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GRRC Technical Report 2011-01

Comparison of Workspace Reduction Between Kinematically Redundant Serial and Parallel Manipulators Under Joint Failures

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Date Completed: March 7, 2011

UNCLASSIFIED: Dist. A. Approved for Public Release

Report Documentation Page

Form Approved
OMB No. 0704-0188

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1. REPORT DATE 28 FEB 2011	2. REPORT TYPE Technical Report	3. DATES COVERED 06-09-2010 to 11-01-2011	
4. TITLE AND SUBTITLE GRRC Technical Report 2011-01 Comparison of Workspace Reduction Between Kinematically Redundant Serial and Parallel Manipulators Under Joint Failures		5a. CONTRACT NUMBER W56H2V-04-2-0001	
		5b. GRANT NUMBER	
		5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Brandon Moore		5d. PROJECT NUMBER	
		5e. TASK NUMBER	
		5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) GROUND ROBOTICS RESEARCH CENTER, UNIVERSITY OF MICHIGAN, 2350 HAYWARD STREET, ANN ARBOR, MI, 48109-2125		8. PERFORMING ORGANIZATION REPORT NUMBER ; #21526	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000		10. SPONSOR/MONITOR'S ACRONYM(S) TARDEC	
		11. SPONSOR/MONITOR'S REPORT NUMBER(S) #21526	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited			
13. SUPPLEMENTARY NOTES Submitted to Ground Robotics Reliability Center			
14. ABSTRACT The purpose of this study is to do a basic evaluation of the usefulness of various kinematically redundant planar manipulators when a seizure of an actuated joint results in the loss of a degree of freedom. Two serial (single kinematic chain) manipulators, one planar parallel kinematic machine (PKM), and one spatial PKM are considered. This study involves only kinematics and uses the size of the reduced workspace under a joint failure as a measure of a manipulator's useful redundancy.			
15. SUBJECT TERMS			
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified	Public Release
			18. NUMBER OF PAGES 18
			19a. NAME OF RESPONSIBLE PERSON

Table of Contents

1	Introduction	3
2	Methodology	4
3	Planar Manipulators	5
	3.1 Serial Manipulators	5
	3.2 Parallel Kinematic Machines	5
4	Kinematic Analysis of Planar Manipulators	7
	4.1 Serial Manipulators	7
	4.2 Parallel Kinematic Machines	9
5	Spatial Manipulators	11
6	Conclusions	14

1 Introduction

The purpose of this study is to do a basic evaluation of the usefulness of various kinematically redundant planar manipulators when a seizure of an actuated joint results in the loss of a degree of freedom. Two serial (single kinematic chain) manipulators, one planar parallel kinematic machine (PKM), and one spatial PKM are considered. This study involves only kinematics and uses the size of the reduced workspace under a joint failure as a measure of a manipulator's useful redundancy.

Redundant manipulators have been studied in many contexts. One of the primary advantages of kinematically redundant manipulators (particularly serial ones) is the ability to reach a given pose from many configurations, thereby giving the robot some freedom to avoid collision with obstacles in the environment or to optimize manipulator trajectories (e.g., [15]). Another advantage is the ability to avoid singularities in the manipulator's workspace. Singularities are troublesome in serial manipulators and designs that remove them are beneficial (e.g., [12]) but they are particularly important in PKMs with type II singularities [7] where the manipulator loses rigidity due to a particular alignment of the passive joints [9]. Using kinematically redundant manipulators can also provide an extra degree of freedom in design and allow for a larger workspace than would be possible in a non-redundant manipulator [6].

In addition to the issues of kinematic redundancy, there sometimes exists an advantage to actuation redundancy. In a serial manipulator this would simply be the uninteresting case of coupling two motors to a single joint through a differential gear, but for PKMs usually means the actuation of a normally passive joint or the addition of an extra limb [8] which requires carefully coordinated control of those joints in order to mitigate internal forces within the manipulator structure. This sort of redundancy, however, can lead to increased stiffness of the manipulator and therefore higher performance in terms of positioning accuracy and/or acceleration [3].

An aspect of redundant manipulators which has not received much attention to date is the study of situations in which one of the manipulator's joints has failed. This was done to some extent with serial manipulators in [10] for the case of a seized joint and in [4] for the case of a normally actuated joint swinging free. For industrial robots that operate in a controlled environment and do not experience regular failures, this may not be of particular interest, but for field robots operating outside and often in extreme conditions of dirt, moisture, and temperature, failures of the various components are inevitable [1]. For robots performing time critical missions in situations where they cannot easily be replaced or repaired, it would be a distinct advantage if a manipulator could still perform useful work despite the failure of one component. There are a number of difficulties in studying these situations, however. For example, just because a planar manipulator has three degrees of freedom does not mean that if it loses one degree of freedom that the remaining two will provide enough useful functionality to accomplish similar tasks. The question of usefulness is itself highly subjective and, while we attempt to provide some quantitative measures of this property, in the end the comparison between two alternate manipulator designs is highly dependent on the purpose of the specific robot in question.

2 Methodology

For the purposes of comparison, this study considered a simple workspace and measured how much of this workspace could be reached by each manipulator under likely seized joint failures. The choice of workspace is highly dependent on the tasks the manipulator will perform, but for this study we chose a couple of fairly generic workspaces. For planar manipulators we choose a workspace that consists of the upper right quadrant of a disc with a radius of one meter and centered at the base of the manipulator, i.e., the set

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0\} \quad (1)$$

For spatial manipulators we choose a workspace that consists of a quadrant of a hemisphere with a radius of one meter that is symmetric about the forward (i.e., x) axis, i.e., the set

$$\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1, x \geq 0, |y| \leq x, z \geq 0\} \quad (2)$$

These seemed an obvious choice for generic manipulators expected to perform tasks in a space to their front and above ground (as would normally be the case in a mobile field robot).

Because the methodology outlined below uses numerical evaluation, we discretized this workspace into a manageable number of points. For the planar manipulators the workspace consists of all points on a uniform grid with 1 mm spacing that satisfy (1). For the spatial manipulator the workspace consists of all points on a uniform grid with 10 mm spacing that satisfy (2). For this study we will only consider the position of the end effector and not its orientation, which is a reasonable assumption since manipulators on field robots usually have a wrist mechanism at the end effector to achieve the desired orientation.

