  0  0.00000 North
LON =  72  0  0.00000 West

Second Station : TP5
-----------------
LAT =  40 59 57.42273 North
LON =  72  0  0.11879 West

Forward azimuth  FAZ = 182  0  0.0000 From North
Back azimuth    BAZ = 1 59 59.9221 From North
Ellipsoidal distance   S =        79.5528 m
Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : AP1
----------------
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station : TP6
----------------
LAT = 40 59 55.68868 North
LON = 72 0 10.26582 West

Forward azimuth FAZ = 241 0 0.0000 From North
Back azimuth BAZ = 60 59 53.2651 From North
Ellipsoidal distance S = 274.3200 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : AP1
----------------
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station : TP7
----------------
LAT = 41 0 7.71547 North
LON = 72 0 22.87627 West

Forward azimuth FAZ = 294 0 0.0000 From North
Back azimuth BAZ = 113 59 44.9915 From North
Ellipsoidal distance S = 585.2160 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : AP1
----------------
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station : TP8
LAT = 41 0 0.88759 North
LON = 72 0 0.31392 West

Forward azimuth FAZ = 345 0 0.0000 From North
Back azimuth BAZ = 164 59 59.7940 From North
Ellipsoidal distance S = 28.3464 m

Output from FORWARD
Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : ALPHA
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station : ALPHA TP9
LAT = 41 0 29.64180 North
LON = 72 0 0.00000 West

Forward azimuth FAZ = 0 0 0.0000 From North
Back azimuth BAZ = 180 0 0.0000 From North
Ellipsoidal distance S = 914.4000 m

Output from FORWARD
Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : ALPHA
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station : ALPHA TP10
LAT = 40 59 59.99816 North
LON = 71 59 20.87440 West

Forward azimuth FAZ = 90 0 0.0000 From North
Back azimuth BAZ = 270 0 25.6687 From North
Ellipsoidal distance S = 914.4000 m

Output from FORWARD
Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, \(1/f = 298.25722210088\)

First Station: ALPHA
--------------
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station: ALPHA TP11
--------------
LAT = 40 59 30.35816 North
LON = 72 0 0.00000 West

Forward azimuth \(FAZ = 180 0 0.0000\) From North
Back azimuth \(BAZ = 0 0 0.0000\) From North
Ellipsoidal distance \(S = 914.4000\) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \(a = 6378137.0000\)
Polar axis, \(b = 6356752.3141\)
Inverse flattening, \(1/f = 298.25722210088\)

First Station: ALPHA
--------------
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station: ALPHA TP12
--------------
LAT = 40 59 59.99816 North
LON = 72 0 39.12560 West

Forward azimuth \(FAZ = 270 0 0.0000\) From North
Back azimuth \(BAZ = 89 59 34.3313\) From North
Ellipsoidal distance \(S = 914.4000\) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \(a = 6378137.0000\)
Polar axis, \(b = 6356752.3141\)
Inverse flattening, \(1/f = 298.25722210088\)

First Station: ALPHA
--------------
LAT = 41 0 0.00000 North
LON = 72 0 0.00000 West

Second Station: ALPHA TP13
--------------
LAT = 41 0 29.64180 North
LON = 72 0 0.00000 West

Forward azimuth \(FAZ = 0 0 0.0000\) From North
Back azimuth \(BAZ = 180 0 0.0000\) From North
Ellipsoidal distance \(S = 914.4000\) m
APPENDIX B. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP BRAVO

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: BRAVO
-------------------
LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station: AP1
-------------------
LAT = 13 0 2.57688 North
LON = 144 0 1.51719 East

Forward azimuth FAZ = 30 0 0.0000 From North
Back azimuth BAZ = 210 0 0.3413 From North
Ellipsoidal distance S = 91.4400 m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: BRAVO
-------------------
LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station: AP2
-------------------
LAT = 13 0 1.14938 North
LON = 144 0 1.95073 East

Forward azimuth FAZ = 59 0 0.0000 From North
Back azimuth BAZ = 239 0 0.4388 From North
Ellipsoidal distance S = 68.5800 m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: BRAVO
LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station : AP3

