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14. ABSTRACT
This project considers the dynamic spectrum management (DSM) problem whereby multiple users sharing a common frequency band must choose their transmit power spectra jointly in response to physical channel conditions including the effects of interference. The goal of the users is to maximize a system-wide utility function (e.g., weighted sum-rate of all users), subject to individual power constraints. The proposed work will focus on a general DSM problem formulation which allows correlated signaling rather than being restricted to the

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Report Title

Complexity Analysis and Algorithms for Optimal Resource Allocation in Wireless Networks

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This project considers the dynamic spectrum management (DSM) problem whereby multiple users sharing a common frequency band must choose their transmit power spectra jointly in response to physical channel conditions including the effects of interference. The goal of the users is to maximize a system-wide utility function (e.g., weighted sum-rate of all users), subject to individual power constraints. The proposed work will focus on a general DSM problem formulation which allows correlated signaling rather than being restricted to the conventional independent orthogonal signaling such as OFDM. The general formulation will exploit the concept of 'interference alignment' which is known to provide substantial rate gain over OFDM signalling for general interference channels. We have successfully analyzed the complexity to characterize the optimal spectrum sharing policies and beamforming strategies in interfering broadcast networks and developed efficient computational methods for optimal resource allocations in such networks.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received	Paper
2012/09/01 1: 4	Gennady Lyubeznik, Zhi-Quan Luo, Meisam Razaviyayn. On the degrees of freedom achievable through interference alignment in a MIMO interference channel, IEEE Transactions on Signal Processing, (02 2012): 812. doi: 10.1109/SPAWC.2011.5990463
2012/02/06 2: 6	Qingjiang Shi, Meisam Razaviyayn, Zhi-Quan Luo, Chen He. An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel, IEEE Transactions on Signal Processing, (09 2011): 0. doi: 10.1109/TSP.2011.2147784
2012/02/06 2: 5	Ya-Feng Liu, Yu-Hong Dai, Zhi-Quan Luo. Coordinated Beamforming for MISO Interference Channel: Complexity Analysis and Efficient Algorithms, IEEE Transactions on Signal Processing, (03 2011): 0. doi: 10.1109/TSP.2010.2092772

TOTAL: 3

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

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TOTAL:

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Received		Paper
2012/02/06 2:	8	Meisam Razaviyayn, Maziar Sanjabi Boroujeni, Zhi-Quan Luo. Linear transceiver design for interference alignment: Complexity and computation, 2010 IEEE 11th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2010). 2010/06/19 01:00:00, Marrakech, Morocco. : ,
2012/02/06 2:	7	Enbin Song, Qingjiang Shi, Maziar Sanjabi, Ruoyu Sun, Zhi-Quan Luo. Robust SINR-constrained MISO downlink beamforming: When is semidefinite programming relaxation tight?, ICASSP 2011 - 2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). 2011/05/21 01:00:00, Prague, Czech Republic. : ,
2012/02/06 2·	3	Ya-Feng Liu, Yu-Hong Dai, Zhi-Quan Luo. Max-Min Fairness Linear Transceiver Design for a Multi-User MIMO Interference Channel, ICC 2011 - 2011 IEEE International Conference on Communications. 2011/06/04 01:00:00, Kyoto, Japan. : ,

TOTAL: 3

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

1. 2010 Farkas Prize from the Institute for Operations Research and the Management Sciences, for outstanding contributions to the field of optimization.