With the desired workspace defined, we evaluated the manipulators using the following procedures:

1. Determine joint values used under normal operation: In order to avoid biasing the results by evaluating joint failures at positions the manipulator is unlikely to experience, we first determine the range of joint values necessary to reach every (discretized point) in the desired workspace. The particular inverse kinematic algorithm used for each manipulator is described in more detail in Sections 3 and 5.

2. Determine workspace of manipulator under all possible single joint failures: In this step we consider a single joint failure for each joint. Using the range of normal joint values determined in the previous step, we lock each joint at different values and test to see which of the points in the desired workspace the manipulator can still reach.

3. Evaluate overall metrics: Here we generate the following measures of the usefulness of the manipulator under a single fixed joint failure.

(a) Histogram for each joint showing the likelihood of reaching a given point in the desired workspace under a failure of that joint (assuming an equal probability of failing at each of the tested joint values).

(b) Plot showing the region of the desired workspace that the manipulator can reach for *any* of the tested failures (i.e., the set of workspace points that the manipulator can always reach).

(c) Plot of the percentage of the desired workspace that is reachable versus the position at which a joint failed.

3 Planar Manipulators

The following planar manipulators were tested for this study:

3.1 Serial Manipulators

For planar serial manipulators we considered just the standard three link and four link manipulators shown in Figure 3.1. In order to treat all manipulators fairly, their physical parameters were chosen so that they are just able to reach the edge of the desired workspace. Thus the three link manipulator has link lengths of one third of a meter each while the four link version has link lengths of one fourth of a meter each.

The forward and inverse kinematic equations for planar serial manipulators of this type are quite simple and are available in any robotics textbook and so they are omitted here. Since the manipulators are redundant there is an infinite set of joint angles that will place the end effector at a specified point and so we made the following choices. For the fully functional three link manipular we calculated the inverse kinematic solution in order to minimize the deflection of joint two. For the four link manipulator we simply chose to set the angles for joints one and three to zero. With these choices in place these manipulators can be treated as two link planar manipulators for which the inverse kinematic equations are simple and straight forward (e.g., see [14]). Using these methods, all the points in the desired workspace can be reached using joint angles (in degrees) of $\theta_1 \in [0, 150]$, $q_2 \in [-120, 0]$, and $q_3 \in [-180, 0]$ for the three link manipulator, and $\theta_1 \in [0, 180]$ and $\theta_3 \in [-180, 0]$ for the four link manipulator.

The inverse kinematic solutions for these serial manipulators under a stuck joint failure are also straight forward. The three link and four link manipulators effectively become two link and three link manipulators, respectively, for which we already have inverse kinematic solutions in place.

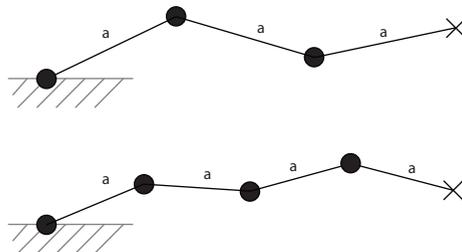


Figure 1: Three link version and four link serial manipulators. Black circles are actuated revolute joints. The x marks the position of the end effector. Joints are numbered starting at the base. For the three link manipulator $a = \frac{1}{3}$ m and for the four link manipulator $a = \frac{1}{4}$ m.

3.2 Parallel Kinematic Machines

The primary PKM configuration used in this study appears in Figure 2. It consists of three legs attached to both the base and to a secondary arm by passive revolute joints. This

manipulator appears in [11] (without the secondary arm). The first joints of the limbs are collinear, the end joints of the limbs are also collinear, and the distance between the two is directly determined by the extension of actuated prismatic joints. For this manipulator the

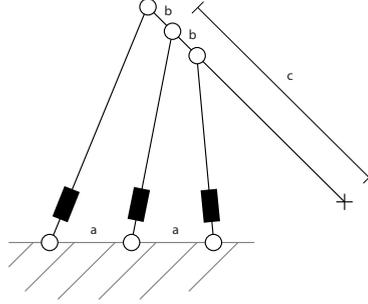


Figure 2: Three legged planar PKM. White circles are passive revolute joints. Black rectangles are actuated prismatic joints. The x marks the position of the end effector. Joints 1, 2, and 3 are the left, middle and right legs respectively. $a = 0.1\text{m}$, $b = 0.05\text{m}$, and $c = 0.5\text{m}$.

relationship between the end effector position (x, y) and orientation θ and the lengths of the adjustable legs q_1, q_2 , and q_3 is given by

$$(x - (b + c) \cos \theta + a)^2 + (y - (b + c) \sin \theta)^2 = q_1^2 \quad (3)$$

$$(x - c \cos \theta)^2 + (y - c \sin \theta)^2 = q_2^2 \quad (4)$$

$$(x - (c - b) \cos \theta - a)^2 + (y - (c - b) \sin \theta)^2 = q_3^2 \quad (5)$$

For this study we used the following dimensions for this manipulator: $a = 0.1\text{m}$, $b = 0.05\text{m}$, and $c = 0.5\text{m}$. The position inverse kinematics for the fully functional manipulator are calculated by setting the middle leg to $q_2 = 0.5\text{m}$ so that the manipulator can just reach the extreme edge of the workspace at it's full extension. (We note that there are singularity problems with this when the manipulator tries to reach it's furthest extension, i.e., it would require an infinite holding force, but at this point we are only concerned with making some kinematic calculations to determine the potential for various manipulators.) With this choice, we can effectively treat this PKM as a two-link planar manipulator (middle leg and secondary arm) to come up with an inverse kinematic solution for the leg lengths q_1 and q_3 . Using this method this PKM can reach any point in the desired workspace with leg lengths $q_1, q_3 \in [0.42, 0.58]$. As for failures, if either leg one or three becomes stuck, we can then calculate the inverse kinematic solution in a similar manner using the two remaining legs.