LAT = 12 59 59.38938 North
LON = 144 0 1.71083 East

Forward azimuth       FAZ = 110 0 0.0000 From North
Back azimuth          BAZ = 290 0 0.3849 From North
Ellipsoidal distance   S =       54.8640 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station : AP4

LAT = 12 59 59.45220 North
LON = 144 0 0.23713 East

Forward azimuth       FAZ = 157 0 0.0000 From North
Back azimuth          BAZ = 337 0 0.0533 From North
Ellipsoidal distance   S =       18.2880 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station : AP5

LAT = 12 59 57.41287 North
LON = 143 59 59.90787 East

Forward azimuth       FAZ = 182 0 0.0000 From North
Back azimuth          BAZ = 159 59.9793 From North
Ellipsoidal distance   S =       79.5528 m
Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, $a = 6378137.0000$
Polar axis, $b = 6356752.3141$
Inverse flattening, $1/f = 298.25722210088$

First Station: BRAVO

<table>
<thead>
<tr>
<th>LAT</th>
<th>LON</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>144</td>
</tr>
</tbody>
</table>

Second Station: AP6

<table>
<thead>
<tr>
<th>LAT</th>
<th>LON</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>143</td>
</tr>
</tbody>
</table>

Forward azimuth $FAZ = 241.0.0.0000$ From North
Back azimuth $BAZ = 60.59.58.2091$ From North
Ellipsoidal distance $S = 274.3200$ m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, $a = 6378137.0000$
Polar axis, $b = 6356752.3141$
Inverse flattening, $1/f = 298.25722210088$

First Station: BRAVO

<table>
<thead>
<tr>
<th>LAT</th>
<th>LON</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>144</td>
</tr>
</tbody>
</table>

Second Station: AP7

<table>
<thead>
<tr>
<th>LAT</th>
<th>LON</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>143</td>
</tr>
</tbody>
</table>

Forward azimuth $FAZ = 294.0.0.0000$ From North
Back azimuth $BAZ = 113.59.56.0088$ From North
Ellipsoidal distance $S = 585.2160$ m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, $a = 6378137.0000$
Polar axis, $b = 6356752.3141$
Inverse flattening, $1/f = 298.25722210088$

First Station: BRAVO

<table>
<thead>
<tr>
<th>LAT</th>
<th>LON</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>144</td>
</tr>
</tbody>
</table>

Second Station: AP8
LAT = 13 0 0.89098 North
LON = 143 59 59.75654 East

Forward azimuth FAZ = 345 0 0.0000 From North
Back azimuth BAZ = 164 59 59.9452 From North
Ellipsoidal distance S = 28.3464 m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station: BRAVO
LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station: BRAVO AP9
LAT = 13 0 29.75526 North
LON = 144 0 0.00000 East
Forward azimuth FAZ = 0 0 0.0000 From North
Back azimuth BAZ = 180 0 0.0000 From North
Ellipsoidal distance S = 914.4000 m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station: BRAVO
LAT = 13 0 0.00000 North
LON = 144 0 0.00000 East

Second Station: BRAVO AP10
LAT = 12 59 59.99951 North
LON = 144 0 30.34380 East
Forward azimuth FAZ = 90 0 0.0000 From North
Back azimuth BAZ = 270 0 6.8259 From North
Ellipsoidal distance S = 914.4000 m
Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,  a  =  6378137.0000
Polar axis,  b  =  6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : BRAVO
---------------
LAT =  13  0  0.00000 North
LON = 144  0  0.00000 East

Second Station : BRAVO AP11
---------------
LAT =  12 59 30.24472 North
LON = 144  0  0.00000 East

Forward azimuth  FAZ = 180  0  0.0000 From North
Back azimuth  BAZ =  0  0  0.0000 From North
Ellipsoidal distance  S =     914.4000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,  a  =  6378137.0000
Polar axis,  b  =  6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : BRAVO
---------------
LAT =  13  0  0.00000 North
LON = 144  0  0.00000 East

