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A structural complexity measure that is useful for generating morphological feature detectors is described. The complexity measure is evaluated using two-class in the search for generalizable feature detectors. The work described in this paper is part of a program to automate the design of pattern recognition systems. We use stochastic search techniques to generate or synthesize morphological feature detectors based on morphological erosion operators and hit-or-miss operators [1]. These operators utilize structuring elements to probe input images for geometrical and topological features. A structuring element is a collection of pixel specifications that serves as a template scanned over the entire input image. A hit-or-miss structuring element is one which contains both foreground and background pixel specifications. The structuring element systematically marks the location for each correspondence between the template and the input image, thus creating secondary output images. When the structuring element is small, a layer of boundary points from the foreground-defined cluster of foreground pixels, the erosion operator erodes a cluster of foreground pixels, the erosion operator erodes a layer of boundary points from the foreground-defined objects. The secondary output image in our experiments indicates the presence or absence of positive correspondence somewhere in the input image. A structuring element when used in an erosion operation can be interpreted as a binary feature detector.

In recent work [2, 3], we have investigated resource allocation strategies that improve the efficiency of the search technique. These strategies are generic because they depend on detector response to a training set of images and not on particular attributes of the detector producing the responses. Often the various attributes which may affect performance of the structuring elements are loosely referred to as complexity. In this paper we address the question of how to assess a complexity measure and to investigate its correlation with performance measures. Factoring this type of information into search strategies offers the promise of more efficient algorithms for designing structuring elements. Two other basic questions are addressed below: What are the optimal performance levels for single detectors? How does the performance generalize when a detector is confronted with new samples of handwritten letters?

COMPLEXITY MEASURE

A complexity measure for structuring elements provides a single parameter to rank the elements based on geometrical characteristics of the two-dimensional distribution of pixels. Structuring elements may have different numbers of pixels and sizes within certain pre-defined limits. For example, the structuring elements used in the experiments described in this paper fit into a 31 x 31 matrix and contain less than 32 pixels.

We define a complexity measure \( C \) to be linearly dependent on the number of pixels in the structuring element \( N \) and on a characteristic dimension \( R \). We let \( C = N \cdot R \cdot f \), where \( f \) is a function of the geometrical distribution of the pixels. In defining \( f \), it is convenient to use polar coordinates \((r, \theta)\) in the center of mass of the structuring element. In this case, we let the characteristic dimension \( R \) equal the maximum radial coordinate.

The distribution function is taken to be the sum of two terms which characterize the angular and radial dispersions of pixels in the structuring element. The angular dispersion is computed with respect to eight angular sectors. The radial coordinates weight the pixel occupancy in each sector so that one derives a total weight \( W_i \) for each sector \( (\bar{S}_i, i=1,8) \):

\[
W_i = \sum_{r} r_i \quad \text{where} \quad r_i \in S_i \quad (i=1,2,3,\ldots,8)
\]

The weights \( W_i \) are used in turn to compute an angular entropy \( E_\theta \) defined by

\[
E_\theta = -\sum_{i=1}^{8} \left[ W_i \cdot \log \frac{W_i}{T} \right]
\]

where \( T = \sum W_i \) and the maximum value for \( E_\theta \) is 3, which is obtained by substituting \( W_i = \frac{T}{8} \) in the equation for \( E_\theta \). The radial distribution is characterized by the standard deviation \( S \) of the radial coordinates. In this paper we investigate the following definition:

\[
f = \left( \frac{E_\theta + S}{6 R} \right)
\]

where we use the normalization factors of 6 and \( R \) to keep the values of \( f \) between 0 and 1.

Eight sample hit-or-miss structuring elements and their corresponding complexity measures are shown in Figure 1. The structuring elements are arranged in order of increasing complexity. As the displacements between the
pixels increase, the measure $R_{max}$ increases. Also the entropy increases as the pixels become more evenly distributed in the structuring element.

**PERFORMANCE MEASURES**

The basic recognition rates used to measure performance are:

\[
\alpha = \% \text{ of targets correctly identified as targets}
\]

\[
\beta = \% \text{ of non-targets correctly identified as non-targets}
\]

\[
\gamma = (\alpha + \beta) / 2.
\]

In our target recognition experiments, we use 25 non-targets to every target. There are eight target images in the training set and 200 non-target characters. In general, we want target recognizers to have the ability to recognize the target before we take interest in their ability to discriminate non-targets. The search algorithms used in our experiments allow up to two target recognition errors; hence, the three allowed values for the training set $\alpha$ are 1.0, 7/8, and 6/8. Within these three classes, the relative performance is determined by the value of $\beta$, $\gamma$, which is the average of $\alpha$ and $\beta$, is used as an overall performance measure. $\gamma$ is defined so that correct identification of targets is given more weight than the correct identification of non-targets.