An alternate design which replaces the prismatic joints with revolute ones and moves the actuated joints to the base of the manipulator is shown in Figure 3. This is a version of the common 3DOF planar manipulator of [6] in which the joints at the base and on the secondary arm are collinear. The inverse kinematic equations for this manipulator can be derived trigonometrically using the law of cosines as

$$q_i = \text{atan2}(Y_i, X_i) \pm \arccos \left(\frac{A_i^2 + D_i^2 - B_i^2}{2A_i D_i} \right) \quad (6)$$

where

$$X_i = x - c \cos \theta - (a - b \cos \theta)(i - 2) \quad (7)$$

$$Y_i = y - c \sin \theta + b \cos \theta(i - 2) \quad (8)$$

$$D_i = \sqrt{X_i^2 + Y_i^2} \quad (9)$$

Using the design parameters $a = 0.1\text{m}$, $b = 0.05\text{m}$, $c = 0.5\text{m}$, $A_1 = A_3 = B_1 = B_3 = 0.29\text{m}$, and $A_2 = B_2 = 0.25\text{m}$ and by treating the manipulator like a two link serial manipulator (i.e., middle leg links at full extension) it can reach the entire workspace with range of joint angles (in degrees) of $q_1 \in [27, 180]$, $q_2 \in [0, 156]$, and $q_3 \in [62, 180]$. The inverse kinematic solution for this manipulator uses the same equations but determines the orientation angle θ necessary based on the position of the seized joint.

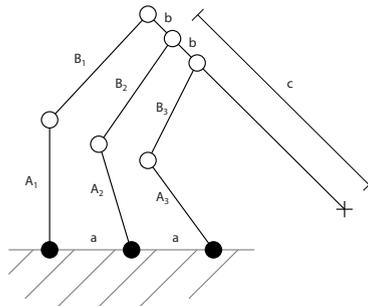


Figure 3: Three legged planar PKM. White and black circles are passive and actuated revolute joints, respectively. The x marks the position of the end effector. Joints 1, 2, and 3 are the left, middle and right legs respectively. $a = 0.1\text{m}$, $b = 0.05\text{m}$, $c = 0.5\text{m}$. $A_1 = B_1 = A_3 = B_3 = 0.29\text{m}$, and $A_2 = B_2 = 0.25\text{m}$.

4 Kinematic Analysis of Planar Manipulators

4.1 Serial Manipulators

Figure 4 shows histograms of how likely the three link serial manipulator will be able to reach a given point in the workspace for failures of each joint. There is no point in the discretized workspace that can be reached regardless of which joint fails and at what position due to the case where joint three is stuck in the fully collapsed position of $q_3 = -180$ degrees. (There is a set of points in the continuous workspace that is one third of meter away from the origin, but this set is of zero measure). Figure 5 shows the percentage of the desired workspace that can be reached for particular joint failures. The best case failure scenario for this manipulator is when joint one fails at $q_1 = 45$ degrees (reaching approximately 90.7% of the workspace). This makes intuitive sense as the first joint in a serial manipulator has the biggest impact on the position of the end effector and this failure places the end of the stuck first link in the middle of the desired workspace. As expected, the four link serial manipulator fares much better than its three link counterpart. Figure 6 shows histograms of how likely the four link

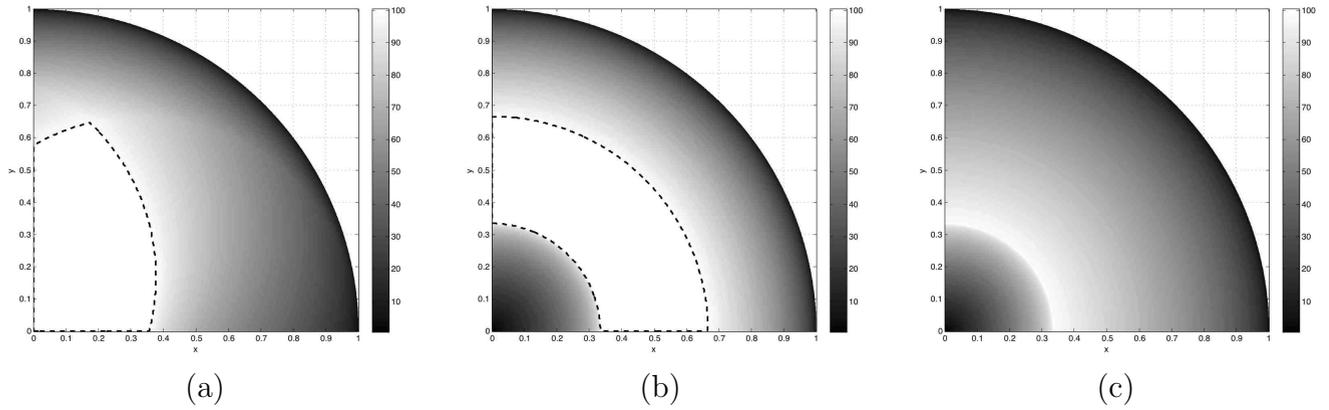


Figure 4: Histogram of the percentage of joint failure positions for which the three link planar manipulator can still reach a given point in the desired workspace. Dashed lines enclose the regions that are reachable for all the considered failures. Histogram (a) corresponds to failures of joint one and (b) to failures of joint two and (c) to failures of joint three. The manipulator is able to reach 26.5% and 33.3% and 0% of the workspace points for any of the considered failures of joint one, two, and three respectively.

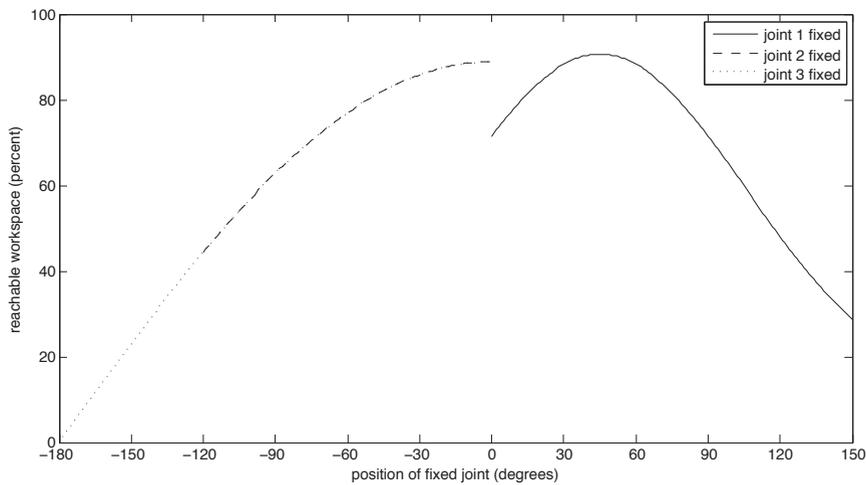


Figure 5: Percent of the desired workspace that is reachable by the three link manipulator versus the position at which a joint is fixed (in degrees). Solid line is for joint one fixed, dashed line is for joint two fixed, and dotted line is for joint three fixed.

planar manipulator will be able to reach a given point in the workspace for failures of joints one and three. Part (c) of this figure maps those points of the discretized workspace that can be reached regardless of which joint fails and where (approximately 25.0% of the workspace). As shown in Figure 7, the four link manipulator also outperforms the three link manipulator in that there is at least one failure ($q_3 = 0$ degrees) for which the entire workspace is still reachable. In addition, the worst case scenario of joint three stuck at $q_3 = -180$ degrees still leaves approximately 25.0% of the workspace reachable.