Second Station : BRAVO AP12
---------------
LAT =  12 59 59.99951 North
LON = 143 59 29.65620 East

Forward azimuth  FAZ = 270  0  0.0000 From North
Back azimuth  BAZ =  89 59 53.1741 From North
Ellipsoidal distance  S =     914.4000 m
APPENDIX C. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP CHARLIE

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,  \( a = 6378137.0000 \)
Polar axis,  \( b = 6356752.3141 \)
Inverse flattening,  \( 1/f = 298.25722210088 \)

First Station : CHARLIE
----------------
LAT = 60 0 0.0000 South
LON = 87 0 0.0000 West

Second Station : AP1
----------------
LAT = 59 59 57.44119 South
LON = 86 59 57.05039 West

Forward azimuth  \( FAZ = 30 0 0.0000 \) From North
Back azimuth  \( BAZ = 209 59 57.4456 \) From North
Ellipsoidal distance  \( S = 91.4400 \) m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,  \( a = 6378137.0000 \)
Polar axis,  \( b = 6356752.3141 \)
Inverse flattening,  \( 1/f = 298.25722210088 \)

First Station : CHARLIE
----------------
LAT = 60 0 0.0000 South
LON = 87 0 0.0000 West

Second Station : AP2
----------------
LAT = 59 59 58.85867 South
LON = 86 59 56.20749 West

Forward azimuth  \( FAZ = 59 0 0.0000 \) From North
Back azimuth  \( BAZ = 238 59 56.7156 \) From North
Ellipsoidal distance  \( S = 68.5800 \) m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,  \( a = 6378137.0000 \)
Polar axis,  \( b = 6356752.3141 \)
Inverse flattening,  \( 1/f = 298.25722210088 \)

First Station : CHARLIE
**AAPSilver System Performance Validation**

---

**Second Station : AP3**

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LAT</td>
<td>60 0 0.60632 South</td>
</tr>
<tr>
<td>LON</td>
<td>86 59 56.67384 West</td>
</tr>
</tbody>
</table>

Forward azimuth  \( FAZ = 110 \ 0 \ 0.0000 \) From North  
Back azimuth  \( BAZ = 289 \ 59 \ 57.1195 \) From North  
Ellipsoidal distance  \( S = 54.8640 \) m  

**Output from FORWARD**

Ellipsoid : GRS80 / WGS84 (NAD83)  
Equatorial axis,  \( a = 6378137.0000 \)  
Polar axis,  \( b = 6356752.3141 \)  
Inverse flattening,  \( 1/f = 298.25722210088 \)  

**First Station : CHARLIE**

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LAT</td>
<td>60 0 0.00000 South</td>
</tr>
<tr>
<td>LON</td>
<td>87 0 0.00000 West</td>
</tr>
</tbody>
</table>

Second Station : AP4

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LAT</td>
<td>60 0 0.54395 South</td>
</tr>
<tr>
<td>LON</td>
<td>86 59 59.53899 West</td>
</tr>
</tbody>
</table>

Forward azimuth  \( FAZ = 157 \ 0 \ 0.0000 \) From North  
Back azimuth  \( BAZ = 336 \ 59 \ 59.6007 \) From North  
Ellipsoidal distance  \( S = 18.2880 \) m  

**Output from FORWARD**

Ellipsoid : GRS80 / WGS84 (NAD83)  
Equatorial axis,  \( a = 6378137.0000 \)  
Polar axis,  \( b = 6356752.3141 \)  
Inverse flattening,  \( 1/f = 298.25722210088 \)  

**First Station : CHARLIE**

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LAT</td>
<td>60 0 0.00000 South</td>
</tr>
<tr>
<td>LON</td>
<td>87 0 0.00000 West</td>
</tr>
</tbody>
</table>

Second Station : AP5

---

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LAT</td>
<td>60 0 2.56914 South</td>
</tr>
<tr>
<td>LON</td>
<td>87 0 0.17914 West</td>
</tr>
</tbody>
</table>