2. SIAM Fellow, 2011, for the development of novel applied mathematics ideas and methods for signal processing and digital communication.
3. 2011 IEEE Signal Processing Society Best Paper Award for the paper: Sidiropoulos, N.D., Davidson, T.N., and Luo, Z.-Q.,
``Transmit Beamforming for Physical Layer Multicasting," IEEE Transactions on Signal Processing, Vol. 54, No. 6, pp. 2239--2251, 2006.
4. 2011 EURASIP Best Paper Award, for the paper: Luo, Z.-Q. and Pang, J.-S., ``Analysis of Iterative Waterfilling Algorithm for Multiuser Power Control in Digital Subscriber Lines",
published in 2006 in the EURASIP Journal on Applied Signal Processing.
5. Semi-plenary speaker, 2011 IEEE joint CDC-ECC conference.
6. Best Paper Award, 2011 IEEE International Conference on Communications, for the paper: Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, ``Max-Min Fairness Linear Transceiver Design for a Multi-User MIMO Interference Channel".
7. 2009 IEEE Signal Processing Society Best Paper Award for the paper: Luo, Z.-Q. and Zhang, S.,
``Dynamic Spectrum Management: Complexity
and Duality," IEEE Journal of Selected Topics in Signal Processing,
Special Issue on Signal Processing and Networking for Dynamic Spectrum Access, Vol. 2, No. 1, pp. 57--73, February 2008.

Graduate Students

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Meisam Razaviyayn	0.50	
Maziar Sanjalbi	0.50	
Yao Huang	0.50	
FTE Equivalent:	**2.00**	
Total Number:	**4**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
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FTE Equivalent:	**1.00**
Total Number:	**1**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	2.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......	2.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:......	1.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):	1.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	1.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME
Alireza Razavi
Yao Huang

Total Number: 2

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Summary of the most important results

1. Distributed optimization in an energy constrained sensor network

2. On the Degrees of Freedom Achievable Through Interference Alignment in a MIMO Interference Channel

3. Linear Transceiver Design for Interference Alignment: Complexity and Computation

4. Coordinated Beamforming for MISO Interference Channel: Complexity Analysis and Efficient Algorithms

5. A Generalized IterativeWater-filling Algorithm for Distributed Power Control in the Presence of a Jammer

6. An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel

Technology Transfer

ARO Final Report
August 2012

Complexity Analysis and Algorithms for Optimal Resource Allocation in Wireless Networks

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Contents

1 Statement of the problem studied

This project considers the dynamic spectrum management (DSM) problem whereby multiple users sharing a common frequency band must choose their transmit power spectra jointly in response to physical channel conditions including the effects of interference. The goal of the users is to maximize a system-wide utility function (e.g., weighted sum-rate of all users), subject to individual power constraints. The proposed work will focus on a general DSM problem formulation which allows correlated signaling rather than being restricted to the conventional independent orthogonal signaling such as OFDM. The general formulation will exploit the concept of 'interference alignment' which is known to provide substantial rate gain over OFDM signalling for general interference channels. Three theoretic aspects of the DSM problem will be studied:

- use optimization theory to analyze the structure of optimal spectrum sharing policies. Game theory will be used to analyze the impact of a hostile jammer to the optimal power allocation strategies and system performance.
- use computational complexity theory to characterize tractability of computing optimal spectrum sharing policies, and identify sub-classes which are polynomial time solvable.

2 Summary of the most important results

The problem of optimal resource allocation in interference limited networks is studied. Significant progress has been made on several fronts:

2.1 Distributed optimization in an energy constrained sensor network

We introduced the concept of estimation diversity in the context of distributed sensing and established its tradeoff with energy efficiency and have shown that for distributed optimization in an energy-constrained network, digital communication scheme is far more energy efficient than analog communication scheme. The gap in communication energy consumption can be exponential.We consider a distributed optimization problem where n nodes, S_l, $l \in \{1, \ldots, n\}$, jointly minimize a common strongly convex function $f(\mathbf{x})$, $\mathbf{x} = [x_1, \ldots, x_n]^T$, and suppose that node S_l only has control of variable x_l. The nodes locally update their respective variables and periodically exchange their values over noisy channels. Previous studies of this problem have mainly focused on the convergence issue and the analysis of convergence rate. In this work, we focus on the communication energy and study its impact on convergence. In particular, we study the minimum amount of communication energy required for nodes to obtain an ϵ-minimizer of $f(\mathbf{x})$ in the mean square sense. We consider both analog and digital communication schemes. For the former, we study a class of distributed stochastic gradient type algorithms implemented using certain linear analog messaging schemes. Our analysis shows that the communication energy to obtain an ϵ-minimizer of $f(\mathbf{x})$ must grow at least at the rate of ϵ^{-1}. We derive a specific design which attains this minimum energy bound within a factor of at most 3 to the minimum communication energy for convex quadratic functions. For digital communication scheme, we introduce a distributed algorithm based on gradient projection method which requires $\mathcal{O}(\log \epsilon^{-1})^3$ communication energy. Furthermore, the algorithm provided for the digital communication scheme converges linearly compared with the algorithm for the analog communication scheme which has a sub-linear convergence rate (Figure 1). Thus, asymptotically digital communication schemes are far more energy efficient than analog communication schemes for distributed optimization. This does not mean that the digital framework outperforms the analog framework in terms of energy for any value of mean squared error. For example, in our simulation, the analog framework requires less energy to obtain an ϵ-minimizer of $f(\mathbf{x})$ compared with digital framework for $\epsilon \geq 10^{-6}$ (Figure 2). For value $\epsilon \leq 10^{-6}$, the digital framework consumes less energy than the analog framework.