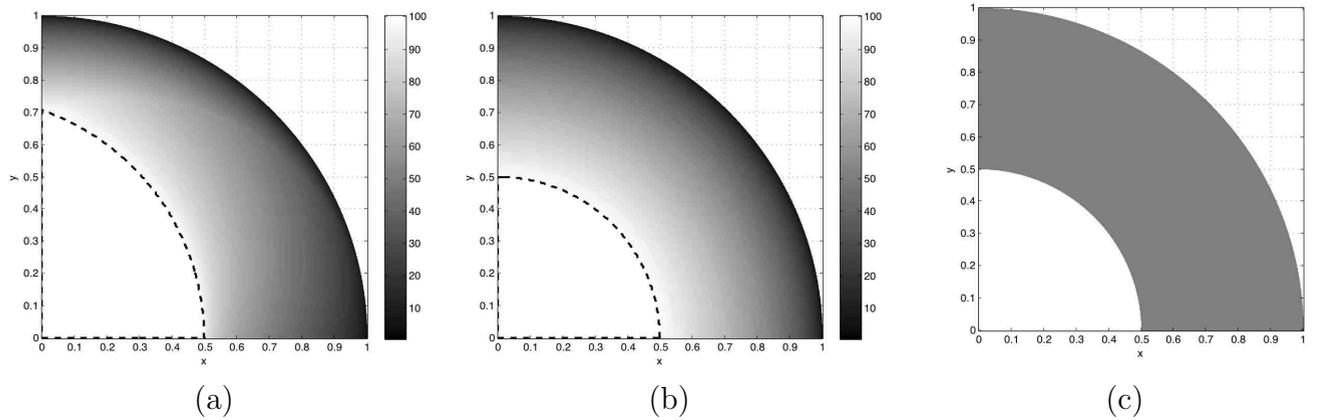


Figure 6: Histogram of the percentage of joint failure positions for which the four link planar manipulator can still reach a given point in the desired workspace. Dashed lines enclose the regions that are reachable for all the considered failures. Histogram (a) corresponds to failures of joint one and (b) to failures of joint three. The manipulator is able to reach 32.9% and 25.0% of the workspace points for any of the considered failures of joints one and three respectively. The white region of the workspace shown in (c) is reachable under any of the considered failures and represents 25.0% of the desired workspace points.

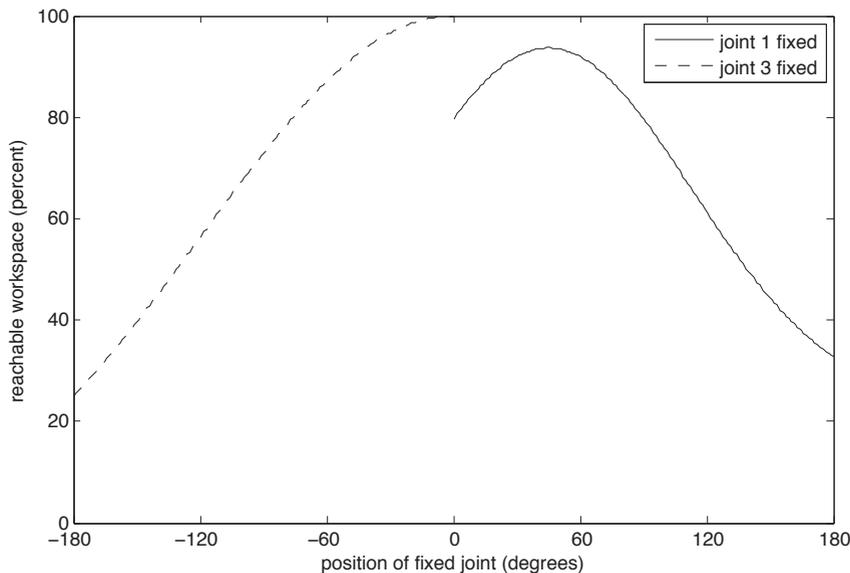


Figure 7: Percent of the desired workspace that is reachable by the four link manipulator versus the position at which a joint is fixed (in degrees). Solid line is for joint one fixed and the dashed line is for joint three fixed.

4.2 Parallel Kinematic Machines

In the terms of this study, the first PKM we considered appears to drastically outperform the serial manipulators in terms of useful kinematic redundancy. Figure 8 shows histograms

of the likelihood this manipulator will reach a given point in the workspace under failures of joints one and three. At least 81.6% of the desired workspace remains reachable for a failure of each joint considered individually, and, as shown in part (c) of that figure, 75.6% of the discretized workspace can be reached regardless of which joint fails and where. The PKM also performs better in terms of best and worst case failure scenarios. There are a significant set of joint failures $q_1 \in [0.55, 0.58]$ m for which the entire desired workspace is still reachable. In the worst case joint failure of $q_1 = 0.42$ m, approximately 81.6% of the workspace is still reachable.