**EXPERIMENT**

In this experiment, we have limited our search to finding a single detector that can distinguish a target letter from the remaining letters in the alphabet. We scan handwritten letters of more or less the same size into a 32 x 32 binary matrix (Figure 2). These images contain a large amount of distortion and some differences in scale. There is no need to center the images since the morphological operators are shift invariant. The objective of these investigations is to optimize the performance of a single extended hit-or-miss (erosion) structuring element. A large population of structuring elements is generated by a stochastic search technique driven by performance measures described above.

In Figure 3, $\alpha$, $\beta$, and $\gamma$ plots of performance as a function of complexity are shown for the target letter A. The $\alpha$ graph shows structuring element complexity as a function of target recognition rate on training images. Recall, only three levels of performance are acceptable: 8/8, 7/8 and 6/8. Notice the slight increase in the range of complexity as the performance level decreases. This effect is even more noticeable in the $\alpha$ plot that shows the complexity-performance interaction of the same structuring elements applied to an independent set of test images. $\alpha$ is not constrained so all levels of performance (0-8) appear. These plots indicate that the training and

![Sample Structuring Elements and Related Complexity Measure](image-url)
Figure 2. Sample Handwritten Training and Test Sets.
test performances on target images is inversely proportional to complexity. The $\beta$ and $\beta'$ plots show the relationship between complexity and non-target recognition for training and test images. There is no constraint on non-target recognition rates so performance levels range between 0 (0/200) and 1.0 (200/200). Visual inspection of $\beta$ and $\beta'$ shows that structuring elements with complexities above 50 have the potential to achieve good levels of performance with respect to non-target recognition. Non-target recognition appears to be proportional to the complexity of a structuring element. The $\gamma$ plots show the weighted performance of the structuring elements applied to training and test sets. Typically, the combined recognition rate $\gamma$ ranges between 0.5 and 1.0. When the structuring elements are applied to the test set, the range of performance is shifted down to 0.35-0.85. This downward shift is not unexpected and indicates that the structuring elements are not capable of recognizing some of the variations present in the test set. The more interesting phenomenon is the behavior of performance as it relates to different levels of complexity. Since the complexity of each individual structuring element is the same in $\gamma$ and $\gamma'$, the number of structuring elements with complexity greater than 100 is the same in both plots. The downward shift in performance between $\gamma$ and $\gamma'$ is less dramatic for structuring elements with complexity less than 100. This behavior suggests that complexity influences the ability of structuring elements to generalize.
The γ plot shown in Figure 3 incorporates a restricted α (≥ 0.75) and unrestricted β. In Figure 4, γ is plotted with the restriction that both α and β on the training set must be greater than or equal to 0.75. Using the same structuring elements shown in Figure 3, the new restriction on β eliminates individuals with performance below 0.75 on the training set but does not significantly alter the behavior of γ. Only the structuring elements that produce training and test set performances above 0.75 are shown in the second pair of γ-γ’ plots (see Figure 4). These results clearly reveal the location of a bounded complexity band (complexity = 10..125) that contains the optimal detectors.

Figure 5 shows complexity bands (γ-γ’) for the letters H, I, and Y. For these letters, the stochastic search process is able to generate structuring elements with recognition rates of 0.85+ for the training images and 0.75+ for test images. The position and size of the complexity band varies for different characters. The complexity bands for the letters H, I, and Y are approximately (25, 175), (25 to 225) and (25, 275) respectively.

**SUMMARY**

Extended structuring elements can be readily customized to discriminate target letters from non-target letters. There are a few letters that are difficult to customize, such as the letter T. We have examined the relationship between complexity measures and recognition rates for letters of the alphabet and have presented typical results which characterize these experiments.

Opposing forces appear to be operating during the structuring element generation process. Target recognition rates on training and test sets improve as structuring element complexity decreases while non-target recognition rates improve as structuring element complexity increases. The complexity range overlap that produces good recognition rates for target and non-target images defines a small complexity band that may be useful in the construction of structuring elements capable of responding to invariant features.

The complexity measure, C, defined for this experiment is not unique, however, it does depend on certain properties characteristic of a structuring element's...
Figure 5. Performance of Letters I, H, and Y. The filtered $\gamma$ performance for structuring elements applied to the training and test sets are shown.
complexity. The general relationships presented in this paper between $C$ and performance can be used to improve the efficiency of search strategies used to automatically generate structuring elements. Because the relationship between complexity and performance measures is consistent and understandable, our definition of $C$ provides a baseline for further study of these issues.

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