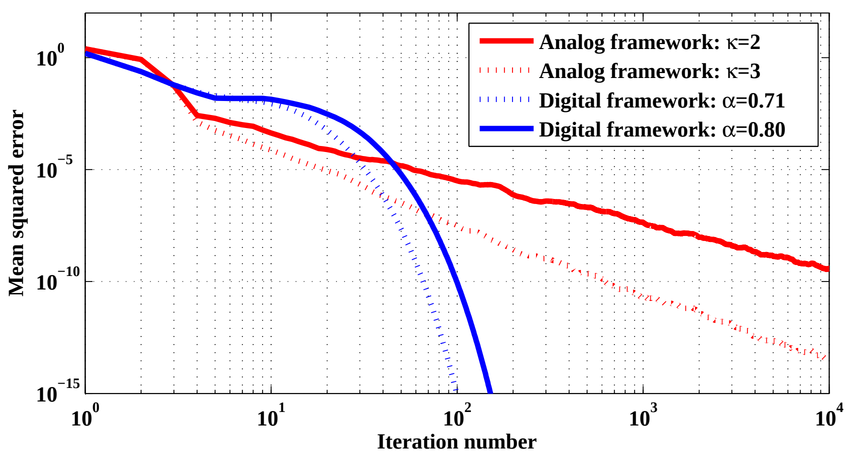


Figure 1. Mean squared error versus iteration number

2.2 On the Degrees of Freedom Achievable Through Interference Alignment in a MIMO Interference Channel

Consider a K user MIMO interference fading channel where each transmitter and receiver are equipped with M and N antennas respectively. If channel extension is allowed and is statistically independent across frequency, the recent work of Cadambe and Jafar suggests that the total achievable degrees of freedom can be maximized via interference alignment, and it grows linearly with K for fixed M and N. In this paper we establish a general upper bound on the total degrees of freedom achievable by any interference alignment scheme for a

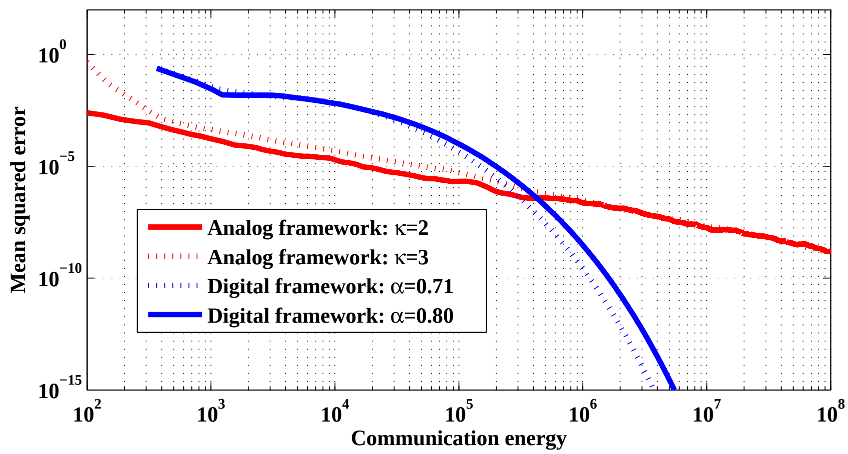


Figure 2. Mean squared error versus communication energy

MIMO interference channel without channel extension. This upper bound implies that the total achievable degrees of freedom cannot grow linearly with K, and is in fact strictly less than $M + N$. We also show by example that this upper bound is in fact tight.