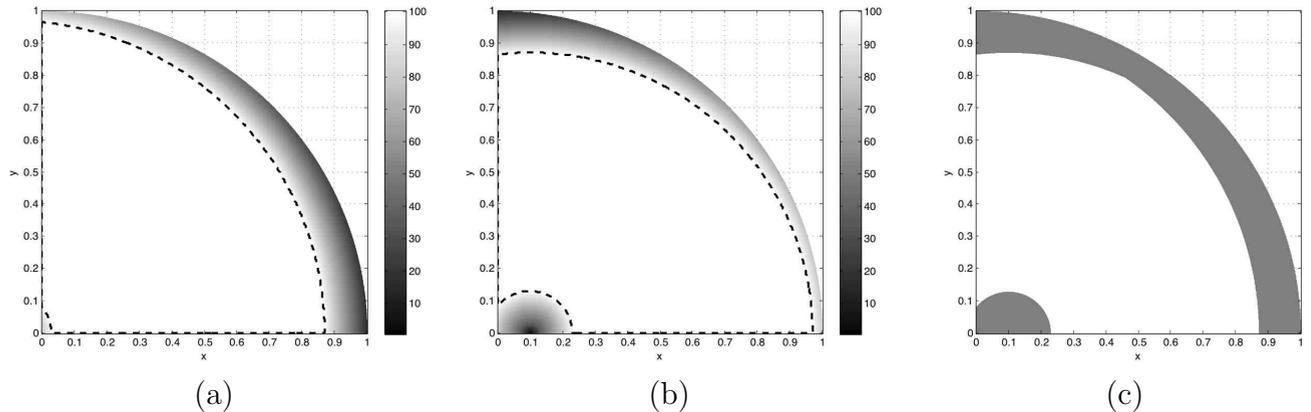


Figure 8: Histogram of the percentage of joint failure positions for which the three legged planar PKM can still reach a given point in the desired workspace. Dashed lines enclose the regions that are reachable for all the considered failures. Histogram (a) shows failures of joint one and (b) shows failures of joint three. The manipulator is able to reach 81.6% and 83.6% of the workspace points for any of the considered failures of joint one and joint three respectively. The white region of the workspace shown in (c) is reachable under all the considered joint failures. This represents approximately 75.6% of the desired workspace points.

Since the use of prismatic joints is sometimes problematic due to a limited range of extension, we note here that with the exception of a very small number of outliers, the leg lengths calculated to reach these reduced workspaces all fell between 0.25m and 0.8m, with most much closer to 0.5m. This is a somewhat large range, but not unreasonable depending on how the leg extension is accomplished mechanically.

The PKM design with all revolute joints did fairly poor compared to the design with extendable legs. Although there were some failures which left the manipulator with a large workspace, none of the workspace points are guaranteed to be reachable regardless of which failure occurs. The best case scenario occurs when joint 2 is stuck at 74 degrees and leaves 91.7% of the workspace points still reachable, which is relatively good, but the worst case scenario of joint 1 stuck at 180 degrees leaves only 13.1% of the workspace reachable. We can attribute this poor performance to the much higher loss of mobility resulting from a stuck revolute joint but also to moving the actuated joint to the base. On the other hand, if the joint in the middle of each leg was actuated, then this manipulator would be functionally equivalent to the one with extendable legs (i.e., position of the joint varies the distance between the base and the secondary arm) and so would have similar performance.

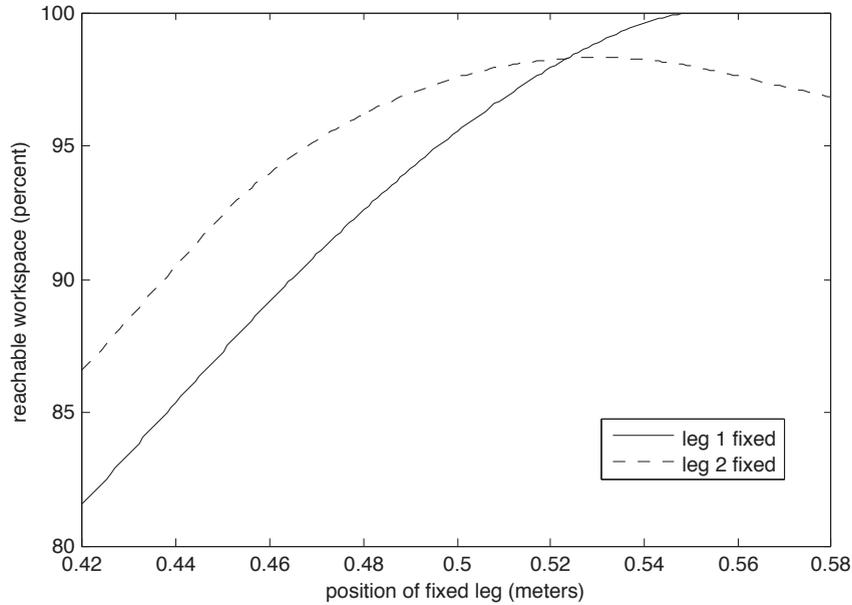


Figure 9: Percent of the desired workspace that is reachable by the PKM versus the position at which a joint is fixed. Solid line is for joint one fixed, dashed line is for joint three fixed.

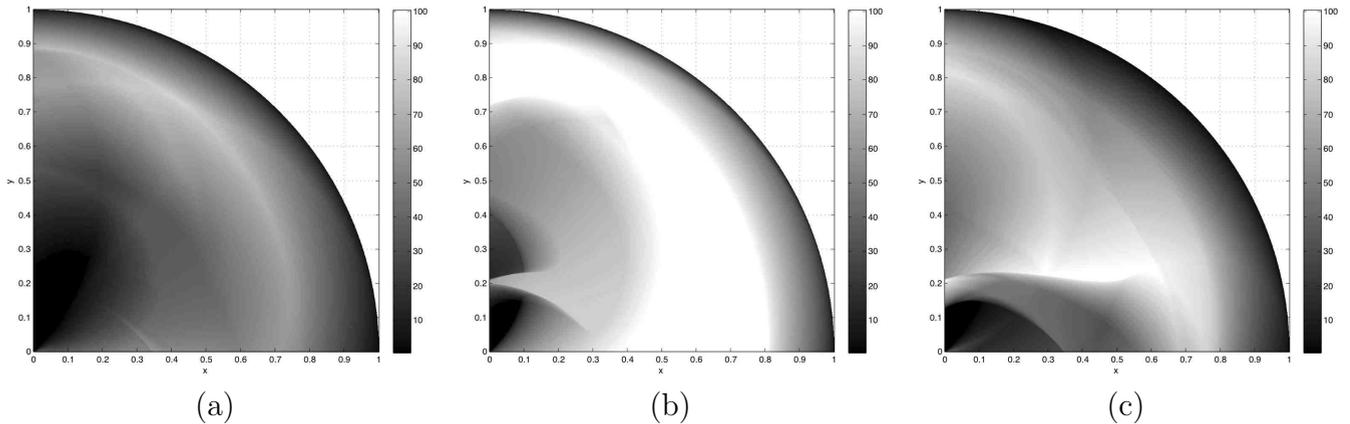


Figure 10: Histogram of the percentage of joint failure positions for which the PKM with actuated revolute joints can still reach a given point in the desired workspace. Histogram (a) shows failures of joint one and (b) shows failures of joint three. There is no point in the workspace guaranteed to be reachable under the tested failures.