Forward azimuth  \( FAZ = 182 \ 0 \ 0.0000 \) From North  
Back azimuth  \( BAZ = 2 \ 0 \ 0.1551 \) From North  
Ellipsoidal distance  \( S = 79.5580 \) m
Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station: CHARLIE
-------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP6
-------------
LAT = 60 0 0.29707 South
LON = 87 0 15.47963 West

Forward azimuth FAZ = 241 0 0.0000 From North
Back azimuth BAZ = 61 0 13.4058 From North
Ellipsoidal distance S = 274.3200 m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station: CHARLIE
-------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP7
-------------
LAT = 59 59 52.30746 South
LON = 87 0 34.48948 West

Forward azimuth FAZ = 294 0 0.0000 From North
Back azimuth BAZ = 114 0 29.8684 From North
Ellipsoidal distance S = 585.2160 m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station: CHARLIE
-------------
LAT = 60 0 0.00000 South
AAPSilver System Performance Validation

LON = 87 0 0.00000 West

Second Station : AP8

LAT = 59 59 59.11527 South
LON = 87 0 0.47332 West

Forward azimuth  FAZ = 345 0 0.0000 From North
Back azimuth     BAZ = 165 0 0.4099 From North
Ellipsoidal distance  S = 28.3464 m
APPENDIX D. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP DELTA

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station:
-----------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station: DELTA AP1
-----------------
LAT = 9 59 57.42259 South
LON = 170 0 1.50121 East

Forward azimuth \( \text{FAZ} = 30 0 0.0000 \) From North
Back azimuth \( \text{BAZ} = 209 59 59.7393 \) From North
Ellipsoidal distance \( S = 91.4400 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station:
-----------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station: DELTA AP2
-----------------
LAT = 9 59 58.85038 South
LON = 170 0 1.93018 East

Forward azimuth \( \text{FAZ} = 59 0 0.0000 \) From North
Back azimuth \( \text{BAZ} = 238 59 59.6648 \) From North
Ellipsoidal distance \( S = 68.5800 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)
AAPSilver System Performance Validation

First Station :

LAT = 10 0 0.0000 South
LON = 170 0 0.0000 East

Second Station : DELTA AP3

LAT = 10 0 0.61074 South
LON = 170 0 1.69282 East

Forward azimuth FAZ = 110 0 0.0000 From North
Back azimuth BAZ = 289 59 59.7060 From North
Ellipsoidal distance S = 54.8640 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station :

LAT = 10 0 0.0000 South
LON = 170 0 0.0000 East

Second Station : DELTA AP4

LAT = 10 0 0.54791 South
LON = 170 0 0.23463 East

Forward azimuth FAZ = 157 0 0.0000 From North
Back azimuth BAZ = 336 59 59.9593 From North
Ellipsoidal distance S = 18.2880 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station :

LAT = 10 0 0.0000 South
LON = 170 0 0.0000 East

Second Station : DELTA AP5

LAT = 10 0 2.5876 South
LON = 169 59 59.90884 East

Forward azimuth FAZ = 182 0 0.0000 From North
Back azimuth BAZ = 2 0 0.0158 From North
Ellipsoidal distance S = 79.5528 m
Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.2572210088 \)

First Station:
----------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station: DELTA AP6
----------------
LAT = 10 0 4.32856 South
LON = 169 59 52.12203 East

Forward azimuth \( FAZ = 241 0 0.0000 \) From North
Back azimuth \( BAZ = 61 0 1.3681 \) From North
Ellipsoidal distance \( S = 274.3200 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.2572210088 \)

First Station:
----------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station: DELTA AP7
----------------
LAT = 9 59 52.25264 South
LON = 169 59 42.44586 East

Forward azimuth \( FAZ = 294 0 0.0000 \) From North
Back azimuth \( BAZ = 114 0 3.0479 \) From North
Ellipsoidal distance \( S = 585.2160 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.2572210088 \)

First Station:
----------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East
Second Station : DELTA AP8