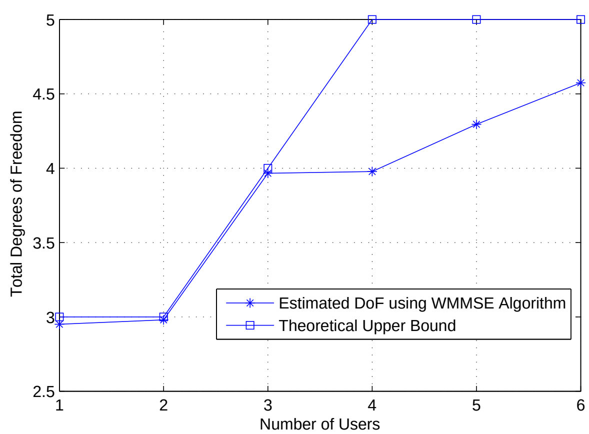


Figure 3. Achievable DoF and theoretical upper bound

2.3 Linear Transceiver Design for Interference Alignment: Complexity and Computation

Consider a multiple input-multiple output (MIMO) interference channel whereby each transmitter and receiver are equipped with multiple antennas. An effective approach to practically achieving high system throughput is to deploy linear transceivers (or beamformers) that can optimally exploit the spatial characteristics of the channel. The recent work of Cadambe and Jafar suggests that optimal beamformers should maximize the total degrees of freedom and achieve interference alignment in the high signal to noise ratio (SNR) regime. In this paper we first consider the interference alignment problem without channel extension and prove that the problem of maximizing the total achieved degrees of freedom for a given MIMO interference channel is NP-hard. Furthermore, we show that even checking the achievability of a given tuple of degrees of freedom for all receivers is NP-hard when each receiver is equipped with at least three antennas. Interestingly, the same problem becomes polynomial time solvable when each transmit/receive node is equipped with no more than two antennas. We also propose a distributed algorithm for transmit covariance matrix design that does not require the DoF tuple pre-assignment, under the assumption that each receiver uses a linear MMSE beamformer. The simulation results show that the proposed algorithm outperforms the existing interference alignment algorithms in terms of system throughput.