5 Spatial Manipulators

While the kinematic redundancy results look good for the first planar PKM discussed above, there exist substantial difficulties in translating these results to spatial PKMs. The primary reason the three legged PKM was able to achieve such a large workspace was that by moving the location of the upper platform and its orientation it mimicked a two link serial manipulator with two physically separated rotational joints. Constructing a spatial manipulator

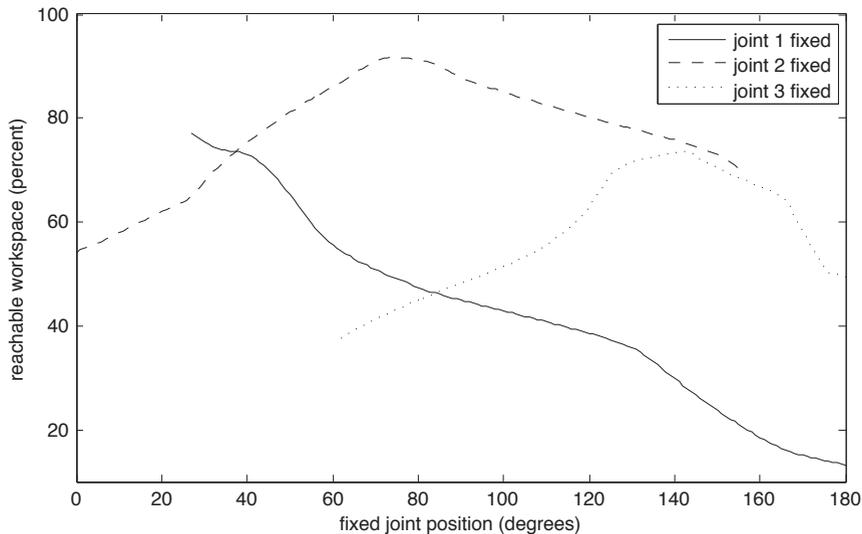


Figure 11: Percent of the desired workspace that is reachable by the PKM versus the position at which a joint is fixed. Solid line is for joint one fixed, dashed line is for joint two fixed, and the dotted line is for joint three fixed.

to do a similar thing in three dimensions proves more problematic. One of the biggest difficulties is that, as shown in [5], a 4DOF PKM built from identical limb structures can only achieve two distinct types of motion of the platform. The first is three mutually orthogonal translations and one rotation about an axes perpendicular to the base, while the second is three orthogonal rotations about a common point and a translation perpendicular to the base. Neither of these is very suitable to imitating the standard spatial serial manipulator arm consisting of a two link planar manipulator on a rotating waist joint. In addition, each of the four legs in these manipulators has four passive joints, which presents difficulties in that these manipulators have a large number of singularities and multiple possible forward kinematic solutions to a single set of joint values depending on the assembly mode of the structure.

The non-uniform spatial PKM that we consider appears below in Figure 12 and consists of a planar trapezoid mechanism that determines the lateral (y -axis) displacement and forms the middle leg of a structure that is similar to the three legged planar PKM of section 3. The upper cross bar of the trapezoid is kept parallel to the base and the rotation of the secondary arm is kept in the x - z plane by the particular arrangement of the various universal joints of the manipulator in a manner similar to that employed by [13] to limit a three limbed spatial manipulator to translations only without employing the parallelogram structure of the more common *Delta* type robot developed in [2].

The inverse kinematic equations for this manipulator for a desired position (x, y, z) and

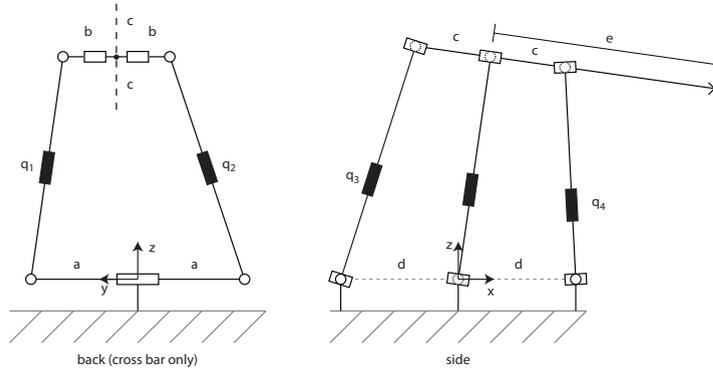


Figure 12: Diagram of 4DOF spatial PKM. Left view shows middle cross bar section and right view is from the side.

orientation in the xz -plane are

$$(x - e \cos \theta)^2 + (y + b - a)^2 + (z - e \sin \theta)^2 = q_1^2 \quad (10)$$

$$(x - e \cos \theta)^2 + (y - b + a)^2 + (z - e \sin \theta)^2 = q_2^2 \quad (11)$$

$$(x - (e - c) \cos \theta + d)^2 + y^2 + (z - (e - c) \sin \theta)^2 = q_3^2 \quad (12)$$

$$(x - (e + c) \cos \theta - d)^2 + y^2 + (z - (e + c) \sin \theta)^2 = q_4^2 \quad (13)$$

The numerical analysis that follows used the parameter values of $a=d=0.10\text{m}$, $b=c=0.05\text{m}$, and $e=0.50\text{m}$. In order to determine the joint angles used in normal operation (i.e., no broken joints), we kept the effective length of the middle "leg" (i.e., the distance from the origin to the middle of the cross bar at 0.50m in order to mimic a convention serial spatial manipulator arm. Since the PKM achieves a displacement in the y direction by translating rather than rotating (as with a serial manipulator waist joint), this means, however, that the PKM cannot reach all of the desired hemisphere quadrant described in section 2. The nominal workspace of this manipulator is approximately 83% of the desired workspace, but still represents a reasonable workspace for a field robot (see Figure 17 below). The range of joint values necessary to reach all of this workspace is $q_1, q_2 \in [0.4, 0.6]\text{m}$ and $q_3, q_4 \in [0.42, 0.58]\text{m}$.