LAT =  9 59 59.10883 South
LON = 169 59 59.75910 East

Forward azimuth FAZ = 345 0  0.0000 From North
Back azimuth BAZ = 165 0  0.0418 From North
Ellipsoidal distance S =  28.3464 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : DELTA

LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station : AP9

LAT =  9 59 30.23861 South
LON = 170 0 0.00000 East

Forward azimuth FAZ =  0 0 0.0000 From North
Back azimuth BAZ = 180 0 0.0000 From North
Ellipsoidal distance S =  914.4000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : DELTA

LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station : AP10

LAT =  9 59 59.99962 South
LON = 170 0 30.02425 East

Forward azimuth FAZ =  90 0 0.0000 From North
Back azimuth BAZ = 269 59 54.7863 From North
Ellipsoidal distance S =  914.4000 m
Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: DELTA
-------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station: AP11
-------------
LAT = 10 0 29.76138 South
LON = 170 0 0.00000 East

Forward azimuth \( FAZ = 180 \ 0 \ 0.0000 \) From North
Back azimuth \( BAZ = 0 \ 0 \ 0.0000 \) From North
Ellipsoidal distance \( S = 914.4000 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: DELTA
-------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East

Second Station: AP12
-------------
LAT = 9 59 59.99962 South
LON = 169 59 29.97575 East

Forward azimuth \( FAZ = 270 \ 0 \ 0.0000 \) From North
Back azimuth \( BAZ = 90 \ 0 \ 5.2137 \) From North
Ellipsoidal distance \( S = 914.4000 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: DELTA
-------------
LAT = 10 0 0.00000 South
LON = 170 0 0.00000 East
Second Station : AP13
-------------
LAT = 9 59 30.23861 South
LON = 170 0 0.00000 East

Forward azimuth FAZ = 0 0 0.0000 From North
Back azimuth BAZ = 180 0 0.0000 From North
Ellipsoidal distance S = 914.4000 m
APPENDIX E. NATIONAL GEODETIC SURVEY FORWARD OUTPUT FOR BPP ECHO

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: ECHO

\[
\begin{align*}
\text{LAT} & = 60^\circ 0' 0.00000 \text{ South} \\
\text{LON} & = 87^\circ 0' 0.00000 \text{ West}
\end{align*}
\]

Second Station: AP1

\[
\begin{align*}
\text{LAT} & = 59^\circ 59' 59.99996 \text{ South} \\
\text{LON} & = 86^\circ 59' 54.10065 \text{ West}
\end{align*}
\]

Forward azimuth \( FAZ = 90^\circ 0' 0.0000 \) From North
Back azimuth \( BAZ = 269^\circ 59' 54.8910 \) From North
Ellipsoidal distance \( S = 91.4400 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: ECHO

\[
\begin{align*}
\text{LAT} & = 60^\circ 0' 0.00000 \text{ South} \\
\text{LON} & = 87^\circ 0' 0.00000 \text{ West}
\end{align*}
\]

Second Station: AP2

\[
\begin{align*}
\text{LAT} & = 59^\circ 59' 59.99909 \text{ South} \\
\text{LON} & = 86^\circ 59' 30.50323 \text{ West}
\end{align*}
\]

Forward azimuth \( FAZ = 90^\circ 0' 0.0000 \) From North
Back azimuth \( BAZ = 269^\circ 59' 34.4550 \) From North
Ellipsoidal distance \( S = 457.2000 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: ECHO
AAPSilver System Performance Validation

----------------
LAT =  60  0  0.00000 South
LON =  87  0  0.00000 West

Second Station : AP3
----------------
LAT =  59 59 59.99634 South
LON =  86 59 1.00645 West

Forward azimuth        FAZ =  90  0  0.0000 From North
Back azimuth           BAZ = 269 59 8.9101 From North
Ellipsoidal distance     S =      914.4000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a   =    6378137.0000
Polar axis,         b   =    6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : ECHO
----------------
LAT =  60  0  0.00000 South
LON =  87  0  0.00000 West

Second Station : AP4
----------------
LAT =  59 59 59.98536 South
LON =  86 58 2.01292 West