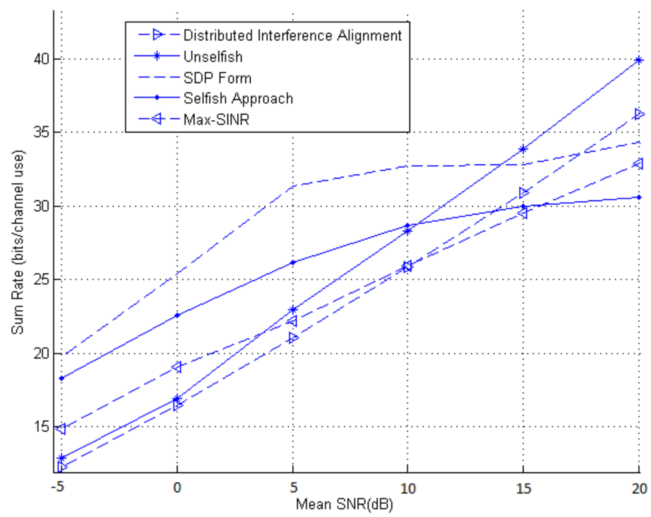


Figure 4. Sum-rate vs. SNR: $K = 4, M = 3, d = 1$

2.4 Coordinated Beamforming for MISO Interference Channel: Complexity Analysis and Efficient Algorithms

In a cellular wireless system, users located at cell edges often suffer significant out-of-cell interference. Assuming each base station is equipped with multiple antennas, we can model this scenario as a multiple-input single-output (MISO) interference channel. In this paper we consider a coordinated beamforming approach whereby multiple base stations jointly optimize their downlink beamforming vectors in order to simultaneously improve the data rates of a given group of cell edge users. Assuming perfect channel knowledge, we formulate this problem as the maximization of a system utility (which balances user fairness and average user rates), subject to individual power constraints at each base station. We show that, for the single carrier case and when the number of antennas at each base station is at least two, the optimal coordinated beamforming problem is NP-hard for both the harmonic mean utility

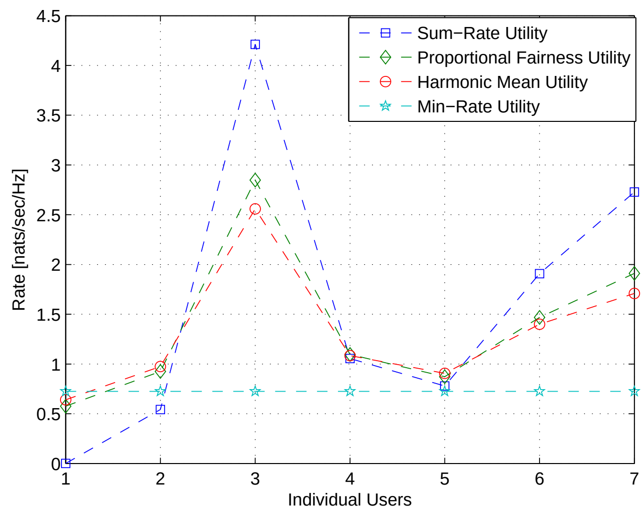


Figure 5. Individual rate distribution with $K = 7, L = 4$ and $P = 30$ dBm.

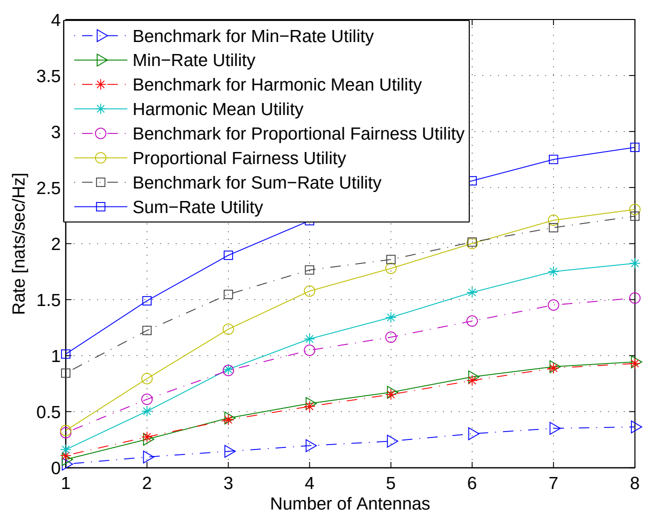


Figure 6. Performance comparison of coordinated beamformers and space matched beamformers with $K = 7, P = 30$ dBm.

and the proportional fairness utility. For general utilities, we propose a cyclic coordinate descent algorithm, which enables each transmitter to update its beamformer locally with limited information exchange, and establish its global convergence to a stationary point. We illustrate its effectiveness in computer simulations by using the space matched beamformer as the benchmark.

2.5 A Generalized Iterative Water-filling Algorithm for Distributed Power Control in the Presence of a Jammer

Consider a scenario in which K users and a jammer share a common spectrum of N orthogonal tones. Both the users and the jammer have limited power budgets. The goal of each user is to allocate its power across the N tones in such a way that maximizes the total sum rate that he/she can achieve, while treating the interference of other users and the jammer's signal as additive Gaussian noise. The jammer, on the other hand, wishes to allocate its power in such a way that minimizes the utility of the whole system; that being the total sum of the rates communicated over the network. For this non-cooperative game, we propose a generalized version of the existing iterative water-filling algorithm whereby the users and the jammer update their power allocations in a greedy manner. We study the existence of a Nash equilibrium of this non-cooperative game as well as conditions under which the generalized iterative water-filling algorithm converges to a Nash equilibrium of the game. The conditions that we derive in this paper depend only on the system parameters, and hence can be checked *a priori*. Simulations show that when the convergence conditions are violated, the presence of a jammer can cause the, otherwise convergent, iterative water-filling algorithm to oscillate.