Histograms showing the reachability of each discretized point in the workspace are shown in Figures 13, 14, and 15. The size of the reduced workspace under failure (as a percentage of the original workspace) with respect to the value at which each joint may fail is shown in Figure 16. In the best case scenario for each joint, the manipulator can still reach nearly 100% of the original workspace. The worst case scenario occurs when joint 4 seizes at its lower limit of 0.42m and in this case 65% of the original workspace is still reachable. Overall, the manipulator is guaranteed to reach approximately 51% of the original workspace despite any of the considered joint failures (see Figure 17 below).

While the above the analysis makes the spatial PKM in question look promising in terms of kinematic redundancy this PKM is highly idealized and there are a number of practical physical difficulties with building such a manipulator. First is the problem of self-collision. In reality, the legs and platform of this manipulator will have a non-zero thickness and a substantial number of the positions calculated for the above analysis may not be possible without the physical overlap of various links. This is particularly likely for the positions

in which the manipulator is stretched out horizontally. Second, the PKM design includes four universal joints, which also have a fairly limited range of motion, making still more configurations of the manipulator impossible to achieve. Third and foremost, the above analysis considered only the kinematics of this manipulator without regard to various forces necessary to hold the manipulator in different positions. Both this manipulator and the planar PKM from Section 3 suffer from singularities when they are fully extended in the horizontal plane, i.e., since the pivots of all the legs lie in the same plane, when all the legs lie in this plane as well their prismatic joints cannot exert any vertical force to raise the end effector against the force of gravity. Altering the design so that the axes of the prismatic joints do not intersect with the leg pivots might help address this problem, but it is not clear how to do this without creating problems with collision between the manipulator and the body of the robot. In addition to that singularity there exist other, less obvious, singularities that define the branching points between two forward kinematic solutions (i.e., points where the manipulator could follow two different paths in response to the same variation in joint values). One of these for the planar PKM is when the lines defined by the three legs all intersect at a common point [11]. Other likely singularities include positions where the universal joints align in a manner where they cannot resist one of the undesired rotations of the upper platform. Adding to the problem is the likelihood that not all of the singularities have analytical expression by which they can be identified.

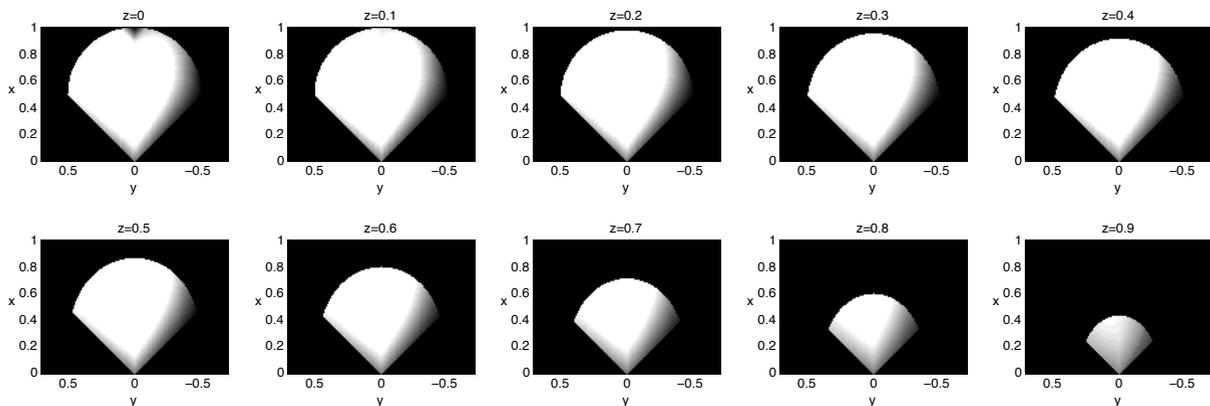


Figure 13: Histogram of reachable workspace for a joint 1 failure (a joint 2 failure appears the same with the y -axis flipped). White indicates positions that are reachable under all tested failures (from $q_1 = 0.4\text{m}$ to $q_1 = 0.6\text{m}$), while black indicates the position is not reachable under any of the tested failures.

6 Conclusions

This study has shown that at least for some manipulators and the considered workspaces that some parallel kinematic machines (particularly those with extendable legs) have the potential to provide greater useful kinematic redundancy in the event of seizures of the actuated joints, in so far as they have a larger portion of the desired workspace that is reachable under likely joint failures as well as having better performance under their worst

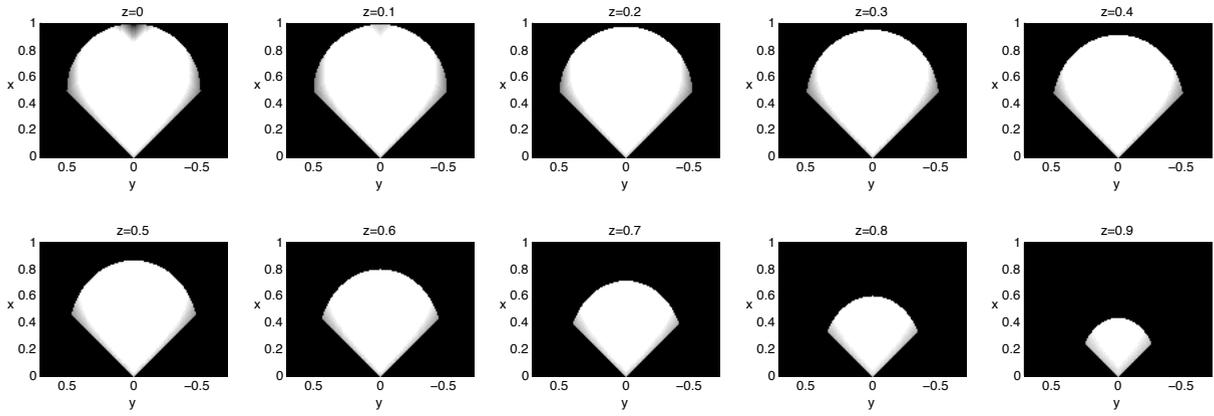


Figure 14: Histogram of reachable workspace for a joint 3 failure. White indicates positions that are reachable under all tested failures (from $q_3 = 0.42\text{m}$ to $q_3 = 0.58\text{m}$), while black indicates the position is not reachable under any of the tested failures.