Forward azimuth        FAZ =  90  0  0.0000 From North
Back azimuth           BAZ = 269 58 17.8202 From North
Ellipsoidal distance     S =      1828.8000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a   =    6378137.0000
Polar axis,         b   =    6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : ECHO
----------------
LAT =  60  0  0.00000 South
LON =  87  0  0.00000 West

Second Station : AP5
----------------
LAT =  60  0  2.95465 South
LON =  87  0  0.00000 West

Forward azimuth        FAZ = 180  0  0.0000 From North
Back azimuth           BAZ =  0  0  0.0000 From North
Ellipsoidal distance     S =      91.4400 m
Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a \) = 6378137.0000
Polar axis, \( b \) = 6356752.3141
Inverse flattening, \( 1/f \) = 298.2572210088

First Station: ECHO
-------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP6
-------------
LAT = 60 0 14.77323 South
LON = 87 0 0.00000 West

Forward azimuth \( FAZ = 180 \) 0 0.0000 From North
Back azimuth \( BAZ = 0 \) 0 0.0000 From North
Ellipsoidal distance \( S = 457.2000 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a \) = 6378137.0000
Polar axis, \( b \) = 6356752.3141
Inverse flattening, \( 1/f \) = 298.2572210088

First Station: ECHO
-------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP7
-------------
LAT = 60 0 29.54645 South
LON = 87 0 0.00000 West

Forward azimuth \( FAZ = 180 \) 0 0.0000 From North
Back azimuth \( BAZ = 0 \) 0 0.0000 From North
Ellipsoidal distance \( S = 914.4000 \) m

Output from FORWARD

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a \) = 6378137.0000
Polar axis, \( b \) = 6356752.3141
Inverse flattening, \( 1/f \) = 298.25722210088

First Station: ECHO
-------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP8
AAPSilver System Performance Validation

---

LAT = 60 0 59.09287 South  
LON = 87 0 0.00000 West

Forward azimuth  FAZ = 180 0 0.0000 From North  
Back azimuth  BAZ = 0 0 0.0000 From North  
Ellipsoidal distance  S = 1828.8000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)  
Equatorial axis,  a = 6378137.0000  
Polar axis,  b = 6356752.3141  
Inverse flattening, 1/f = 298.25722210088

First Station : ECHO  
---

LAT = 60 0 0.00000 South  
LON = 87 0 0.00000 West

Second Station : AP9  
---

LAT = 59 59 59.99996 South  
LON = 87 0 5.89935 West

Forward azimuth  FAZ = 270 0 0.0000 From North  
Back azimuth  BAZ = 90 0 5.1090 From North  
Ellipsoidal distance  S = 91.4400 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)  
Equatorial axis,  a = 6378137.0000  
Polar axis,  b = 6356752.3141  
Inverse flattening, 1/f = 298.25722210088

First Station : ECHO  
---

LAT = 60 0 0.00000 South  
LON = 87 0 0.00000 West

Second Station : AP10  
---

LAT = 59 59 59.99999 South  
LON = 87 0 29.49677 West

Forward azimuth  FAZ = 270 0 0.0000 From North  
Back azimuth  BAZ = 90 0 25.5450 From North  
Ellipsoidal distance  S = 457.2000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)  
Equatorial axis,  a = 6378137.0000  
Polar axis,  b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : ECHO
----------------
  LAT = 60 0 0.00000 South
  LON = 87 0 0.00000 West

Second Station : AP11
----------------
  LAT = 59 59 59.99634 South
  LON = 87 0 58.99355 West

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : ECHO
----------------
  LAT = 60 0 0.00000 South
  LON = 87 0 0.00000 West

Second Station : AP12
----------------
  LAT = 59 59 59.98536 South
  LON = 87 1 57.98708 West

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, a = 6378137.0000
Polar axis, b = 6356752.3141
Inverse flattening, 1/f = 298.25722210088

First Station : ECHO
----------------
  LAT = 60 0 0.00000 South
  LON = 87 0 0.00000 West