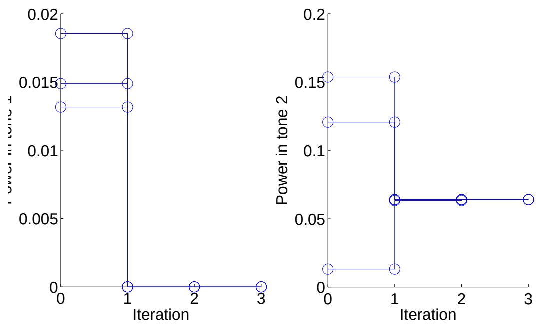
2.6 An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel

Consider the MIMO interfering broadcast channel whereby multiple base stations in a cellular network simultaneously transmit signals to a group of users in their own cells while causing interference to each other. The basic problem is to design linear beamformers that can maximize the system throughput. In this paper we propose a linear transceiver design algorithm for weighted sum-rate maximization that is based on iterative minimization of weighted mean squared error (MSE). The proposed algorithm only needs local channel knowledge and converges to a stationary point of the weighted sum-rate maximization problem. Furthermore, the algorithm and its convergence can be extended to a general class of sum-utility maximization problem. The effectiveness of the proposed algorithm is validated by numerical experiments.

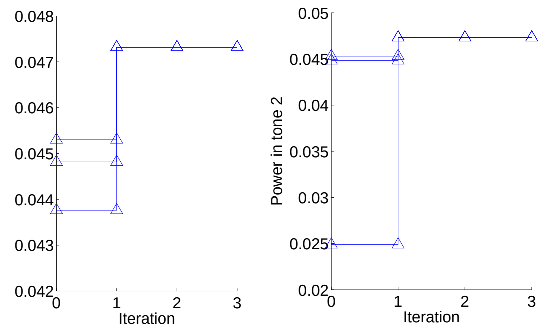
3 Published papers

1. Razavi, A., Zhang, W. and Luo, Z.-Q., "Distributed Optimization in an Energy-constrained Network: Analog Versus Digital Communication Schemes," Accepted for publication in *IEEE Transactions on Information Theory*, 2012.

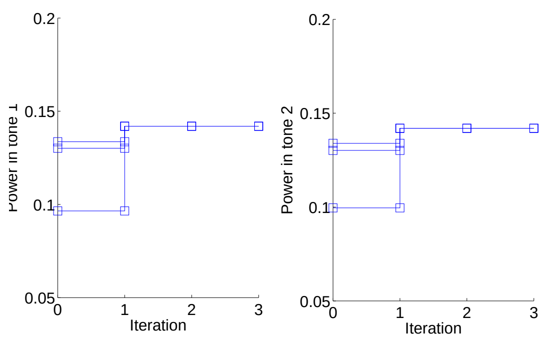
2. Razaviyayn, M., Lyubeznik, G. and Luo, Z.-Q., "On the Degrees of Freedom Achievable



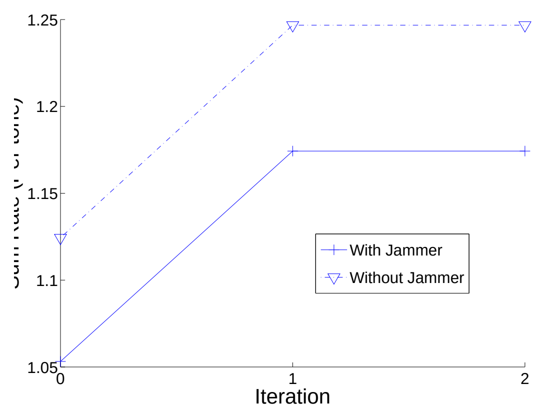
(a) Power allocations of User 1



(b) Power allocations of User 2



(c) Power allocations of the jammer



(d) Sum rate with and without a jammer

Figure 7. The power allocations of Users 1 and 2 are marked by 'o' and '△', respectively, whereas the power allocations of the jammer is marked by '□'. The GIWFA iterates converge to a unique Nash equilibrium irrespective of the initial power allocation.