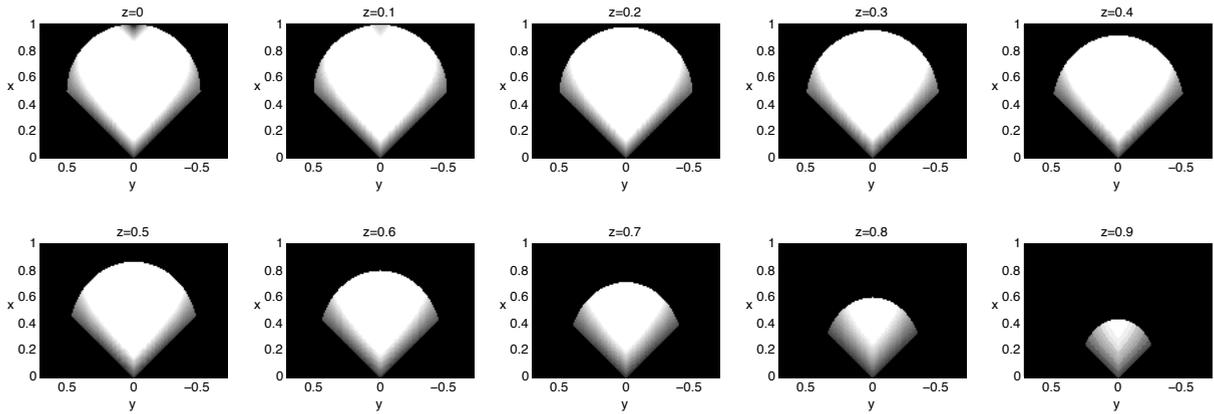


Figure 15: Histogram of reachable workspace for a joint 4 failure. White indicates positions that are reachable under all tested failures (from $q_4 = 0.42\text{m}$ to $q_4 = 0.58\text{m}$), while black indicates the position is not reachable under any of the tested failures.

and best case failure scenarios. A kinematic analysis for a spatial parallel kinematic machine also showed that that manipulator could maintain a large useable workspace in the event of a joint failure, but because of the reasons listed in Section 5 (i.e., issues of required force in certain positions, self-collision of the legs, and problematic singularities) we have decided that such a manipulator is not practical in reality and have thus terminated this line of research. In short, if one desires kinematic redundancy of a manipulator, it is probably much more profitable to focus on highly redundant serial manipulators.

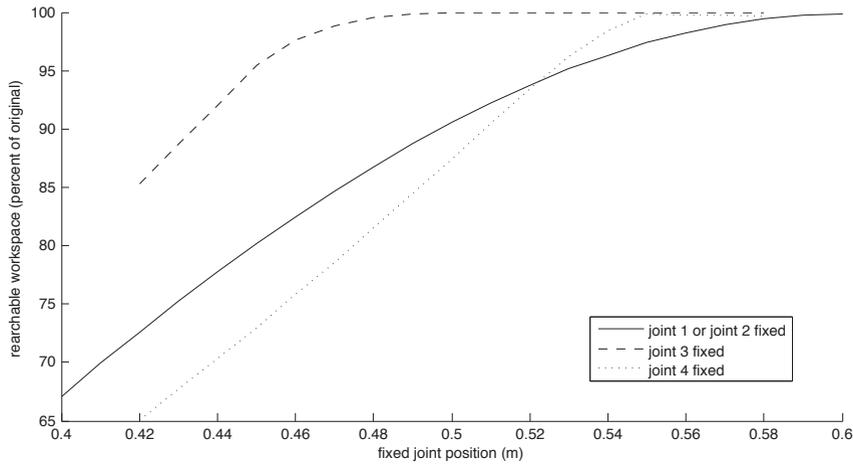


Figure 16: Percent of its original workspace that is reachable by the spatial PKM versus the position at which a joint is fixed. Solid line is for joint one or two fixed, dashed line is for joint three fixed, and the dotted line is for joint four fixed.

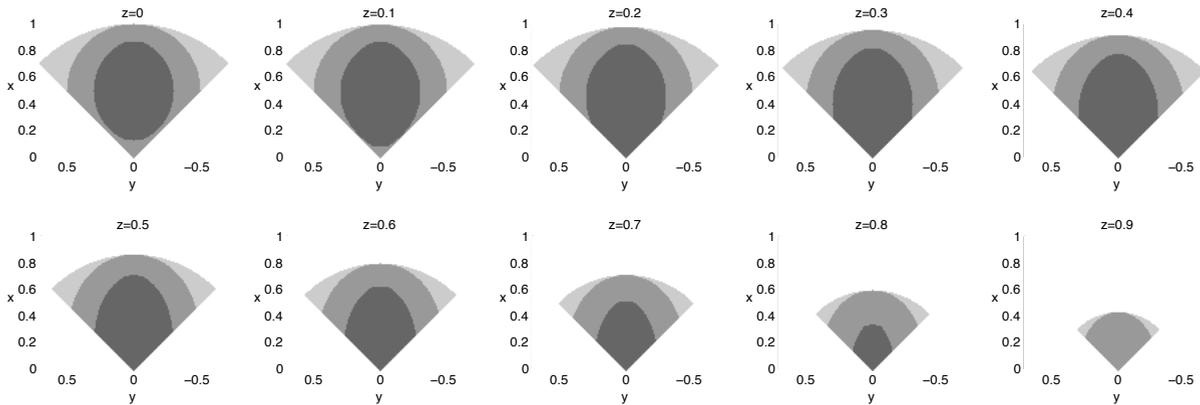


Figure 17: Workspace reachable in the event of any of the tested joint failures. Reduced workspace (dark gray), original workspace (medium gray), and full hemisphere quadrant (light gray). The manipulator is guaranteed to be able to reach approximately 51% of the original workspace despite any of the considered joint failures.

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