Second Station : AP13
----------------
  LAT = 59 59 57.04535 South
  LON = 87 0 0.00000 West

Forward azimuth FAZ = 270 0 0.0000 From North
Back azimuth BAZ = 90 1 42.1798 From North
Ellipsoidal distance S = 1828.8000 m
Ellipsoidal distance \( S = 91.4400 \text{ m} \)

**Output from FORWARD**

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: ECHO
--------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP14
--------------
LAT = 59 59 45.22676 South
LON = 87 0 0.00000 West

Forward azimuth \( FAZ = 0 0 0.0000 \text{ From North} \)
Back azimuth \( BAZ = 180 0 0.0000 \text{ From North} \)
Ellipsoidal distance \( S = 457.2000 \text{ m} \)

**Output from FORWARD**

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: ECHO
--------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station: AP15
--------------
LAT = 59 59 30.45351 South
LON = 87 0 0.00000 West

Forward azimuth \( FAZ = 0 0 0.0000 \text{ From North} \)
Back azimuth \( BAZ = 180 0 0.0000 \text{ From North} \)
Ellipsoidal distance \( S = 914.4000 \text{ m} \)

**Output from FORWARD**

Ellipsoid: GRS80 / WGS84 (NAD83)
Equatorial axis, \( a = 6378137.0000 \)
Polar axis, \( b = 6356752.3141 \)
Inverse flattening, \( 1/f = 298.25722210088 \)

First Station: ECHO
--------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West
Second Station : AP16
----------------
LAT =  59 59  0.90698 South
LON =  87  0  0.00000 West

Forward azimuth    FAZ =  0  0  0.0000 From North
Back azimuth       BAZ = 180  0  0.0000 From North
Ellipsoidal distance    S = 1828.8000 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a =  6378137.0000
Polar axis,         b =  6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : ECHO
----------------
LAT =  60  0  0.00000 South
LON =  87  0  0.00000 West

Second Station : AP17
----------------
LAT =  59 59 57.04580 South
LON =  87  0  0.10296 West

Forward azimuth    FAZ = 359  0  0.0000 From North
Back azimuth       BAZ = 179  0  0.0892 From North
Ellipsoidal distance    S = 91.4400 m

Output from FORWARD

Ellipsoid : GRS80 / WGS84  (NAD83)
Equatorial axis,    a =  6378137.0000
Polar axis,         b =  6356752.3141
Inverse flattening, 1/f =  298.25722210088

First Station : ECHO
----------------
LAT =  60  0  0.00000 South
LON =  87  0  0.00000 West

Second Station : AP18
----------------
LAT =  59 59 45.22901 South
LON =  87  0  0.51473 West

Forward azimuth    FAZ = 359  0  0.0000 From North
Back azimuth       BAZ = 179  0  0.4458 From North
Ellipsoidal distance    S = 457.2000 m
Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, \( a \) = 6378137.0000
Polar axis, \( b \) = 6356752.3141
Inverse flattening, \( 1/f \) = 298.25722210088

First Station : ECHO
--------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station : AP19
--------------
LAT = 59 59 30.45801 South
LON = 87 0 1.02932 West

Forward azimuth \( FAZ = 359 \) 0 0.0000 From North
Back azimuth \( BAZ = 179 \) 0 0.8914 From North
Ellipsoidal distance \( S = 914.4000 \) m

Output from FORWARD

Ellipsoid : GRS80 / WGS84 (NAD83)
Equatorial axis, \( a \) = 6378137.0000
Polar axis, \( b \) = 6356752.3141
Inverse flattening, \( 1/f \) = 298.25722210088

First Station : ECHO
--------------
LAT = 60 0 0.00000 South
LON = 87 0 0.00000 West

Second Station : AP20
--------------
LAT = 59 59 0.91598 South
LON = 87 0 2.05814 West

Forward azimuth \( FAZ = 359 \) 0 0.0000 From North
Back azimuth \( BAZ = 179 \) 0 1.7823 From North
Ellipsoidal distance \( S = 1828.8000 \) m