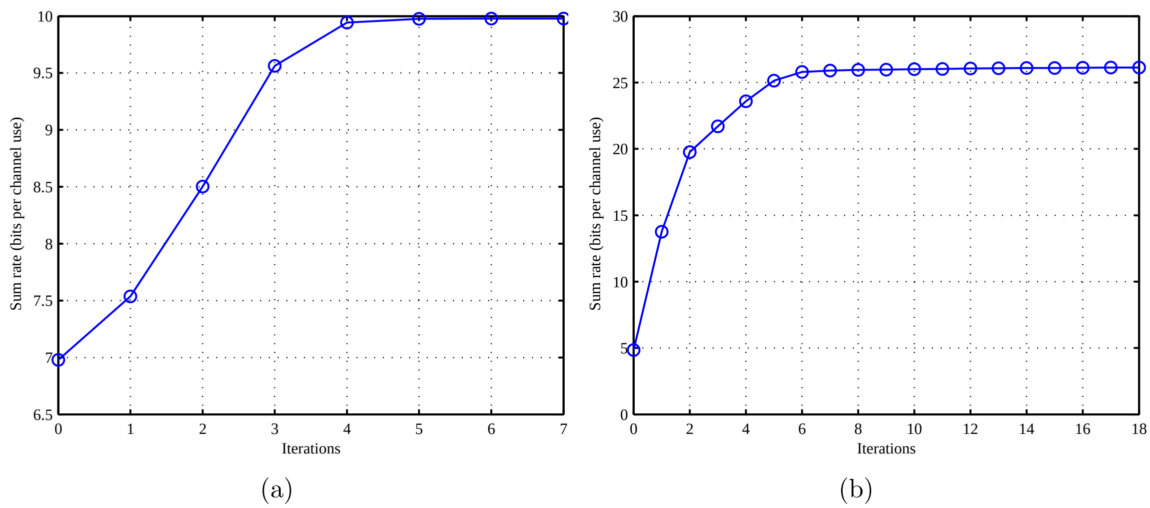


Figure 8. Convergence examples of the WMMSE algorithm: (a) SISO-IFC, $K = 3$, $\epsilon = 1e - 3$; (b) MIMO-IFC, $K = 4$, $T = 3$, $R = 2$, $\epsilon = 1e - 2$.

Through Interference Alignment in a MIMO Interference Channel," *IEEE Transactions on Signal Processing*, Vol. 60, pp. 812–821, February 2012.

3. Shi, Q.J., Razaviyayn, M., He, C. and Luo, Z.-Q., "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Transactions on Signal Processing,* Vol. 59, pp. 4331–4340, March 2011.

4. Razaviyayn, M., Sanjabi, M. and Luo, Z.-Q., "Linear Transceiver Design for Interference Alignment: Complexity and Computation," Accepted for publication in *IEEE Transactions on Information Theory*, 2011.

5. Liu, Y.-F., Dai, Y.-H. and Luo, Z.-Q., "Coordinated Beamforming for MISO Interference Channel: Complexity Analysis and Efficient Algorithms," *IEEE Transactions on Signal Processing,* Vol. 59, No. 3, pp. 1142 –1157, 2011.

6. Gohary, R., Huang, Y., Luo, Z.-Q. and Pang, J.-S., "A Generalized Iterative Water-filling Algorithm for Distributed Power Control in the Presence of a Jammer," *IEEE Transactions on Signal Processing*, Vol. 57, No. 7, pp. 2660–2674 July, 2009.